Introduction to Logic

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Lecture - 43
Important Theorems in First Order Logic

Welcome back, in the last lecture we discussed natural reduction method. And then before that we discussed very several discussion procedure methods such as, semantic tableaux method etcetera. So, using those particular kinds of methods we can discuss, we can decide whether a given well form formula is valid or when two groups of statements are considered to be consistent etcetera. So, in this lecture I will be focusing my attention on some of the important and celebrated results in the First Order Logic.

And these are usually their many theorems, which are widely used in the first order logic celebrated results and all. One of the important results in this first order logic. And in that context I will be dealing with three important theorems. I am not going to say, when we say that something is a theorem, it has to find a proof and all. I may not able to produce proof of all the theorems that I am going to talk about in this course. But, basically I will be giving you some kind of idea, about these three important theorems.

So, I will be talking about completeness theorem, which is due to a famous logician good Gödel. And then I will be talking about one of the important theorems, which is known as compactness theorem. And third one which is discussed widely in the literature of logic and no course of logic would be complete without this, celebrated result. So, that is the Gödel's incompleteness theorem. So, Gödel has been created for both the completeness theorem, as well as the negative result that is the incompleteness theorem. Incompleteness theorem is used widely used misused and even sometimes even abused also. So, we will be talking about some of the basic ideas of these particular kinds of theorems in this lecture.

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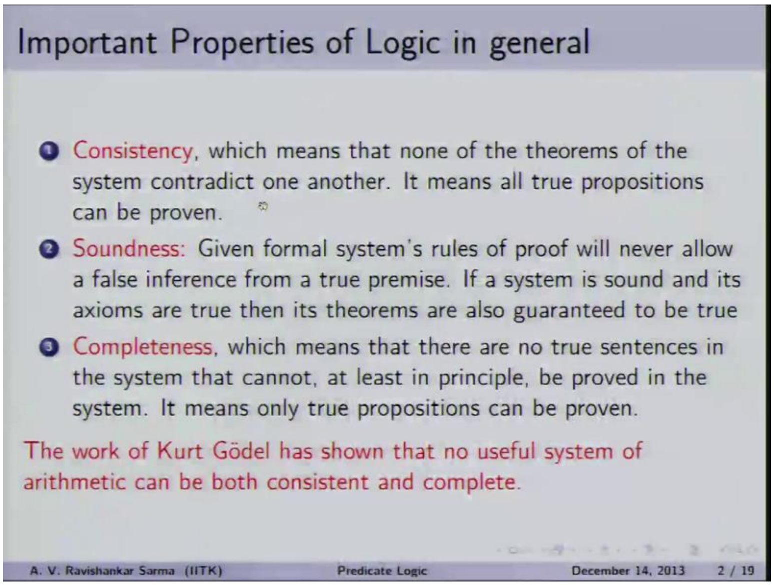

To start with before going any further, we need to know something about this three important logical properties. So, any logical formal system, any logicians dream is to have this three particular kinds of properties. So, in this course is all about the first order logic, which includes predicate and prepositional logic. The one of the important characteristics of first order logic is this that, first order logics are considered to be consistent, they are consider to be sound, they are also considered to be complete.

When we come to second order logics etcetera and all, it is very difficult to establish the completeness property. So, now what are these important logical properties, we will just go through some kind detail about this, important logical properties. All the time we are using these three properties, while we are discussing about semantic tableaux method or natural reduction method, etcetera.

So, consistency the first property is consistency. Consistency means that, none of the theorems of your formal system contradict one another. So, that means, you are not in a position to derive p and north p and all. If you do both p and north p your system is considered to be consistent. So, that means, all true propositions can be proven. For example, if something is a, true propositions or a true formula or it is tautology. All tautologies are obviously, valid formulas.

And all valid formulas we have to find a proof for all the valid formulas. In that contest we use natural reduction in some other semantic tableaux method etcetera. We have already done those things. Second important property is soundness. The soundness property tells us that a given formal systems of rules of proof will never allow us false kind of inference from the true premises. If we start with the true premises, you almost end up with the true conclusion.

There is no way you can generate, the contradiction when we start with the database. So, that is the reason why we are cleverly choosing axioms into consideration. And then these axioms are transformed by means of substitution rules etcetera and all, more respondance etcetera, and then it will become theorem. All the theorems are considered to be true. Each step of your proof is considered to be true, the final step of your proof is also considered to be true.

Soundness ensures that, you start with the axioms which are obliviously true tautologies, you never end up with contradictions. Suppose, if we start with up tautologies and end up with contradictions is something wrong with your formal system. If something wrong with you proof or there is something which is mistaken in the proof, mistakenly considered in the proof. The third important property is completeness, which means that there are no true sentences in the system.

That cannot at least in the principles be proved in the system. Suppose, if you say that something is a true proposition it has to find a proof. That means, we are also ensuring that only two propositions will find a proof. Suppose, if I give you a contradiction, then ask you to prove and there is no way in which you can prove that particular kind of contradiction, but if I give you a tautology like p r p, p r not p etcetera. Obviously, you can find a proof for that.

So, now these are some of the important virtues every logician would dream of... But, we will be seeing very soon, that the works of Kurt Gödel. He showed that no useful system of arithmetic. Gödel was looking for these interesting properties in the context of arithmetic. You could find an important result, that is incompleteness theorem, with which he could say that no useful system of arithmetic can be both consistent at the same time it can be completed on.

So, it has to be in consistent to be complete or it has to be incomplete to be consistent etcetera. These interesting things which you see, it will be treated. So, now, we will be focusing on three important theorems.

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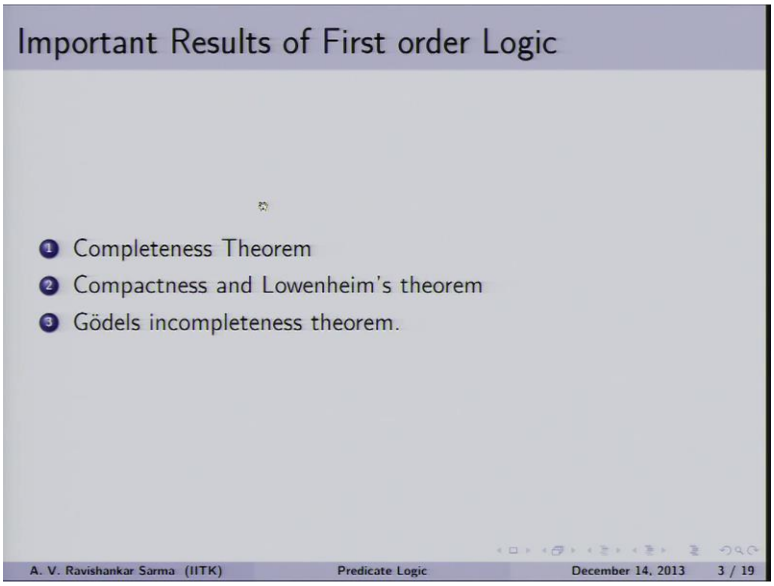

First is completeness theorem, second is compactness theorem, which has consequence to another important logician Lowenheim’s, Gödel Lowenheim’s theorem. And then will end up with Gödel’s incompleteness theorem and it is impact on in the area of philosophy.

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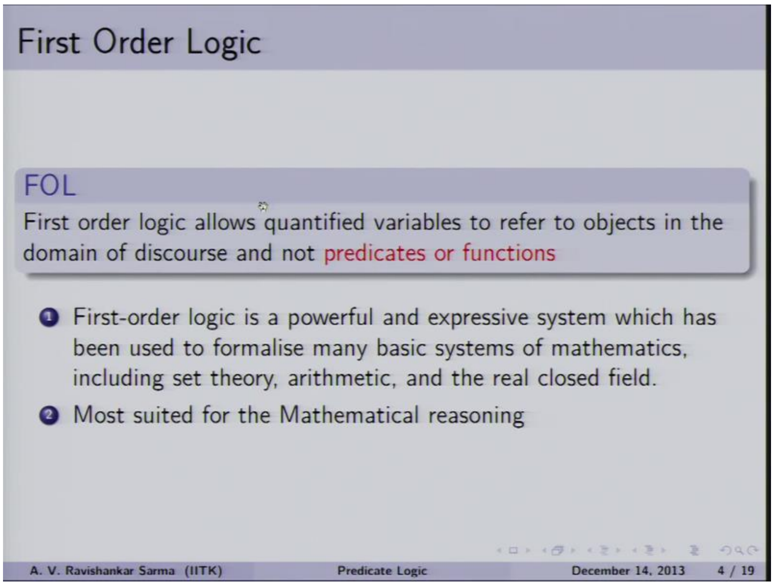

So, before going further is need to know something about the first order logic. So, when we say first order logic it means, this first order logic only allows quantified variables to refer to objects in the domain. In the domain we have objects, predicates, functional symbiosis, etcetera and all. So, a quantified variables, refer to only objects in the individual in the domain of discourse. But, not the predicates are functions.

