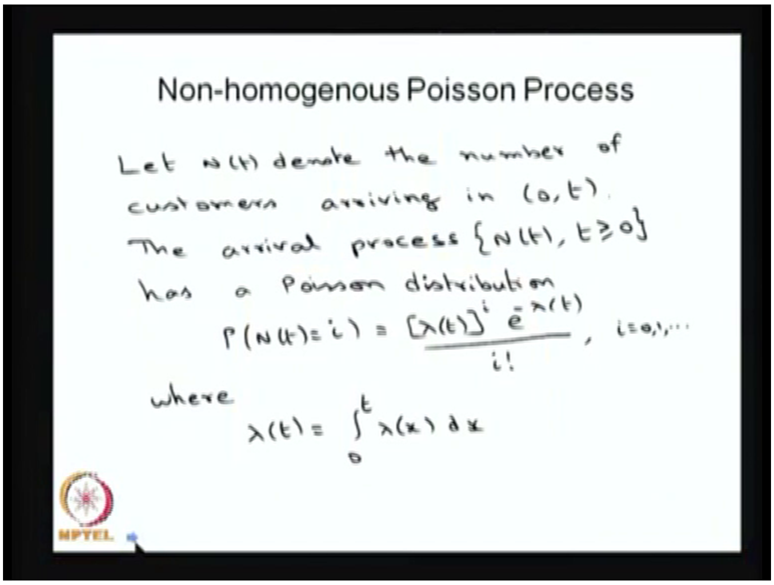
Introduction to Probability Theory and Stochastic Processes

Prof. S. Dharmaraja
Department of Mathematics
Indian Institute of Technology, Delhi

Lecture - 84

Next I am going to give some more process related to the Poisson process.

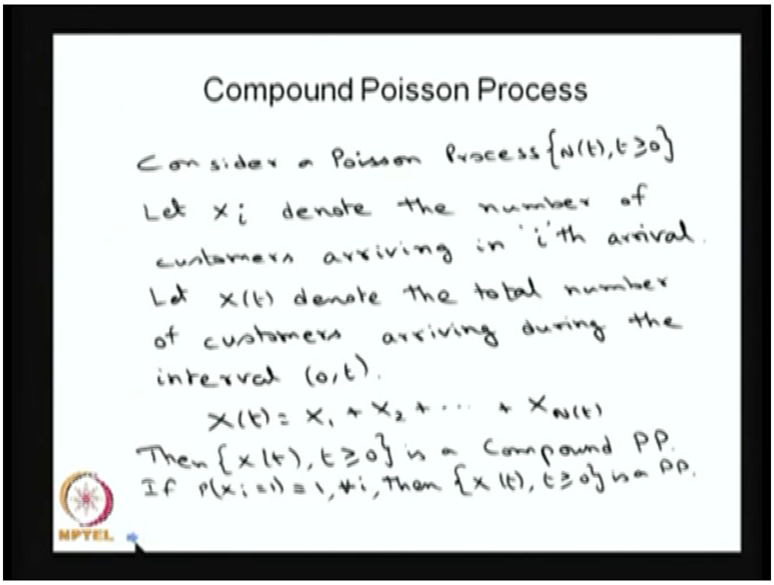
(Refer Slide Time: 00:08)



The first one is the non-homogeneous Poisson process. Let N t denote the number of customers arriving in the interval 0 to t, the arrival process has a Poisson distribution. But here the change instead of the mean arrival rate is a constant. Mean arrival rate is a constant lambda, but here it is a function of t, lambda t is the cumulative rate till time t. That is a change from the Poisson process then this stochastic process is called a non-homogeneous Poisson process.

Instead of mean arrival rate is a constant, here the lambda t is a function of t. Therefore, this stochastic process is called the non-homogeneous Poisson process. Second one compound Poisson process.

(Refer Slide Time: 01:02)



Using Poisson process, one can develop a little complicated stochastic process related to the arrival that is called a compound point Poisson process.

