Symbolic Logic
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Lecture - 36
First Order Predicate Logic
Introduction
Syntax

Hello welcome back, we are on the last week of our lessons this is a module 36 and we are towards and of this course. So, we have finished a quiet a bit actually.

Now we are into the first order predicate logic where we are going to look into just the bear overview of the syntax of first order predicate logic and we are going to learn about predication in a slightly different way. So, today's module I introduced to you what this first order predicate logic is and what are the most elementary syntax in this syntax as you always know is the way to learn how this logic moves, what are it is priorities all of this you shall able to find in the syntax itself.

So, you can say that this is our introductory lesson to first order predicate logic you are going to see some many, similarity in the way the categorical logic things, but that was the whole idea to give you a sort of have the categorical logic as a preamble to understand first order predicate logic why are we going towards first order predicate logic again we need to remember, what I told you this that we left preposition logic behind right the whole idea was to get inside the structurally simple propositions. When I said there would be two way to look into the insights of the structurally simple propositions one is the categorically logic way or that is unway another is the first order predicate logic way.

So, now, we stand in the first order predicate logic and we are still talking about, how to analyze the structurally simple propositions.

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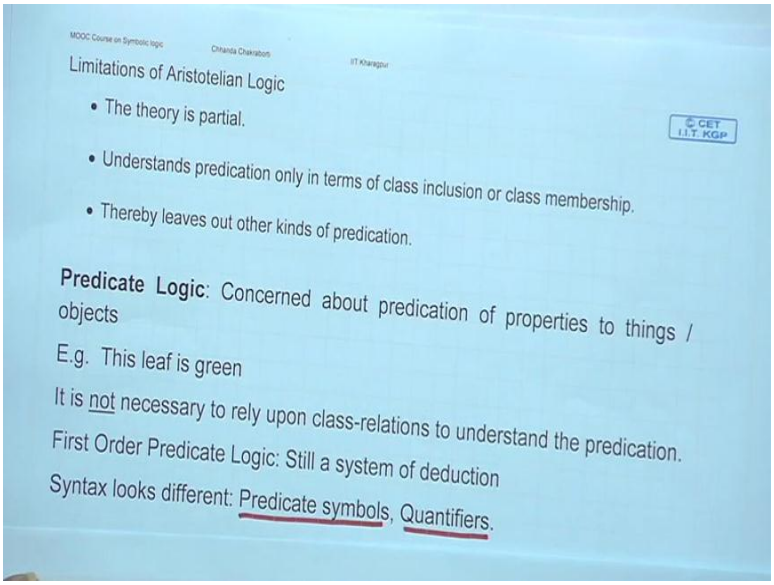

What was problem with Aristotelian logic do we why could not we stop there. So, let us barely touch upon that see Aristotelian scheme is a wonderful scheme it give you statement forum it is a four kinds it tells you the structural properties and the formal validity and formal invalidity and also all that is good. But it looks in a certain way at propositions and that is what makes it a partial theory it only looks into certain subset of propositions namely the categorical propositions and that too it tries to only approach the categorical proposition analyses through predication in terms of class inclusion or absence or negation of class inclusion.

So, that is the lens that it uses and that becomes a definite limitation because there are so many other kinds of predication possible. So, this is the reason why we are leaving categorical logic behind and then we thought supplementing or understanding by the first order predicate logic also when you are in predicate logic we still talking about predication which I have touched upon in the categorical logic also predication. Wherever I use this word I told you it means ascribing the properties to some things or objects. So, you are assigning properties to some objects and things and this is exactly what categorical logic did except they treated the properties also as the class the things and object also as a class which is not going to happen in this kind of predicate logic.

For examples, take a look into this, this leaf is green this is the object which as the properties of green, but this object first order predicate logic will tell you need not be a class need not be a seen as a class it is an individual leaf a very particular leaf, just like this pen is the very individual and unique object it is a particular object. We do not need to thing the pen has the class by itself in order to understand the properties that it as in order to predicate some properties to this. For example, this redness that this pen has in order to say that the pen the cap is red we do not have to go into class terms inclusions to talk about that this pen is red, this is what we are learning from first order predicate logics. So, how are we going to understand it what is our take on this is what we are discussing?

