Data Analysis and Decision Making - III
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Lecture 31

Welcome back my dear friends very good morning, good afternoon, good evening to all of you who are taking this course and as you know this the data analysis and decision making free course under the NPTEL MOOC series. And this course basically which deals in the, which is the third part in the DADM series. Basically deals with the optimization, operation research, linear programming, the primer dual problem, the concept of sensitivity analysis.

Then we will as I had mentioned we will discuss about the transportation problem and then slowly we will go into the areas of nonlinear programming, quadratic programming, reliability programming, and robust programming and so an so. So, this total course duration is for 12 weeks which when broken down in the total contact hours is 30. And the total number of lecture is 16 number, which means that each lecture is for half an hour. And as you know that in each week we have 5 lectures, each being for half an hour and as you can see from the slide as you will see later on.

So we are going to start, we have already finished 30 lectures that means we have completed 6 weeks of classes which is we are almost half way through and we will be starting the 31st lecture. And after each week you take 1 assignments. So we have already completed 6 assignments, in total we will have 12 assignments. And after the end of the course we will be taking the final examination.

So, if you remember we were discussing about the general concept of how the transportation problem would be formulated in the sense you have some origins of the ware houses from where you are going to transport goods to the destination and the number of origins and number as destination need not be the same. So, number of origins can be M in number M as in Mumbai and the number of destination is N which is in N is in Nagpur.

As you remember we use the suffix of I changing from 1 to m, and G changing from 1 to n. Now, in the general problem formulation if you remember we have a cost matrix, cost matrix means that you are transporting from the first origin to the destination. So the costs are given by the C with the corresponding suffix and if I consider the overall metrics which would be of size M cross N each cell value which is C 11, C 12 if you go through the rows they would be the corresponding cost which would be incurred when you transport or ship. Whatever number of goods it is that we have to find out but this cost are given per unit when you ship. See for example the first origin to the N number of destination so C 11, C 12, C 13 till C 1N.

Similarly, in the second row we would have the values of C 21 C 22, C 23 till C 2N and the corresponding in the last row would be C M1 till C MN. So M1 to MN are the suffixes now given the cost we need to find out the total quantum of goods which have been supplied. And these would be given correspondingly the values or the variables will be given as X with the corresponding suffix as you have already are aware so it will be X suffix 11 till X suffix 1N and the last row would be X suffix M1 to X suffix MN.

Now our main concern is basically to find out the total cost. So, the total cost if you remember will come to that within few minutes is basically to find out the sum of the total factors which is basically C 11, X11 till C MN into X MN. But the interesting part is that we have considered the right most column which is there is basically the total, if you add up all the cell values in the right most column they would be add up to the total quantum of goods which is been supplied from all the destination combined together which is M in number.

And if you consider the bottom of stoke it will be the total number of goods which are being transported or transferred to each and every destinations which you have which the suffixes are given accordingly. So, if I consider that I am going to come to that later within few minutes you have basically the transportation problem. Now this is the balanced transportation problem in the sense the total sum of all the elements in the last column would be exactly equal to the total sum of all the elements in the last row.

Such as that equality signs holds which means that the exact number total number of goods which is being supplied from all the destination should be exactly equal to the total number of good which are being transported from all the origins would be exactly equal to the total number of goods which is being received at the destination.

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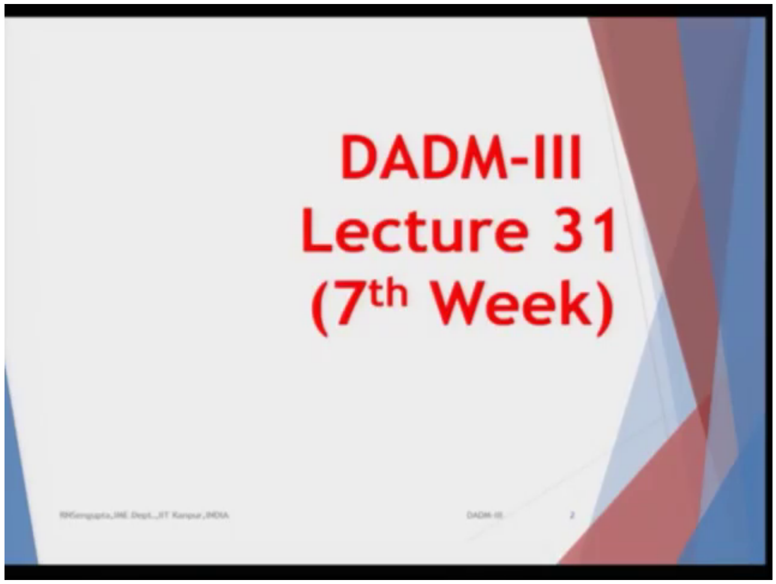

So, as you know this is the, we are starting the 7th week and this is the 31st lecture as I mentioned just before starting the class.

