Six Sigma
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Lecture No: # 29
ANOVA in DOE

Good afternoon, we resume our discussion of design of experiments. This is part of our this special sequence of lectures on six sigma that we are conducting for you. I have been discussing the simpler methods for conducting designed experiments, factored experiments and also the data analysis method that is used for it. I discussed a simple method.

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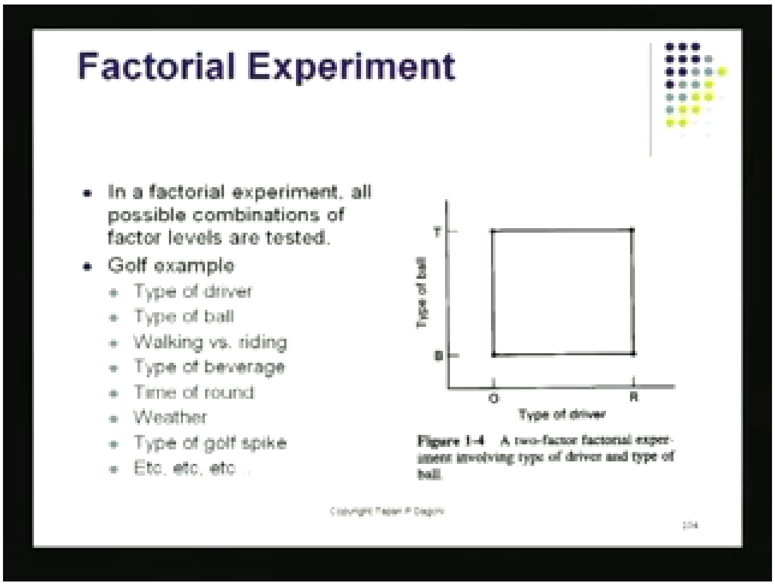

Which is basically adding or subtracting certain numbers and you would end up the main effects and what we call the interaction effects. If you look at the slide, if you look at what we began with, we have the various types of factors involved.

Some of those we decided to include in our experiment. We had the type of driver involved in this golf game,we had the type of different types of balls involved, we had walking versus riding and also the kind of drinks we are using while you are playing golf. Then there were these time of round, was it in the morning that we were playing this game or was it at lunch time, after lunch or was it in the evening, what was the weather like and was it changing during the day, the type of golf spike we used that also could affect the score of the thing and there are many of course, many other factors the nature of grass and so and so, that also might be affecting your score and you wanted of course, as a good golf player you wanted to keep your score as low as possible.

Now, getting a low score is the game is really the goal of playing golf. Now, it could be affected by some external factors of course, a big one is my own skill in hitting the ball and getting into the hole. The other is the effect of any of these factors the driver, the ball, the walking versus riding or the type of beverage. Now, suppose we are not controlling these other factors, but they are changing they are the nuisance factor they are changing as time progresses.

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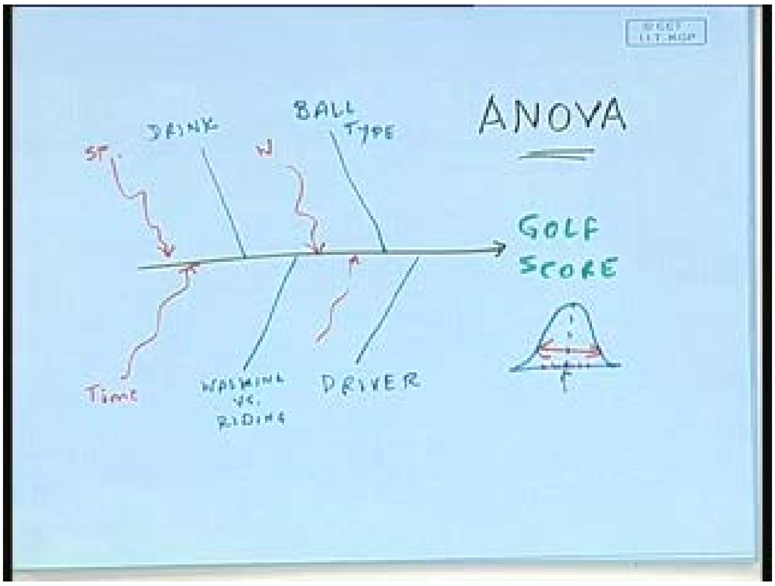

So, let me show that to you. Let me show this total thing, total picture in the form of a cause and effect diagram, the fish pond diagram. The effect of course, is my score golf score. That is my response and what are some of the control factors.

Let us list out some of the control factors. The one that we have decided to use in our experiment those are going to be the driver, this is the stick with which you hit the ball. Then of course, the ball type. This is also going to be one of those variables that you might like to optimize as you are playing golf to try to keep your score low, then walking versus riding this could be another factor and of course, the drink that you are using. So, drink, but like I said there are many other factors that we have ignored that we have left out of the thing there and I am going to put them down as noise factors.

These might also be affecting my score and this could be weather, this could be the kind of spike I use, this could be the time at which the game is played and so on and so forth. There are many other factors perhaps and these are we are not controlling, these are going to be the noise factors the result is going to be instead of getting a very a sharp score every time I play a round of golf I will end up with a variation there. So, what would this variation be like? This variation, I can probably draw a picture of it could be a variable quantity like this. So, instead of getting a sharp score like this, I will not be getting be getting a sharp score like this. This, will not be the one that I will be observing. I will be actually seeing some variation.

This variation is what we will be observing if I play many different rounds. I will have my scores would be varying all over the place here. Now, this is going to be possible then at any of these corners go back to our diagram, again it could be that I will end up with a variation here, I will end up with a variation here, I will end up with a variation here, also I will end up with a variation here. So, instead of getting one sharp number I will end up with a set of data that will show some variation. In the presence of such variations can I still find whether changing the ball type or changing the driver or changing my drinking you know material or walking versus riding can I say with some confidence that I can still say that playing with ball type b is going to be better than playing with ball type t.

I will score less with ball type b. How do I do that in a noisy environment, how do I really do this in this noisy environment? Look, at the noisy environment and my response is going to be like this can I still say is there a still is there any method or variable that can now look at this kind of data, you can still get me some information about the effect about the ball type or driver or walking and drinking and so on, While, these noises are all going to be active that technique of course, is called ANOVA and I am just going to write it down. First, I am going to write it down very clearly here. So, you see what that technique is ANOVA stands for analysis of variance. This is very important statistical technique, it looks at variable data like this and then it tries to see does ball have an effect that is significant when I compare that to the effect of noise, does driver have an effect that is significant when again I compare that to the effect of noise, the background noise. That is actually all this is done by this technique of ANOVA and this is going to be the subject of this hour’s talk.

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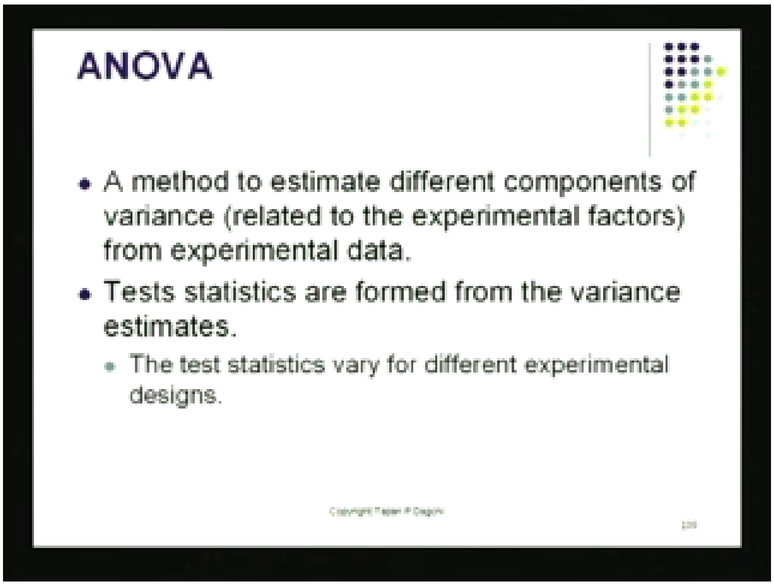

I start by saying it is a method to estimate different components of variance. Now, variance components are being caused now by these different experimental factors. They, will cause a variance to the mean score from the average. So, some may try to reduce that score, some may try to increase that score. Each factor setting is now going to cause a variance in that response there. When we are constructing what we call a test statistic and we will be comparing that test statistic as the check whether or not the effect of a particular factor or the interaction between two or three factors is that significant, when I compare that to the background noise. This is going to be our approach in the ANOVA method.

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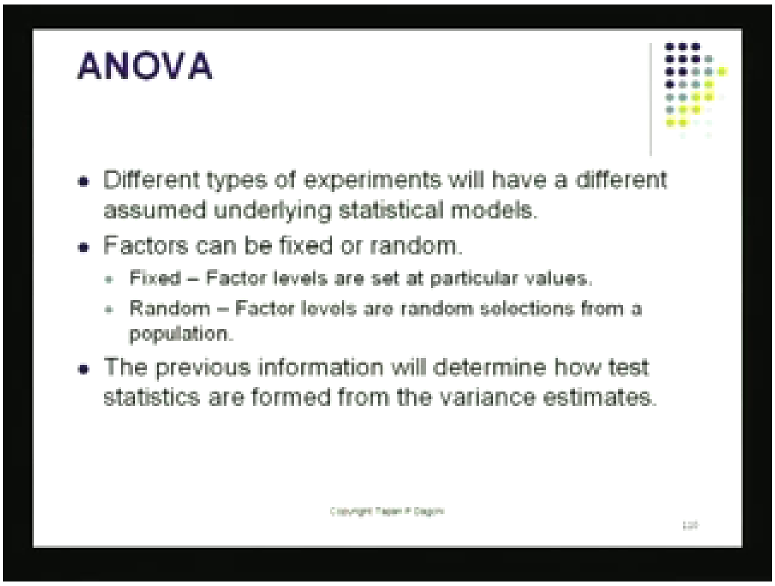

So, different types of experiments actually will have different types of effects underlying and therefore, we will have different statistical models that we utilize for different types of experiments and we will see examples of some of them and again we have the fixed situation and then we got the random factor situation. And of course, our goal is going to be to try to perform some statistical test on the data that we that we work out that we figure out and we will begin all of this using the single factor experimental statistical model that is the one we will be using. So, in place of having all of these things being active together.