But first of all this is what we touched upon that in order to approach predication of this kind or any kind this leaf is green leaves are green whichever way you want to go predicate logic say that it is not necessary, that we have to always use the lenses of class relationship or membership of the inner class, but said all that said and done we are still into the deductive system and what you will find we are going to carry everything that, we have learned in the propositional logic and will bring them here. But first order predicate logic is going to have it is own unique kind of syntax the symbols are going to look a little different because, apart from the proposition symbols you are going also have predicate symbols. Now this is a new earlier in proposition logic you saw only the proposition letters.

Now, you are going to see some symbols which are going to predicate letter also. So, naturally your propositions are going to look kind of different right. So, be prepared to appreciate the difference plus there is going to be another kind of entity operators called quantifiers. What they are I will explain as we go along in this next module and so on. But we are just learning the ropes here and the syntax I said it is going to look little different. So, now, let us precede with the syntax first thing in a language is the constant the names. So, the individual constants for first order predicate logic they do have a place their function is to pick up unique particular objects from the domain.

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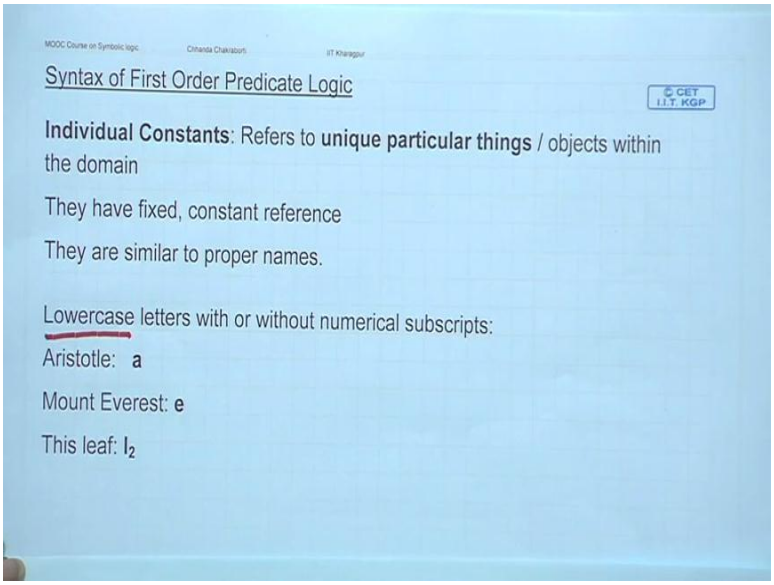

In a way the individual constants work like proper names you know when you are sitting in a class whole lot of people, but your name is called you respond is not it. So, in way your name picks you out from the entire class room and that is why you are respond to it. So, similarly think of the individual constants as proper names of things in the domain, in your domain where you are working. So, individual constants they are going to have a fixed constant reference to tied up to some particular thing in your domain.

Now, visually they are going to be lower case alphabets lower case and there is case sensitive here lower case letters. Now you may have subscripts numerical subscripts with them you may not have, but it has to be lower case letters alphabets suitable lower case letters. Let me show you some for example, your domain includes Aristotle then, you might think of assigning this small a lower case a to Aristotle. So, whenever a is used in your logic predicate logic predicate logic this refers to Aristotle and Aristotle only.

Similarly, suppose Mount Everest is there in your domain and we have picked e lower case e to refer to Mount Everest. So, the reference remains fixed and whenever you are using e it will pick up only Mount Everest and nothing else this leaf. So, many leafs are there. So, this leaf let us give it as subscripts of numerical subscripts. So, l2 arbitrarily speaking you can call it l14, if you want or l one anything as I said you can even call it

just l that is also, but in this context this reference will remain fixed and that is the job of what we call the individual constants. So, please understand the term very well constant means reference constants fixed individual as in this will pick individual objects or things entities from your domain individual as opposed to groups' classes large clusters and so on. Then we come to discuss about the predicate symbols. So, this is something new.

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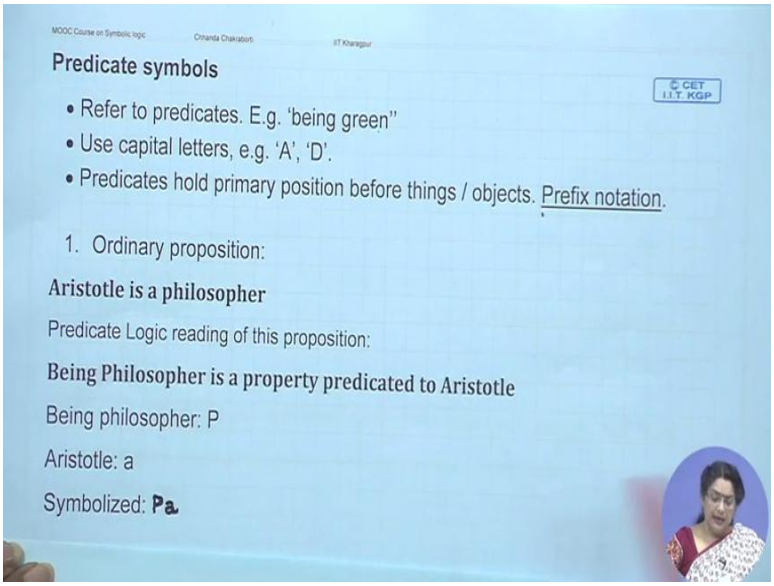