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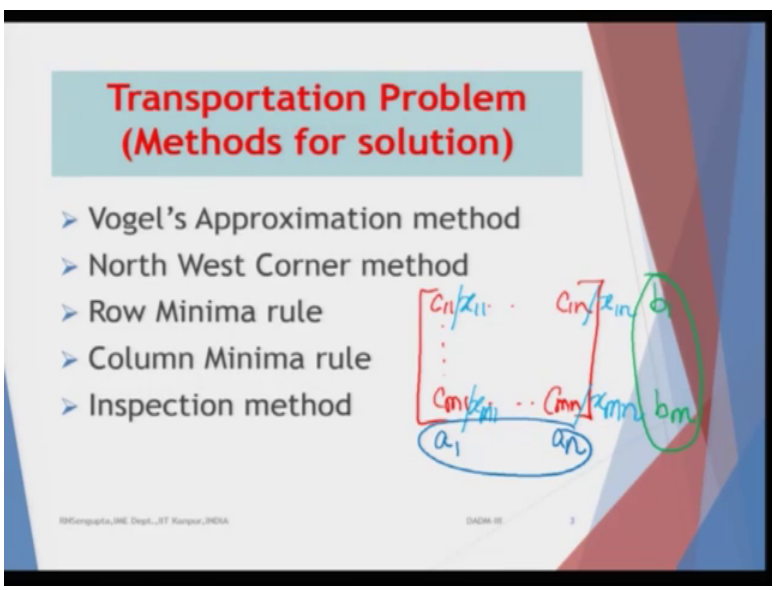

Now if you consider, before I come to the problem, so I will just do restate the problem is I have a matrix where all the costs are given C 11 till C1N, CM1, CMN so this is the cost structure which is given and I want to minimize it. And If I consider the total quantum of goods so in place of Cs I am basically replace them with X. So, obviously each Cs I will use a different color so it is easy for us, so each Cs would basically have X11.

So quantum of cost for transporting, from the first origin to the first destination is C11 into X11. Similarly, the total cost of transporting the goods from the first origin to the Nth destination would be C1N into X1N. Similarly, if I follow the logic the last row would be basically CM1 would be multiplied by XM1 and the multiplication factor would basically mean the total cost of transporting XM1 amount of goods from the Mth origin to the first destination multiplied by the cost which is CM1 per unit.

And finally the last cell value which is CMN would be multiplied by XMN where XMN is the total quantum of goods which is being supplied from the Mth origin to the Nth destination and each unit cost is CMN. So, if I basically multiply them there is a linear function I want to basically minimize. Because our main concern is basically to minimize the total transportation cost. Now, for all the problems we are going to consider that the total sum of all the X's, the total quantum of goods which can be supplied by the first destination would be given by either A suffix or B suffix whatever it is, it is given by B so I will basically mark it with a different color it looks better.

So, b1 till the last one it is bm and here it will be a1 to an, so the sum of all this is exactly equal to the sum of this. So, I basically need to optimize it, and obviously the constraints which you know that the supply from all the origin should be exactly equal to the supply of all the destinations. Now, we will use five different methods, so I will basically lay more stress on the first two method, the third and the fourth I will mention conceptually and when I come to the solving of the problem by the first and the second method, the third and the fourth will come out automatically, how we basically use the logic to solve.

So these are, remember these are just the initial approximation methodology based on which we will start our algorithm and try to find out the approximate solution then try to improve them. So first is the Vogel’s Approximate method, the second is the North West corner method, now as the word North West corner method means that we basically start at a particular point. So if you basically see the coordinate system, so the north goes up, the south goes down and the east and west are correspondingly given.

