Symbolic Logic

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Lecture - 37
Quantifiers
Categorical Propositions in Quantified Form
Learning to Use Quantifiers

Hello we are in module 37 of Symbolic Logic Course and we are discussing this first order predicate logic.

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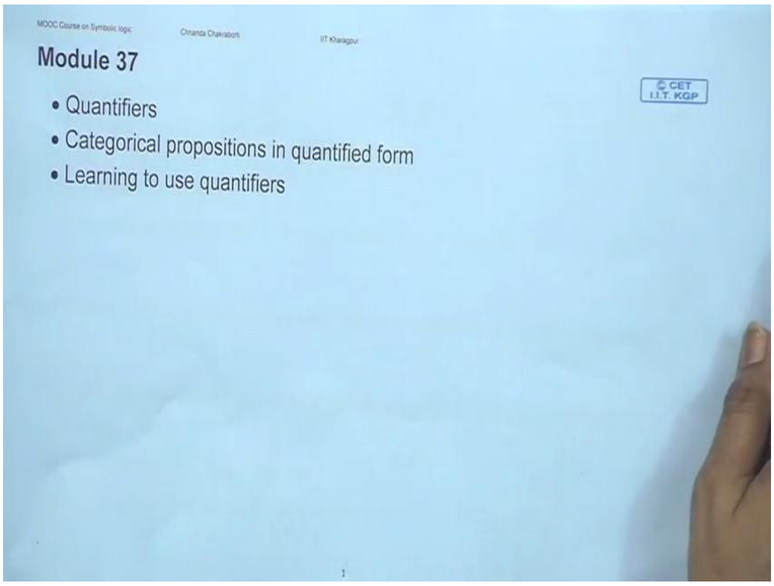

What we did in the last module was to sort of get you (Refer Time: 00:30) with the basic syntax of this first order predicate logic, but now today we are going to talk about the quantifiers which is an interesting addition in this first order predicate logic. You will see the idea has link with the categorical logic. So, it will be not be difficult for you to grasp it, expect the symbolic representation of it that is what you have to learn.

And then we are going to take on the categorical propositions and will try to apply the quantifiers with them and in general we are going to learn how to use the quantifiers. So, this is what we are going to discuss in this module 37. Quantifiers the name itself should tell you what they are and why we are introducing them will become clearer as we start the operation with them, but first note that the quantifiers are legitimate terms sorry expressions in first order predicate logic which talks about how many the quantity of the individuals that you want to refer too. Quantity, are you referring to all of them, are you referring to just one of them, are you referring to just majority of them and so on and so forth. These are all quantity terms that we have learnt in categorical logic also. The job of covering that quantity is by the quantifiers.

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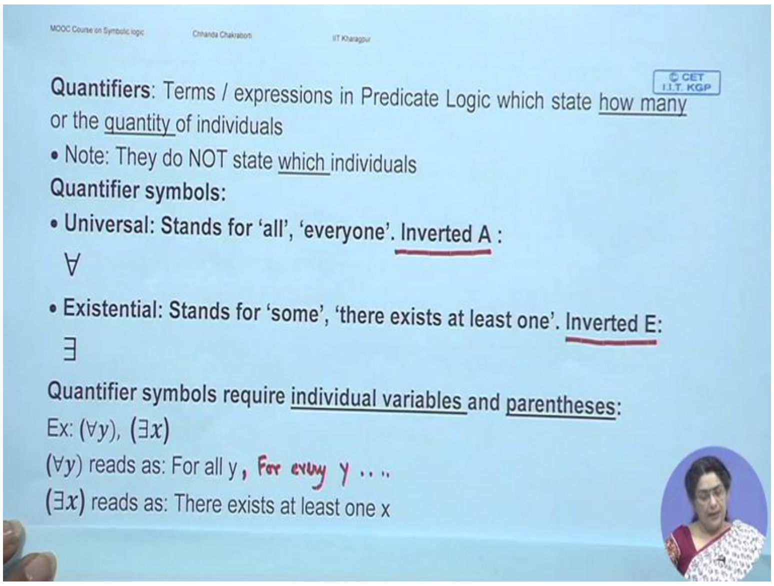

Note they speak about how many the quantity of individual, but they do not identify which individuals. So, they will not the mark there, they are not like names which will tell you this, this, this, are included in this rather than they will simply talk about how many of the individuals you want to refer too in your propositions this is when we use quantifiers. Simons, well we are coming there now quantifier symbols are composed of many different components.