So, pay some attention to this see we are going to deal with the properties that we will predicate. For example, being green this leaf is green there is being green predicated to this leaf this is how it is going to read.

So, what we will do use capital letters assign capital letters suitable capital letters to predicates remember, we use to assign capital letters to structurally propositions and propositional logic now here the capital letter are going to refer to only to predicates or properties and this should also tell you the emphasis on more on the properties or predicates rather than on the individuals. Because this is where you can see the case sensitivity showing up the upper case or the capital letters are being assign to predicates and they hold primary positions in our logical considerations. We will show you how to

read sentences. So, before things the predication comes the predicates comes before the things and that is known as prefix notation I will show you how to do this.

Let us take an a ordinary proposition Aristotle is a philosopher right and this is what is the sequence, but in first order predicate logic the best way to read that would be to start at the property. So, you are reading and this reading I thing you should practice this also because that is the key to understand the syntax of this logic predicate logic reading of this proposition is going to be this that being philosopher is a property predicated to Aristotle take a look. So, primary position is that of the property which is being predicated to the individual who is Aristotle get it, now let us bring to our mind that we have decided to have capital letters for the property and for the individual we have the individual constants right.

So, once more reminding you being a philosopher is let say capital p and Aristotle is lower case a. Now Aristotle is the philosopher is read like being philosopher is a property predicated to Aristotle and the symbolization would look like this p comes first that is the prefix notification I am talking about followed by lower case a this is the reading of Aristotle is a philosopher your philosopher being philosopher is assigned or predicated to a. So, this is what this propositions, are going to look like that still a structurally simple proposition, but notice that we have gotten inside it and we have found the predication inside that is happening between an individual and the property, but no class reference.

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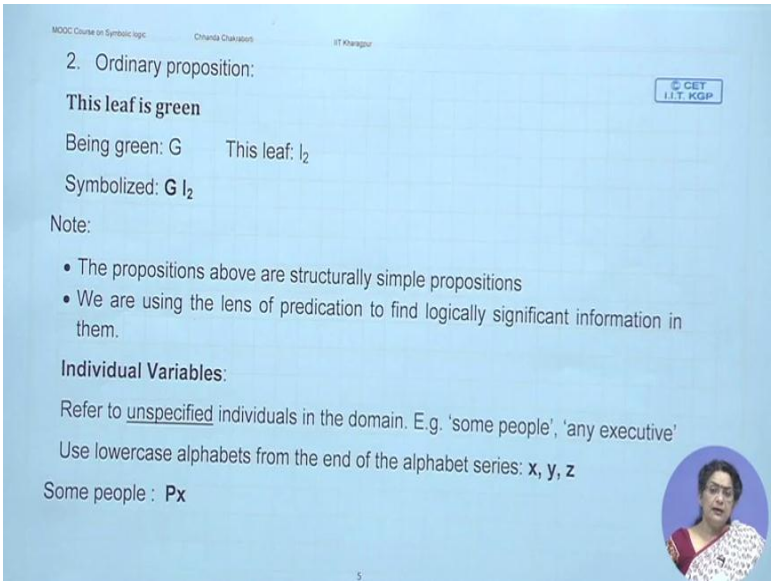

Let us take another example; for example, we saw this leaf is green fine. So, being green is g and this leaf is l, 2 we decided arbitrarily. So, the translation of this is going to be one it is going to be g l, 2 g is first being green is predicated to l, 2 which is this leaf this leaf as in this leaf. Now notice that each of this propositions that you have seen.

So, far as I said are structurally simple proposition what we have done is to use the lens of predication to find logical information inside. So, we have analyzed them the structural simple proposition we have open them up and we try to look inside them and, but the lens this time is not class inclusion, but that of predication.