If you also take in to consideration predicates and functions etcetera, you are talking about higher order logics, second order, third order etcetera. First order logic is usually considered with powerful and expressive system, which has been used to formalize basic systems of the mathematics, arithmetic etcetera. Set theory and any other real closed field, which are related to these two fields. It is mostly suited for the mathematical reasoning.

You should not have the impression that all kinds of reasoning can be captured within the scope of first order logic that is mistake. So, first order logic is not supposed to capture everything you know. Basically, as far as possible it has to capture mathematical reasoning. So, now to start with we will begin with the compactness theorem.

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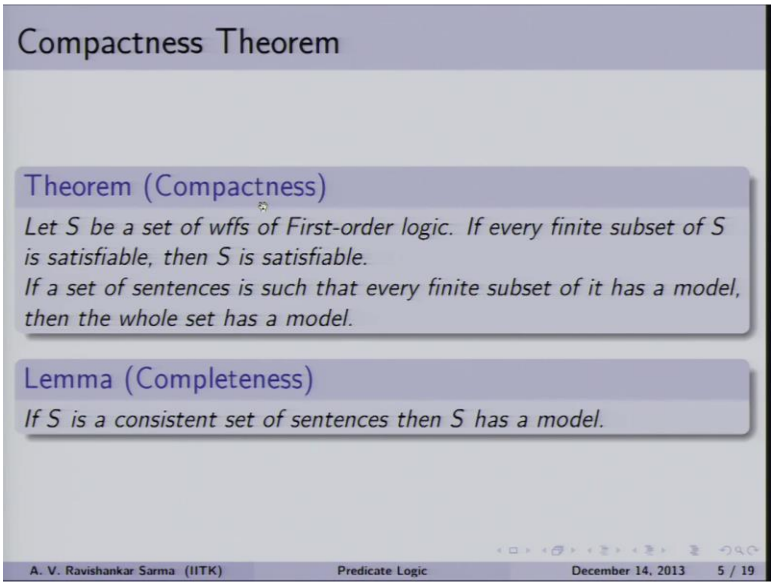

Compactness theorem tells us this thing. Suppose, if assuming that S is considered to be set of well form formulas of first order logic. We are seeing more, how we constitute well form formulas for the first order logic. So, now if you take S to be the well form formulas, then every finite subset of S, if it is satisfiable. Then, that particular kind of set S is also considered to be satisfiable. So, the S is considered to be some kind of set of well form formulas.

And then let us, assume that S 1, S 2, S 3 are subsets of S. If every finite subset of S that means, S 1, S 2, S 3 were all are satisfiable. Then obviously, the conjunction S 1, S 2 and S 3 will also considered to be satisfied. That is, if a set of sentences is such that, every finite subset of it has a model. Then, the whole set is also having a model. So that means, you do not have to worry much about all the sets of formulas and all. But, even if you have a finite subset of set, which satisfies this particular kind of thing.

Then, you can extend it to the whole set, you can say that whole set is having the model. One lemma for this one compactness theorem is this thing, if S is consistent set of sentences, then obviously S has to have a model.

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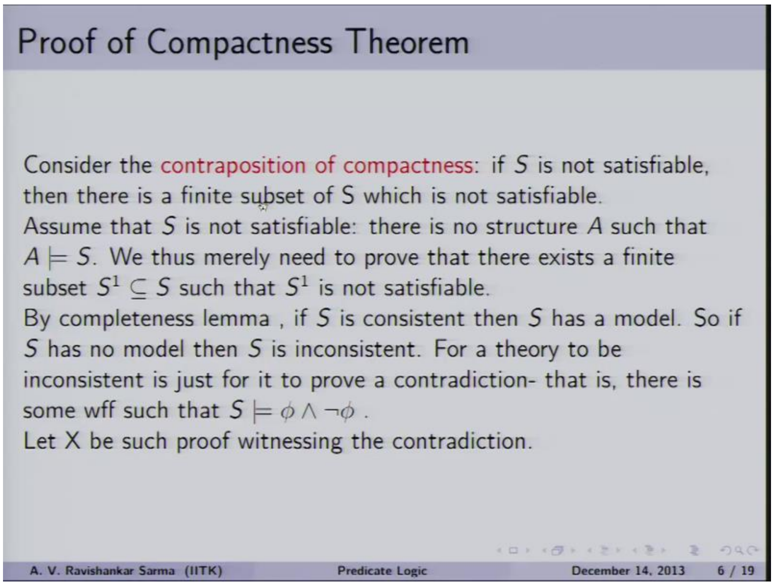

So, proof of compactness theorem goes like this, just I am giving you general description of a proof. So, we need to know that each and every theorem should find a proof and all. Although I am not producing proofs of all the theorems that I am trying to discuss. My intention is to talk about only the general idea of this proofs, where it is later you can see where it can be applied and all. So, proof of compactness theorem goes like this.

Let us consider the contraposition of the compactness. That means, you take the negation of the statement that you have seen in the theorem of compactness. That means, if S is not satisfiable. Then, there always exists some kind of finite subsets of S, S 1, S 2 or anything. And that also should not be satisfiable and all. If each and every finite subset is satisfiable, then S is considered to be satisfiable. If suppose if one of the important use of this compactness theorem is that.

If you can check the satisfaction with respect to subsets of a given set. And that will serve our purpose. So now, let us assume that S is not satisfiable. That means, in the context of predicate logic we need to have a structure A. Such that, S is not a logical consequence of A, A does not A models does not model S. So, we just merely need to prove that, there exist a finite subset S kind, which is a subset of S such that S prime is not satisfiable.

So now, we have a completeness lemma, which says that if S is consistent. Then obviously, S has to find a model. So, that we will make use of it here and we say that, so if S has no model. Then obviously, S has to be inconsistent. If you do not find any model of that one obviously, in all the interpretation that formula is false. That is why it is considered to be inconsistent. So, for a ((Refer Time: 11:26)) to be inconsistent, it is just for it to prove that, it leads to some kind of contradiction.

That means, they have come up with the case where you have a formula and it is negation, as an outcome of your that particular kind of set, set of well form formulas. That means, let that particular kind of proof, which consists of this contradiction, we sequence of steps, which is represented by X.

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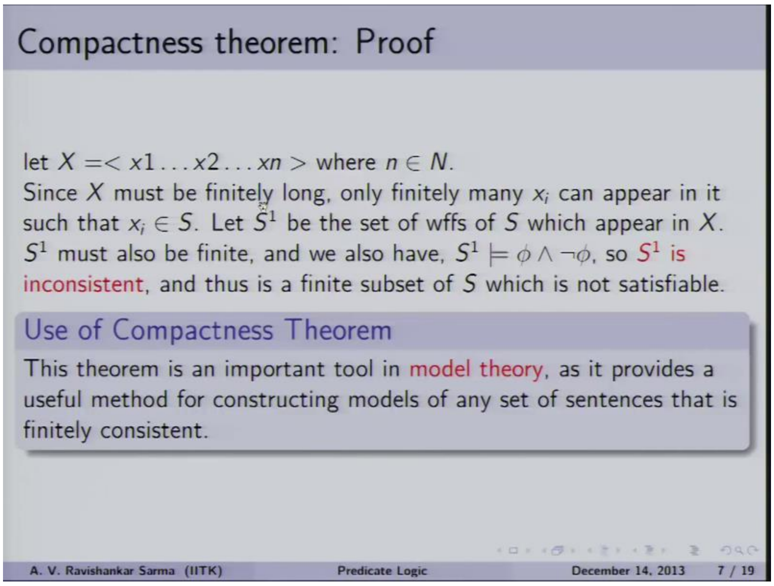

So, that X has this thing first step is x 1, x 2, x n and the n step is something like phi n not phi. That is the contradiction that you got in your proof, where N stands for N is the natural number. That means, finite steps this proof ends. That means, since X must be finitely long and only finitely many x i's can obviously, appear in it. So that means, you came across some kind of contradiction. That means, you could have got it through finite number of steps.