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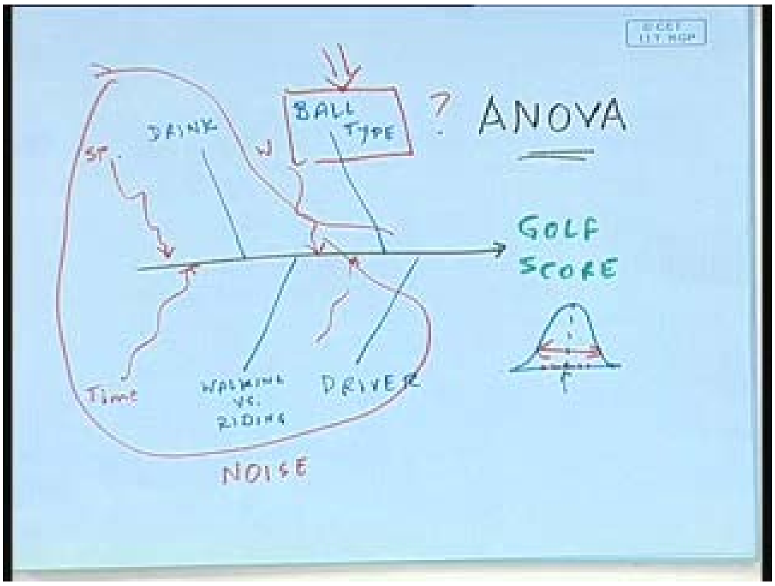

We will basically look at the effect of may be perhaps only ball type where as this is the one we will be focussing on. This is the one we will be focussing on we will try to see if I treat everything else as noise, all the other factors as noise. If I treat all of these as noise can I see in the response can I see the effect of change in ball type. Can I see that? Can I see that effect? This is what we would like to be able to do using this technique called ANOVA

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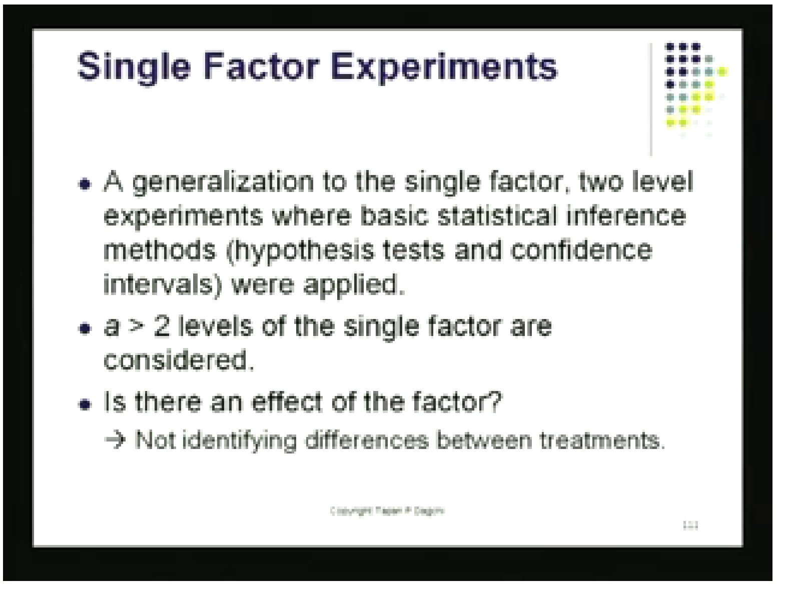

Let us see how we go on doing that. One thing I have to do is I have to make sure that I have more than two settings available; that means, like more than two ball types available. This makes this statistical analysis easy and straight forward. This is something I would like to be able to do.

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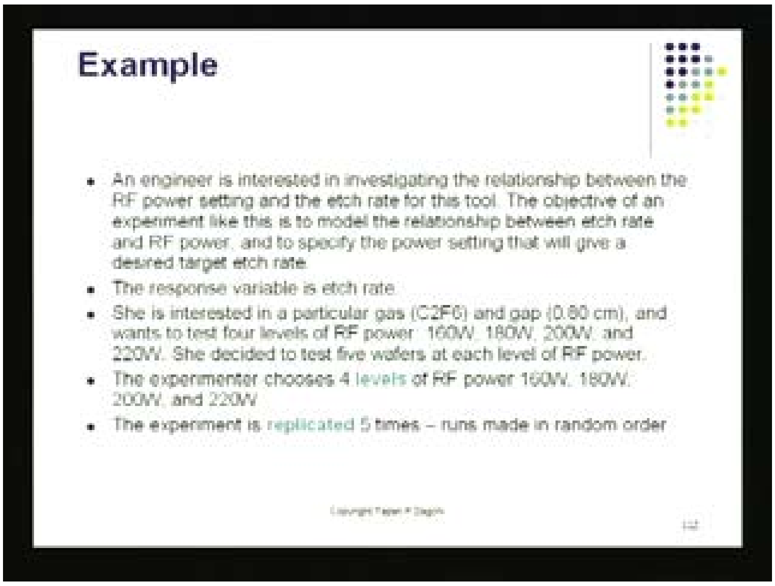

And here is an example it is a little complicated example. This lecture by the way is going to be more complicated than what we have done so far than what we read it in the past hour.

This one is going to be looking at some engineering systems and also the data analysis method is going to be more fancy and please be prepared for it because it is going to be little more complicated than what we have done so far. Again,

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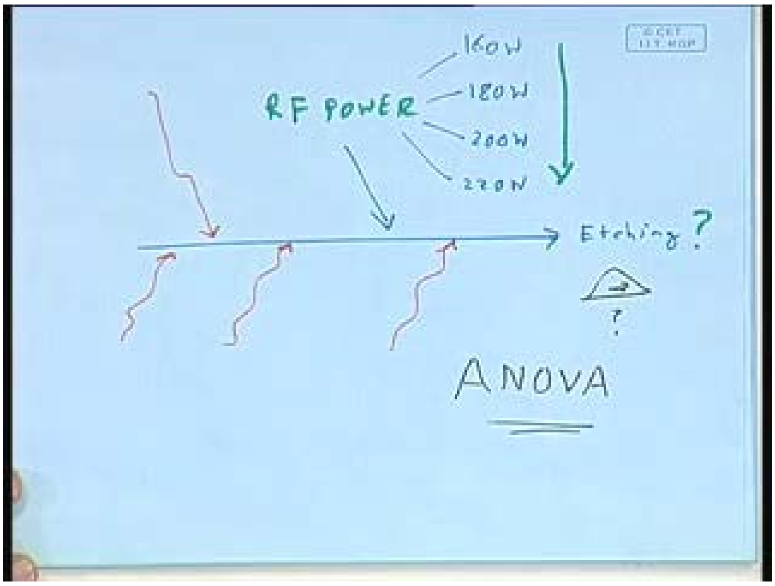

If I draw the cause and effect diagram for this system. I have the effect and the effect is going to be in this case, we are interested in the engineering system and we would like to be able to see the effect of certain process factors. These are going to be engineering process factors on Etching. I should tell you something about this process, in our most of these electronic devices like for example, this one this digital watch for example, this digital watch has chips inside and those chips contain transistors and other semi conductors and these are of course, not fabricated by using soldering guns. He has not done that may be because this is at a very micro level, very tiny level.

So, here what we do is we come up with a substrator and that is made of silicon on that we deposit various types of foreign elements and these elements are brought in as gases and they are etched on the surface and the result is we ended up, we end up forming what we call junctions. Junctions or you know just a position of two different set of materials and so on. By diffusing certain amounts of other materials such as antimony and arsenic and so on, those are diffused into this substrator which is made of silicon and thereby we form these transistors.

Now, this process requires a controlled environment and lot of conditions they are changed when you are trying to do this if you look at the system, if you look at the physical system where you produce these silicon. These silicon joints they turn out to be in an environment, that is very controlled and very precisely controlled. So, you get the right amount and the number of junctions formed and then lot of physical properties and electronic properties have to be there. They also have to be ensured when you are trying to come up with the chips that go inside these digital watches and lot of other solid state devices for example.

Now, what we are going to be looking at is we are going to be looking at the R F power this is the radio frequency power which is a pride on the system. So, that is going to be one process variable. So, I am going to be putting down here as the process variable and that is going to be R F power. R F power that is going to be my process variable. Everything else in the system is going to be treated as noise. There may be a whole bunch of other factors that could also be affecting this etching quality, but I am going to be treating all of them as noise. So, these are all going to be noise factors. I will be interested in knowing, whether changing these settings of this R F power the set becomes statistically visible when I look at the effect in the presence of this noise. So, the noise is going to be rare. My goal is going to be to try to understand whether if I change these R F settings at different levels and what are those levels? I have been given certain levels 160 watt, 180 watt, 200 watt and 220 watt.

Notice here I am changing them. So, I am changing basically what I am doing here is I am changing these settings. So, R F power is changing as I go this way. Does, that produce an effect on this? And, I have got to find this out in the presence of noise. In the presence of this noise, I am going to be finding this out and like I said to you before this technique is called ANOVA. The technique that allow you to do this is called ANOVA. So, you can see this is a pretty powerful method. It looks at a noisy background and in spite of that noise being there it tries to tell you whether or not changing these settings and these are called treatments of a certain factor on this thing, if this is significant. When you change these settings when we change the treatment of this particular factor this could be, this is one experimental factor. Does that actually show up here? And, can you tell that there is an impact when I change power from 160 to 180 to 200 to 220. Does that actually show up here?

When this itself is variable caused by these noise variables, this response itself is variable and I would like to, you will be able to tell that I would like to you to be able to tell this In fact, in the presence of noise I would like to really know whether R F power really, changing R F power really has an effect. Does it shift? Does it shift the process in anyway? Does, it shift that etching response in anyway? Now, because this noise is going to be there because this noise is going to be there we are going to be replicating the experiment. So, whatever I do at each of these settings I will be doing five replications I will be doing five replications in order to make sure that these noise thing does not fool me. It is not that it is high when I am running at 160 and it is low when I am running at 200 for example, that chance should not be left to nature.

We are going to be randomizing and also we are going to be running five replicates at each setting these what we will be doing then of course, we would be doing our averaging and we will be doing these statistical data analysis to be able to say whether or not that factor had an effect.

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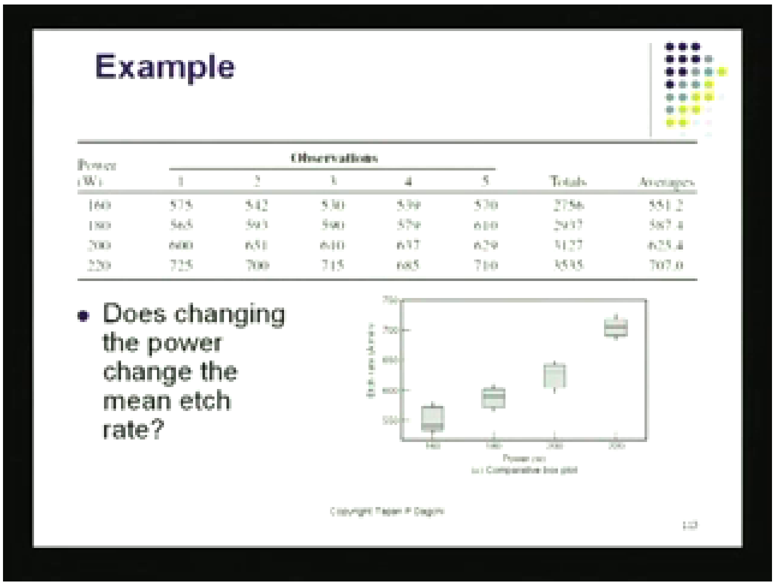

Let us see how we go about doing that. First of all on the screen, I show you the results. I show you the results which is the be the effect of etching, the etching that I produced on those silicon wafers and I have got 5 replicates of each when power setting was 160 I got 575, then I got 542 and I have got 530, 535, 539, 570 and the average turned out to be 551.2

When I change power setting to 180 I have got an average of 587, when I changed it to 200 I got is, I have got an average of 625 and when I put power setting at220 and I conducted 5 trials I ended up with an average of 7 all 7 0 7. These are the effects that are turning out to be measured on that etching. Etching is really the process effect that I would like to be able to see. If we see it in another way that etching actually is showing up as a little band here at 160, notice here this is not a there is of course, a mean value there, but there is a band. There is a variation there. Of course, a band of variation in response.

