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**Lecture No. #05**

## Probability Theory

Hello and welcome back to the lecture on Applied Econometrics or we are continuing with module I and we are going to talk about probability theory in this particular lecture. (refer time: 00:37)

Now the items that we are going to cover are, we will introduce the probability theory. There are many vocabularies, and as you already understood that vocabulary is really very important. When we talk about uh econometrics, we need to talk in the language of an econometrician. So we will familiarize ourselves with the vocabularies associated. Then we will go into specific probability theory, which is classical probability.

And we will also learn associated vocabularies. Then we will look into couple of examples of classical probability theory. And we will also discuss the defects of classical probability theory. So that is basically the ambit of this lecture. (refer time: 01:17)

Let us first introduce probability theory. Now probability, the way we understand is that it is a measure of likelihood of certain event to happen. So for example, let us say probability of rain, say you estimate probability of rain and it says that it is 0.5. And then you make another estimate of probability of rain and it says 0.8. So in your second estimate says that the likelihood likelihood of rain is actually higher than the first estimate.

In other words, probability is also is a measure of trustworthiness. You can see that how trustworthy the fact that we will have rain. So there are different ways of measuring probability. One is prior probability and the other is posterior probability. So the prior probability is something that you really do not need to sort of depend on the event to give you data or information for for you to compute the probability.

So we will just come to that. Whereas the posterior probability is something that actually you generate evidence from the experiment itself. Now prior probability are actually of two types. One is the classical approach, and another is the frequentist approach. So we will talk about the classical approach, where we will actually estimate the probability even before the event happens.

Whereas in frequentist approach, we actually depend on historical data to compute the probability. So we will see that. And whereas for posterior probability, we will only discuss about the Bayesian probability. And in this case, the data for the for the computation of the probability will actually depend on the event that actually has happened within that experiment.

So we will see how we actually compute Bayesian probability. So with this, let us actually uh explain some of the vocabularies associated with classical probability. (refer time: 03:16)

So now we will come to the vocabularies associated with the probability theory. So the first term that we will learn is random experiment. We will learn what is a trial, we will learn what is a random variable, we will talk about sample space, we will understand the difference between outcome and an event. So let us say the first term is random experiment. So what is random actually?

This is a term that we will often come across. So when you say random, it means there is a chance involved. So that is all it means, there is a chance factor involved. Sometimes people actually confuse, by random they feel that the chances are equal or maybe or equally probable, but that is not correct. So any kind of chance, uncertainty involved, whenever it is involved, we call it random, alright?

Now what is an experiment in econometrics or statistics for that matter? Now an experiment is that where you can repeat certain act under a given circumstances, under a given conditions. So you can repeat again and again. When you can do that, that is what you call a random experiment. Now, only experiment. Now when I say random experiment, then we can we can basically define it as an experiment, you can run again and again under certain given conditions.

And in that experiment, there is a chance factor involved. So you really do not know the outcome of that event beforehand, okay. So that is what is random experiment. The second important term that we should have a clarity about is trial. And I already used the word trial before. Now trial is basically one run of that experiment or one repetition of that experiment.

So you if you say like I toss a coin and the first, in your first toss, you get a head. So you say in my first trial I got a head, right? Or you can also say that okay I have n trials. So which means that you have tossed the coin n times. So that is how we understand the term trial, okay. Now the other term that you will come across often is random variable. Now what is a random variable?

Random variable to under to sort of understand what random variable is, is any real valued variable uh that is associated with a random experiment. So basically, when you have the outcomes of a random experiment, you represent it to a variable. For example, if I say that I am uh tossing a coin say n times or maybe I am tossing two coins n times and my random variable is the variable that actually says number of head appears, okay.

So it can be like, if it is one coin, so then you will have number of head 0, 1, 2 or up to n. So then your random variable is it can take values from 0 to n. So that is what is called a random variable. The other term that we come across is the sample space and this is this is a pretty simple definition. And the sample space essentially is that all the outcome of a random experiment that constitute the sample space, okay.

So for example, if you are tossing two coins, you have HH, HT, TH and TT right? So you have head head, head tail, tail head, tail tail. So these basically are all the possible outcomes. And when you have all the possible outcomes with you and you create constitute a space that is called your sample space. Another uh definition that you should know and you should know the distinction between these two terms.

Often they kind of come pretty close to each other, but there is a difference and that is the definition of outcome and an event. Now for that, for example, let me just use the whiteboard for a while. (refer time: 07:06)

Say for example, you have you are tossing a coin, alright? And in your first toss so you have this HT and then you are tossing again. So you can have H, you can have T, you can have H, you can have T, right? Now when I say outcome, outcome essentially means that the last result okay the last result that you get, so here you have HH, HT, TH and TT right?

