Humanities and Social Sciences
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Lecture - 40
Semantic Tableaux Method for Predicate Logic

Welcome back in continuation to the last lecture where we discussed about Semantic Tableaux Method we discussed some examples and we discussed about when their considered to be when they are going to be valid etcetera. So, we will talk about some more examples in this class. So, that we will get our self familiarized with this particular kind of technique.

So, this technique occupies and till position for this course. So, that is why we have spending little bit of more time on this particular kind of method. So, as I said in the last lecture, Semantic Tableaux Method is all about finding some kind of counter example. Suppose, if you are trying to check the validity a given well form formula of a predicate logic what we are trying to do is that.

First we indicate the, formula and then you construct a tree based on the tree rules that we have discussed in the last class. And then if are the negation of the formula leads to the branch closer then we said that negation of that formula is unsatisfiable and whenever, not x is unsatisfiable x is considered to be valid. So, that is 1 thing which we have been doing, and then the second thing is that if you want to talk about consistency of set of statements in the predicate logic.

Then, what we need to do is you construct a tree diagrams for these 2 sentences, and then when the branch a branch does not close in all; that means, it satisfies the formula the given formulas and hence these 2 formulas are set to be consistent. For example, if you want to talk about consistency of these things.

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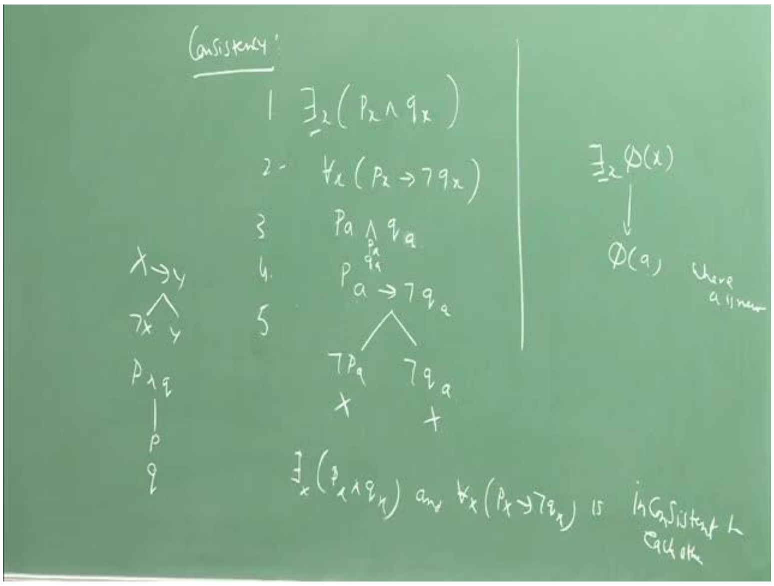

For example if you take into consideration px and qx and then any other thing for all x px implies not qx. Let us considered, that let us assume that these are the 2 statements that are given to you. So, now you like to see whether these 2 are consistent to each other or not using Semantic Tableaux Method. So, the first thing we need to do in the Semantic Tableaux Method is that, always handle the formula which consist of existential quantifier.

So, now first you eliminate this quantify using these particular kinds of rules suppose, if you have a formula like this in your tree. Then, if you remove this existential quantifier then you are replacing it with some kind parameter a. And then each time when you remove this existential operator existential quantifier you have to use a new parameter where a is new. So, now if you remove this particular kind of thing then this will become pa and qa.

So, this is 1 of the instances of this particular kind of formula, so now we are checking further consistency; consistency of these 2 formulas. Now, fourth 1 so now, this px implies not qx holds for all x in particular. So, that is why it holds for even this particular kind of thing 1 instance of this 1 could be even this consistency not qa. So, now we use the same rule x in plus y is not x and y.

So, you apply this particular kind of rule for this 1 and this will become not pa and not qa. So, now pa and qa can be written in this sense I am just writing it here itself qa. Whenever, you have 2 formulas like this p and q the tree diagram for this 1 is simply this 1 p and q it looks like trunk. So, p and q followed by that you have to write like this in the tree diagram.

So, now in this 1 you have pa here and not pa here this branch closes I mean, these 2 are contradicted to each other a literal and its negation is found here that is why this branch closes. And there is no way in which you can go beyond this 1 and you have qa and not qa here even this branch also closes. So that means, you will list out this statements 1 after another and you construct a tree diagram and all the branches closes.

So that means, there exists some x px and qx and this particular kind of formula for all x px implies not qx is set to be inconsistent to each other. Why? Because, if you take both the statements and construct a tree it leads to the branch closer so that means, Unsatisfiability. So, it is not satisfiable any 1 this interpretation so because, all the branches closes. So, in that sense there exists some x for x px implies not qx there set to be inconsistent to each other.

So, we can replace it with some kind of proposition for p we can replace p with some many other kind of thing in the natural language yourself can see that. If you state for all x px implies not qx and at the same time you say that, there exist some x px and qx, then these 2 statements are set to each other. So, let us consider another example and see whether these 2 formulas are said to be consistent or not. So, let us for the sake of understanding way I am taking this simple example, then later I move on to check the validity of a given predicating logical formula.

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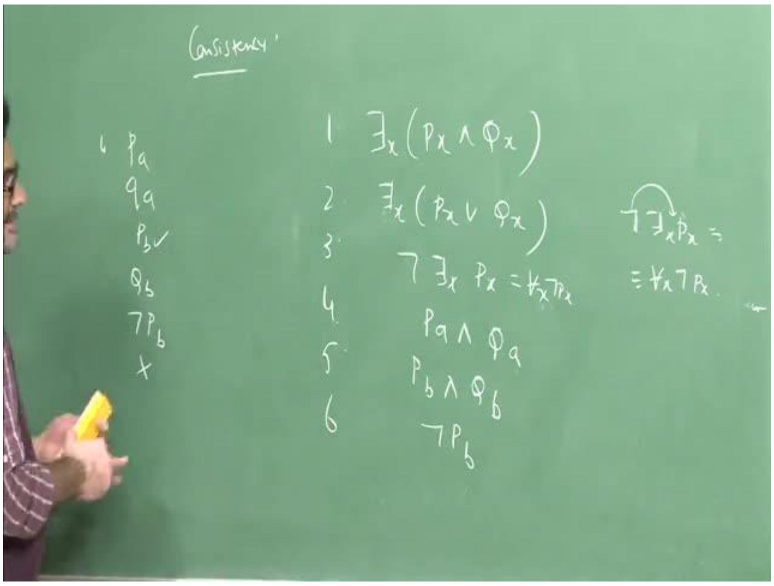

Let us there exists some x px and qx this is the first statement and you considered another statements such as px or qx, and then the third 1 take any other thing such as there exists some x it is not the caser their exist some x px. So, just for a sake of what I being you take this 3 things. So, now you want to check whether these 3 statements are consistent to each other or not.