Consider a Poisson process N of t, then you define a random variable X i denote the number of customers arriving at the ith time point of arrival. X 1 denotes how many arrivals takes place at the time of first arrival, first arrival time point. And X 2 will be what is the second time of arrival how many arrival takes place. Therefore, I am making a new random variable X of t, that involves a t that denotes the total number of customers arriving during the interval 0 to t; that means, it is going to be how many arrival takes place in the first time point X 1, how many arrival takes place at the second time of arrival that is X 2 and so on plus X of t X of X of X N t. Here N t is a random variable and how many arrival takes place, that is a X i's. altogether that is going to be the total number of arrivals.

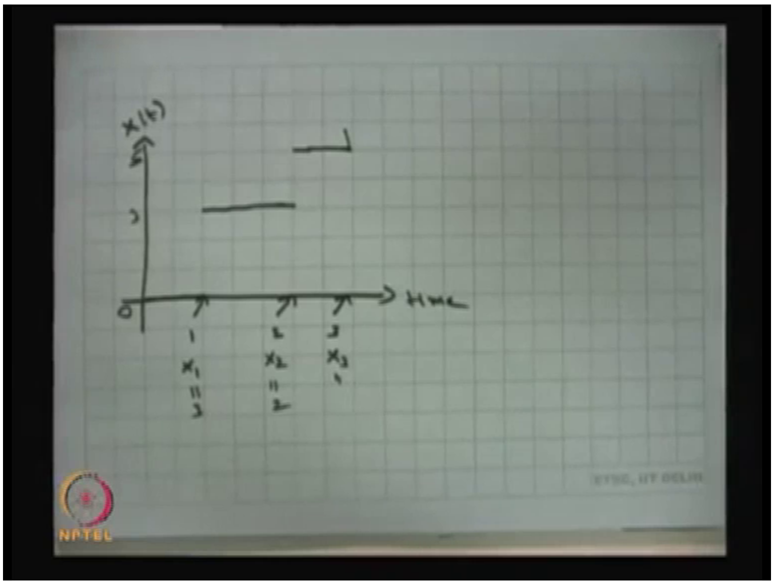
The X i's are independent and identically distributed random variables with some distribution function g independent of the Poisson process N of t. So, this is nothing but a random sum because these are all the random variable. And how many random variables you are going to add that depends on the value of N of t over the t. Therefore, this is a random sum of X i's with N of t.

Obviously these 2 are independent X i's are independent of N of t, and since it is a number of customers arrival during the in the ith time point therefore, X i's are discrete random variable. X i's are discrete random variable, and N of t is also Poisson process; therefore, X of t is going to be a discrete state continuous time stochastic process. And we are using a Poisson process to get this stochastic process therefore it is called a compound Poisson process.

One can deduce a Poisson process from a compound Poisson process by substituting each X i takes a value only one unit; that means, when the number of customers arriving at the ith time point is going to be only one; that means, if I make a P probability of X i takes the value only 1 that probability is one for all i. Then i will have a only one value possible till N of t, then this is going to be a Poisson process.

Suppose the probability of X i is a suppose the X is are going to be a discrete random variable with the possible values 0, 1, 2 and so on, then the X of t is going to be a Poisson process. I can make a simple sample path for the compound Poisson process.

(Refer Slide Time: 04:32)



This is over the time and this is over the N X of t

Suppose these are all the time points in which arrival time point. So, this is the first arrival time point, and this is a second arrival time point and this is the third arrival time point. It can be anywhere in the in a continuous time therefore, this is called a discrete state continuous time stochastic process. So, here I am relating with the random variable X 1, this is X 2 random variable, this is X 3.