So, such that x i’s some x i that you take into consideration, which has that particular kind of contradiction that belongs to S. Now, let S prime be a set of well form formula surface, which appear in the proof that you are trying to talk about, where you have a contradiction. Then, S prime should also be finite like the one which, we have you have seen in the case of S. That means, we also have phi and not phi, which is which comes as an outcome of S prime. That means, S prime has to be inconsistent.

And thus the final event of finite subset S is also considered to be not satisfiable. So, the essential thing we need to notice is that, we want to show that S is not satisfiable. You can show that, you can take some finite subsets of S and you can show that it is not satisfied and that will serve our purpose. In that sense we call a formal system to be compact. Each and every finite subset of S has to be consistent or satisfiable, so that the whole set is going to be satisfiable.

Wherever, one particular kind of subset of S is not satisfiable. Then, the whole set S is not going to be satisfiable. What is the use of this compactness theorem? This theorem is considering importance to especially in the context of model theories, model theories is important branch in the area of mathematically logic. It provides us with the useful methods of constructing, models of any set of sentences that is finitely consistent. So, it is one of the important applications of this compactness theorem.

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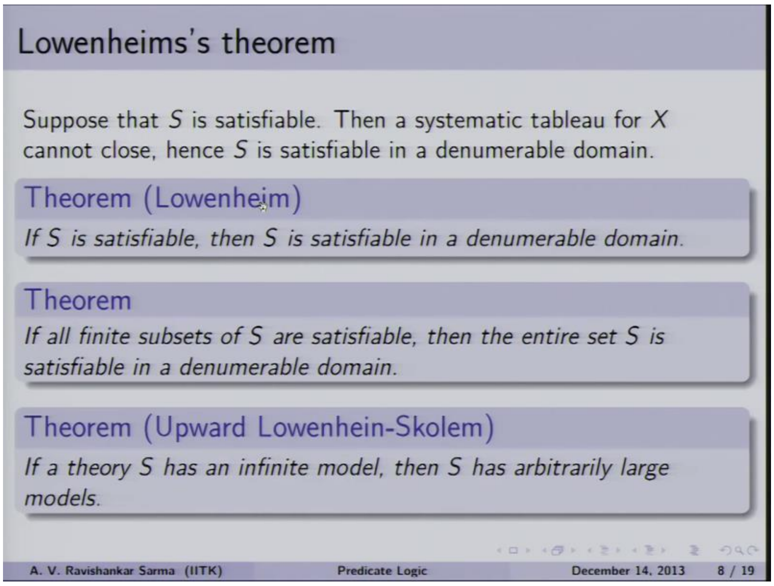

Now, let us talk about another important an interesting result in the first order logic. That is the Loweheim's theorem. Suppose, you take S to be a set of well form formulas, that is considered to be satisfied. That means, at least one interpretation the formula is true. Then, obviously, if it is satisfiable then it has to find some kind of systematic tableau for X with that S which obviously, cannot close.

That means, all the open branches are leading to satisfiability, means if you have a formula in the predicate logic you consider tree diagram. And you have open branches, that means that open branch tells us that the given formula is satisfiable in that particular kind of interpretation. That means, S to be satisfiable in a denumerable domain. So, this Loweheim's theorem tells us this thing. If S is considered to be satisfiable that means, it finds an open branch after using the three rules.

Then, S is satisfiable in a denumerable domain. That means, you can number steps should be finite in number. It is satisfiable in a denumerable domain. And the corresponding theorem is this thing, if all finite subsets of S are considered to be satisfiable. Then, the entire set is considered to be satisfiable in a denumerable domain. So, there is other interesting corresponding theorem in the context of Loweheim's, it is called as Loweheim's upward Loweheim's columns theorem.

So, this tells us this thing, if a theory S the formal system S has an infinite model. Then, S this particular S has arbitrarily large kind of models. If it is in the uniform mode it will have, it will take time it will take infinite number of steps. So, thus has infinitely large models.

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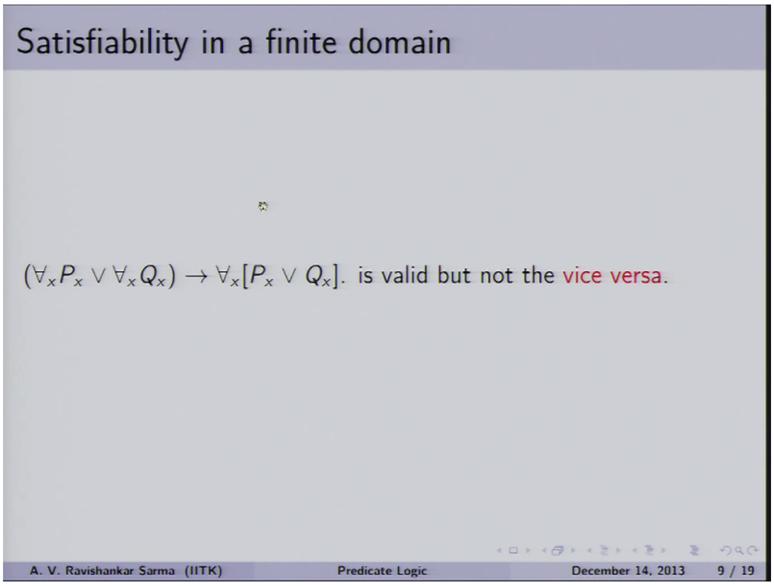

Example if you want to see, the one of the applications of this particular kind of a theorem it is like. This formula for all x P x for all a Q x implies for all x P x of Q x is going to be tautology, hence it is valid formula. But, if you take the vice versa of that one, that is for all x for P x or Q x implies individually for all x P x or for all Q x that is considered to be in valid kind of formula. And if you want to establish that it is unsatisfiable.

And if you construct a t diagram in finite number of sets it will end. And then you will have some kind of open branches. That means, if you deny the formula, you will have the open branches. This example we have taken into consideration in the last few lectures. So, we can easily construct from the open branch as counter example. And if you see that counter example, you only require of some kind of finite domain and all, which involves one or two elements.

One or two parameters a r b to establish this particular kind of thing. So, you just require finite domain pure whether it is satisfiable, unsatisfiable.

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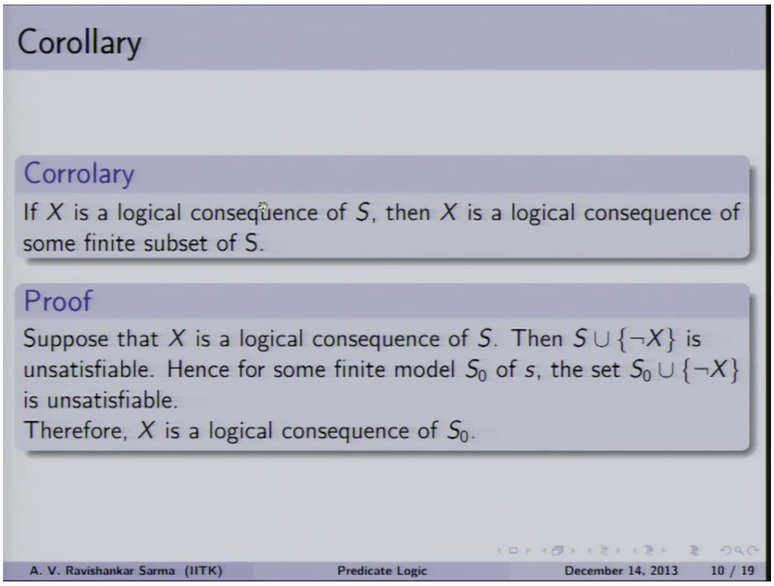

One importance corollary of this particularly kind of theorem is this. Suppose, is S is a logical consequence of setoff well form formulas S, then X is also consider to be logical consequence some finite of subset of S. So, you have some logical subset of formulas. And then S 1 consist of some kind of set of formulas, which are subset of this particular kind of S. Then if X is a logical consequence of S, then X is also logical consequence of so finite subset of S.