So, now you start constructing the tree diagram for these things, first you eliminate this existentially quantified it will be pa and qa. So, now first you eliminated this 1 existentially quantified, and then 1 instance of this 1 is going to be this 1. Now fifth, now this can be written as not there exists some x px is nothing, but this negation goes inside and negation of existential quantifier will become universal quantify. And you have to push this negation say and this will become this 1.

So now, you can write straight away like this for all x not px. So, now here in the second 1 if you eliminate this existential quantifier, then you need to ensure that it is replaced by a parameter which is not use earlier. So, a is the parameter which is used here, so we are not suppose to use it again here next, when you remove this existential quantifier. We need to use another parameter let say b other than this a; this is going to be Qb.

Now, so now the next 1 is going to be this 1 for all x not b x. So, this is going to be true for all x and all irrespective of whatever you substitute whether a or b it is going to be the case. So, that is why it is going to be the case not pb. So, now Pa Qa it can be written in this sense qa so that is the 4th 1.

Another you can write and the second 1 is pb and qb and then you have not pb. Once again, since you have pb here and not pb here it is branch closes it transfer to be the case that these 3 statements there exist some x px and not no px and q x there exist some x p x or qx here this is r. So, here not suppose to close like this.

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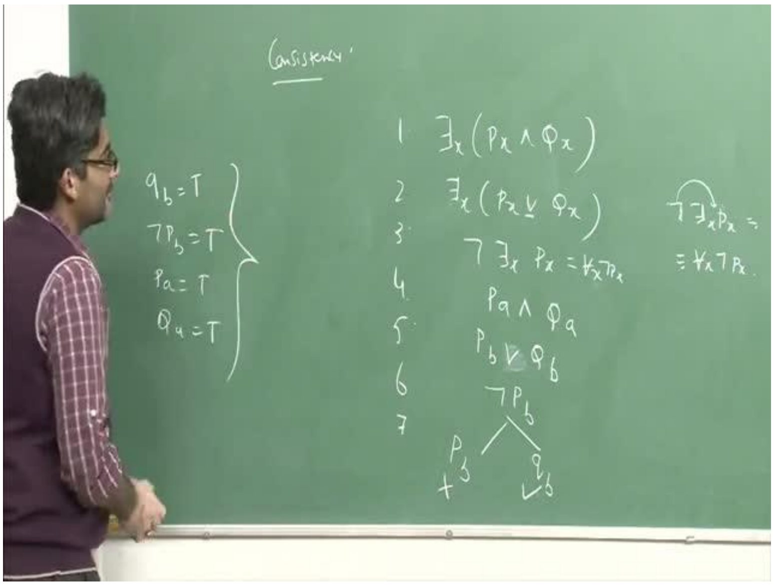

So, this is going to be this 1 pb or Qb. So, now this is going to be like this this is pb and qb. So, this we expanded it and then it will become pb and qb since their or connective is a which i did not notice it. So, now not Qb and pb closes, and then this branch is open. So, this branch is open the sense that you have Qa here, but you have qb here. So, there is no way in which you can cancel close you can close the branch that means, this branch is open.

So, now from the open branch open branch is the 1 which satisfies this particular kind of this formulas; satisfies the formula means, the value set that are going to be there here satisfies that we that makes this 3 formulas true. So, what are these things when qb is T and not pb becomes T and then both pa T Qa T and this is going to satisfy these 3 formulas.

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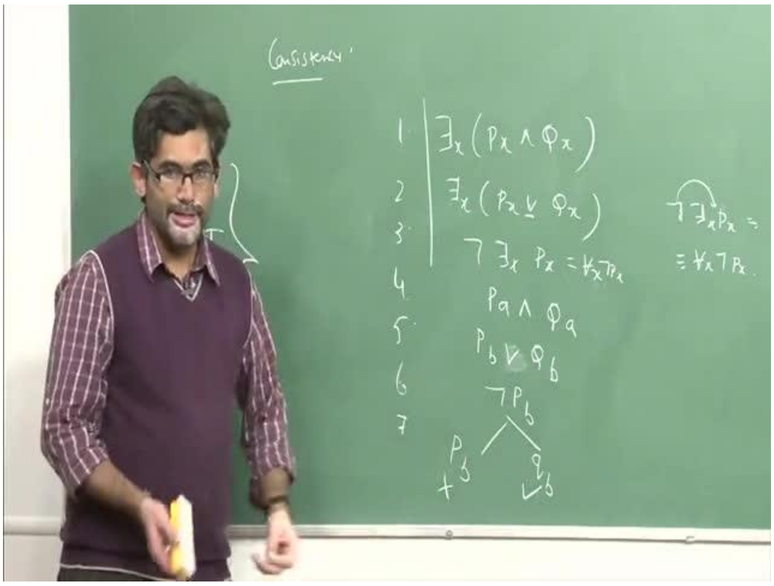

That means, that it going to make this 3 formulas true. So, it is in that sense these 3 formulas are set to be consistent to each other. So, the only 1 thing which need to note that is: the list out all these formulas 1 by 1 after and other, then constructed tree diagram and if at least 1 branch is open; that means, the formula given formulas are set to be consistent.

But in the earlier case, when we constructed a tree diagram for the given formulas all the branches close; that means, is run satisfiable. So, in this case at least in 1 instant it is satisfiable, then it is considered to be these 3 sentences are considered to be consistent to each other. So, this is the way to check whether it given statements are consistent to each other or not. So, now, let us talk about some more examples which respect to validity.