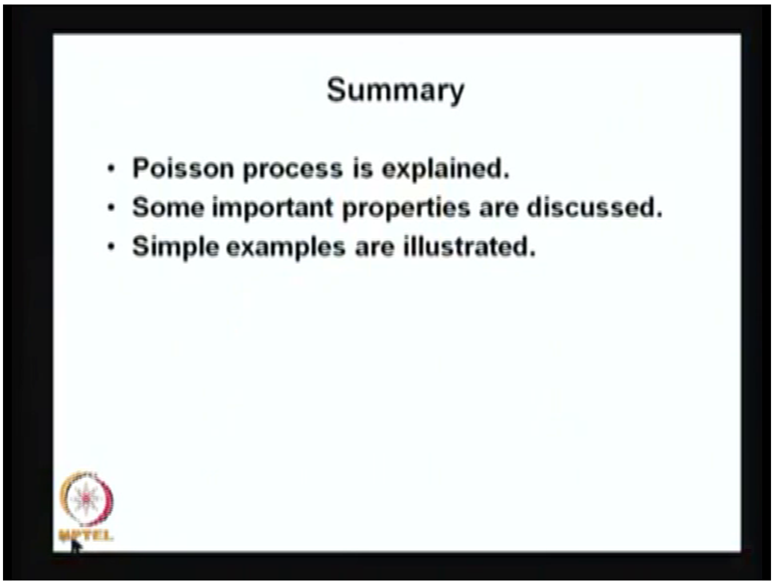
So, till the first arrival till the first time of arrival, the number of customers in the system is 0. At the first time of arrival the X 1; suppose, you think you make the assumption X 1 takes the value 3 X 1 takes the value 3, therefore, this will be incremented by 3 till the second arrival. At the time of second arrival suppose you assume that this takes a value 2 with some probability of X 2 takes the value 2 is greater than 0. So, you have a assumed value 2, it can take any other value also.

So, it is incremented by 2 till it takes the third arrival the value is ah; so, this is 0 this is 3 then 3 plus 2 5. At this time whatever be the number of arrival accordingly this can take some value.

So the difference between the compound Poisson process and the Poisson process in the Poisson process the increment will be only 1 unit increment over the time whenever the time at which the arrival occur arrival time epochs. Whereas, here wherever the time of arrival time approached the number of customers entered that need not be 1, it can be more than or equal to 1 so that is the way the job goes therefore, this is called the compound Poisson process.

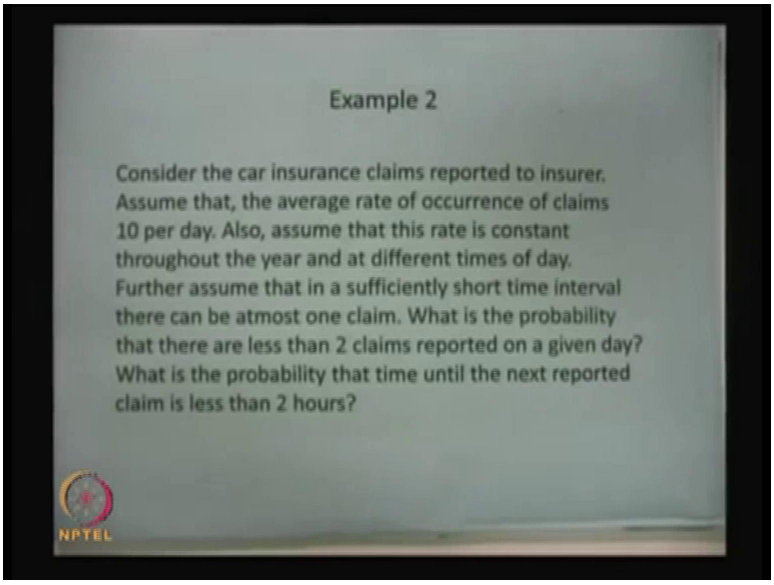
So, we have seen 2 variations of Poisson process one is a non-homogeneous Poisson process the other one is a compound Poisson process.

(Refer Slide Time: 06:54)



So, before I go to the; I complete let me give the solution for the first second example which I started. That is discussing the car insurance problem.