First of all notice we have borrowed from Aristotle the thought about universal. The quantity when it is universal it refers to all or every, when that happens notice the symbol is Inverted A, Inverted As upside down A which looks like this. Take a look, it looks like A, but only inverted upside down that stand for the universal quantity, get it. Now there is also what categorical logic talked about was particular, whenever you wanted to refer to 'some' at least one you called it particular. Here please note, there is a lot of theoretical reason behind that, but in first order predicate logic will call it existential; as in you are claiming that there are, there exists such things. In universal there is no claim about existence, existence as things that are actually occupying space in your world, but whenever you are talking about the 'some' there is at least one there is a claim of existence attached to it.

Notice this is the reason why it is symbolized with Inverted E; the E refers to existence, Inverted E it looks like this get it. This is Inverted A for universal, this is Inverted E for what used to be call the particular please note in that particular with first order predicate logic there is a default existence claim. So, whenever you say 'some' it will read as this Inverted E reads as there exists at least one. So, this reading is something you need to remind yourself again and again that there is a default existence claim with the 'some' from now on and it captured by this Inverted E.

Now, together with these symbols we are going to require individual variables. The quantifier symbols to work with individual variables not with constants, with constants he will never ask how many, but with the variables which is unspecified with them you can ask how many of those, right. So, individual variables are they come with along with the quantifiers and they formed the part essential part of the quantifier symbols there will be also parentheses or brackets.

For example, this is the complete look of the quantifier symbol. Here is the parentheses and here is your universal quantifier upside down A followed by y which is a variable. This whole thing is the universal quantifier, get it or take a look into this - this is the whole thing is your existential quantifier; the whole thing, starting with the bracket closing at parentheses here is the Inverted E followed by a variable x. So, x y z anything you want to pick, but it will be preceded by the quantifier expression itself, but the whole thing is the quantifier symbol; did you understand. Now for this for example, will read as for all y or for every y and so on. You can put here also x no problem but that, but all I am saying is that the quantifier expression must contain variable, individual variable and that individual variable could be x y z any of your choice. But the reading is for all y or for every y such that. So, will show you how after the quantifier what come etcetera.

This is on the other hand they existential quantifier which reads as there exists at least one x, such that etcetera. Notice there exists at least one x which is not the case in terms of for all y, you are not saying for existing y's, you are not talking about existing y's, but for all y's such that, but when you are using existential quantifier there is a default existence claim attached to that, and keep that in mind.

Now, we are going to if we have understood this universal quantifier and the existential quantifier now what we will do is to apply them to represent the four basic type categorical propositions in our first order predicate logic. So, you remember A E I O, right. So, we are going to simply take them and symbolize them in our first order predicate logic that is the first thing to do.