So all these four, these are you know the final results that you have gotten out of this experiment right, out of this random experiment. So these are all outcomes, these are all outcomes. But whereas, if you say event, event is something that you can basically define any phenomena in that random experiment. For for example, if I say that uh my the event I define that say at least at least one head, right?

So this is my event, at least one head. My event is defined as at least one head. So if that is how I define this event, then I will have these three cases, which are actually representing this event, right? So you see that all the outcomes are kind of event, but then you have, event you can define in your own way. So you know whichever way you define, that will be the event, right?

So that is basically the difference between the two terms, outcome and event. Now we will come to the definition of definition of definition related to the classical approach. So now we will talk about the classical approach. And when I talk about classical approach, so we are basically talking about if you remember, we are talking about the prior probability.

So in so in, in case of prior probability, we said that there are two types of prior probability, one is classical another is frequentist approach. So in in class, when we talk about classical approach, we again need to familiarize ourselves with certain definitions. So let us let us look into that. (refer time: 09:34)

So the first definition that we learn is mutually exclusive. The events need to be mutually exclusive. So what it means uh when I say mutually exclusive is that occurrence of one event means non-occurrence of the other event, for example, if I toss a coin, and if I get a head, so it automatically entails that I do not get a tail, right? So that is what is the definition of or understanding about mutually exclusive events.

Now the other term that we need to understand is the term exhaustive. So by exhaustive I would mean that it essentially talks about all the events or basically all the possible outcomes actually constitute, you know like, give you an exhaustive list of outcomes, for example. So for if I if I am tossing two coins, the exhaustive list of outcomes are going to be and it also means that at least one of these event has to happen, at least one of the outcome has to happen.

So here you can note that I am using the term event and outcome interchangeably. So it is not a problem because I am actually defining the last result in result as event or as an outcome. So here I can you know like when I say exhaustive, then I am insured, I need to ensure that at least at least one of them necessarily necessarily occurs. The third definition that we need to sort of be familiar with is equally likely.

What equally likely means? The very definition itself says that, it is essentially talking about equal equiprobable. So events are equiprobable. When you talk about classical probability, we consider the results, the outcomes, they are basically equiprobable. The fourth term that we need to be familiar with is that events, outcomes favorable to an event, outcomes favorable to an event. Now what does that mean? So let us say, let us say I will use a new page here. (refer time: 12:16)

Let us say I have an event A and my event A is say, just the this the definition that we have heard about this event, at least one head, one head from this uh two coin toss, HH, HT, TH, TT, at least one head. Now which are the outcomes favorable to that event. So the outcome favorable to that event are this, this one, and this one, right. So that is how we define at least, that is how we define the outcome favorable to an event.

Now now we have definition of all these four terms, which are essential to define the classical probability. Now a classical probability of an event A could be defined as probability of A could be defined as a ratio of outcomes favorable to that event, outcomes favorable to event A.

Whereas in your denominator, you will have all outcomes of the random experiment, experiment where the outcomes are, they are actually fulfilling the condition uh conditions that they are uh mutually exclusive. They are mutually exclusive, exclusive, exhaustive, and equally likely.

So in these examples, in this example uh, in the coin toss, we will have outcomes favorable to event is 3, whereas I am assuming this all these different outcomes that we have gotten they are actually, you know like, they are equally likely. These are unbiased coin. So there is no event which is given preference over the other. Then they are mutually exclusive.

So you know one event basically means that the non-occurrence of the other event and they are exhaustive. We are basically taking into account all outcomes. So it is 3 by 4, right? So that is how we actually define classical probability. And note that you do not have to even actually run the experiment. You actually know, you actually have in your mind, what are the different outcomes that you can possibly get.

And from that knowledge, you can actually get this probability 3 by 4, right? Now let us do couple of examples to sort of understand how we really do not need to get the data from real life to actually uh, you know compute the classical probability. (refer time: 15:12)

Let us say that there are 10 students, there are 10 students and you want to sort of arrange in a way, in a way that two students are always together, are always together. What is the probability of this event? Let us say this is my event. What is the probability? What is the probability that that two students are always together, that two that two students, that two students are always together? So let us solve this.

And we really do not have to run an experiment. We really do not have to get an historical data to kind of get this result. So let us just do that. So let us have another page. So let us say how we do that. (refer time: 16:14)

So there are 10 students 3, 4, 5, 6, 7, 8, 9, 10. Now you choose any two of them, because so that two students have to be together. So you choose two of them. So let us say this, and then you will have rest 8, right? Here, there are 8. And here is, so basically bunch these two students as one group. So essentially, we have 9. And then so essentially, you can actually uh do all the possible arrangement with a 9 factorial way.

Whereas the for the two students, you can, because they can change places, you can get 2 factorial, right? And you can do the same thing for all other positions, right? You can take this guy and this guy, you can take this guy and that guy. So there are all possible ways of doing it. And so that is how you actually get this 9 factorial. So you choose any two, and then the rest eight. So altogether, it will make nine.