(Refer Slide Time: 07:17)

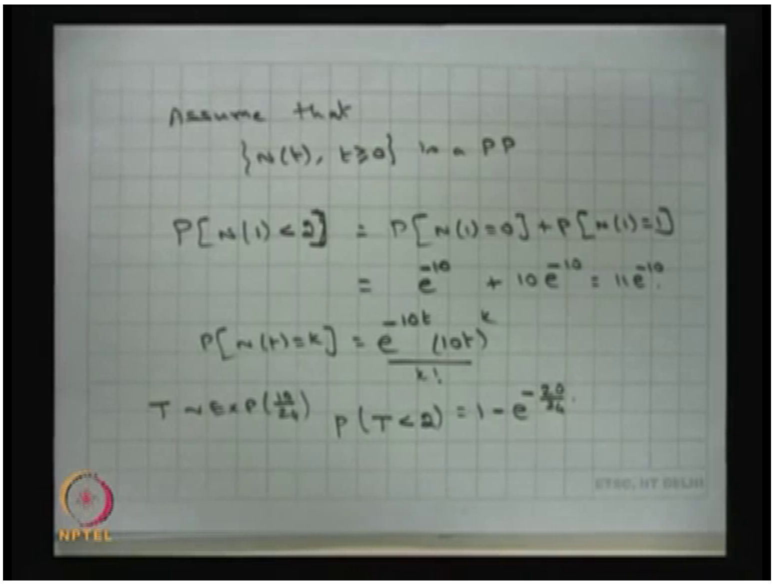


We have discuss the 2 problem the first problem is related to the bus stand bus arrival issues and this is the car insurance problem.

So, in this problem, we have not assume the Poisson process, but the problem is related to the Poisson process one can assume it is a form of Poisson process, because you see the assumptions the average rate of occurrence of climes is a 10 per day, also the rate is a constant. And in a very small interval of time at most one claim can happen. The questions are, what is the probability that there are less than 2 claims reported on a given day?

Since the increments are stationary so, any day you can think of with the only the interval. What is the probability that the time until the next reported claim is less than 2 hours? So, this is related to use the exponential distribution because the inter arrival times are exponential distribution.

(Refer Slide Time: 08:37)



So, for the first question, you can assume that the N of t is nothing but a number of insurance car insurance claims reported to the insurer that has a Poisson process. He can assume that N of t is a Poisson process based on the assumptions given in the problem.

Once you assume that this is a Poisson process the question is, what is the probability that there are less than 2 claims reported on a given day? So, the given day 2 days you can shift into 0 to 2 days itself because of the increments are stationary. So, the question is nothing but what is the probability that N of 1 is less than 2 in a given day a day.

So, what is the probability that N of 1 is a less than 2? That is nothing but what is the probability that N of one equal to 0 or N of 1 equal to 1 therefore, the probability is added. So, you substitute since it is say N of t is a Poisson process the probability mass function of N of t is equal to k. That is e power minus lambda here the lambda is a 10 per day 10 times t and a 10 times t power k by k factorial.

So, this is the probability mass function for the random variable N of t for fixed t. Therefore, N of 0 N of one is equal to 0 that is nothing but e power minus 10; here the t is 1 day. Plus, N of 1 is equal to 1, you substitute here therefore, you will get 10 times e power minus 10. So, the answer is a 11 times e power minus 10, numerically you can get what is the value. So, the probability that there are 2 claims reported on a given day is a lambda times e power minus 10.

The second question what is the probability that time until the next reported claim is less than 2 hours. So, this is equivalent of the next reporting claim is less than 2 hours; that means, the residual time of the next claim, that is going to happen the one claim is going to happen less than 2 hour; that means, you can use the inter arrival time, that is exponential distribution with the parameter lambda here the lambda is a 10 or 10 by 2 hours.

Therefore, you should convert the values so, it is a 10 divided by 24 claim can happen at any day throughout the 24 hours. Therefore, 10 per day therefore, it is 10 divided by 24 per hour; so, that is exponentially distributed with the parameter 10 by 24.

Now, the question is what is the probability that the time until the next report claim is less than 2 hours? that means, what is the probability that T is a less than 2. That is nothing but that is nothing but since it is exponential distribution and you know the CDF of the random variable T. So, the probability of T is less than 2 is nothing but a 1 minus e power minus 2.

So, 2 times so, 20 by 24 so, the answer is a 1 minus e power minus 20 by 24, that is the probability that the next report a claim is going to be occur before 2 hours. So, with this I have completed the 2 examples also. So, in this lecture we have discussed a Poisson process, and we have illustrated 2 examples for the Poisson process also some important properties also discussed in this.

Thanks.