There is a band at 180, there is a band at 200 and there is a band at 220. These bands these are these are variances that had been caused because all the different noise factors which have which actually also changed when I was replicating my experiments. So, because of that I end up with these little bands there in all these four settings there. In the presence of such variation can I still say that changing power had an impact on response, can I say that? And, I would like to be able to say that with a certain level of confidence this is what I would like to bring in statistics I would like to conduct a statistical data analysis.

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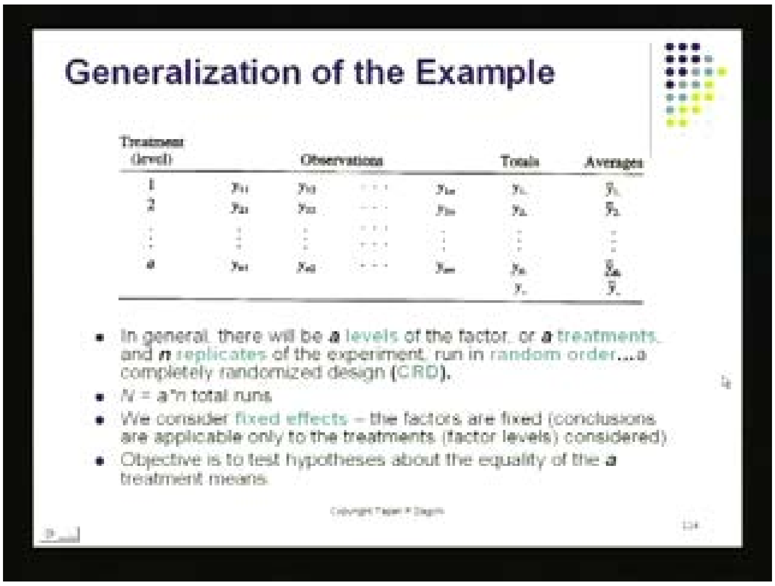

Let me give you the frame work for this of data analysis, treatment levels are like those four settings that I had 160 watts 180 watts, 200 watts and 220 watts. Here, let us say that I have got 12 3 4 up to small a, little a different levels of treatment and I have replications which go from one to n, these are replications possible. And the average here is going to be y 1 bar is going to be the average response noted at treatment level one. There is only one factor that is being varied from one factor that has been called a factor a is being changed from 1 2 3 through this these number of settings there 1 2 3 4 5 6 7 8 up to a little a.

The average is turned out to be y 1 bar here, y 2 bar here, y three bar and so on. I have got y a bar y a dot bar there. The overall average is going to be y bar double bar. That is going to be the average of everything. How many data points do I have? I have a this way and I have n this way. So, a times n is the total number of data points I have. In fact, that is the total number of runs I made when I conducted this experiment, this involved only one factor at a different levels. So, one factor the treatments changed from one to a little a different levels and each trial was replicated n times. So, I have got a times n this is the total number of trials I have run in doing this experiment. That is what I have gathered. This is the data on which I will be performing my statistical analysis.

Let us see how we go about doing this. This is of course, a complete randomized experiment and I mentioned to you I have to randomize whenever I am not controlling noise factors and noise factors themselves they may actually impact they may end up impacting the outcomes.

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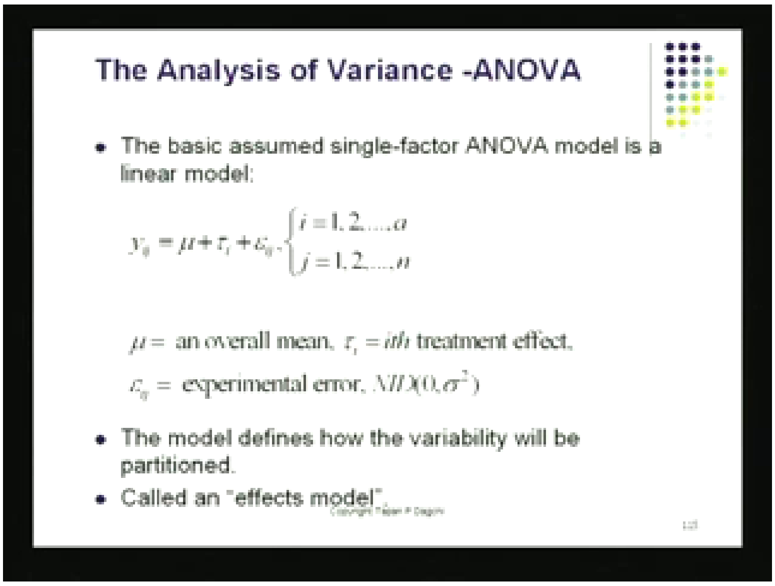

I will probably construct a little model there. So, I have got my response and the response in the etching case was this R F power would be factor a this, R F factor is my factor A.

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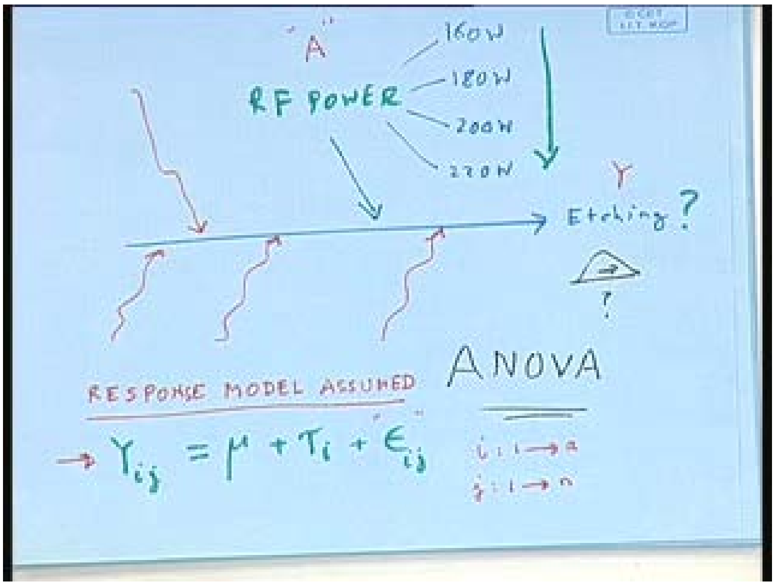

And I have got my response here response is going to be measured as Y. Y is the response there. Therefore, I can really write here Y and let me use a different colour pen here Y is the effect i treatment j replication this is equal to there is an overall average, there is an overall average where I am going to call mu plus there is going to be an effect due to the treatment which I am called tau i and there is going to be a random component random effect due to all the other factors which I am not controlling. This I am going to i j because this is going to change from run to run, these tau i, tau i levels actually they are coming from when I am changing my setting for this factor a from one to a. So, there are a different settings they are possible and I have got this. So, i here is going. Just note this down i here is going from one to little a and j is going from one to little n and I have a total of them a time n trials. I have got Y i j n different a times n different values of this, each will be a this thing.

Now, this is my model this is going to be my theoretical response model. I am just going to write them down as response model assumed is this. This is my response model assumed. There is an overall average, average response that is the effect due to treatment of R F power which is the factor I am changing and of course, anything else that does not get changed contributes an error. We call this error and this is coming at for each reading I will have for, each trial I will end up with a E i j epsilon i j component that is going to be the error term there. So, this is what I will be calling actually the error term the background noise, if I do that.

If you come back to my slide again you will notice here, I have got the model there Y i j equal to mu plus tau i plus epsilon i j, i varying from one to a, j varying from one to n, mu is the overall mean, tau is the ith treatment effect, epsilon i j is the experimental error and this is supposed to be, this is assumed to be to be normally distributed with a mean 0 and some background variance which we call sigma square. This is supposed to be relatively small, but it is acknowledged to be there and it is there, this model now defines how the response is going to change when I change treatment.

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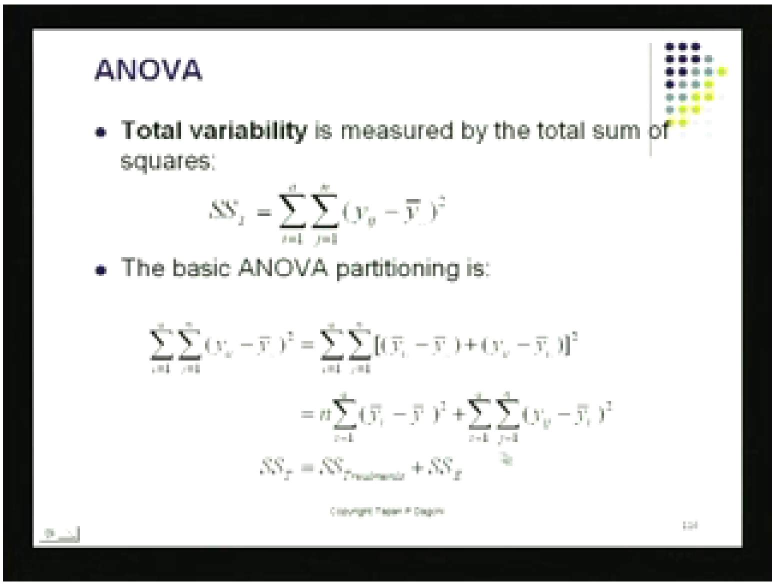

If you look at the total variability, what is the total variability? It is actually if you look at our quantities, our quantities have these y i j. So, some point here something will be called y i j that is the response of my treatment being at level i and my replication number being at j, the difference between y i j and this grand average. This is going to be a delta there, what is this contributed by what is this difference attributable to? One is of course, the treatment variation, the other is the effect of noise. Both of them they contribute to the variation that is that exists between y i j which is here and the overall average and this exactly what you will see when you see the variation there.

Now, this is my delta, this is my deviation. This is a deviation of the i j i j-th data upon y i j minus the overall average as square they become because sometimes this delta this deviation may be plus something it will be minus. So, to sum them all I end up with this summation first it is done over the replications, then it is done over the treatment errors. I have got y i j minus y bars whole square this thing is squared, that is my summed. When I sum them up I end up with my total sum of squares. This is a very important quantity and this is really the overall variation that is that exists in the data that is there.