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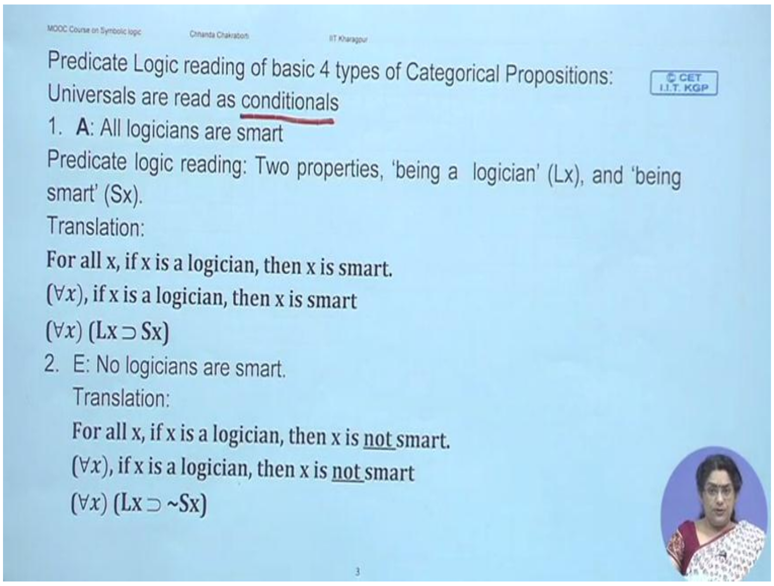

Notice that the universals are going to be read as conditionals, conditionals means if then statement. They are rather quickly put, remember there is no existence claim and there is also the hypothetical or the conditionality attached to them they are to read as conditionals, how? Let me show you.

For example, let us take all logicians are smart as an example of arbitrary a proposition, all logicians are smart. Can you see the properties that are predicated that being an logician, being smart, can you see that? Now, the reading of the predicate logic is different is not it, but will show you that, but first notice that the two properties are to be identified. So, being a logician let us see Lx, x is a logician; being smart x is smart is represented by Sx. How are we going to read all the logicians are smart? In first order predicate logic we are going to read them like this, this is how as a conditional. First of all this all we need the quantity specified it is universal, so we are going to use the universal quantifier for all x or if you want to for all y, no problem. For all x here comes the reading that you are going to train yourself in, if x is a logician then x is smart. If x is a logician then x is smart this is what all logicians are smart will mean, for every x in your domain if that x happens to be a logician then that x is smart.

Note there is no claim that there is such an x it exists or not there is no such claim, more over this is just left as a conditional. If this property belongs or predicated to x this property also gets predicated to x, that is the way the logicians would keep it, and again we cannot go into great details, but there is a big theoretical debate about this and George Bull sort of intertwined are provided solution to give this kind of reading to the universals and so. That is not the debate that we are going to cover here, but I am telling you that the outcome is that universals are going to read as this kind of weak conditionals, if then statements. So, translated now, we are going to translate this - for all x becomes this universal quantifier and this part is now going to be translated how if x is a logician then x is smart. How am I going to write that? Well like this, like this. So, Lx horseshoe Sx, remember I said all the truth functional connectives truth functional compounds are still very much part of our vocabulary of our syntax. So, this is the translation for all logicians are smart. Here comes, first comes the quantifier they followed by the whole sentence, alright.

Now, why we need the brackets will try to explain you in the next module when we talk about the scope of a quantifier, but right now take it in that this is the translation in first order predicate logic of all logicians are smart. I have taught you two things, one - that this is a categorical proposition, but we can capture it in our first order predicate logic. Second I said all universals are going to be read as conditional, so remember this.

Same thing applies to your E where no logicians are smart is a random example fine. Once more same properties, being a logician and being smart these are two properties, but this time we have a No, now No refers to what? Universal quantity, right, we have established that in our during categorical logic module. So, No, where we need to start again with the universal quantifiers, so our first paraphrase before we even go into the symbolization make it a habit to paraphrase in this quasi (Refer Time: 13:31). So, this is our reading of this sentence first of all. So, for all x that stands for No, but you are saying No is a negative where is the negative, we are coming there.

First of all what is the quantity that you want to give it, that is universal quantity therefore we need to start it like this for all x, if x is a logician once more, if x is a logician. So, it is not guaranteed that x is a logician, but if it has the property of being a logician, then what? Then according to this sentence x is not smart and that is true about every x in your domain, get it. So, once more for all x if x is a logician the x is not smart and this translation is again once more this is for all x and here comes this. Now how to translate that will like so, x is not smart the tilde appears here, exactly as we have said it if x is a logician then x is not smart preceded by the for all x that is your universal quantifier; this is my E proposition, alright.