So we basically start from the left top most corner and basically follow a policy that how the allocation would be done. In the row minima rule and the column minima rule we basically concentrate on the row and do the allocation in such a way that the rows are mainly are concerned that we take care of the row allocations first taking into consideration the columns would be done in the secondary stage and in the column minimization rule we will basically follow just the reverse where the main emphasis would be on the columns and then basically we do the secondary approximations in the rows.

And the inspection matrix is just a very brief thumb rule based on which we basically do the allocations depending on where we think. The minimum cost would basically be possible for us to allocate. So first we will basically start with the Vogel's Approximation method, I first talk about the rule then with an example I basically (())(11:09) explain every step of the algorithm of the rule.

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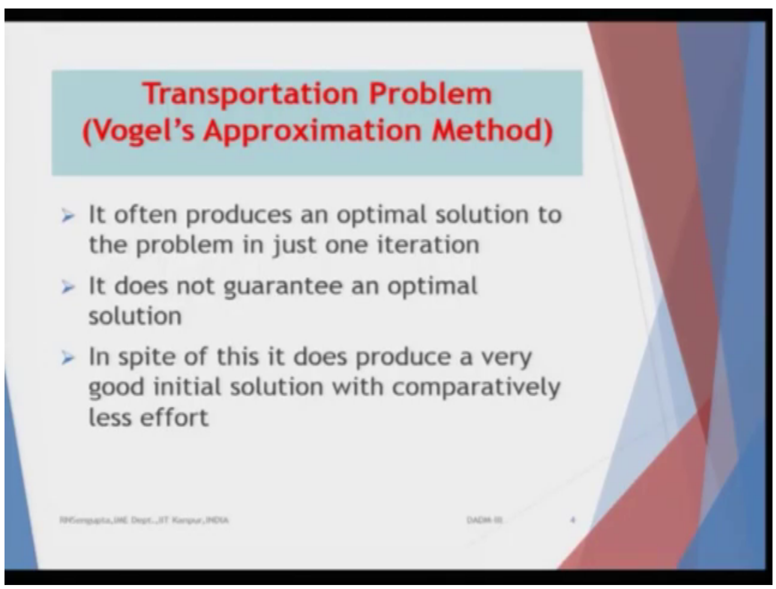

Now, this transportation problem when we solve in the Vogel's Approximation method so it states that it often produces an optimal result provided the allocation has been done in such a way such that it is optimal and any change in the value of the allocation would definitely not reduce the cost, but it will keep at the same level or basically it will increase the cost.

So, technically we will consider the Vogel's Approximation may be possible that at the first allocation arrive at the optimum solution which means that technically the Vogel's Approximation method actually if it does not give us the optimum solution, it will basically give us a basic feasible solution based on which we can produce to basically reach the corner point where our total cost is minimized because this is the minimization problem.

Now, as I mentioned in the first point this methodology does not guarantee an optimum solution. So, if it does well and good, if it does not basically you have to optimize it using the rules based on which we can do allocations accordingly so that minimum cost is achieved. In spite of this fact it does produce a very good initial solution that is what I am saying that it does gives a very good initial solution with competitively less efforts such that starting from that point and trying to reach the optimum solution is relatively easy.

So, if you consider the basic methodology of the simplex method which you start so generally what we do either in the trying to basically add the slag or the surplus or trying to add the artificial variables. We basically are insuring on trying to basically start off with such a matrix the identative values, if you remember when you are utilizing the Gauss Jordan method that A matrix which you basically try to start with would basically give you the basic feasible solution for those values of the x, now x is a vector, it will basically consider the actual variables which we want to optimize the slag, the surplus, these artificial variables all together if we consider.

So, it will be considering that all the slag and the surplus are positive such that once we start the iteration it will move from the corner point to corner point till we reach the optimum position such that any further approximation would not give us any better solution so we stop there. So, generally it means that the Vogel's method does give us some corner points which may be optimum or may be in one of the corner points which is in between as we proceed from that particular corner point which is given by the Vogel's Approximation. We may be able to reach the optimum solution as fast as possible.

Now, what are the steps of Vogel's approximate methods? So I will read them, explain them and then come to the problem, so we basic, now it is a minimization problem. So, what we will do is that, we will calculate the difference between the two lowest costs for each row and column. So, what does the cost mean? So, if you are considering the column wise so it means that at the destinations I am trying to find out the difference between the minimum two costs. Because trying to basically minimize the cost is the main motivation.