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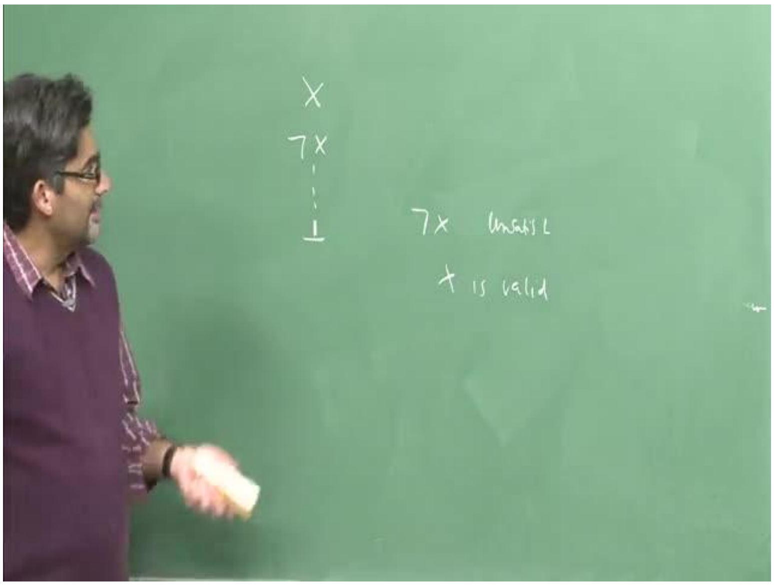

For validity what you will do is for example, if you have given a formula X what you need to do is you need to construct a tree diagram for not x and then if all the branches closes; that means, you land up with a contradiction all the branches closes. Then, not x is considered to be unsatisfiable and that ensures as that x is going to be valid.

Such as what we will be doing in the case of checking the validity of a given well form formula in the predicate logic this is an important decision procedure method. Because, so in this method ah this is also serves as a proof in the sense that any proof is considered to be considered to ending in finite steps in finite intervals of time.

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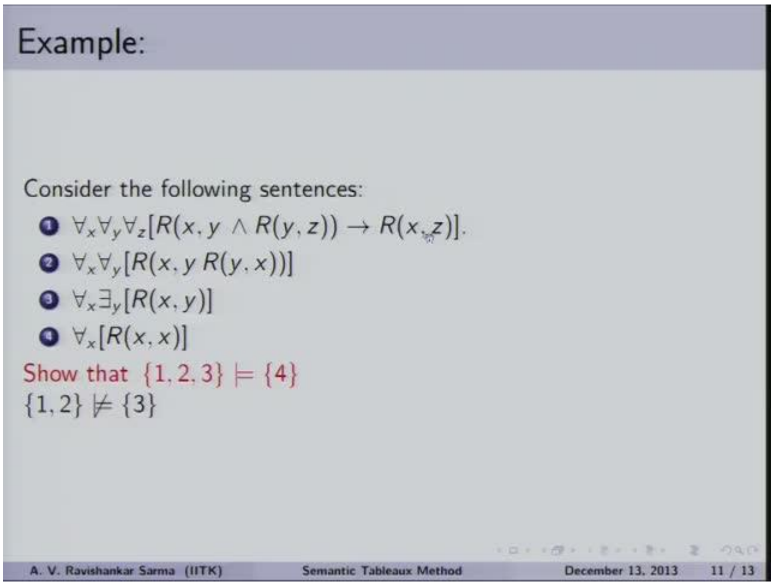

So, now let us consider this example for all x for all y for all z R x, y R y, z R x, z etcetera and second statement is for all x for all x R x, y R y, x. And third statement for all x there exists some y R x, y and the fourth 1 for all x R x, x. So, now you want show whether 1, 2, 3 the first 3 statements leads to the fourth statement or not; that means, fourth statement is considered to be semantic consequence or logical consequence of 1, 2, 3.

So, for that what you will be doing simple is that list out all the 3 statements 1 after another and you take the negation of the conclusion. And then start constructing the tree and we turnout that all the branches close. Then, the negation of the conclusion is unsatisfiable that means, the given conclusion is considered to be the correct kind of conclusion from these 3 premises.

So, now let us considered this particular example and then we will see. So, why we are doing, all this is because you have to for getting our self familiarize with this particular kind of technique we are solving these particular kinds problems.

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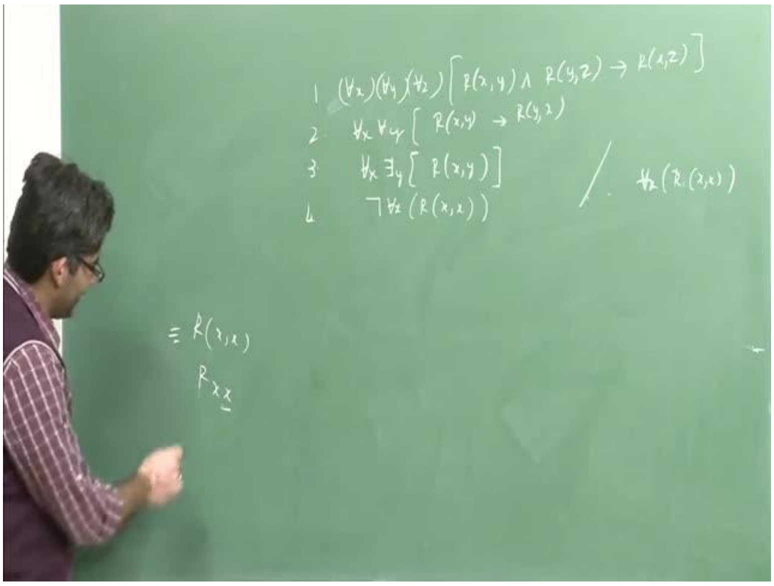

So, now the statements are given statements in the predicate logic are like this first 1 for all x for all y for all z this is the case R x, y and R y, z implies R x, z this is the kind of some transitivity property. Second we have for all x for all y we have R x, y implies R y, x. 3 for all x there exists some y R x, y fourth 1. So, now this is considered to be the conclusion for all x R x, x.

So, now we want to check whether this particular kind of statement follows from these 3 or not using the technique of Semantic Tableaux Method. So, for that what you need to do is you take into consideration the negation of this formula. So, that is not for all x R x, x there is a different kind notation that is being used here. Sometimes I write R x, x in some other text books it will simply written as R x, x.

So, just to separate the predicates with the individual variables subscript and superscript and subscript you write it in this way. So, it does not matter whatever way you write R x, x means, this x and x are in some kind of order it can be written in this sense or forward by x x or you can written in this sense, so it is used interchangeably.