Now, this is contributed by two components one of which comes from the experimental factors, my factor R F, my factor there. This is contributing one component, this will be the treatment component of the total variation that will be contributed by this. Then of course, is the noise all these noise factors they are also contributing in component and this is going to be those little epsilons and this is called the sum of squares 0 errors. So, if you compared to the slide again you will find that this total variation here can be split up algebraically between two components one is the sum of squares of deviations due to treatment effects and the sum of squares due to errors which is like the any other factor that might also be a affecting the process.

This sum total variation is deploy is the summation of sum squares, sum of squares of treatments and the sum squares of errors. This result by the way was found by Sir Ronald Fisher in the 1940s. He, is the one who first you know he worked out this algebra for the first time, that was a very important contribution because it let us separate out the effect of the experimental factor from the rest of it. I looked at this total variation Ronald Fisher actual looked at this total variation and he was able to split it out mathematically between the contribution due to this treatment factor, the experimental factor and the rest of it. This is this component is sum of squares due to treatment and this contribution this component of the total variation there is called the sum of squares error sum of squares, that is what this is. These two the sum of squares due to treatment, when it is errors sum of square they together constitute the ANOVA analysis and we will see how that is done.

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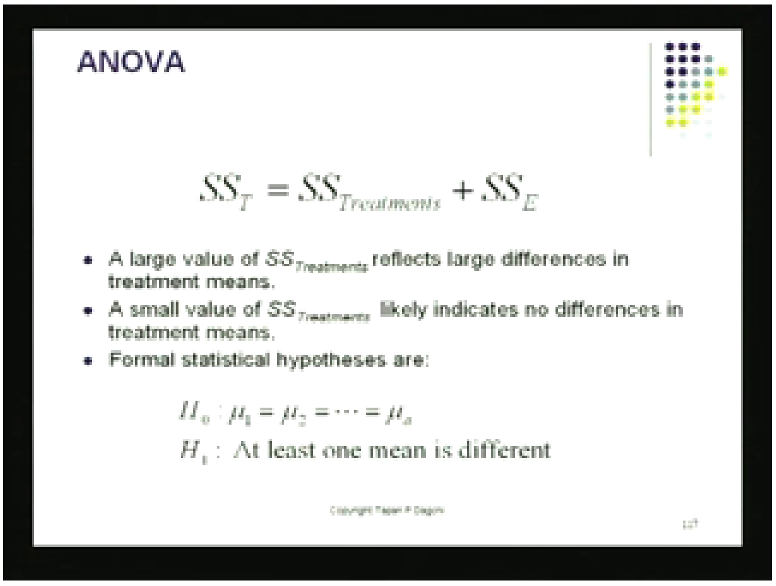

So In fact, Ronald Fisher’s identity is shown here and again you can see that sum of squares, they will do treatment plus sum of squares they due to these errors these together they form the total sum of squares and that is what is shown here. Now, what we do is we setup a statistical hypothesis. This, comes under the motion of statistical test of hypothesis and our hypothesis going to be cheap and it has no effect varying R F power has no effect on basically the etching, basically initially now this is going to be our initial hypotheses which is just a guess and a speculation. What we are saying is initially we assume, we have assumed that this R F power contribution contributes no variation to etching that is going to be our H 0 of the null hypothesis. The alternate of course, is going to be at least one of these settings produces a deviation that is different from what is produced by the other effects there. At least one of them produces a produces an effect that is going to be a significant.

When I compare that to what we call background noise. So, effect H 0 which is your null hypotheses turns out to be mu one which is the effect due to first treatments, second treatment mu 2 third treatment and so on up to mu a. We assume that all of those treatment effects are going to be equal and h one is the alternate one. There is the H 0, H 1 is the is the hypotheses then at least one of those means is different. So, now I am going to do some data analysis. I am going to do some calculations using these sum of squares due to errors and these have been produced by our data analysis, data measurements and so on. I will be testing the acceptability of this hypotheses, the treatment errors indeed are absent.

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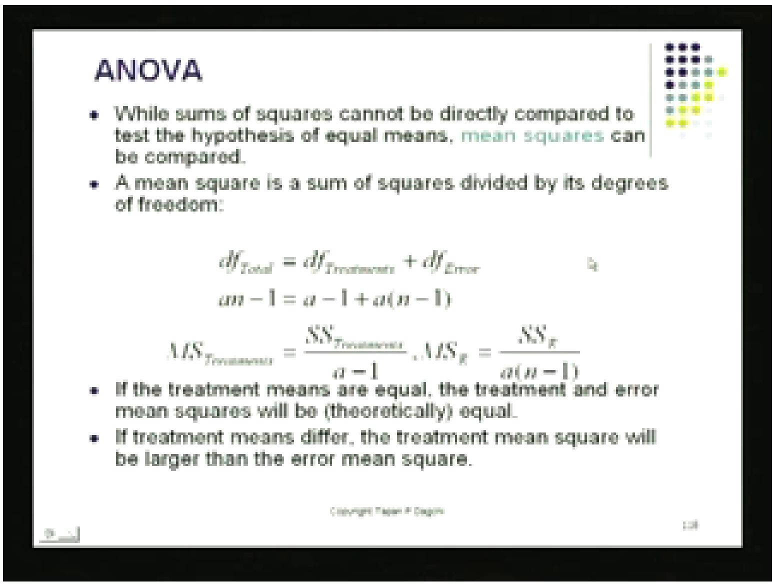

How do I do that for that? For that I need another quantity called the degrees of freedom because it turns out the sum of squares by themselves they are really not comparable unless we convert them into what we call mean sum of squares. Mean sum of squares turn out to be comparable to what we call variance, mean sum of squares are the sum of squares divided by degrees of freedom or degree by the divided by the appropriate sample size. Just like we have sample variance, we have got the total sum of squares from the estimated mean and I divide that by sample size minus 1, I do that I do the same sort of thing when it comes to calculating my mean sum of squares.

So, the mean sum of squares due to treatments is going to be sum of squares due to treatments divided by its own degrees of freedom and that mean sum of squares due to errors is going to be sum of squares due to errors divided by its own degrees of freedom. How did I find the error degrees of freedom? How did I find that? Well, I have got this total degrees of freedom. Now, the total number of data points we had was a times n. Why is it, why is the total degrees of freedom a times n minus 1 when it turns out there in order for me to find the all the different deviations I had to find at least one quantity, I had to measure at least 1 quantity in the data, from the data that are then used to find all the different deltas all the different deviations.

That one quantity is going to be the overall average the y bar dot dot. The y bar dot dot is the quantity. Let me show you show you where it stands, the y dot dot bar is standing right there. So, that is the quantity that I calculated in order if I will be able find all the deviations, all the different deviations. I had to find this quantity because I have done that I have lost one degree of freedom and that is why the total degrees of freedom available to me is n minus 1. Then of course, I have got the treatment sum of squares and I have got its corresponding degrees of freedom that turns out to be a minus 1 because again one . The one of the treatment because the sum of the treatments sum of the all the treatments is basically 0, One of those turn out to be, In fact, dependent on the rest of them and. In fact, it turns out that that is what forces you to lose 1 degree of freedom. So, I have got 1 degrees of freedom lost here I have got treatment degrees of freedom turned out to be a minus 1 and if you subtract a minus 1, if you subtract this quantity a minus 1 from a n minus 1 you are left with what we call error degrees of freedom that is what I have got there.

Now, I am able to calculate these quantities the mean square treatments and the mean square errors. I am are able to compare these two quantities. Now, let me you know talk about this a bit intuitively. We again go back to our picture here, it turns out suppose indeed R F power had no effect what so ever on etching. Suppose, it had no effect then the effect that we would measure here would be affected by all these noise factors. So In fact, it turns out if R F power had no effect what so ever on the response, we will probably find the mean sum of squares due to treatments which is like mean deviation that I produce, sum of all the deviations that I produce by varying the experimental factor over this full range of treatments. It would turn out that mean sum of squares should be comparable to what we have here from the errors and the ratio of mean sum squares due to treatment and you know mean sum of squares due to errors should be something like approximately equal to one. Mean sum of squares due to treatment divided by mean sum of squares due to error, this ratio should be at order of one. If mean sum squares are not significant, if they are really if there is no treatment to treatment variation then between two readings it would be of the same order as just plain simple error in experiments.

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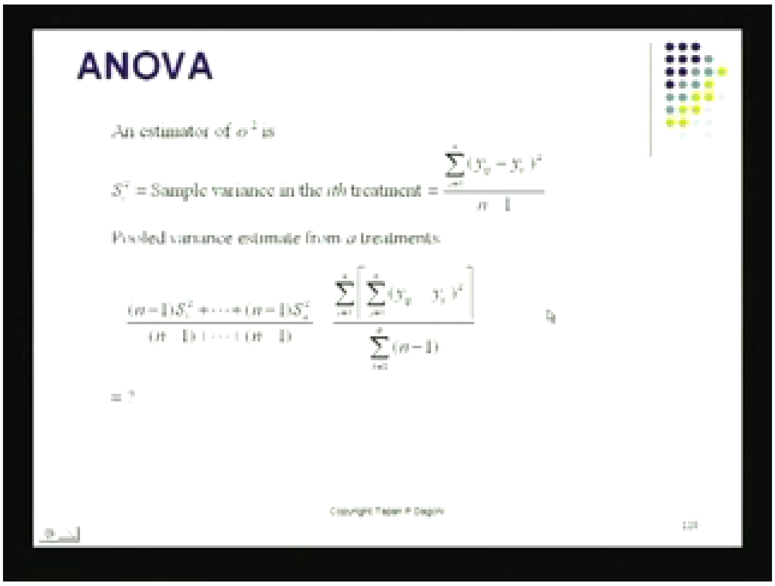

When I do that I end up with a ratio and that ratio it turns out that ratio turns out to be an f distribution and I am going to be skipping some of these slides you can see them in the text, these are going to lead you these are going to eventually lead you to the estimation of a of an f factor

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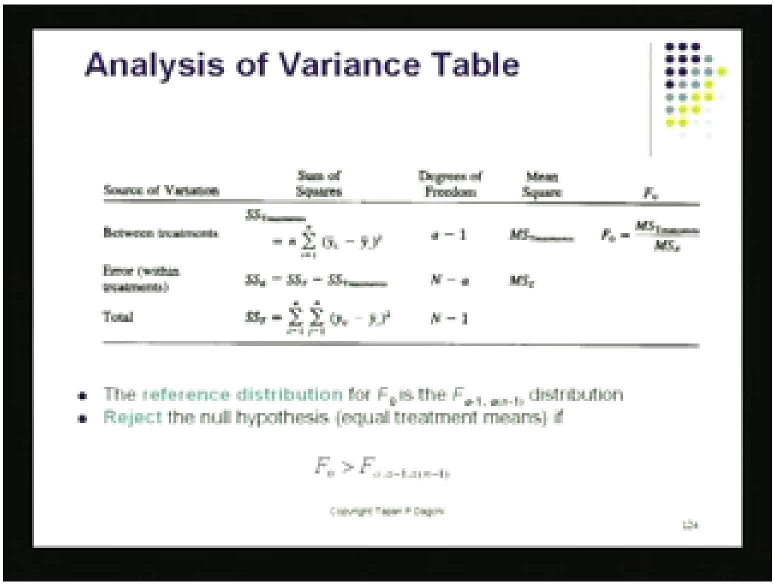

And that F factor is summarised on this analysis of variance table and what does this show to us, it makes a summary of first of all the different sum of squares. There are degrees of freedom and the mean treatments and the columns here, the first column is the sum of squares due to treatments, this is going to be between treatments and the error sum of squares this is within treatment, one within one treatment as I go from 1 to n when I replicate the experiments, when I replicate the experiments at the same treatment level the variation that I ought to see is due to the errors only and that is what turns out to be here. The error is within treatment within a treatment, I will have this sort of variation there.