So, we will basically consider the difference between the minimum cost in each and every column and also we will try to basically find out the minimum cost at each and every row. And we will pick up the minimum, do the allocation likewise till it is exhausted. So, the exhaustion what we mean that we exhaust is basically the total number of goods which are to be supplied from the origin. If they are all finished which means the origin is done that means you cannot transport any more goods from that origin or if their total allocation is already achieved in the destination it means no further goods can be transported to the destination.

Now, it may so mean that in the best possible condition all the goods are transported from any one of the origins and they are utilized by any one of they are all transported from anyone origins and they are all utilized but anyone of this destination, so our life is done. But in case it is not done, if it is not possible, so what we will do is that, we allocate at each and every stage. The total quantum of the goods which is needed by that destination, if any (())(15:59) and extra is left, for that origin it goes to the next destination and basically we allocate it there depending on the lower cost point 1.

In case if the origin is already exhausted, exhausted means all the goods have been supplied so what we will do is that in order to fill up that destination which is already eaten up from the first origin we will try to basically take up from the second origin, the total cost based on the fact that we are going into the second higher cost. That means we start from the lowest level and we basically go the second higher level and basically fulfilled accordingly. So, as we do that any origin, any destination if it is done with that means all the allocations has been done, we basically remove that from our further consideration.

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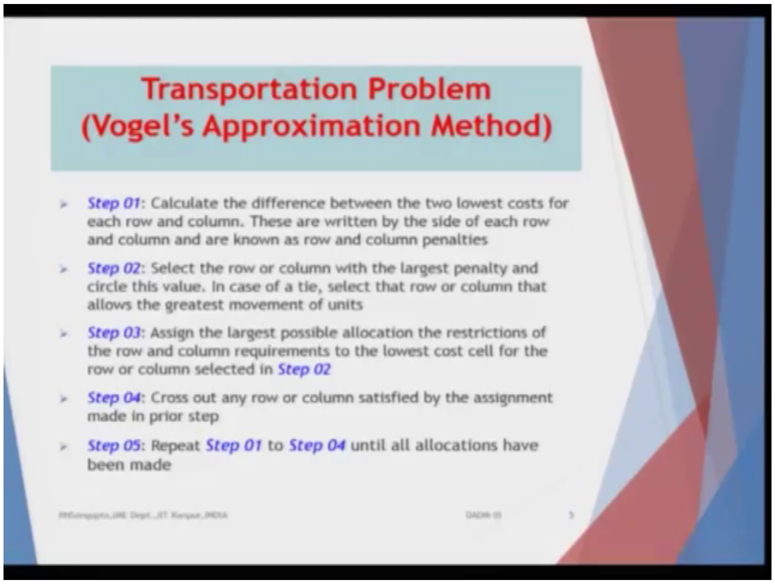

So, this is how it works calculate the difference between the two lowest cost for each row and column they are written by the sides so basically you have you will be by the way, when I am talking about the cost I am only talking about the cost matrix it is nothing to do with the x values that is the allocation which you are going to do. So, we will basically utilize the cost and do the allocations accordingly because our actual variables based on which you are going to take the decisions are the x not the c, c are given to us they are fixed. So, they would be written as mentioned the x value would be written by side of each row or column.

And they would be given known as the penalties, penalties in the sense that means we have already exhausted this numbers, we have increased our total cost because our main motivation is to reduce the cost. And basically we will consider these penalties in such a way that we basically exhaust, utilize these penalties and then go to the next level. Next level means depending on how the exhaustion has been done. In step 2 you will select the row or the column with the largest penalty, circle this value, in case of a tie you will select the row or the column that allows the greatest movement of the units that is inimportant point.

Say for example I have the lowest cost, say for example the lowest cost is 10 rupees per unit. Now, if it is 10 rupees per unit we will try to basically utilize that cost to transfer the maximum number of goods from any of the origins to any of the destination depending on the cost structure is minimum for that particular origin.

Why? Because if you are able to transport at the minimum cost, the maximum amount of goods then will basically be ensuring that we are trying to keep the total cost as minimum as possible. So, we will try to reduce it as possible, as far as possible, try to transport all the goods to any of the destination maximum as possible at the lower cost, and basically go one step at a time such that in case the cost increases the total number of goods which are to be transported are already taken care of in the previous step.