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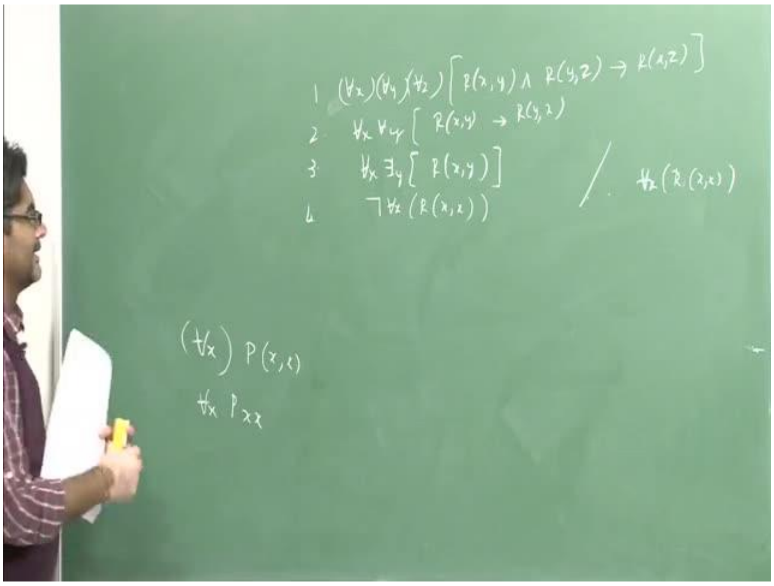

The other thing is that, quantifies in some text books you put parenthesis like this just to separate this 1 for example, you can write like this. In some other text books, it is simply this parenthesis is ruled out and then you can simply write x and x. So, this is only for our convention in all the all these things correct all correct kind of correct way, so representing the same thing.

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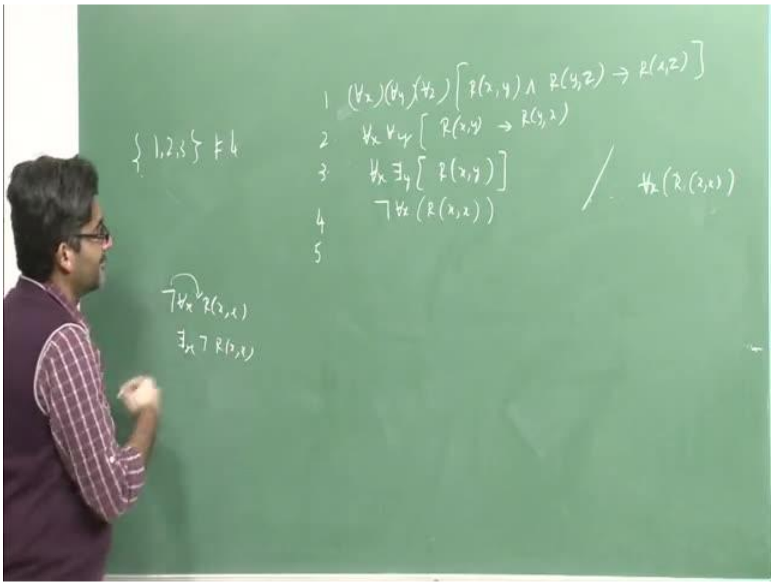

So, now, we are showing that 1, 2, 3 is the set of propositions leads to 4. So, means these 3 things leads to this particular kind of thing. So, now for the Semantic Tableaux Method you start with the negation of the conclusion. So, now not for all x R x, x means this 1 if you simplify this 1 then this is their exist some x, and then you push this negation inside it will become R x, x. So, that is what you are going to write here.

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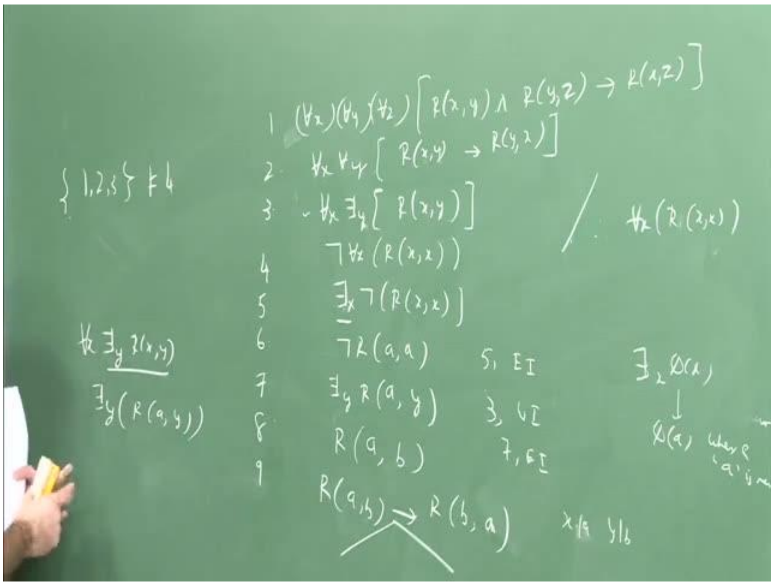

So, this is there exists some x not R x, x, so this brackets needs to be clear and all so. Now, this strategy for this Semantic Tableaux Method is that first you need to eliminate this existential quantifier before handling the universal quantifiers. So, the best thing to handle is this fifth 1. So, that is when you replace this when you eliminate this existential quantifier and we have a rule is something is true this than phi of a where you have to replaced this in with phi of a where a the parameter a is mu.

So, now, this is going to be like this not of R a, a this is the first that we will be trying to look. So, now in the same way 6 1 7, so now coming back to this 1 for all x there exists some y R x, y; that means, there exists s some y R x, y holds for all x I mean it hold for even when you substitute a for x that also is going to whole for that particular kind of thing also. So, this is for all x there exists some y R x, y.

So, now this particular kind of thing holds for all x; that means, there exists some y r even if you replace it x with a that is going to hold and all. So, now this is what you are going to write here there exist some y. So, how did you get this 1 5 existential instantiation for all y r, so this is a y this is this 3 universal instantiation because, you are removed this universal quantifiers.

So now, 8 since this is the only thing which we have in this 1 which starts with the existential quantifier we settle with this thing and then we move on to the universal quantifiers. So, now, there exist some y R a y if you remove this particular kind of existential quantifier you have to ensure that when you replace y with any other parameter, that parameter should not figure out in any one of this things above this particular kind of formula.

So that means, when you remove this y it has to be b rather than a anything other than a you can substitute it for this 1. So, this is going to be R a, b rather than R a, a it is a is already exhausted here. So, this is 7 existential instantiation 9, so now coming back to this 1. So, this is over and this is now coming back to this 1 for all x for all y R x, y implies R y, z you take any is substitute any values x and y , hat R x, y implies R y, x holds.