So, how do you proof this particular kind of thing, if it is goes like this. Suppose that X is a logical consequence of S that is what is given to us. That means, if you had not X to S that leads to unsatisfied means, something is this particular kind of things S is X.

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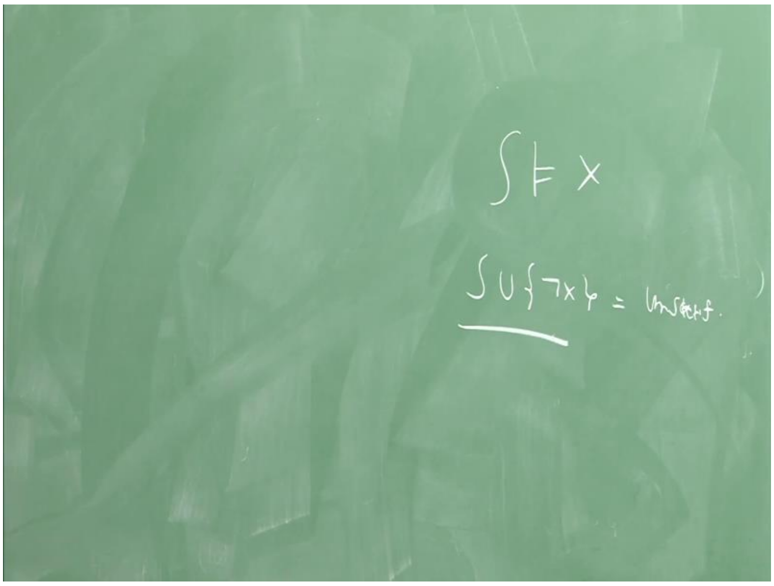

Then add not X to it is and this becomes unsatisfied. You started with the permission, you denied the conclusion obviously, leads to consideration that means it is unsatisfiable. So, S union not of X obviously, has to be unsatisfiable. Hence if it is unsatisfiable ((Refer Time: 18:45)) then if constructive t diagram using tabular proofs obviously, tends in finite steps one. Hence it will have finite model S 0 of s capital S that is even S 0 union not X is not consider to be unsatisfiable.

So, therefore obviously, X is also a logical consequence S 0. So, if you want to show that S is constructive unsatisfiable, we can take subset of S, and we can show that even from subset of S also that X is has to be logical consequence.

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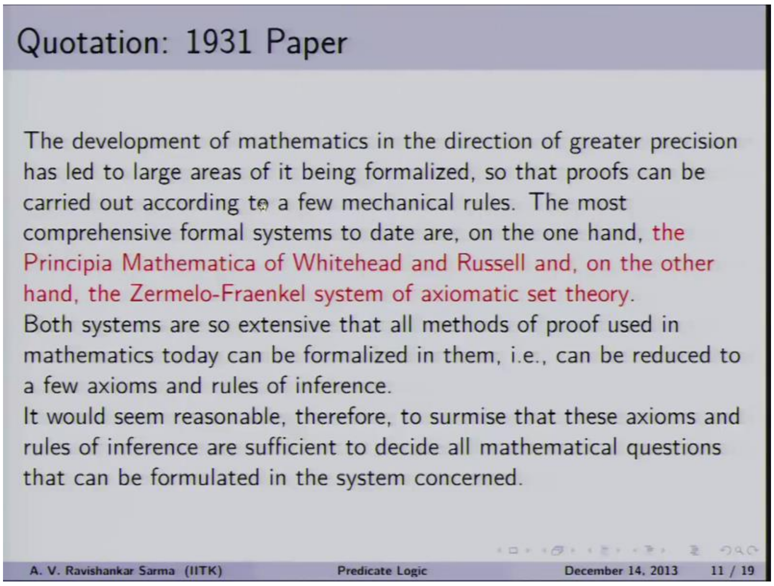

So, this what is compactness sign Lowenheim’s. Now, lest us comeback the celebrated result in the first order logic. That is Gödel’s incomplete in theorem, before talking about Gödel’s incomplete theorem. It is talk about this particular startling kind of passage, which will find it in one of the important part braking papers by Gödel. It is on some kind of formally undesirable proposition. So, it is in important question in worth discussion, so Gödel’s say particular kind of things.

So, he is of the view that he says, so development mathematics in the direction of greater precision has let to large areas of it being is formalize. So, that proof can be carried out according to just few mechanical rules. That means, you started with few axioms that is what we have seen in the case of Russell whitehead axiomatic system or Hilbert axiomatic system of etcetera.

You started with some four or five kind of rules and then you have some substitution rules and some definitions that key mass and then generated numbers of theorems and all, numerous theorems and all. So, which just simply requires few mechanical the rules and all and then axioms of course to start with the most comprehensive with formal system to date accordingly Gödel on the one hand are this. One is principle mathematic of whitehead and Russell.

And other hand we have Zermelo-Fraenkel system of axiomatic, which we did not discuss in this course. But, the context of set theory is very important. Now, this is the two examples of that he take into consideration on. Now, is talk about some important merits of these things. The both the system are, so expensive that all methods of proof used in mathematics today. We can formalized with in this two frame works. That is it can reduce to a few of axioms to and rules of inference.

The grand programs is reduce is the mathematics to logic. And all the notions are mathematics if it is express in the language of logic. That means you have axioms rules etcetera on. Then you in a way you are reducing mathematics to logic. In that sense arithmetic and reduce to logic are geometric reduce to logic. So, it would same reasonable according to Gödel. Therefore, to surmise that, these axioms and rules of inference are sufficient to decide all mathematical questions that can be formulating system in concerned. So, it is placing this thing in this way.

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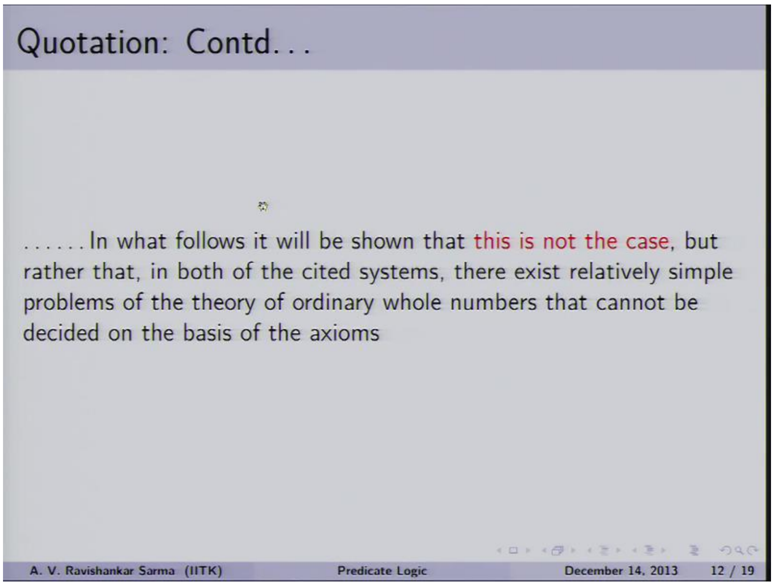

But, goes on in say that in what follows is that, it can shown that obviously, is not the case, this grand programs might fail. But, rather that in both the side cited systems there exist relatively simple problems of the theory of ordinary whole numbers. But that cannot be decided on the basis of these axioms. He will not position to decide whether, x can be provide or not x can be proved and all, that leads to undesidability are incompleteness.

If all the truths are provable or all the thing which can proved or true then your system consist complete. But, we can come off with simple example, where life is not easy an all. So, come off with simple example where at least go to the shot that formal system is going to incomplete. So, now, before talking about Gödel's negative result that is the incompleteness theorem. Let us, talk about some of the important theorems, due to that is the completeness theorems.