And that turns out to be SS T sum of squares total minus sum of squares due to treatment and the corresponding degrees of freedom is shown here and of course, from this I can calculate my mean square error and of course, this is going to be my test statistic. My test statitistic is going to be the ratio of mean squares sum of squares due to treatment divided by and the means sum of squares due to error mean sum of squares due to treatment divided by mean sum squares due to errors, this turns out to be the f calculated. It turns out this being a ratio of 2 square variables or two variances this being the ratio of two variances or 2 square variables, it has the F distribution and because it has the F distribution I can then go back and do my test on hypotheses. This right hand side is coming from a table and this table actually allows for a set amount of type one error in test on hypotheses. I use my calculated ratio, I compare that to what we call the critical value at the chosen level of significance which is at the chosen level of type one error which is alpha.

For the appropriate degrees of freedom which is a minus 1 and a times N minus 1. If this quantity is bigger than this quantity; that means, my ratios are my ratio my this ratio means sum of sum of squares due to treatment and M S E this ratio is greater than the critical value and if it is critical, if it is greater than the critical value I will be rejecting the hypotheses that mu 1 equal to mu 2 equal to mu 3 equal to mu 4 equal to mu a. This hypotheses I will be rejecting. In fact, what I will be saying is that changing R F power, changing R F power did in fact, impact the response which is etching there. So, changing R F power this is initially what we started out to do. We changed the treatment levels we went from 160 watt and look at the picture here, we look at looked at 160 watt, then we changed this setting to 180, then looked at 200 and looked at 220 and we each time we did this we kept looking at our etching and of course, we replicated the trials each time treatment was changed we replicated the trials and I ended up with my data there. Then to find out whether or not the variation here is as caused by these treatment whether it is significant when I look at the variation that is caused by noise only, that is this little this test statistic and that turns out to be this F quantity there, if this F quantity is greater than the critical value of that F quantity for the same degrees of freedom and the level of type one error that is acceptable to me I will say the treatments indeed had an impact.

What happened in this case there. Remember the data that I showed you? Let me just show you the data again, this was my experimental data and my experimental data was like this, just by looking at this data we could not really say whether or not treatments had an effect, but when I convert this data when I converted this data into an ANOVA table, the ANOVA table looks like this.

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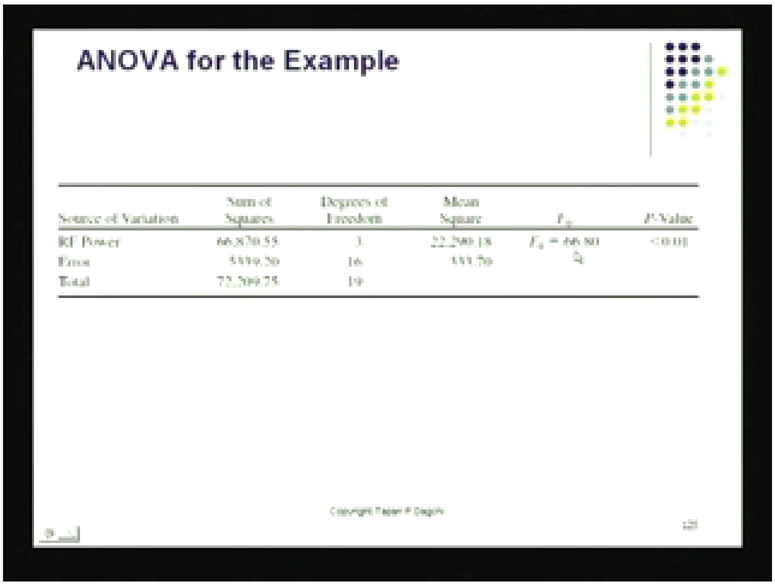

This is ANOVA table for the same example. Low and behold I find a value of F naught which is the ratio of the mean sum of squares caused by R F power divided by the error mean sum of squares, that turns out to be quite large. In fact, this is considerably large when you compare the critical value of it

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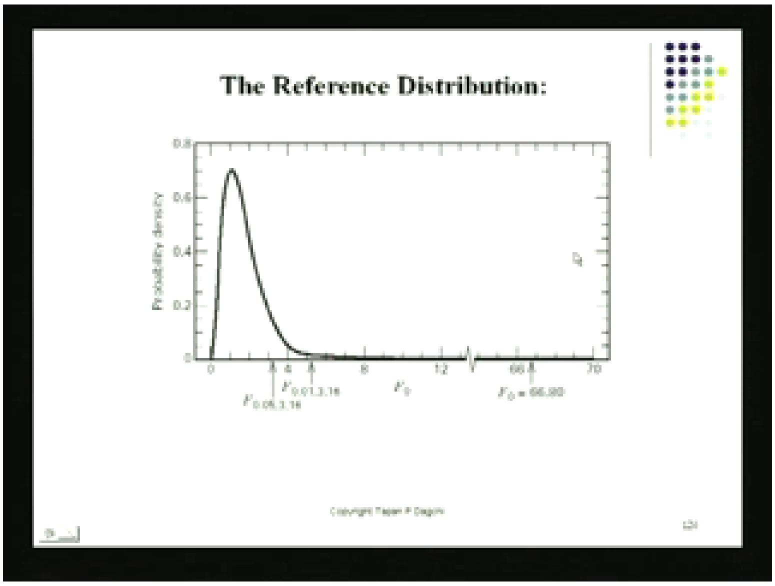

The critical value as I show in this picture here, is quite small. The critical value at 5 percent alpha level is here, the critical value at 1 percent alpha level is here and the calculated value of F which is the ratio of the mean sum of squares due to treatment and mean sum of squares due to error, that ratio is 66.80 which is much bigger than the quantities that would you know that would allow me accept the hypotheses that indeed treatment levels had no effect at all. So, this is a very powerful method and it is a method that is you know that works even in the presence of all the different noise that is there. Notice here all the different noise. All these red factors they are causing their own disturbance there, but what I have been able to do with the F test is I have planned my trials in a such a way that I take replicated experiments. I take replicated readings at each of these treatment settings.

I calculate the total some of squares, I split it up into the power that is caused by the treatment and whatever remains is the error sum of squares. I work out the mean treatment sum of squares and I work out the mean error sum of squares I look at the ratio of those two that is my F test statistic I compare the F statistic on this diagram. On this diagram I compare that the calculated value, I compare that to the critical values critical theoretical values of my F this is way to the right hand side which is like really in the rejection region and therefore, I would say that R F power indeed has a significant effect on etching that is like one of the things I wanted to establish empirically. I should tell you I should again go back a little bit I will tell you that such results are not possible to be obtained even at this time using only theory. Theory cannot tell us this.

We have some empirical models that can tell us this, but if you come up with the new process and you got new factors involved as far as these process factors are concerned, as far as noise factors are concerned. We end up with some new factor there new this thing there which will be true for any new r and d process. If you do an f test if you do ANOVA, if you do ANOVA and if you an F test you will able to say with some confidence that yes changing a particular design factor or a process factor has a significant effect or the output there are certain tests that you could do and I will just give you an example of some of the tests there one of the things is take a look at the errors

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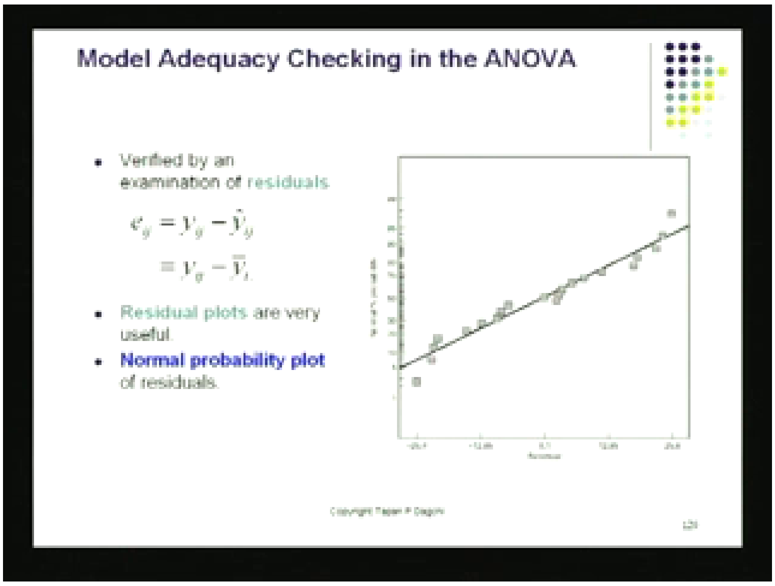

And what are errors? Errors are the observed value minus the model value. The model value was the response that we saw if I would bring up the model value again first of all let us just go back and see where I got these error measurements from. Let me first show you where these model values are, the model values are exactly what we began with and I am just going to show you the model. The model is going to be right here, this is the model. Notice, this is the model I have got an overall mean then I have got the treatment effect then I have got the error effect there.

The model assumes that these errors are normally distributed they have 0 mean and they have got some small variance. This is my model and this is going to be the the representation of a model, a treatment setting i at replication number j that is what is this model like. Now of course, I have got an actual observations. So, the model will assume that the response is going to be mu plus T i this is going to be assume to be expected value of this is 0. So, the expected value of this quantity for the i-th treatment is going to be a mu plus tau I, that is what I use as my model and I when I evaluate my errors and I evaluate my errors.