So that means, if you substitute x for a y for b then also that is going to hold. So, in that sense ah this is going to be R a, b so you substitute x for a and y for b and this going to be the case, and then R for y you substitute it b and for a substituted a. So, this is R a, b implies R b, a. So, how did you get this 1? You substituted x for x for a y for b this is what we have.

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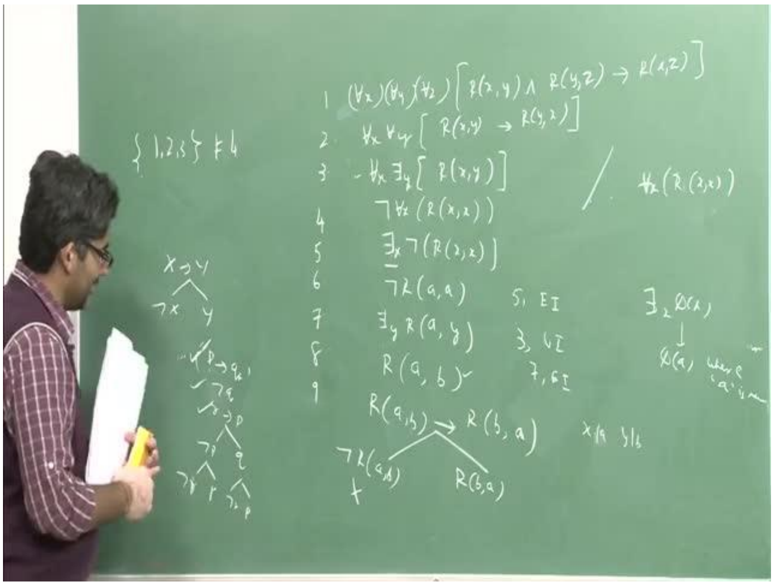

Now, you expand this 1, so whenever you have formula x implies y it is like this x implies y is not x and y. So, now apply this on this 1 it will become not R a, b and then R b, a problem is little bit lengthy and all. So, 1 needs to have little bit patience to check this validity of this particular kind of formula. So, 3 might be little bit big, but it still manageable it ends in finite intervals of time in finite steps.

Now, this is what we have now observe this particular kind of thing R a, b and you have not R a, b this exactly constricted to each other it is like x and not x. Now this branch closes here itself so now, we need expand this particular kind of branch is this branch which is open there is no b a etcetera and all. So, now what is unchecked is this 1. So, you need to note that universal quantifiers whenever a formula starts with the universal quantifier, you can that is no way you can check the formula and all.

In the case of propositional logic for example, if you have p in plus q not q etcetera and all r implies p while constructing the Semantic Tableaux Method. First when you are checking this particular kind of formula, then you write it like this and then you check this formula like this; that means, you are not you are no longer using this same again. But if this formula starts with universal quantifier like this px qx this can be used n number of times recursively you can use for same formula, because it happens for all x.

So, in the case of propositional logic each time when you are expanding the tree with this formulas you are checking this formulas and all next time when you do it when you check this particular find a formula you do like this: not r not r or p not r or p. So, now you check this formula and all the formulas are checked. So, you put tick mark for this particular kind of thing that is not going to happen in this particular kind of situation it can be use recursively and all.

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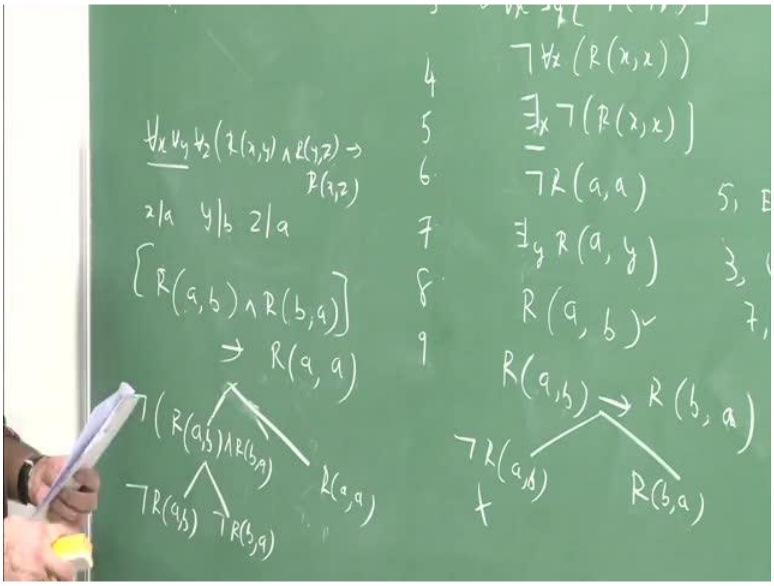

So, now, coming back to this particular kind of formula now, in this formula what you do is this thing you substitute x for a y, for b and z for c z for e. So, wherever you have x you substitute with a and wherever you have y you substitute with b and wherever you have z you substitute again with the a only. That substitution should be uniform, when this formula will become for example, you get you have like this for all x, for all y, for all z R x, y and R y, z implies R x, z.

So, now you are substituted like this x phi a wherever you have a see you substitute with a or you have y substitute with b, and then wherever you have z you substituted with a. So, now this will become r now you are eliminating this now 1 instance of that 1 is this particular kind of thing. So, now, this this will become a b the first 1 and R y, z means, instead of y we have b here b and a implies this 1 R x, z; R x means a, z means a x and z are same, so that is why R a, a.

Why we did like this? Because, we have a term R b, a somehow we need to eliminate this particular kind of term. So, that is set is why we cleverly chosen it is variables to be like this. So, now this is what you substitute it here now this will become r. So, now if you further simplify this 1, so this is x implies y. So, now this will become not of r, so this is not x not R a, b and R b, a, and then this implies to this 1 R a, a.

So, now this is going to be like this not of R a, b is like this is a not of R b, a and this remains as it is. So, now you need to substitute the entire thing here for this open branch. So, now we have just written it down here this remove this particular kind of thing let this branch remains the same. So, now you observe whatever is the open branch.