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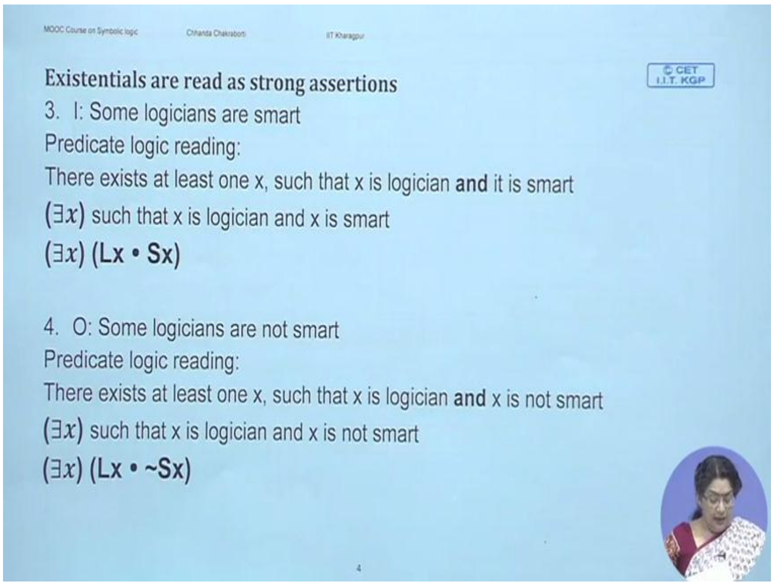

Here comes the other two, now the other two means I and O, I and O which we earlier called the particulars we are now calling them existential. Remember existential I have said carry a default existence claim, whenever you are using them claiming that in your domain there is exist at least one entity to corresponding whatever you are claiming, remember that. So, these are strong assertions, stronger than your universals because with them there is an attached default existence claims. So, let us see how we read them as.

Some logicians are smart, some logicians are smart; how I am going to do that? Well this some which quantity do you want to attach that existential quantity, the moment you do that the reading is going to look sort of difference. You are going to use there exists at least one x for some, what kind of x is that - such that x is a logician and x is smart. Now why did we not say if x is a logician then x is smart, first of all as I said look it this that you are already claiming there is such an x in your domain, not if x is there or not right.

So, in your Venn diagram also when we plotted I and O you may remember we used in x to indicate populated region, non empty region and the whole logic behind that was this modern logic inside that the particulars are existential, alright. Even if you compare that with your Venn diagram representation of A and E you will find that all we have said which areas are empty in them, we have shaded those areas; go back and take a look. But the unshaded areas we have not made any comment about that, unshaded does not necessarily means populated wherever you mean populated you have to use existential and you need to put the x do you understand the Venn diagrams better now.

Go back and revisit your A E I O Venn diagram representation, you will see in case of the universals we never claimed that the unshaded areas are populated. Whereas in I and O the particulars we have put the sign of being populated by at least one entity and that is because we believe in this that existential are strong assertions. So, where you are saying some logicians are smart there is an existent entity who is both logician and is smart, and then the translation looks like this, this is for their existence at least one x such that x is logician and is smart will look like this. And this dot makes a heaven and hell difference in the assertion, this is much more direct and more powerful assertion, there is such an x and that x is both of this both properties get it. So, remember this is what I get the treatment as.

Similarly, for O some logicians are not smart that is going to be read as there exists at least one x such that x is a logician and x is not smart there is such one person. So, here is the translation for that, this is for there exists at least one x and this is for x is a logician and x is not smart. Schematically speaking either AEIO I O representation is going to look like this where you have phi and psi and any two properties.