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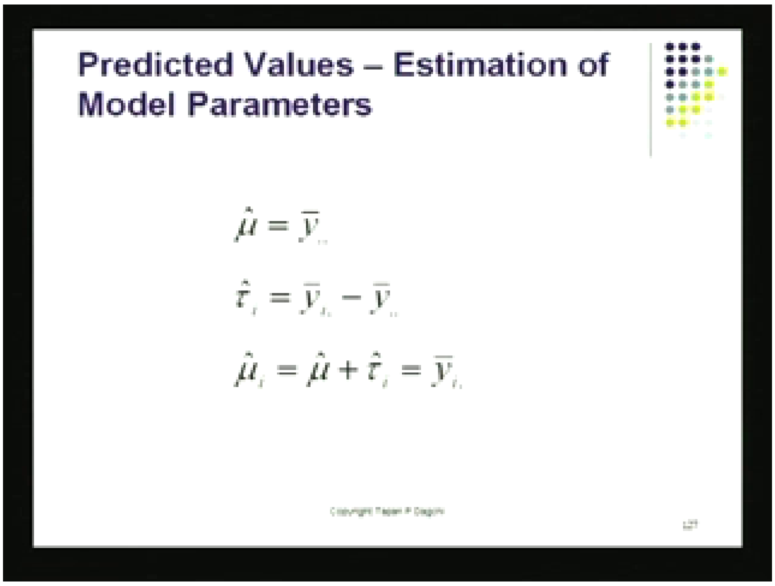

I have here mu plus tau i, mu is the overall mu average plus tau I, this is going to be my expected value of my of the response at my treatment level and the error is going to be now the difference between these two I have here a quantity that is the error quantity, the error quantity is going to be difference between the observed value of y minus the value that is predicted by y, this is the true error.

Now, one of the assumptions that the ANOVA method makes is that these errors if a model is done well if a model has been constructed well which is like this y hat i j. If this has been constructed well, the errors are going to be normally distributed and they will be visible like this as a straight line or a normal probability plot. If you produce a normal probability plot of the errors it will turned out to be straight like this and of course, variance also is going to be small.

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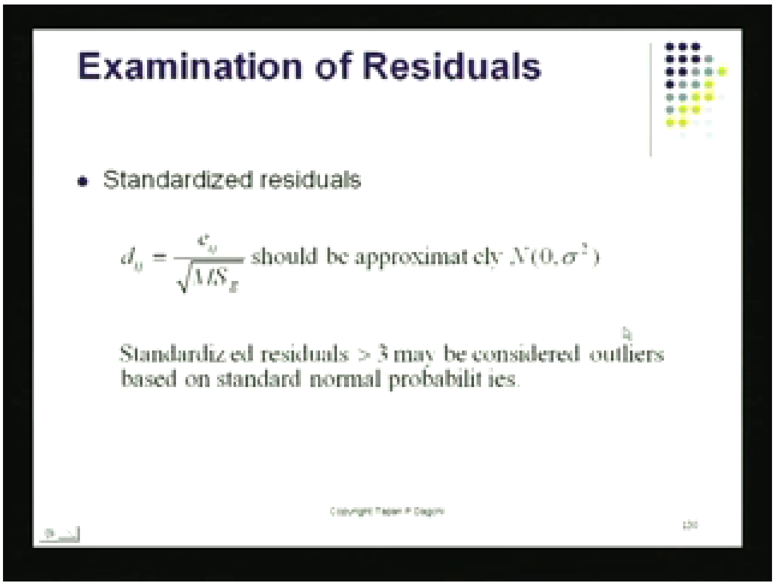

So, that is also something we could do. These are some of the tests that are done on the errors before you accept the results of the of the ANOVA, of an ANOVA and this is quite easy to do once you got your there are some other tests also available those can be looked at.

Let me spend a few minutes to show you one example where you will be able to see for example, what other use can basically design of experiment be put to. I am just going to roll off to a point when I come back to a method called response surface method, that is going to be pretty powerful method in optimizing a function.

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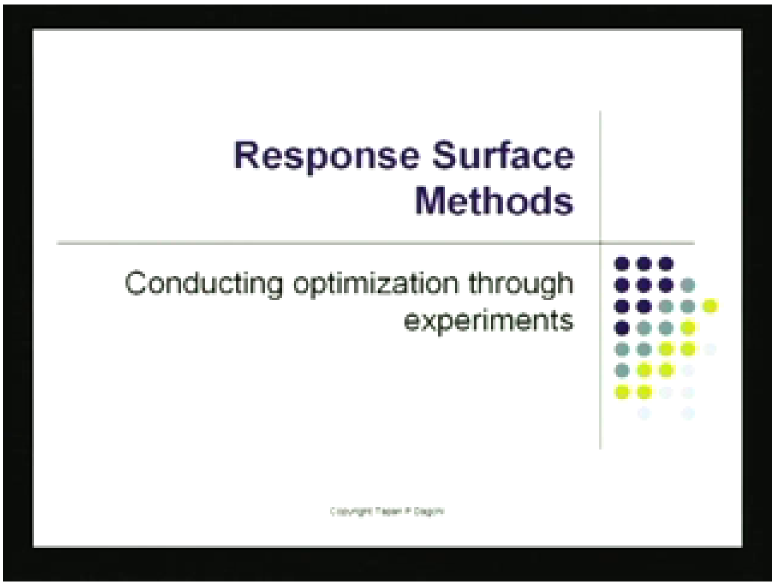

Let us see how design of experiments are used there. The response surface method turns out to be one of those methods which is used directly empirically. You conduct experiments and you study the response . What we did in the case when you are looking at for example, we are looking at ANOVA or we are looking some of the other one's for example, we may look at that golf game there, we changed the settings of the control factors at discrete levels. Now, what we would really like to be able to do is when we look at the response and you see that in a couple of minutes you will see what we mean by the response of particular process. The response itself may turn out to be very crazy function and if you trying to optimize the process if you are trying to reach the peak of that the response surface that is there certain special methods have to be utilized there certain special designs schemes or experimental schemes have to be utilized.

I am going to give you glimpse of that, I am just going to glimpse of it because later on we can pull up a text book and study this technique called r s m response surface method and that will help you empirically optimized different processes.

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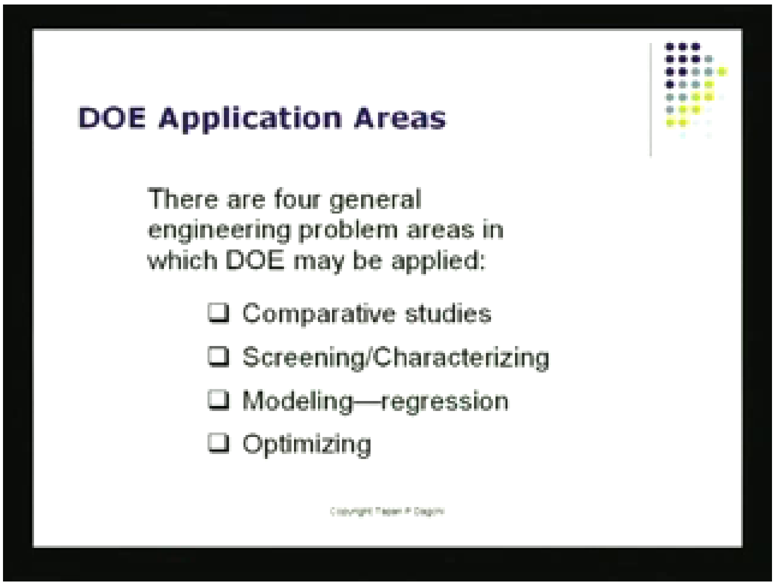

So, what are some of the methods now that we will be utilizing we will be using focusing right now on RSM the response surface method for optimizing a procedure.

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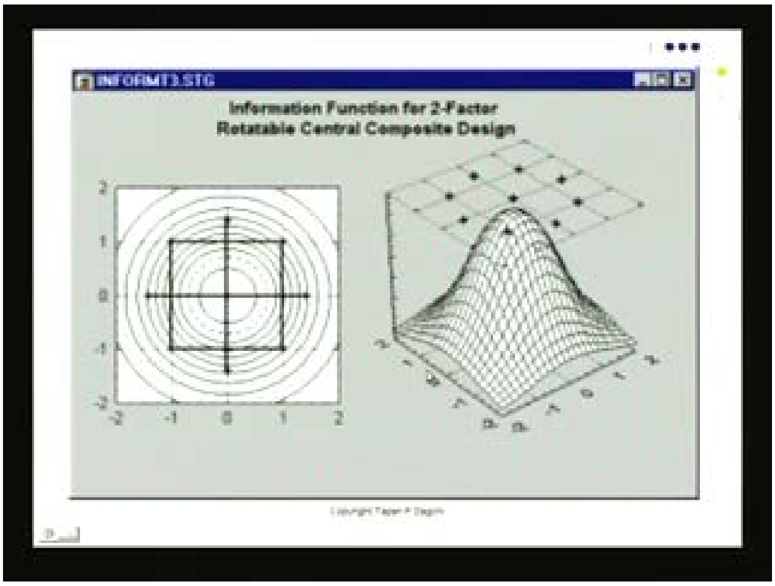

If you look at a process if you look at a two variable process, a two variable process will probably give you a response like this. If you plot the contours of it and if you project them vertically downward the contours may look like this and of course, if you get a 3D picture of the thing will see the doom there, now this might be the real process, but I have no real idea that the process actually is like this.

What I would like to be able to do is I would like to empirically reach this peak there, this is what I would like to be able to do. I would like probably I start somewhere around here or I may start somewhere there, I should be able to slowly climb up to this point this is what I would like to be able to do that, I would be able to optimize find the optimum settings for this design variable one and design variable two. If, I am trying to do process design I should be able to reach this point empirically and that is actually the goal of response surface method.

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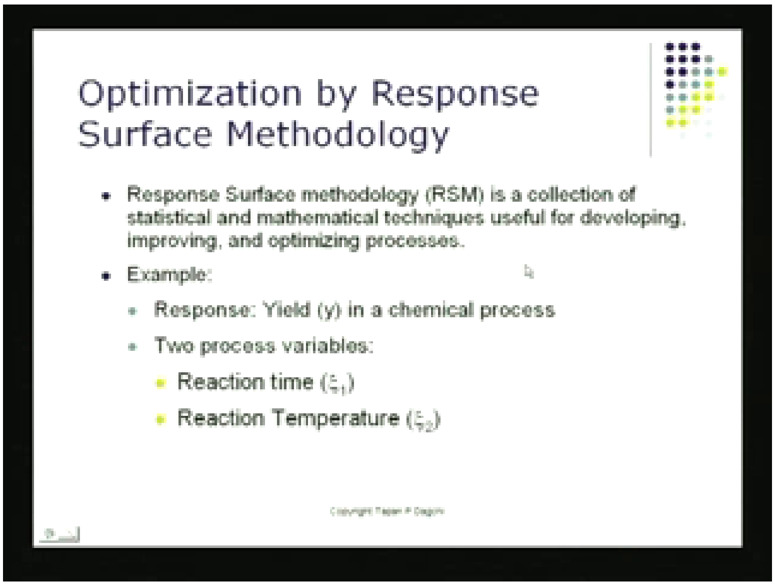

Let us see we how end up doing it. We will take an example and we will take an example of a chemical process where yield is the performance. So, instead of the etching situation that we had before remember I will have a different cause and effect diagram I just going to be sketch of it