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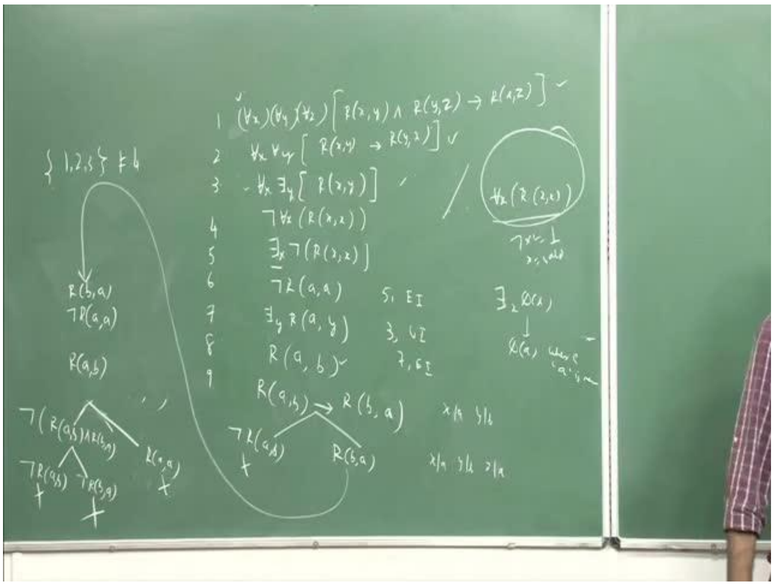

Then you list it out and this particular kind of thing and you have not R a, b and then you have not of R a, a and then all the way down these are the things which we have. So, now observe this particular kind of thing not of R a, a and you have not R a, a is branch process actually this should be like this since we do not have space here. So, we have gone the other way down.

So, not R a, a and then you have R a, a this closes now coming back to this 1 R a, b and not R a, b this branch closes. Now not R b, a that is something called not R b, a where is this here R b, a is there here and then all way down here you have to write this also set of R b, a is there and not R b, a is there even this also closes. So, now all the branches closes.

So, what is it mean? So, we started with this 3 formulas and then this is considered to be the conclusion and we negated the conclusion. And that leads to the branch closer that means, negation of the conclusion is unsatisfiable; that means, x has to be valid. Valid means, it has to be true that means, this is the this is considered to be the original conclusion is considered to be the true kind of conclusion; that means, this follows from these 3 statements.

So, in the same way you can check whether 1 and 2 leads to 3 or 2 and 3 leads to 1 all these things. You can check just you know taking into consideration, the same thing that at first you list out the premises and you take the negation of the conclusion, and then see whether it leads to the branch closer or not. If you leads to the branch closer; that means, the negation of the conclusion leads to Unsatisfiability; that means, negation of x is considered to be contradiction; that means, x has to be a the case x has to be true, x has to be tautology.

So, that is the way to prove to show that a given formula is considered to be valid whether or not a given formula follows from that or not. So, now let us considers some more examples which are considered to be invalid. And those formulas which are invalid, you can construct a counter example within the domain. All the open branches indicate that, it is a kind of counter example within the domain.

So, let us consider some more examples so that, you will get use to this particular kind of technique that is the Semantic Tableaux Method lets 1 or 2 examples which will be considering and then we will end this lecture. So, let us consider let us coming back to the consistency again the problem of consistency.

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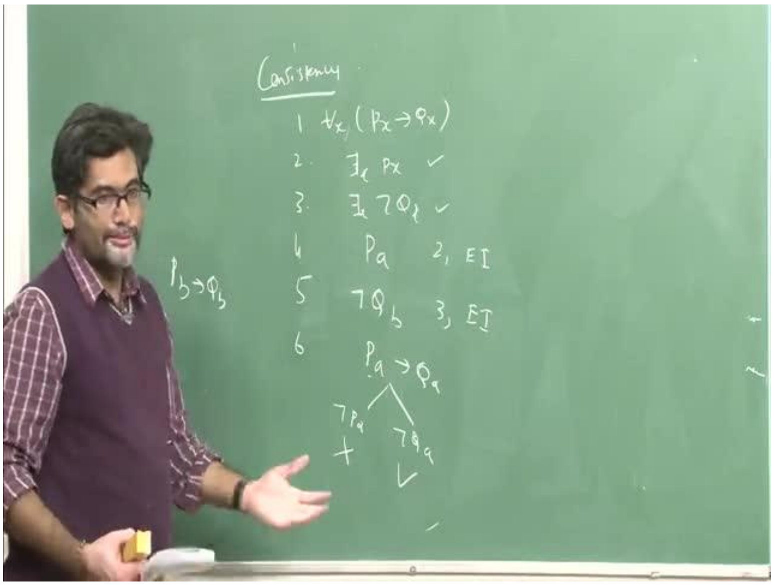

Let us say, we have a set like this px implies qx and then there exists some x px and there exists some x not qx. So, we have then in our set we have these 3 formulas. So, now we are checking whether these 3 formulas are consistent each other or not. So, now start numbering those things 1 2 3 now, you are checking to consistency. So, the first thing that you do is as usual in the Semantic Tableaux Method in the predicate logic is this you have to handle the - existential quantifiers first.

You can handle any 1 of these things now, if you eliminate this existential quantifier here is an instance Pa to existential instantiation. Now, you would not have to jump to this 1 now you to handle this 1. So, now, this is going to be both Q, but you are not suppose to use a it has to be b. So, this is 3 existential instantiation is this 1. So, now sixth 1 px implies qx holds for all x.

So, that is why it has to whole Pa implies Qa it has to be true for even Pb implies Qb you can also use that particular kind of things. So, we have use this Pa implies Qa this is going to be Pa and not Qa. So, Pa and not Pa closes and Qa and you have not Qa and this branch opens; that means, this particular kind of interpretation satisfies this 3 formulas and all that makes this 3 formulas true together.

Let means, these 3 statements are set to be consistent to each other now, if you change this problem little bit. Then, we are trying to see whether..., so now in this case this 3 formulas are set to be consistent now just slightly change this particular kind of problem and then let us talk about the same problem in a different way.