But then there are two guys, and you can actually exchange positions, so they can actually have a 2 factorial. Now I have 10 students right, 10 students. So my all possible outcomes are going to be 10 factorial, all possible outcomes 10 factorial. So the probability that two students are going to be together is this, 2 factorial by 9 factorial 2 factorial into 9 factorial by 10 factorial.

Now look at it, we really did not have to do an experiment to understand what is the what is going to be the result, right? So we already know that, given the fact that the conditions of classical probability is satisfied, this is going to be your answer, alright? Let us do just another problem. I am sure by now you will get it. And this time, I am going to give a pause. And I will allow you to think or do it yourself, before I actually jump into giving you the answer. So let us say there is a maths problem. (refer time: 18:00)

Let me use, let us say a maths problem. Say a maths problem is given, is given to say A, B, and C, they are three students. Now A has a probability of solving a problem, say probability that A can solve a problem. That probability is say 3 by 5 let us say, it is just a random number I am just conjuring up. B can solve probability let us say, we can write probably A can solve, probability B can solve.

Let us say this 2 by 5 and probability say, probability C can solve is say 1 by 5. Now I ask, what is the probability that the problem is not solved? What is the probability that the problem is not solved? It is a very simple problem. And the whole the reason I am giving you this problem is because the of the fact that if you have a pen and paper, you can simply get the answer.

You really do not have to, you know get lot of data to sort of understand it. So now I will I will pause for a moment. And I want you to pause the video and actually do the computation on your own and come up with the answer. Okay, I hope you come up with some answer. So let us say let us now do the problem on our own. So probability that nobody could solve a problem. So that means A could not solve the problem. (refer time: 19:35)

So which means A had the chance of solving the problem that was 3 by 5. So probability A could not solve the problem is 1 - 3 by 5. What about B? So B could not solve the problem. The probability is 1 - 2 by 5. Now what about C? C could not solve the problem is 1 – 1 by 5, right? And now I have not explained it yet, is that if they are trying to solve the problem independently let us say.

So then the events are going to be, you can actually multiply the probability to sort of get the intersection. So nobody could solve the problem. And we are just going to look at the probability rules in a while. And then we will talk about why where from this multiplication has come. So essentially, it will mean 2 by 5, 3 by 5, and 4 by 5. So that is the probability that the problem is not solved.

So which means this is 24 by 125, okay? So essentially again to make the point clear that for classical probability, you do not have to get data from anywhere. It is, you can just simply calculate the numbers with your information you have okay, as long as the assumptions you are, you know like having that is kind of satisfied. (refer time: 20:58)

Okay, so now the last thing about classical probability is that there are certain defects, there are certain defects of classical probability, classical probability. Now let us think about it.

When we are talking about classical probability, we are assuming that the events need to be mutually exclusive, they are exhaustive, and they need to be independent. Now that is kind of restrictive assumptions.

As long as this assumption is satisfied, you can actually have the uh, you know you can actually you know claim that classical probability approach you can actually uh take. But otherwise, you cannot. So that is one of the major restrictions you have. Now secondly, I actually want you to uh see this definition. (refer time: 21:43)

Here I am, what I said is that all outcomes of the random experiment, they have to be mutually exclusive, exhaustive, and equally likely. Now there is a problem with this equally likely um term. Now when I say equally likely, that means equally probable, right? That means equi probable.

Now if the terms are equi probable, and you are already using the term probability while you are calculating probability, so that kind of is you are kind of assuming that you are calculating probability, whereas you have already calculated the probability. So that is kind of not so, that you kind of you know presuming calculating probability before you calculate the probability.

So that is kind of, it does not really make a lot of sense. So that is another big problem uh with the definition of classical probability. The third uh aspect, the third defect of classical probability, is that you often um sort of, you know like, all the things that you do in classical probability is that you live in an ideal world.

You are you are basically, you know tossing a coin or you are picking up a ball from an urn or you are, you know throwing a die. You know all this kind of ideal situation, you can actually handle with classical probability. But suppose you have something more realistic thing that to deal with. For example, an insurance company actually wants to understand how likely a guy who is 30-year-old in India would survive when he is 80, okay?

So do can classical probability solve the problem, perhaps not. So this is, this is the limitation. When it comes to real life classical probability does not really help us. Nonetheless, classical probability is a good beginning to understand the theory of probability. And we have sort of understood the terms and you know like the concept associated, the limitations of classical probability.

And with this, we I will end classical probability. Here what you need to do is you can refer to the textbook of NG Das, the book I referred in my introductory talk. Or you can also look at Levin's textbook. So both these textbooks are good enough to sort of get a grasp on classical probability. So with this uh, let me come conclude the lecture on classical probability.