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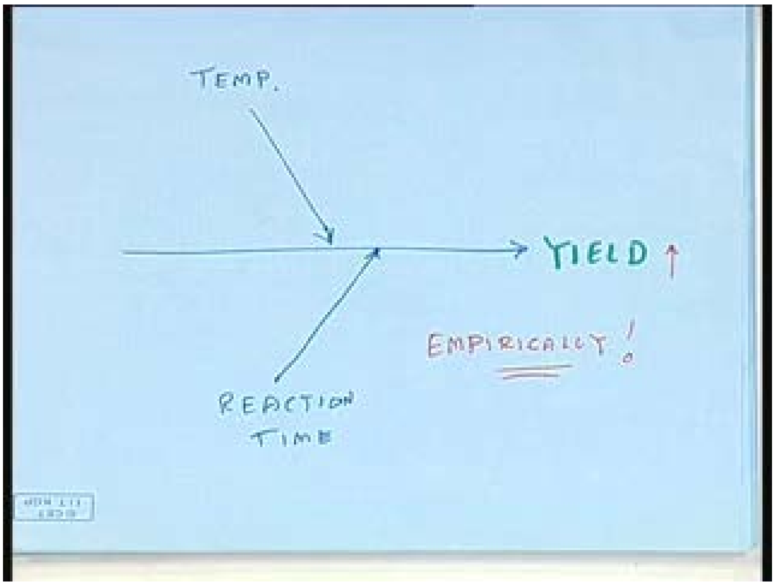

I will have a fish pond drawn again and I will have in this case, yield as one of the variables and as far as the process factors are concerned those are going to be reaction time and the other factors going to be reaction temperature. These are going to be my two control variables and what I would like to be able to do is I would like to maximize, I would like to maximize yield. This is my objective and I would like to do this empirically. I would like to do this and I would like to be able to do this empirically by running a series experiments this is what I would like to be able to do.

So, I will be climbing a hill I will be basically I will try to climb a hill and let us see how we do that. I have these two process variables, process variable one and process variable two.

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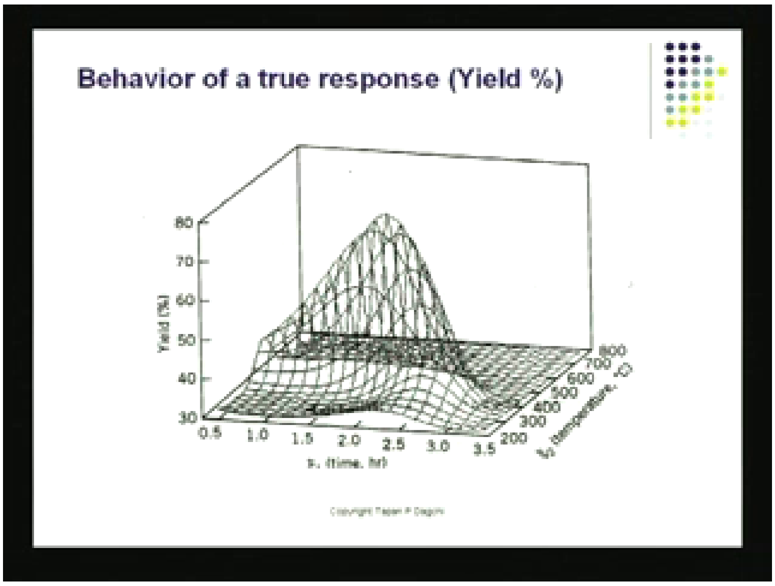

Let us see how we climb that path there. This is the real response and in this case notice here I have got two variables. I have got time there, I have got time on this axis here and I have got temperature on this other axis here. The real process is like this ,my God if you look at that you would say how I am going to be studying this well unless I have the theoretical equation for it is not going to be easy for me to optimize this process. That is going to be pretty well in path way.

How do I then go about doing this well what I will do is I will start somewhere. I will start somewhere and then I will apply the technique of RSM and that will let me slowly rise up to this point and come up to the peak. It will avoid falling into traps and everything because the RSM technique is a pretty fancy method and it will require me to run some experiments in the neighbourhood of the point where I am searching. I will try to find the direction which values are climbing.

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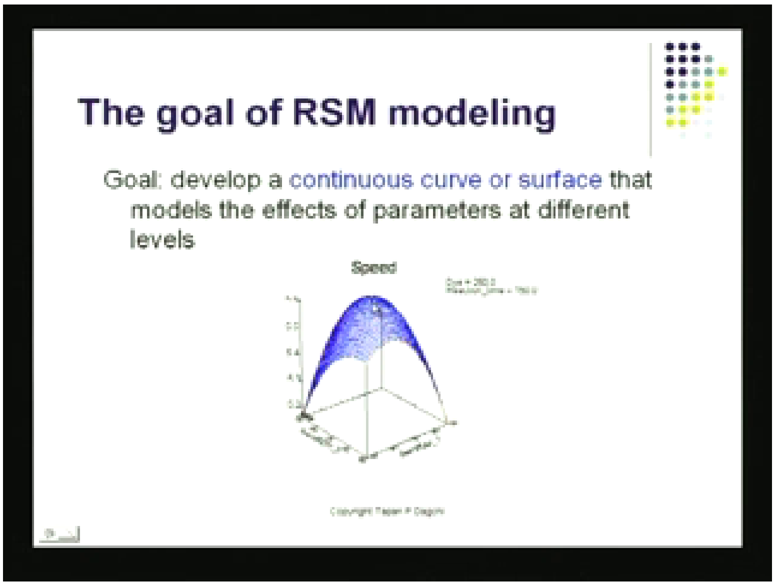

And let us see how we have end up doing that. The goal is going to be if there is a peak, the goal is going to be if there is a peak discover that and find the basically the process settings that corresponding to that correspond to that particular peak there, this is something I would like to be able to discover. I would like to find out at what setting of the of process variable one and process variable two my response peaks this is what I would like to able to do if I am able to that, I am really able to optimize the process. This would I would like to be able to do. So, I would like to be able to develop a continuous curve or a surface that will model the response that will model the response of the parameters.

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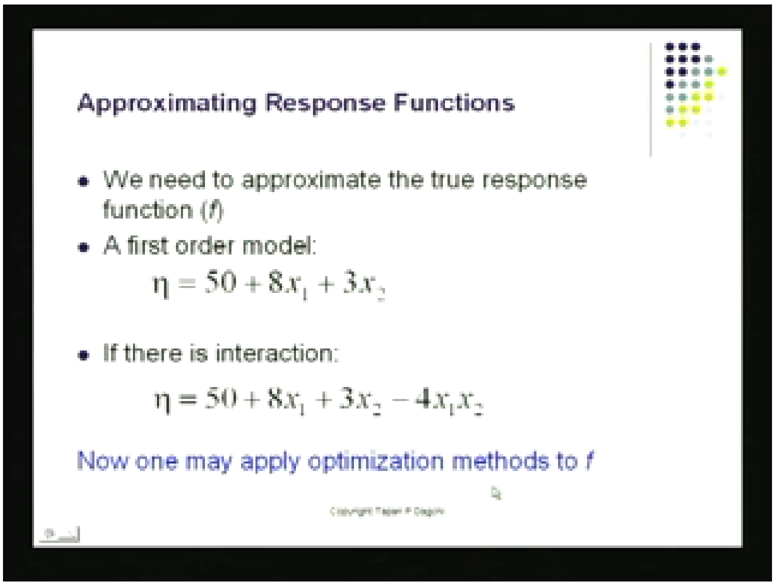

Let us first take a look at some of the possible models, the first order model which is a simple linear model of course, is the one that involves two variables it has a shape like this, it could also involve the interaction. Now, this is just going to be a main factor. First order effect if there is interaction I will probably have this interactive term also. Once, I have this of course, then it is pretty easy for me to once I have these equations available I can apply a lot of different optimization method including something as simple as calculus I could do that.

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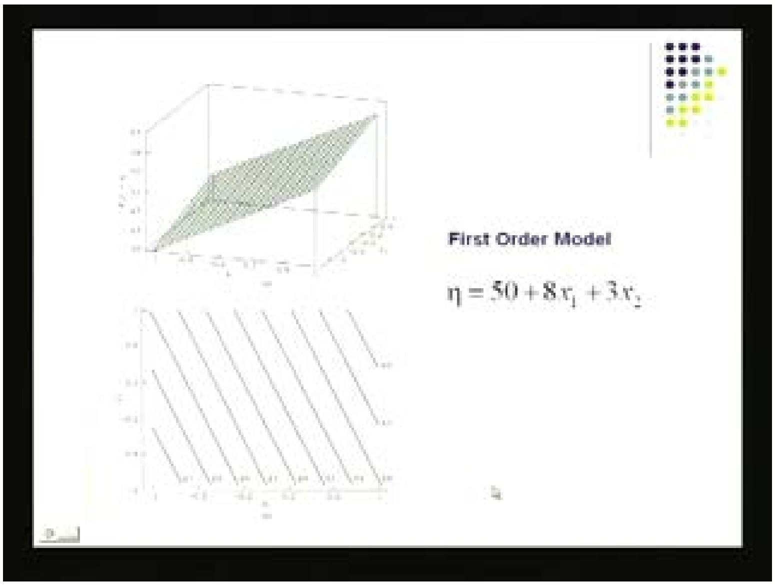

And what does the what do these look like was this is a first order model you can actually see the shape of it. Now, this is sort of like for a known surface I could plot this graph and response is rising this way so in fact, if I have to optimize the process I slowly have to somehow climb this way and we will able to do that. I am just showing it to you pictorially.

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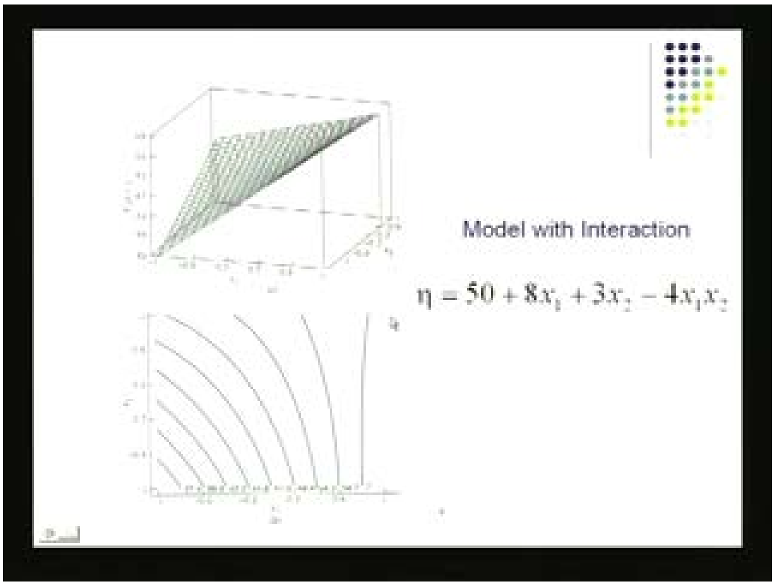

If I have an interaction combined, this is again a first order model and if I combine an interaction term with it there is some non-linearity that comes in and notice here the curvature, notice here the curvature of the thing and this again is a thing.