We will assign the largest possible allocation which is possible as I told, so that was what was told in step 2, that means select the row or column with the largest penalty and circle this value. In case of a tie, you will select that row or column that allows the greatest movement the unit maximum. So, you will assign the largest possible allocation that is the restriction, so the restriction of the row and the column requirement to the lowest cost sell for the row or column is selected as I just mentioned. So, you will try to basically take the lowest cost, lowest cost always and try to basically push or transport maximum number of goods at the combined values of c and x as they are as minimum as possible.

Once this is done you will again go to the step 2, so step 2 what you will see is that is there any amount of allocations still left to be done from the origin or the destination. So, if the origin still has, say for example 30 units of goods still to be transported and the destination has already been exhausted, the first one will utilize that extra amount of goods which is to be transported at the next highest level of cost and then transport that such that any additional amount in the total cost structure for the minimization problem would be happening at the lowest level.

That means the rate of change of the function which is going to happen, that is total quantum of jump which is going to happen for unit of goods which would have been transported would be at minimum level or minimal level. So, assign the largest possible allocation and based on the restriction that the row and the column requirements at the lowest cost sell.

For the row or the column is selected in step 2 and you will basically go ahead like follow this loop, assign, come back if there are still unfilled demand left, unfilled amount of goods to be transferred is left you basically follow this group till that particular row or the particular column which is basically the origin or the destination are fulfilled. Then you go to the next step.

Cross out which is what I just mentioned, cross out any row or column which has been satisfied with the assignment made in the prior step. So, obviously if the destination or the origin has already exhausted its total quantum of goods they would not be considered. You will go into the next higher level and basically repeat it, you will repeat step 1 to 4 until all the allocations is been done.

But, provided you will always be seeing that the total number of allocation which is been done in each cell either along the row or along the column should basically add up to the bottom most row and the right most column. Because the values of b1 to bm and a1 to an should not be violated because the sums would all be the same implying that the total number of good which you are going to supply from any origin or going to any destination cannot basically be exceeded.

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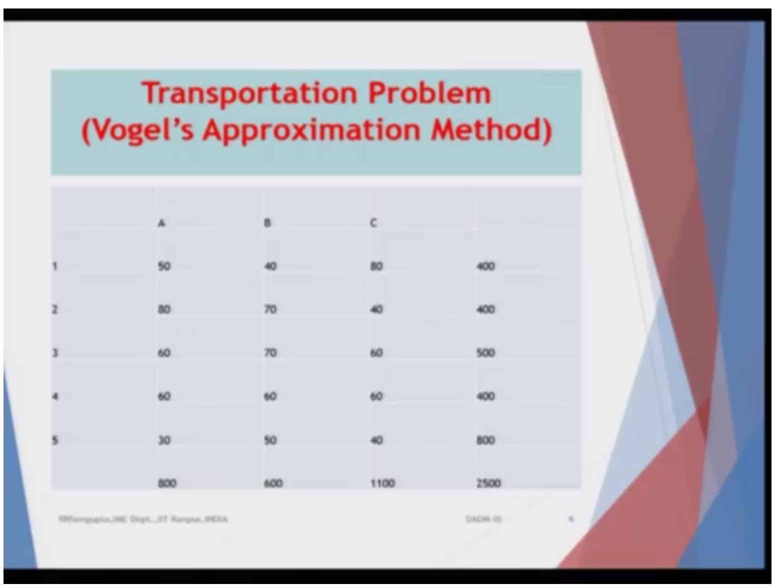

	A	B	C	
1	50	40	80	400
2	80	70	40	400
3	60	70	60	500
4	60	60	60	400
5	30	50	40	800
	800	600	1100	2500

So, let us consider this problem in a very simple manner. So we have A B C we will consider them as the origin and the destinations as 1 2 3 4 5. So, it can be other way around also the number of A B C is 3 and 1 2 3 5 there is a 5, n value is 5 and m value is 3 it can be the other way around also you will basically do the problems accordingly. Now, the cell values which is important to note these are not the total quantum of goods even though it is very-very easy for all of you to understand but I will still repeat that. So, this 50 means that if I am transporting goods between A and 1 whatever the destination, whatever the origin is, I incur 50 rupees or 50 units of costs per unit of transportation.