Our next stop is to know about the individual variables individual variables see again the reference is to individual as a particulars particular things but variables as you know variables are place holders. So, the difference between individual variables and the individual constant is the constants is like a name, it attached to one specific particular thing throughout individual variables is different they work differently they refer to unspecified individuals you are referring to individuals. But not necessary to particular one for example, when you talk about some people you do not have any one particular in mind, but you want to keep the reference slightly unclear and unspecified that is the time to use the individual variables when you are using individual variables in this logic

remember use lowercase alphabets from the end of the alphabets series, xyz uv and so on and so forth.

So, towards the end of the alphabets series choose lower case alphabets to represent individual variables, I will show you will show you enough examples of when to use the variables and how to use the variables. In fact, you will find that quantifiers cannot work without using the individual variables. But at the moment keep in mind that individual variables do not have any fix reference they are like place holders. So, they can be refer to unspecified individuals and the only things is that they, the variables are going to be chosen form the end of the alphabet serious.

But lowercase. So, we have covered individual constants and then we looked into the predicate symbols and we also saw individual variables.

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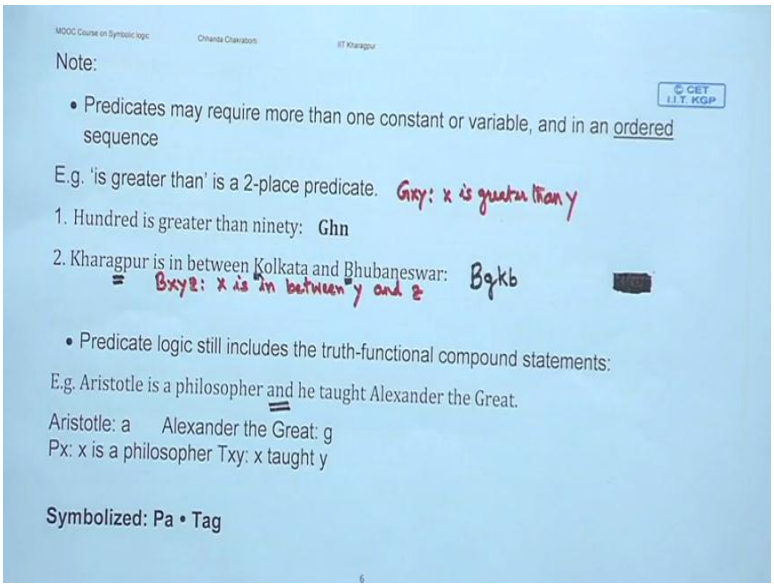

Now notice that, far we kept it rather simple, but there are predicates which deserve and require more than one constant of variable these are called n place predicates nf n for area variable number of simple. So, it can two place predicate it can be three place predicate and so on. But the point is they require more than one their nature is such that they require more than one constant of variables and they are in ordered sequence it is

not necessary opposite. So, these are going to be sort of like ordered. So and in proper sequence, sort of like take a look for example, is greater than is greater than that; obviously, a two place predicate.

Something has be to greater than something else right. So, that is the two place predicates. So, you are going to need either 2 constants here or 2 variables here right. For example, 100 is greater than 90. Now that is a constant 100 is you need a constant for that because, 100 is one specific number 90 is 1 specific number they are like names of number.

So, we have chosen h and n h for 100 n for 90 and this is the representation in first order predicate logic of this English sentence, 100 is greater than ninety the greater being greater than is the g is greater than so, this is going to be put like this that we can put it like. So, x and y are variables. So, for any x y x is greater than y and that is represented in gxy, if that is the case then this is our translation of 100 is greater than 90.

Here is the another example between in between in between requires 3 terms, 3 constants or three variable something has to be in between this place and that place or this thing or that thing. So, three different individuals you need to specify. So, here is the predicate letter for example, b and xyz that is your variable lined up where this means x is in between y and z right given that we are trying to represent Kharagpur is in between Kolkata and Bhubaneswar. Kharagpur is in between Kolkata and Bhubaneshwar. So, Kharagpur let us take that is the place right that is the name of a place. So, we need an individual constant Kolkata, Bhubaneshwar we need individual constants to represent this. So, your translation becomes bg for Kharagpur. Kharagpur is in between that is why I said it is a ordered sequence you cannot change it you cannot bring Kolkata and Bhubaneshwar before.

So, Kharagpur is between Kolkata and Bhubaneshwar g represents Kharapur. So, that is how we are going to handle this kind of n place predicates symbols there is also as I told you that, we are carrying back carry everything back from propositional logics. So, you are going to see truth functional statements also in first order predicate logic let me show you.