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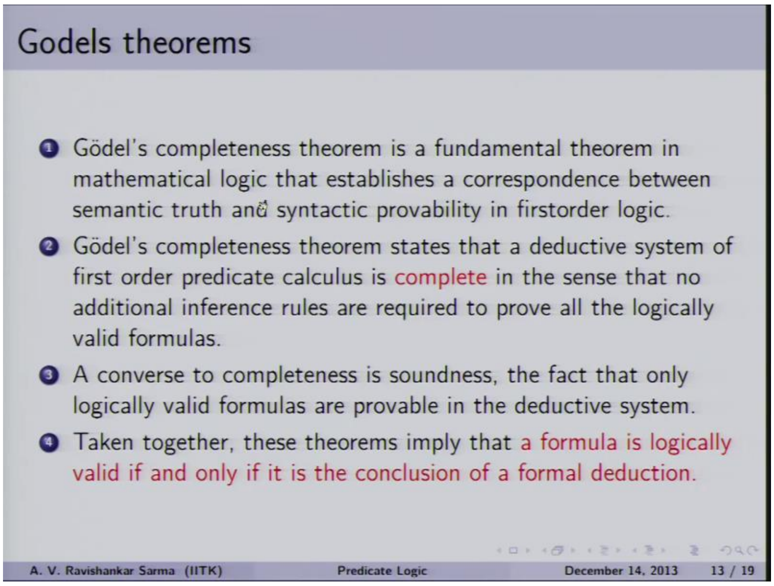

Gödel's completeness theorem is conservative fundamental theorem mathematical logic. In fact, he worked for his p h d thesis this particular kind of problem completeness. The completeness tells as a all the valid formulas are provable and all the provable formulas are hence to be true that valid. Gödel completeness theorem is fundamental theorem the mathematical logic. It establishes a correspondence between semantic truth and syntactic provability in the first order logic. So, on the one hand the we have this particular kind of thing, there two symbols that we have used areas.

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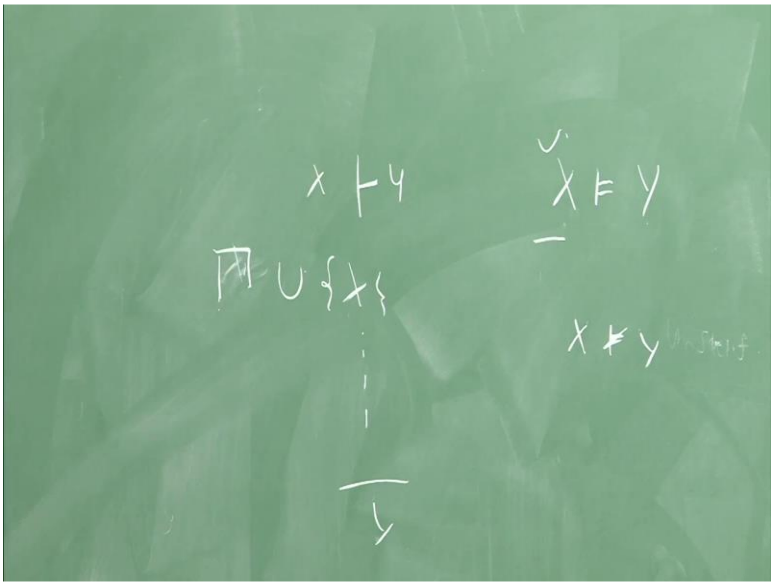

Suppose, if you write like this x and y. It means y is reduce form x that means, x because of a some kind of a axioms etcetera as well formulas. And then you got y got a finite number steps. It is in that since y is reduces from x, they are not trying talk about to mean by x ((Refer Time: 24:22)). So, there is another symbols, which is called has this thing, this means x entails y or y is a logical consequence of x etcetera. So, that means, all interpretation in which x is true, y also has to be true.

This no way in which x is true and y is false and if that is a case, you write is like this. This is the simple thing which we have done in this particular kind of course. So now, first one is the trying talk about probabilities. And the second one the double ton still is different semantic true. Now, Gödel’s completeness theorem states that, a detective system of first order predicate logic. Of course, we need not axiomatic system in the context of first order logic.

But, we can extend the proposition logic form adding some more quantifies ((Refer Time: 25:12)) you can come off with axiomatic system. That particular kind of deductive system of first order predicate logic is considered to be complete. In the sense that, no additional inference rules are required to prove all the logically valid formulas. He start with only axiom and the transformation rules and modules phone etc. And that is all we need to prove to check the valid all the formulas and all.

We do not required any other formula, any other kind of inference rules, which is outside of this things. It is in the that sense within the system, we can show that there complete. So, now converse of the completeness theorem is what is called as soundness. So now, soundness ensures fact that only logically valid formulas are provable. In only those which are considerable true's are provable. Those who are considered to be false obviously, cannot proved enough within the deduction system.

Did ensure that whatever you proved at the end of the day has to be will be true enough. In all the true formulas are again provable and all. So, take in to whether this theorem implies that the formulas logically valid, if and only if conclusion of if an only it is consists conclusion of formal deduction. Some kind of prove procedure method that we has seen natural reduction method, symmetric tableaux method. We should be in a position to establish that it can find a proof her a given valid formulas.

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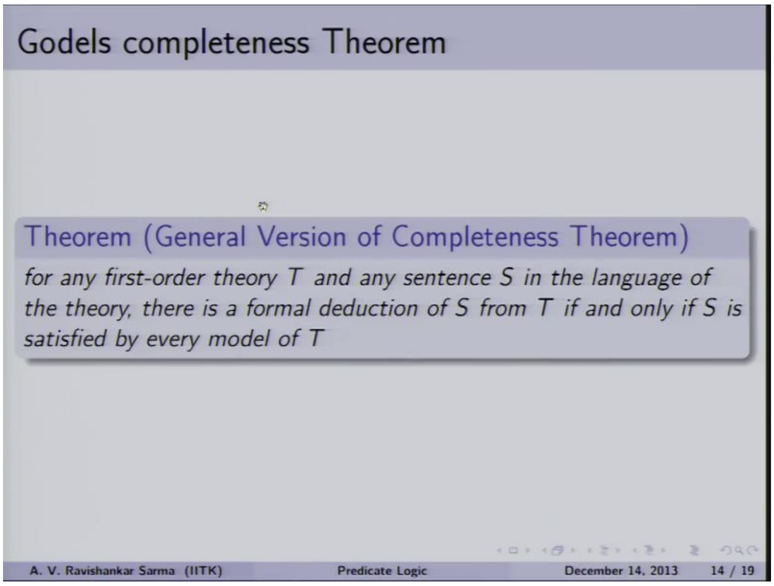

So, now Gödel’s completeness theorem tells us this. This is considerable general version of completeness theorem. For any first order theory T and any sentence S in the language of the particular kind of the theory. There is a formal deduction of S from T, if and you can axiom all the things. If and only if S is satisfied by every model of T, if it is the case then S can reduce from T.

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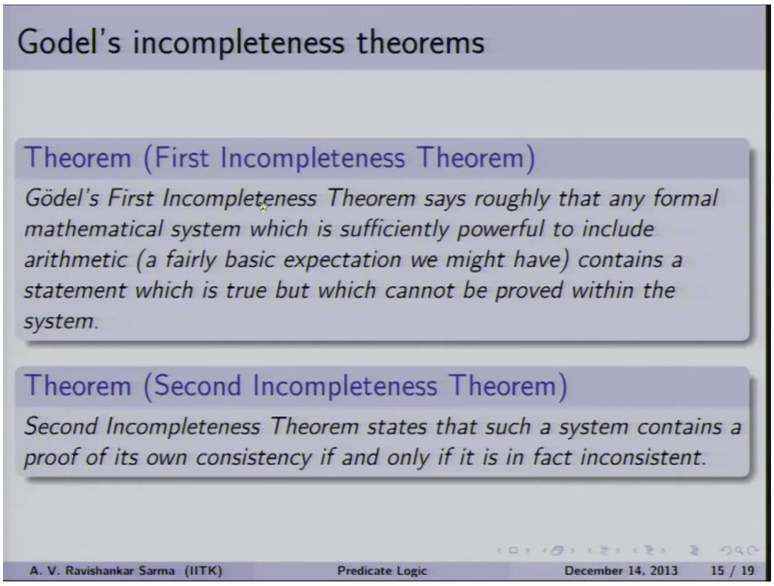

So, now that is come back to the negative result the Gödel's has come off with. So, these are widely discuses in the literature of logic. So, they are the famous to incompleteness theorems. First one is first inconvenient theorems it tells is that it roughly says that any formal mathematical system, which is sufficiently powerful to include for example, arithmetic, geometric, etcetera.