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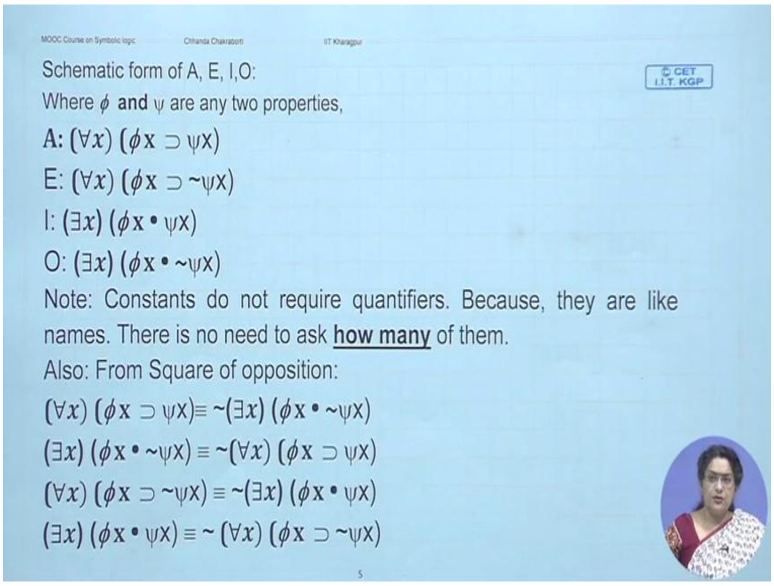

We are using the Greek symbols phi and psi to represent any two properties of your choice then a schematic form would be like so, it is a conditional with the universal quantifier attached to it. The E would look like this with the tilde attached to the consequent of this and we have shown this earlier and I is going to have this kind of a form, where it is with the existential quantifier and with the dot it is a conjunction and O is going to look like this where there is going to be tilde or indication sign at the last conjunct. Pick this is schematic representation of all A E I O propositions in our first order predicate logic.

I do not need to tell you this, but still for the sake of reminder that variables are the once will go with the quantifiers, constants do not require quantifiers because constants are like individual things you do not ask how many of those how many of you are there, there is only one. So, there is no need to even raise the question about how many, but variables they deserves this kind of questions. Also from square of opposition we are going to represent contradiction in this way that remember A O and E I they were contradictories. So, we can represent them like this now that A, A is equivalent to negation of O is that correct or No, the contradiction holes is equivalent to negation of O.

Similarly O is equivalent to negation of a correct, this is E, E is equivalent to negation of I correct because they are contradict and I is equivalent to negation of E did you see that. So, we have kept everything that we wanted to from Aristotle logic and still we have more coming in, but this was how to show that the categorical logic is also carried with this the truth functional will also be there, but I will show you slowly.

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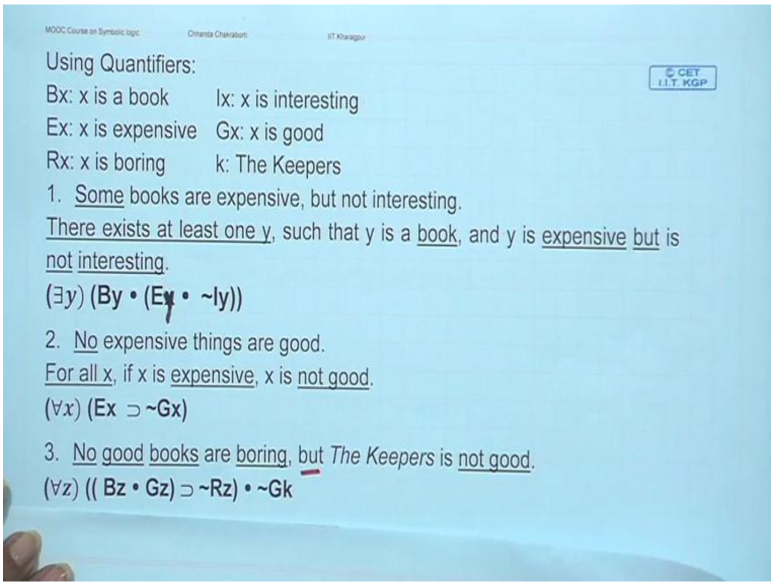