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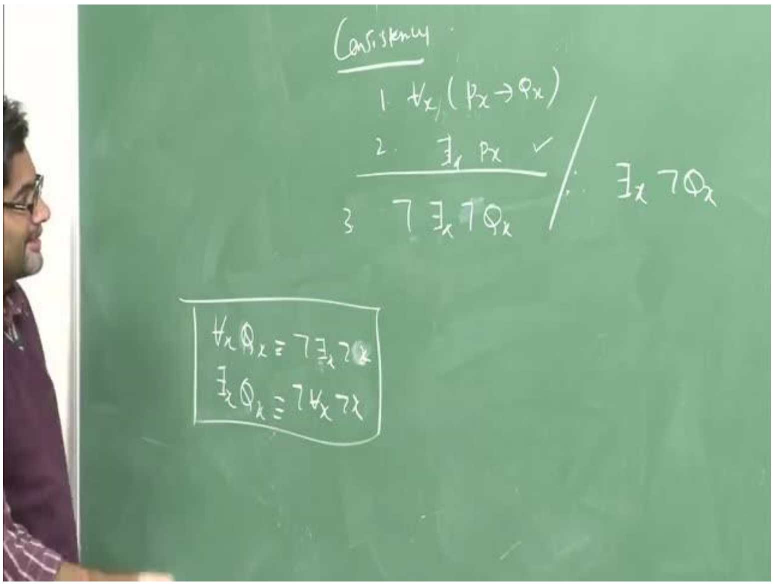

So, now, let us see whether you take these 2 statement into consideration now, whether or not this not Qx follows from these 2 statements or not. So, now we write it in the conclusion. So, for all x Px implies Qx there exists some x Px and then their exist some x not Qx whether this follows are not from these 2 premises and all. So, how do we check? Whether or not, this argument is valid or not.

So, again we use the Semantic Tableaux Method in that the first step includes the negation of the conclusion that is not there exists some x not Qx you start with this particular kind of thing. So, now here we have a definition for all x Qx the same as they does not exist some x not x.

In the same way there exists some x Qx is same as not for all x not x, so this is the standard definitions and all. So, universal quantifier can be defined in terms of existential quantifier and existential quantifier is defined in terms of universal quantifier in this sense, so you use particular kind of thing and then you put it here.

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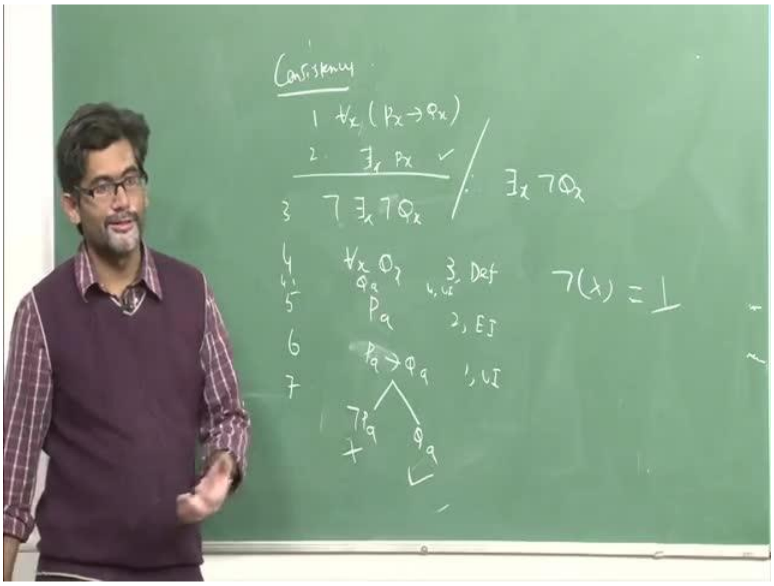

So, this is simply for all x Qx 3, by definition. So, what we are done here? Take and list out the premises you take the negation of the conclusion, and then we are constructing a tree and we are going to see whether the branch closes or not. So, now fifth 1 always handle this existential quantifier when you remove this thing there exists some x Px it is going to be a. So, 2 existential instantiation is Pa.

So, now sixth 1 you can handle any one of this things now we all these 2 starts with for all x something. So, now1 instance of this 1 is going to be Pa implies Qa so this is instance of this 1. So, universal instantiation this 1 is going to be Pa implies Qa. So, this is if you expand this 1 it is going to be like this not Pa and Qa. Now, you have another formula this thing for all x Qx; that means, it has to be true for even a also.

So, that is why you can write it straight away like this 4.1 Qa. So, this is 4 universal instantiation this. So, now Pa and not Pa closes, but this branch remains open; that means, from these 2 premises; that means, negation of the conclusion does not lead to contradiction. So that means, you are not able to we are able to construct a counter example even after denying the conclusion.

So, what we in the context of in the in the basic concepts we discussed about invalidity and invalid argument is an argument, in which you have your premises to be true and the conclusion is false. If we can come off with an example where premises are true in the conclusion is false and that is considered to be considered to be a counter example for the given argument and they hence the argument is invalid.

So, here is an instance where you have even if you deny the conclusion you still have it is still makes this satisfiable and all. That means, true premises and false conclusion is going to be satisfiable in this particular kind of thing. Especially when Qa is to true, Pa is true then it is whole statements are going to be true; that means, you are true premises and a false conclusion that will serves a counter example.

So, open from the open branch you can construct a counter example. So, for this particular kind of thing you can choose a domain to be anything as a set of people, a set of rivers or anything. And then in that particular kind of thing we need to have some kind of relation are in particular predicate. And then you whatever is true here you list it out Qa and Pa are true and based on that you can judge that we now you can easily constrict a counter example for counter example within the domain.

So, if you can come off with a counter example within the domain that obviously, that argument is considered to be invalid. So, in this way we can solve some difficult problems as well. This considered 1 more example and then we will finish it off and all. So, in the in the context of distribution of universal quantifier we asked our self whether universal quantifiers are distributed or not, so that is particular kind of thing.

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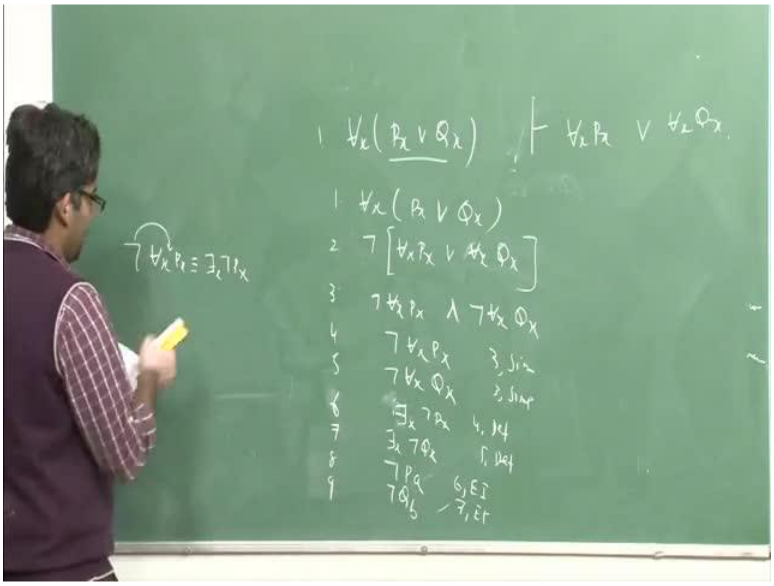

For example, if you have this formula for all x p x or q x. So, from this whether or not, it follows that I means, whether we can derive this particular kind of for all x Px or for all x Qx. If for all x Px implies for all x Px are for all x this is distributed over disjunction universal quantifiers are distributed over the disjunction. So, again I want to see whether this particular kind of thing holds or not.