Contains the particular kind of statement which is true, but which cannot be proved within the system. That means, it incompleteness result in all. So, the particular kind of formula is obliviously true, but it cannot be proved.

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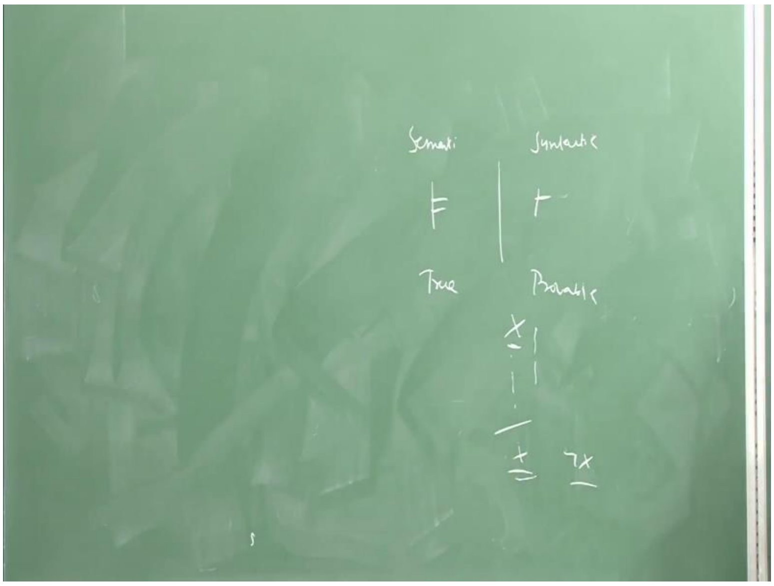

So, this is Gödel’s completeness theorem links this two things, two notions. This is semantic notion and other one is Symantec notion. So, something which is true has to be provable. Now, Gödel has come up with them interesting observation, it is consider path break result in the area of first order logic. That is does all the true statement all the validity forms verified forms true and that is the questions that last.

And in the context of arithmetic especially there are some kinds of obliviously true preposition we talk about some example little bit letter. And there are obliviously true or the feeling says that obliviously true, but there are not provable, that list to incompleteness. That means, what all the valid formulas are consider to all the valid formula find the proof. If are not able to proof it system is called incomplete.

And second incomplete theorem ((Refer Time: 29:00)) is like this is states that such a system contains the proof of it is own consistency. Suppose if you want show that consistency of that particular kind of system you can show it only when if and only if it is inconsistence. It has to be inconsistence to be complete. That seems to be major blue to Russell whitehead grand program of logisisum and even ((Refer Time: 29:34)) axiomatic system.

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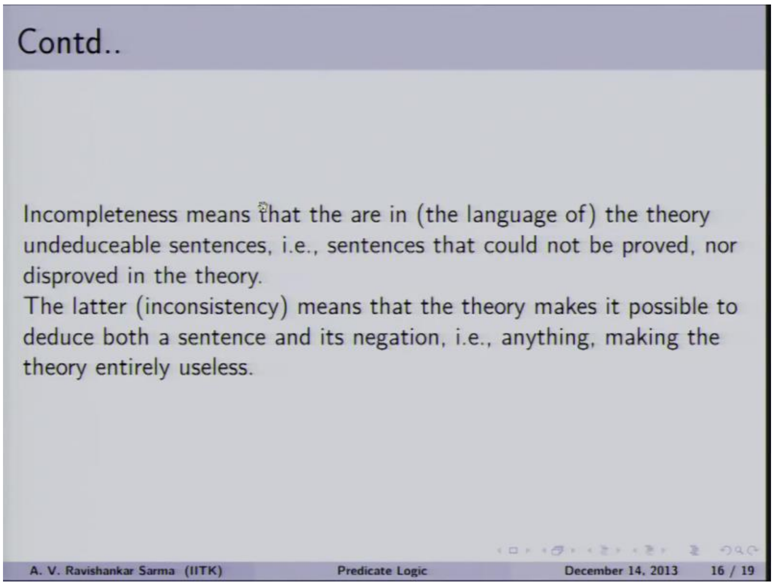

Russell whitehead is the motivated by agreement it as well as whereas, Hilbert acquirement motivated by geometry. So, here incompleteness means that, there are ((Refer Time: 29:53)) in the language, there are uneducable sentences. That means, sentences that could not be proved nor disproved in the theory. There are uneducable deduce in the sentences. That means, there are obviously, true statement of in all, but we are not able to prove ((Refer Time: 30:10)) we know the x is prove and all.

But, we are not able to prove x, we have not able to prove x means, we are not able to the deduce x. And we have same time we are not able deduce even not x also. At least one of things should come as the outcome in your theory. If both the things will be come as outcome obviously, system is considered to be inconsistency. That means, it should be position delay y the x that I should be position delay not x, if both are not delay and all and each to incompleteness.

Now the second incompleteness means that, the theory makes it possible to deduce both sentence that is the alpha and negations. That is if you deduce the x and not x that is contradiction is part and parts full of your systems. And that means, you can delay anything and all. So, this is what we have seen earlier. 2 plus 2 is equal 5 is a human if use this particular kind of example. 2 plus 2 is equal to 5 which is consider of contradiction.

But, as well as show that is consider to be is a pop 2 plus 2 is better than is a pop. So, contradiction anything you can derive. So, if you system has to be inconsistency proved the completeness and there is something wrong in the system that list to incompleteness theorem 2. That means, anything if you can prove anything all then theory is going to be entirely useless.

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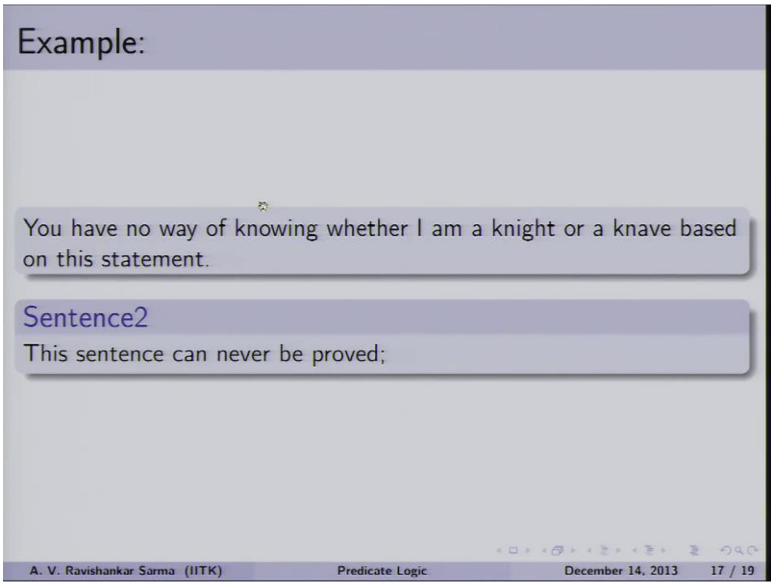

Gödel take in consider a simple examples with which he show that there are some obviously, sentences which you come across within formal a system, which is not consider to be either probable are not provable. So, let us consider to sentences to this sentence can never this proved this is the sentence which we are taken in to consideration.

And then showed that you cannot deduce given x that means, you cannot prove x, that means sentence can never be proved is you can be deduce are that sentence can never be proved that is it is the not the case of sentence never be proved also will come us will outcome of your formal systems. And both cannot be true that means, to incompleteness.

Now, take of the first sentence of you have no way of knowing whether, in the context of the knights and knave puzzles spoiling has nicely put this things in the context of Gödel's incompleteness theorem. Let us take this example you have no way of knowing whether I am a knight or a knave based on this statement, this statement is the statement you have made. That is you have no way of knowing whether I am a knight or a knave based on this particular kind of statement.