Now, let us talk about how to use the quantifiers little bit. See if we give you this kind of properties Bx x is a book, Lx x is interesting, Ex x is expensive, Gx x is good, Rx x is boring, the K that is the constant stands for a book called the keepers, fine. Here is the sentence some books are expensive, but not interesting, how am I going to read that, how am I going to translate that and does this sentence require any quantifier or not, that is the first thing. The properties are all here we can see it from the key also, the quantity term that is given is some. So, you know which quantifier to use, namely the existential quantifier that much you show.

We are going to read it now like the predicate logic wants as to. So, how am I going to read that this means there exists at least one x or one y or one z, any variable of your choice, there exist at least one y let me let us say I have chosen y such that now we are going to assign to that that is the thing in your domain, that is the individual thing in your domain unspecified, but it is an individual which has what certain properties such that y is a book, such that y is expensive, such that y is not interesting, get it. So, this is the property, now once more you do not be attempt to say if y is a book, remember you are dealing with existent individuals there is no we she, was she conditionality about it anymore it as to be strong assertion.

So, existential quantifier means that it as to be non conditional assertion sort of remember that. So, reading an I instant upon that before translation always go through this kind of paraphrase because you are beginners and you need to see this sentence in the lens, with the lens of predication. So, there exists at least one y s such that y is a book and y is expensive, but y is not interesting.

Now, comes the symbolization and that should be very easy there exists at least one y such that y is a book right and exists sorry this should be y. So, y is expensive and y is not interesting. So, this is where our translation will take us to and we have learnt to use the quantifier also. Quantifier variable it rules over the variable you are talking about certain individuals in your domain unspecified which are books which as certain properties.

Let us try another one no expensive things are good, no reference to books no expensive things are good how are we going to translate that. So, let us take a look first of all no means universal quantity, right. So, you are going to use the universal quantifiers. So, you need some for all y, for all x so on, suppose you say that for all x then if x is expensive x is not good. x is what? x is anything in your domain that is what it says, no expensive things are good. So, let us he wants this is done the translation will become easier for all x, if x is expensive x is not good. Why this if then? I have given you the answer already; when you are dealing with universals remember to treat them as a conditional.

So, if x is expensive then x is not good, what does it shows not good the tilde attaches here - that is the classic E proposition I have shown you the schematic form of the E also. So, you can see that is what we have followed. Let us do the third one no good books are boring, but the keepers is not good. So, no good books are boring, but the keepers is not good and you can do this paraphrase first - no good books are boring, but, there is but and then the keepers is not good. So, translation now looks like this - for all x if z is a book this is a z. So, for all z if z is a book and z is good then z is not boring, get it; Rx x is boring, but the keepers that is the constant that is the name of a book. So, the keepers, is not good did you see that. So, what you have here is capturing the whole thing. Why did we leave the K out, why did we have the dot outside? Again I will try to answer this question in our next modules when we talk about the scope of a quantifier. But right now, we understand how to use the quantifier, now remember the quantifier requires the variables to operate on not a constant that is my first answer to your question.