So, Px or Qx and then here it is individually we have written it like this. So, now, again the using the Semantic Tableaux Method whether this argument is follows or not is a 1 which we have trying to check, so you list out the premises like this Px or Qx. And then you start with the negation of the conclusion for all x Px for all x Qx. So, now 3 you simplified this particular kind of thing then it will become for all x Px; Negation of disjunction will become conjunction and then this will be for all x Qx.

So, now this can be written in this sense for all x Px and then if you simplify this thing it will be like this Qx is only 3 simplification you will get this 1 3 again simplification you will get this 1. So, now sixth 1 now you further simplify this 1 not for all x Px is same as there exists some x you put negation inside, and then it will be this thing. It is not the case, so there exists some x not Px.

Then seventh 1 there exists some x not Qx. So, how did we get this 1? 4 by definition and 5 by definition the definition is this 1. So, the problem is not at over, so now we have there exist some x not Px there exist some x not Qx and then we have this particular kind of thing. So, now you always try to eliminate these existential quantifiers first before going to the universal quantifiers.

So, now first time when you eliminate this existential quantifiers and this will become not Pa. So, this is 6 existential instantiation, and then 8 7 if you apply existential instantiation again, then this will be not q it is should not be a it should be b. Now, we have this for all x Px implies a q there is there is going to hold for any value, any parameters you can substituted into it lets going to hold. Let me, it is it satisfies that particular kind of formula.

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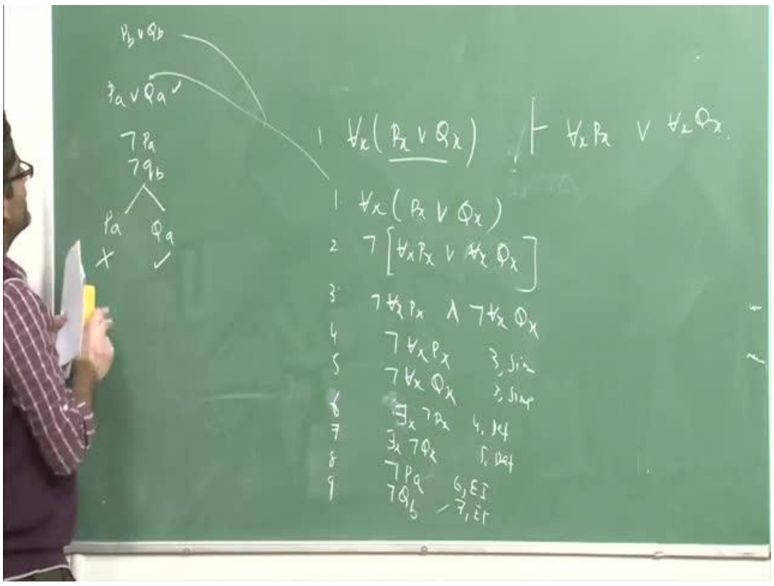

So, now, we have not Pa and we have not Qb and then for example, if you substituted it as Pa, and so 1 instance of this 1 is going to be pa or qa it can very well be like this also Pb or qb also. So, we take this into consideration then this will be like this Pa and Qa. So, now in this case this branch closes and this branch remains open. But if you take the other 1 into consideration instead of Pa Qa you are taken into consideration Pb and Qb.

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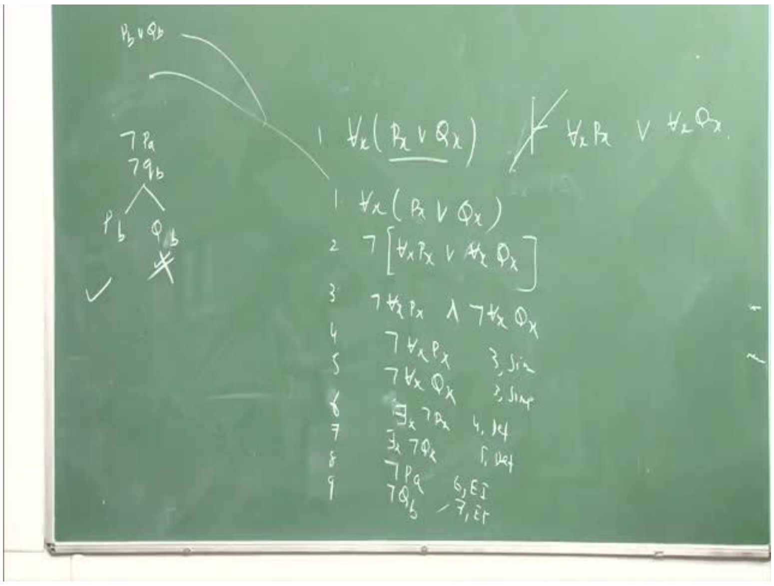

So, then what will happen is nothing which happens this branch closes and this left hand branch remains open. In either cases one of the branches remains open; that means, for all x Px or Qx you will not be able to derive for all x Px or for all x Qx. So, now that means, that they does not imply this particular kind of thing you can check whether you saw, you replace it with there exist some x.

Then, you can construct a tree and you can see whether it distributes over the disjunction or not. So, in this lecture what we have done is we have taken into consideration them Semantic Tableaux Method. And then we discussed in some detail with some examples it is for getting our self families with this particular kind of technique. So, this Semantic Tableaux Method is a simple to use and the rules are very few in number.

Then, it is easy to use and it can be implemented in computers as well. So, there are some of the some important uses for this particular kind of technique. But the problem here is that, are we human being do we use method like this particular kind of thing that is a question that needs to be answer and all is it close to common sense reasoning or the way we reason etcetera and all.

That is a difficult question to answer, but as per as implementation into computers machines etcetera and all it techniques is going to be widely used. So, in that context the 1 which is closer to the human reasoning is what we call it as Natural Deduction Method. So, that is what we are going to take up in the next lecture; the lecture will be talking about the Natural Deduction Method in the context of predicate logic.