Now, what I am really doing is I am sensitizing you to the fact that when I have got non-linearity involved it is no more a easy surface to optimize it is going to be some pretty complicated. And certainly

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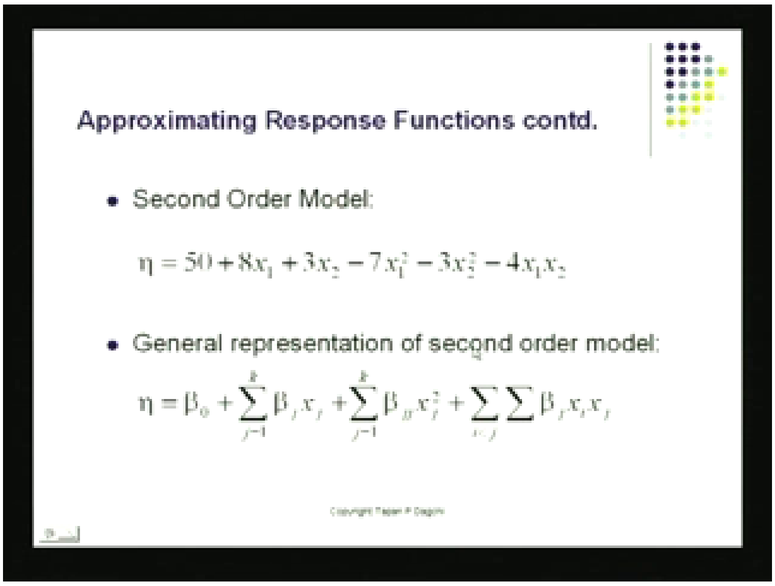

If you look at some of these second order models they have very complex, they have very complex behaviour. It is not going to be easy to visualize how they move, let us take a look at some of these second order models eta which I will let us say I will call this the yield equal to 50 plus 8 times temperature plus three times time plus minus seven times the square of temperature minus three times the square of time minus four times temperature in time that is going to be my response model. It is a second order model because we have got these terms there.

The general representation of course, is going to be something like this, the general representation of the second order model and now notice this is not something as simple as these guys where I could really optimize that once I knew what it was I could do it

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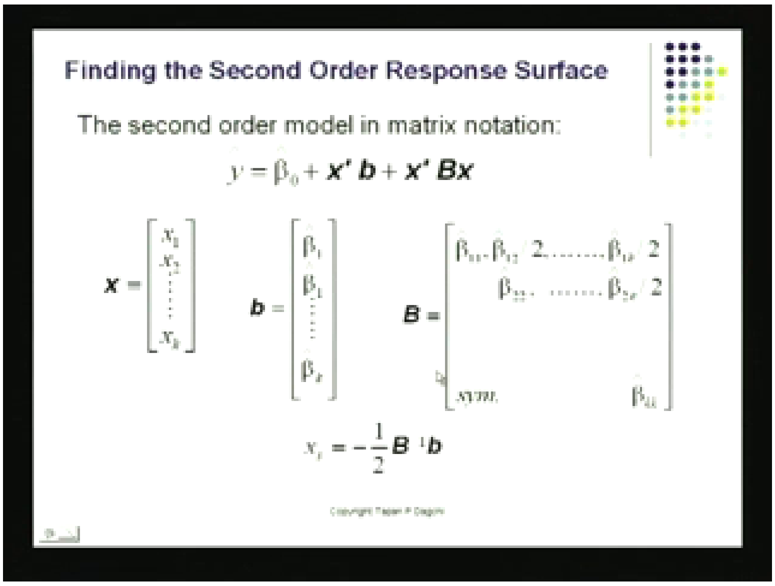

This is going to be a more involved, how do I do that well one of the ways to do that is to build what we call a regression model. You have to build a regression model and for that some matrix algebra is involved and what we end up doing is we have responses that we calculate, we involved a bit of matrices, we will have a vector for a y, we will have a vector for X. We will end up with basically some parameters beta one to beta k and we will end up finding the responses, this is the model there. In the model it is shown here this is going to be one particular type of regression model.

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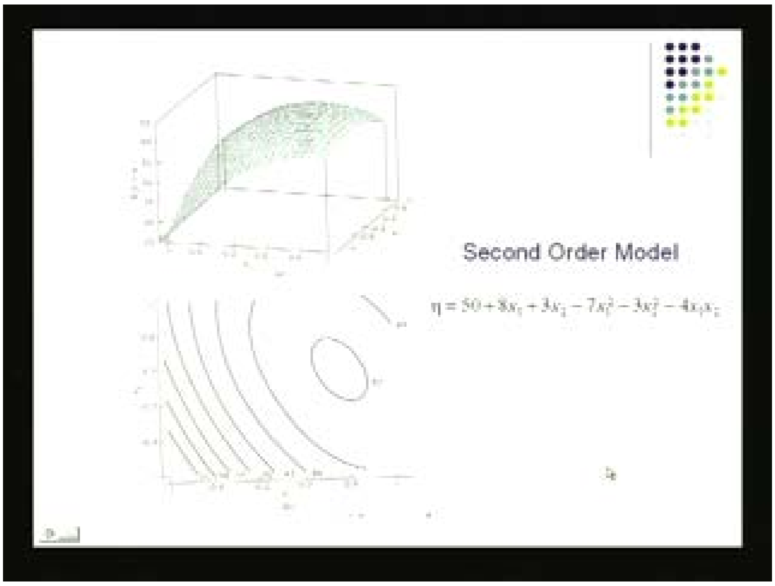

If I do that I may end up with a model like this. Now, how do I really construct? How do I construct this model here? I have to really obtain our various settings of X which now involves k variables, I will have to vary them over the space over the range of which X one can vary, X 2 can vary, X 3 can vary and so on. And for each of these points there I have to observe y. So, I end up with a collection, I end up with a collection of observed data. This is what I subject to multiple regression and I will end up with then I will end up with a model which should be like this, once I have this second order model then of course, they show the surface is going to be like this and I may end up finding the optimum. I may end up finding the optimum that would be easy, but of course, I will have to do experiments to be able to find these parameters I will have to do experiments to be able to find these parameter that going to be a big challenge there.

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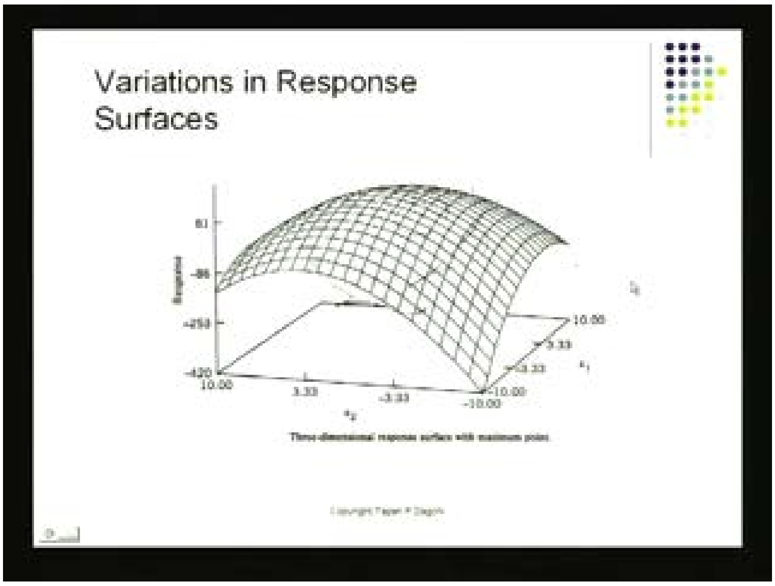

This is like another variation of the response surface. This again is a is the product of lot of experimental work then of course, you end up constructing this.

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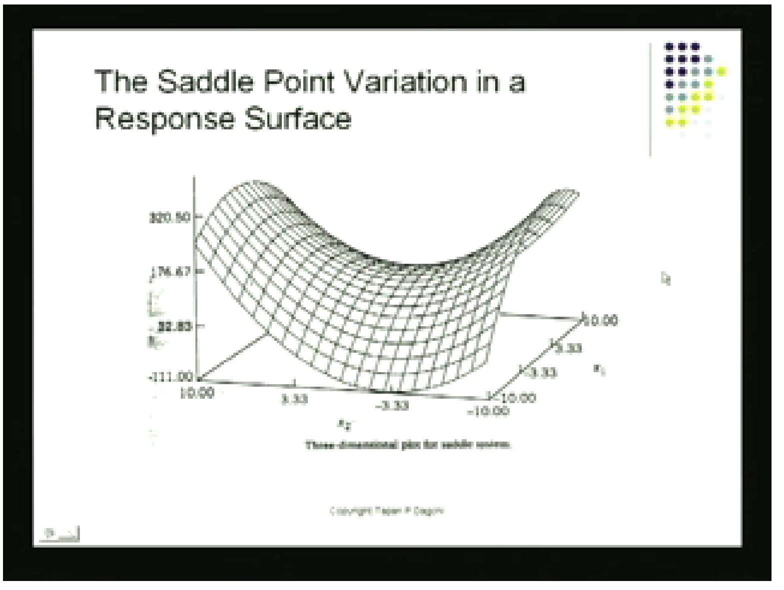

If you look at the other variations this is a third variation of a response surface and notice this one this one has something that we call is saddle point. It is rather important first to realize that not all response surfaces are simple to visualize. Some of these have special mathematical structure. How did I develop the surface? I develop that by collecting real data. Again, in a matrix structure by conducting experiments, special experiments for example, one particular approach is called the box benchen model, the box benchen experiments. These experiments, we conduct those lead a response of the generation of a response surface they help you determine the different parameter the beta 1, beta 2, beta 3 and so on. Once, you have them you end up with the surface like this.

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I have given you couple of references I have given you some responses which you can actually see. These references are there I will leave them on the screen for you. You can click on them, you can go into more details and you will end up finding a lot of things. So, to summarize we reviewed DOE today, we had two sessions. One dealt with finding the main factor effects and the interactions, then the second one work with a noisy environment which was sort of like this. This is the noisy environment and there we utilized the technique of ANOVA the third method that we that we looked at was this technique of response surface what you end up with surfaces like this and that can actually help you optimize the process, which is otherwise very difficult to optimize because you do not have theoretical equations for these surfaces and I cannot really optimize them given the shape of it.

We will continue with this, all this is our march towards six sigma would you like to be able reduce defect, we have to understand how these different processes work and then do something about controlling those control factors and also perhaps control noise either to make them make the process robust or to try to improve the performance of the process. So, that most of our production comes well within, well within the tolerance that is acceptable to the customer. This is what we would like to able to do. That is really the goal of six sigma. So, DOE turns out to be a real important step to move towards six sigma and DOE as I had told you earlier, if you look at the six sigma process DMAIC define measure analyze improve and control, it is the i-th step where you utilize design of experiments. We will continue with this in the next sessions.

Thank you very much.