Similarly, between B and 1 utilize 40 rupees or 40 units of monthly value for each unit which is being transported or transferred. Similarly, for C to 1 or 1 to C is 80, the other corresponding values, if I am basically reading the row wise it is 80 70 40. I am repeating the values so you make of note of that, the third row values are 60 70 60 that means 60 amount of units of rupees is utilized for transporting between A 3, 70 is the amount utilized per unit between B 3, and 60 is basically the amount of money utilized between C 3.

Similarly, for A 4 B 4 and C 4 the corresponding values are 60 60 60 that means they are all equal and the last row values the corresponding values for A 5 B 5 and C 5 are 30 50 40 which means that I incur 30 rupees per unit transportation between A and 5, 50 rupees of transportation cost per unit between B and 5. And 40 rupees of cost is between C and 5. Now, what are the right most values which I have written? So, if these are the total amount of goods either transported or needed depending on what you are trying to basically signify as 1 is 400 in number. Similarly, for 2 whether it is origin or destination that is immaterial for our consideration for the time being is 40.

So, depending on how you basically consider the problem is 400. The total quantum of goods which is required at point 3 which is basically the destination or to be transported from point 3 is 500. Similarly, for 4 it is 400 and last value for 5 is basically 800. So, technically these values are B 1, B 2, B 3, B 4, B 5 and if I go to the bottom most row these values corresponding to A is 800 that means if A is transporting goods maximum it can supply 800 units of goods. Similarly, for B if it transports goods to all 1 2 3 4 5 the total quantum of goods is 600, and if I consider C the total quantum of goods which is to be supplied from C to 1 2 3 4 5 is 1100.

So, add up this 800 600 and 1100 it comes out to be 2500. Similarly, if I add up 2500 which is a balanced one, incase if it is not balanced one, adding up an extra row or an extra column would not matter much because there we will basically have some values which are to be a portioned in such a way that the total values along the bottom most row and right most column should balance.

But in that case we will consider a huge amount of penalty cost to be there such that we cannot transport those goods because the costs are very high. We cannot take 0 because immediately if we consider 0 you will be tempted to basically push the goods in that destination or the origin depending on whether m is greater or n is greater and the balancing values which is 2500 has to be done either along the row or the column.

Now, I will basically follow the rule, so what the rule basically says that I will calculate the difference between the two lowest cost for each row and each column so they would be written by the side of each row and each column. And they would be rows and the rows penalties and column penalties, so, if I consider this I am only considering the penalties. So, the penalty for the minimum cost if I consider A column would be the difference between 50 and 30 which is 20. If I consider the penalty cost between the column B so the values will be 50 and 40 which is 10. So, the first one is basically 20, second is 10.

If I consider the allocation which is happening between C, so the minimum the values would basically be 0 because there are 2 values of 40 so obviously the difference is 0, so when I am considering. But obviously if you take the next higher value later on we will see that it can come out to be 60 minus 40 which is 20. So, these are nothing to do with the extreme right most columns or the bottom most rows. Those total x values which are A and B does not matter.

Now, if I consider the minimum cost along the values of along the rows so they would be 50 minus 40 is 10, then if I consider the difference between the values of 70 and 40 is 30, if I consider the values of the difference between 60 is 0, but if I consider the next higher level it will be 70 and 60 and in the fourth row it will be again 0.

But in any case whatever combination you take it will always be 0 and the last value which we consider, the last row the difference would be 40 and 30 which is 10. And any higher levels would basically be the values between 50 and 30 which is again 20. Now, if I do the allocations the allocations based on the rule, based on each step which you go it will come out to be like this.

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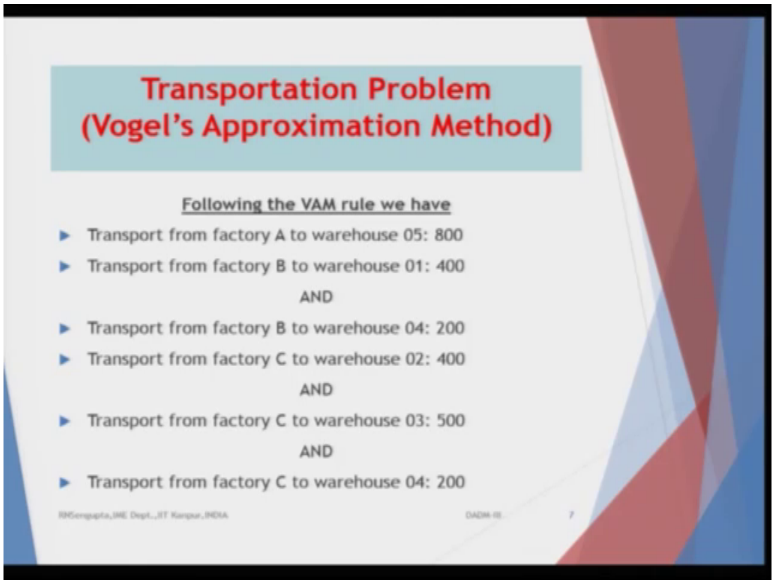