For example, thing about this sentence here Aristotle is the philosopher and he taught Alexander the great once more Aristotle is the philosopher and he taught Alexander the great. Now read the sentence carefully is an sentence, but the conjuncts, now this you understand this is going to be our correct, but the conjuncts take a good look into the conjunction Aristotle is the philosopher you now see the predication happening being philosopher is predicated to Aristotle there is that that has to be to there and look at this conjunction he taught Alexander the great.

So, if we have a property call x taught y hmm. So, that would be the predication that we can now assign to Aristotle where Alexander is the great is the name also. So, we need a constant for that let see what we have suppose, we have the symbols lined up like this where Aristotle is a Alexander the great is small g a lower case g px stand for xis a philosopher txy stands for x taught y then, again we have the 2 place predicate where this is expressed by the variables, but we know what we want to do what would be the translation? Here, Aristotle is the philosopher we have done that is your capital p followed by lower case a right. We have done that this is who taught whom. So, tag right Aristotle taught Alexander the great.

Now, let see the translation therefore, is going to be something like this one conjunct is Aristotle is the philosopher, this is where Aristotle taught Alexander the great and look at this dot that is your end note that this sentence, which may not look to you very familiar if you are still in the propositional logic phase this is the first order predicate logic sentence. But it uses the truth functional connective dot not only that it is the truth functional compound it is going to follow the truth table of the dot it will be true only when both the conjuncts are true false in every other case get this.

So, once more remember what I said that predicate logic will include truth functional compound statements also like. So, so this was an example, but we will see many more as we go along in this. Now finally, we have now slightly better grasp of the syntax, but one question might come to your mind that why it is call the called first order predicate logic. Predicate logic we understand because it is all about predication, but why it is called first order.

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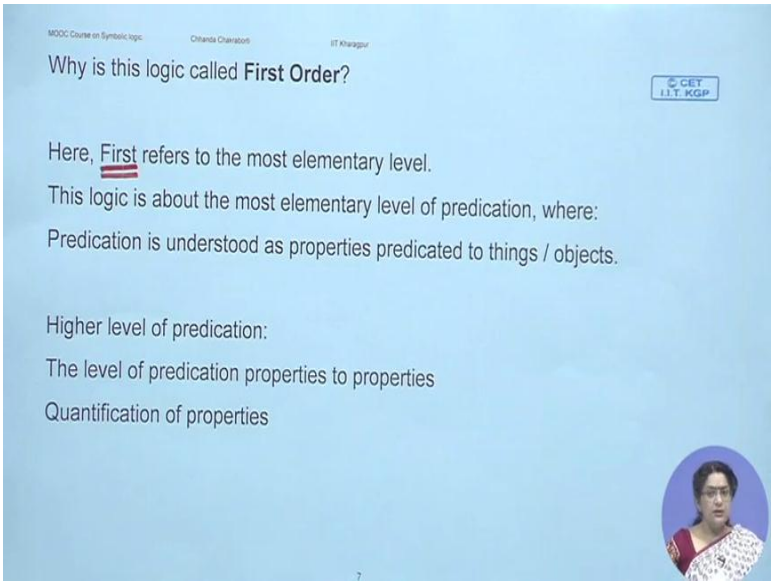

Now, that answer from my side I am going to keep it rather short, but it should be also satisfactory please note the first here stands for being the most elementary all right. So, you the very first in the order of predicate logic this is the first order predicate logic. So, remember the first means here the most basic of most elementary kind of predicate logic elementary in what sense elementary in the sense of predication. It is the it deals with the most elementary kind of predication which is what which is about properties being predicated to things or objects you can remember as I said this pen right this pen or if you want a same red pen here is a red pen or the pen with the red cap fine.

Now, this pen is an object to which properties belong. So, we use the language to talk about predication to this thing in this logic. For example, the cap of this pen is red the body of this pen is somewhat blackish. So, I am using predication language to characterize this pen as long as predication remains at this level of things and their properties. We are still in the first level where as, there can be higher level of predication where we speak about properties of properties or properties of properties of properties. So, the order of predication most elementary is things and properties, but then it can go into more abstract level where even properties of properties can be talked about their quantity can be talked about and so on.

So, this is not our concern we are here and we are into first order predicate logic all right, this where we are going to close the discussion today but we are going to learn more about this logic in our next module.

Thank you very much bye, bye.