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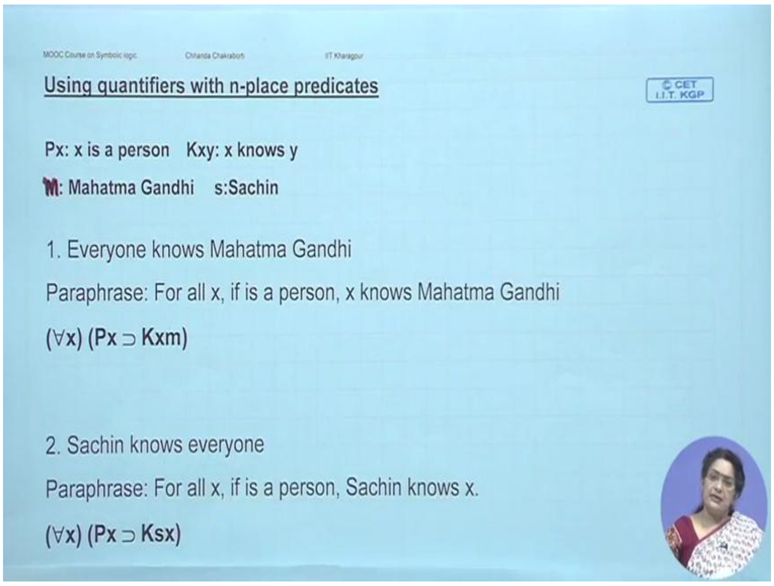

So, there is few more to go along with, these are you saw some kind of predicates, but know let us say we have this kind of two place predicate where x is a person and Kxy x knows y and you have Mahatma Gandhi. So, you have m that is the small m and small s for Sachin. Now let us see, everyone knows Mahatma Gandhi very good. So, everyone that (Refer Time: 28:05) is for all x if x is a person then x knows Mahatma Gandhi that is our translation. So, then the translation looks like this - for all x, x is a person then x knows Mahatma Gandhi look at this ordered (Refer Time: 28:22). So, there is a certain sequence who knows whom, compare with that Sachin knows everyone, Sachin knows everyone - you do cannot say there exists at least one s such that s is Sachin because Sachin is individual already. you do not need a quantifier to rule over Sachin.

On the other hand this everyone you need a quantifier. So, how do you read that the reading is if you want to start here the quantifier will show up for all x if x is a person Sachin knows that person, that is what mean by Sachin knows everyone and the translation is like this for all x if x is a person then Ksx. This is how we will move from here and this is the last one, very last for this module.

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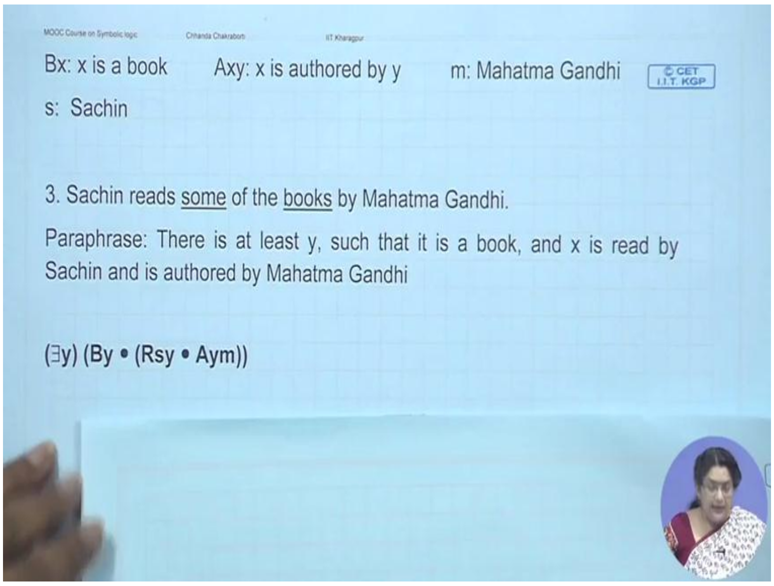

That suppose we have Sachin reads the some of the books by Mahatma Gandhi, Sachin again not quantifiable but the books are, Mahatma Gandhi also not quantified but the books are. So, some books as such that read by Sachin and some books as such by their authored by Mahatma Gandhi. So, this is how we will read it there is at least one y such that it is a book and it is read by Sachin and is authored by Mahatma Gandhi and our translation will look like so.

With that I am going to close this module, we will try to do more in the next module in this, but so far we have covered the quantifiers and I have shown you how to use them.

Thank you.