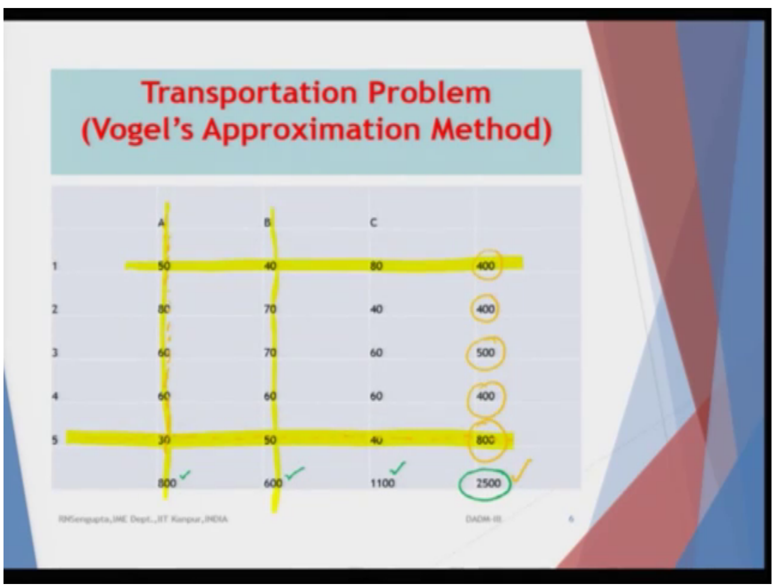

Transportation from factory A to warehouse is 800, transportation from factory B to warehouse 1 and 4 would be done in the combination of 4 is to 2 which is 400 and 200, for C I will basically transport so C is the last most column. So, what I am doing is that if I consider that, I consider the first values and the total amount of combination is basically 800, so what I will do is that I will basically transport from A to 500.

So, let me see why A to 5, because if you see the total allocation is 800 and the total demand or the need is also 800 so I will basically apportion the total amount of 800 at a value of 30 such that at the first step I have ensured the minimum cost. Similarly, when I go to B the total quantum of goods which is applicable and available of B is 600.

Now, check here 5 is already done so 5 has been fulfilled, so if I go to the next level I would basically have the values as shown are basically 1 and 4, why it is 1 and 4? See it very interestingly, 1 would basically, so 50 is already exhausted so we cannot utilize 50, the next values which will be there would basically be 60 and 40. So what I will try to do is that, if it is equal so obviously if it is equal I will basically go for the case where I will basically transport (200 to 4 and 400 to 1) 400 to 1 and (400 to 4) 200 to 4, sorry.

The reason being that, I have 600 so will transport so this is rolled out and A has already exhausted. So, let me use the color, so this is exhausted, this is exhausted. So, when I come to B, B has a cost of 40 for transporting to 1 so I basically divide it into 40 and done. Next what is the total quantum left? Total quantum is 600 minus 400 which is 200. Where should I go? First let I check, should I go to 70, should I go to 60? So, logic would say that I will definitely go to 60. So, I will basically give 200 here as it is, then once this allocation is done which means B is over, 1 has already taken place which is 400 is done. 4 is 200, so 4 has 200 that means 200 is still left, so this 200 still left here we will come to that.

Now, see when it chooses C would basically choose the values in such a way that it will first allocate 400 into 2, let us see, so it basically chooses 400 to 2 because that is the least. And the corresponding values would be because 500 would be needed where? 500 is needed for the sorry, 500 is needed for the cell which is 3 so it will basically allocate the total quantum to C and the rest 200 which was still left will be basically done to 4 because 200 amount is the value which was pending. So, you will use this approximation value and give the results based on that we will try to basically produce and do the problems accordingly. Thank you very much and have a nice day.