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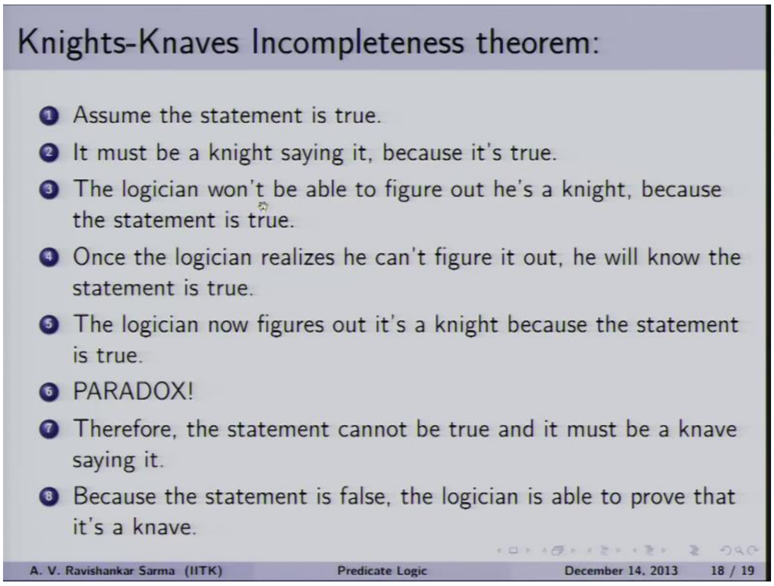

So, this how this leads incompleteness theorem ((Refer Time: 32:59)) Assuming that statement is true. That is you are knowing whether you have item name. Then, if the statement has to be true then the statement has to made by the knight. So, what we trying do is we are discussing load as incompleteness theorem, it context of a knight ((Refer Time: 33:17)). Knight always tells a truth, knave always tells a lies. It is the never the case them knave tell the truth.

So, it must be knight whose saying it that why sentence is true. that means, if the sentence is true than this is the what follow us. That is the logician will not be able to figure out he s a knight, because the obviously sentence is true. The once the logician realizes he cannot figure it out, he will know the statement is true. Then known what happen here is that logician now figure out that it is knight, because the statement is true.

That leads to a paradoxical kind of situation.

Therefore that statement cannot be the true and it must be a new saying it. Because, the statement is false the logician is able to prove that it is a knave. It is able to prove both knight a and both knaves and all.

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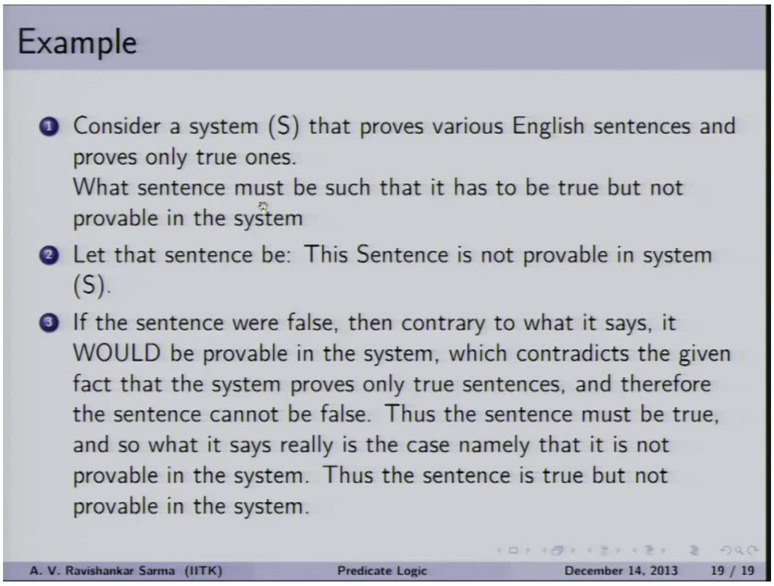

So, that is leads to some kind of problem that let be incompleteness. Go to the all takes in to consider simple examples such has that is consider a formal system S that proves various English sentences. And only true ones, now let us take one simple example. Now, we are asking this, questions what sentences the particular kind of thing be such that it has to be true, but, not provable in the system.

We are taking natural sentences we trying to come of the given particular kind of statement, which is the obviously, we know the true, but it the not provable in the system. He takes the particular of the example. The sentences is no provable in the system, that particular kind of thing is consider a sentence, which represented by S. Now, if the sentence is false that means, it is false that sentences is not provable in the system. Then contrary to what is says, what will happen is it would be provable in the system.

If the sentences if false and all and what is says is true. It contradicts the given fact that the system proves only true sentences and all. Because, here it is we are proving false under this also. And hence therefore the sentences cannot be false. Their assuming that sentence cannot be false the leads to fact to it cannot be false ((Refer Time: 35:44)), because it is consideration. But, it suppose assume it is the true, then what the sentences really says is this things.

The sentence is not provable in the system that means, there are some kind of which are not provable in the system. That means, the sentence is true but not, sentence is obviously, true, but is the not provable in the system. So, in both cases if take this particular kind of sentence that this sentences are not provable in system. That is false there is a problem if it is the true then all there is a problem. So, the system has the sentence has to be true to be not provable and it has to be provable and it has to be false.

That means, all false sentence we are able to prove and all the true sentences it cannot be prove and all. That goes to against this particular kind of things at all the valid forms the should find of proof. All the true formulas should find a true final. But, we are not able to find a proof for this, because even are not able x we are not able not x also. So, in a different context he discussed this particular kind of results.

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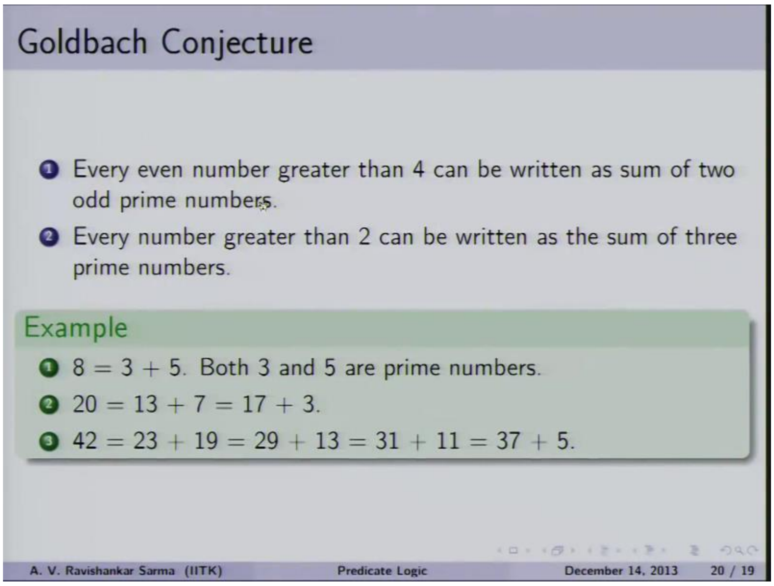

In the context of there are important conjecture in mathematics, which all get feeling tell us which as obviously, true. But, it has no proof. We note that all the theorem should find a proof. If you say that something theory it has no proof and that is not consider thrift. It is simply it can be called a conjunction or some kind of assumption or your belief extra. So, in mathematics proof means it has to sorry theory means it has to have proof.

Now, gold box conjecture is like this, every even number greater than 4 can be written as sum of two add prime numbers. Everyone knows even elementary high school going student all some knows that. The seen to be the case obviously, the case that means, if a take 6 into consideration that is nothing but, some of two add numbers that is 3 plus 3, the 3 is consider to odd prime number. And it is same 14 into if take consideration it is 7 plus 7.

So, we know that every even number of expressive terms of greater than 4 can be express has some of two odd prime numbers, the prime numbers obviously odd only so. So, now every number of greater than 2 this also another way of putting it gold for conjunction. Every number greater than 2 can be written as sum of 3 prime numbers. So, now in the first case, 8 for example, even number can be expressed as 3 plus 5, 3 is odd and 5 is odd prime numbers and 20 for example, 13 plus 7 as written as 7 plus 3 as well.

The second case, even number for example, 42 is written as 23 plus 19 etc. So, our get fill tells that this is the obviously, case whatever number that your taking that consideration even number at can be expressive terms of some of odd prime numbers. But, till to date we do not have any proof for this particular kind of obviously, known factors it consider true, but we do not have any proofs. So, we have some kind of reference it will be get it in the letters by Euler to Goldbach.

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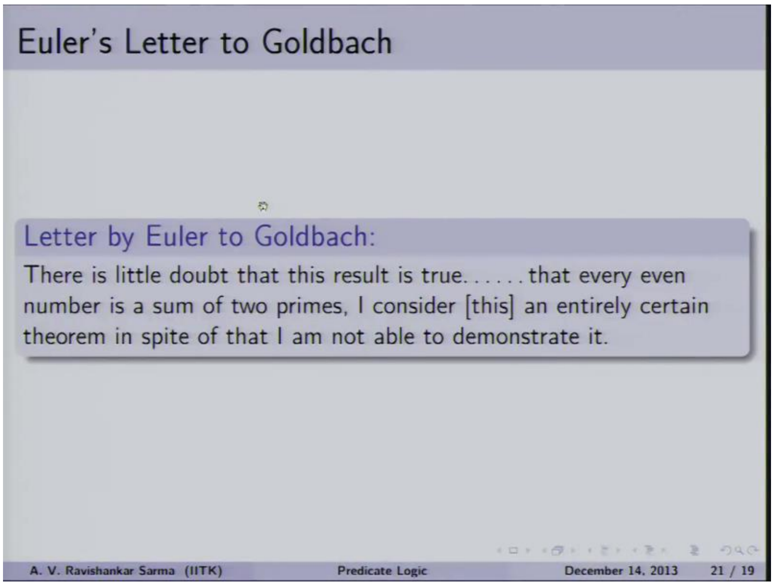

There is obviously little doubt that this particular kind of result, that is true one all. That is no doubt cut feeling says that obviously, true and all. Because, whatever number that even number taking into consideration it is expressed as some of two or prime numbers. But, what is the guaranty that is the exits kind of there is big numbers, where it cannot be expressed as some of odd prime numbers all. We do not know this movement. So, there is it is not proved it, there is little doubt that there result is true.

This is what another either is expressed Goldbach. Every even number is a sum of two prime numbers. I consider this the entirely certain theorem in spite of that I am not able to demonstrate that this is the not able to proof either x, this is the not able to proof not x. So, this these are some of things which poses some kind of challenges as set kind of limits to the first order logic, where we said that all the true preposition are consider to be provable. We lend this lecture with by saying some kind insights from Gödel's incompleteness theorem.

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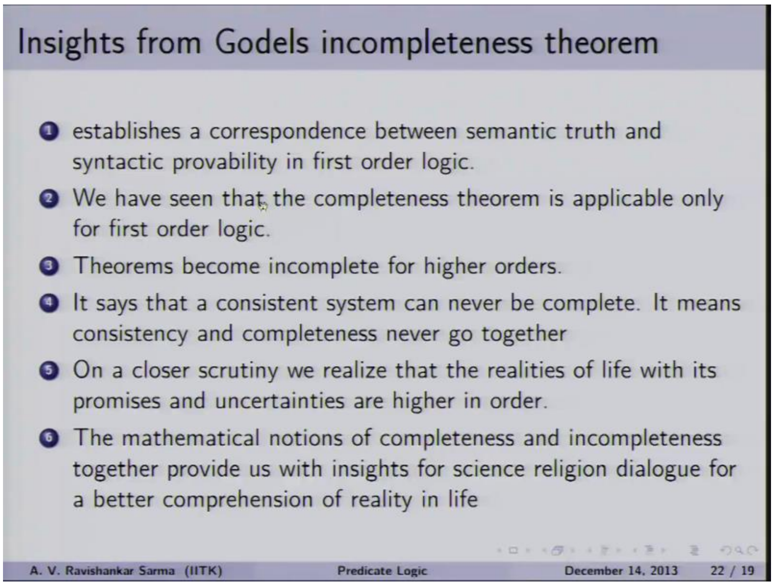

If you note that we are not proved Gödel’s incompleteness theorem it require entirely different kind of course, to talk about the celebrated result of cute Gödel. But, I only given the general idea of this Gödel incompleteness theorem. Some of the insights we can talk about then the, we can end this lecture. The Gödel’s incompleteness theorem as establishes a kind of first of all a completeness theorem, which is mostly ignore by many legation in all. It was consist of one of celebrate result and all.

Mostly people are interesting in the negative result. Incompleteness extra, but completeness also he worked out in p h d thesis. It establishes a correspondence between semantic truth and syntactic provability. Single tense and double trance in the first order logic, whatever is provable to true whatever is true. And we all the scene that completeness theorem is applicable only for first order logic. In case of in talk about variable stringing over padicates function symbols etcetera.

We are talking about second order logic it loses some of the interesting future like completeness consistency etcetera. So, very difficult to establishes this important logical properties. The theorems become incomplete in the case of higher orders logic, third order logic and second order logic etcetera. So, now Gödel’s incompleteness in also says that a consistent system can never be complete. If it has to be complete, it has to be in consist. That means, consistency and completeness can never go together.

Consistency means, we not in a position both x n if are able to derive both an existence system in called inconsistency. Otherwise derive x or derive not x. Completeness tell us that something is true it has find a proof. This both consistency and completeness has never go together. If giving as some kind of impression of that we do not go together. So, now if you observe, if you have some kind of closer scrutiny over our own individuals lives and on. We realize that the realities of life with is promise and uncertainties are etcetera.

And all can be expressive I higher in order. So, that means, there is some kind of incompact ability between consistency and completeness. So, on the things there some gap between consistency and completeness. So, now the real question is comes to has is which will be a rise in available. So, now the mathematical notions of completeness and incompleteness together provide us with insights for science religion dialogue for a better comprehension of reality in life.

The obviously, see that there is some kind of we know that consistency and completeness never go together with as a result of is Gödel’s completeness theorem. Now, feel bet patent question one need to ask oneself, before we end this lecture we as ask this particular kind of questions in the consistence of Gödel’s incompleteness of theorem.

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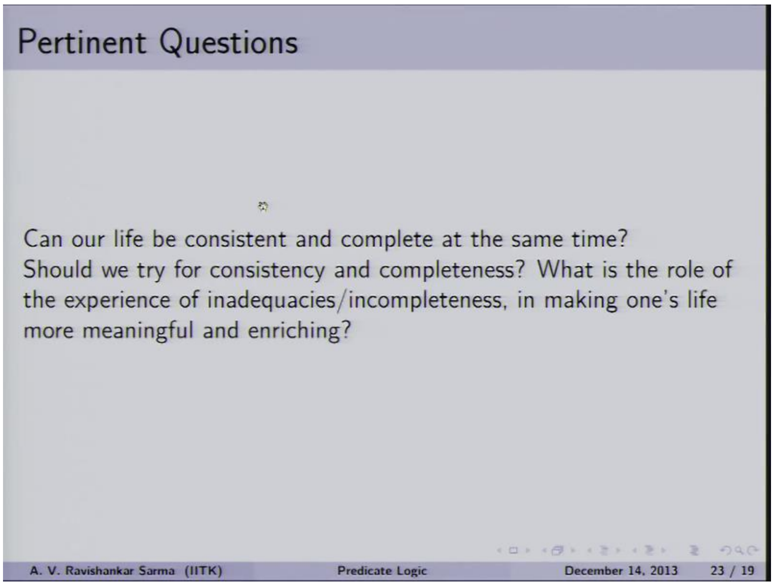

Can our life be consistent and complete at the same time. Taking into consideration higher all complexity etcetera and all. And second question that can we ask is should we try for consistency and completeness together or third question is what is the rule of the experience of inadequacies and incompleteness. In making one's life more meaningful and enriching obviously, our goal list make our life meaningful enriching. But, we celebrated result, where consistency and completeness cannot go together.

So, what sense we can making one's life meaningful and enriching. So, will end this lecture with the summary, we begin with some of the important theorems of first order logic. Some of the important theorems we are not kind cover all the theorem. We started with compactness theorem and then we mood on to completeness theorem. And then we discuss the celebrated negative result by Gödel, which set limited to the grand program of logician. That means, all the reduce to logic. So, we discussed Gödel's incompleteness of theorems and then to certain extremely discussed some of the simpatico in fact of Gödel's incompleteness of theorems. The questions are at rise pertinent question that require valid answers, required satisfaction answers. So, this lecture is not going not taking about answers for these particular kinds of questions.