Scientific Computing Using Python

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Lecture No. 16

Hello, welcome to Scientific Computing using Python. So, today we will start the iterative method to solve the system of equations. So, let's get started.

So far, we have discussed some methods to solve the system of equations, in which we have discussed LU decomposition, Gauss elimination, and Crout's methods. So, all these methods come under the direct method category. Now, what we are going to discuss is the iterative method. So, we will solve the system of equations by iterative method.

So, for that, we have two methods: Gauss Jacobi and Gauss Seidel iterative method. So, we know that to start the iterative method, first of all, we have to give an initial to start. So, an initial solution is what we need for starting, and this solution improves with every iteration. Only then we will say that our method is converging. If it is not improving, then we will say that our method is diverging.

So, let us see how it is. So, what is this system of linear equations? The method that we are adopting is unique solution, and the coefficient matrix is diagonally dominant. We have this condition. It means that this condition is sufficient condition of our matrix, is diagonally dominant. If we apply Gauss Jacobi or Gauss Seidel method, then we will definitely converge. So, we call this thing a sufficient condition. So, diagonal dominance is a sufficient condition for us to solve the system of equations. We have the requirement.

Now, let us see what happens. Now, see what has to happen in the Gauss Jacobi. Now, how do we start the go Gauss Jacobi? For example, if we have a system, then in the system, I wrote that the equation that we have is a11 x1 + a12 x2 up to a1n xn = b1. This is our first equation. Like this, we get our second equation: a22 x2 a2n xn = b2. We have written it like this. And the last equation we have is an2 x2 ann xn= bn. So, we have it.

And in this system, the matrix in it — A — is a n cross n square matrix, with this vector being n cross 1, and the right-hand side vector is also n cross 1. So, we have this system, and we have to solve this system. So, this system can be a very large system, okay? It may be 4 by 4, 5 by 5 or even 100 by 100, million by million. So, this algorithm will work for all. Our condition in this is that our diagonal elements — these diagonal elements — the condition is strictly diagonally dominant, will satisfy it, right? So, we will satisfy it. So, we will say that our matrix is diagonal dominant.

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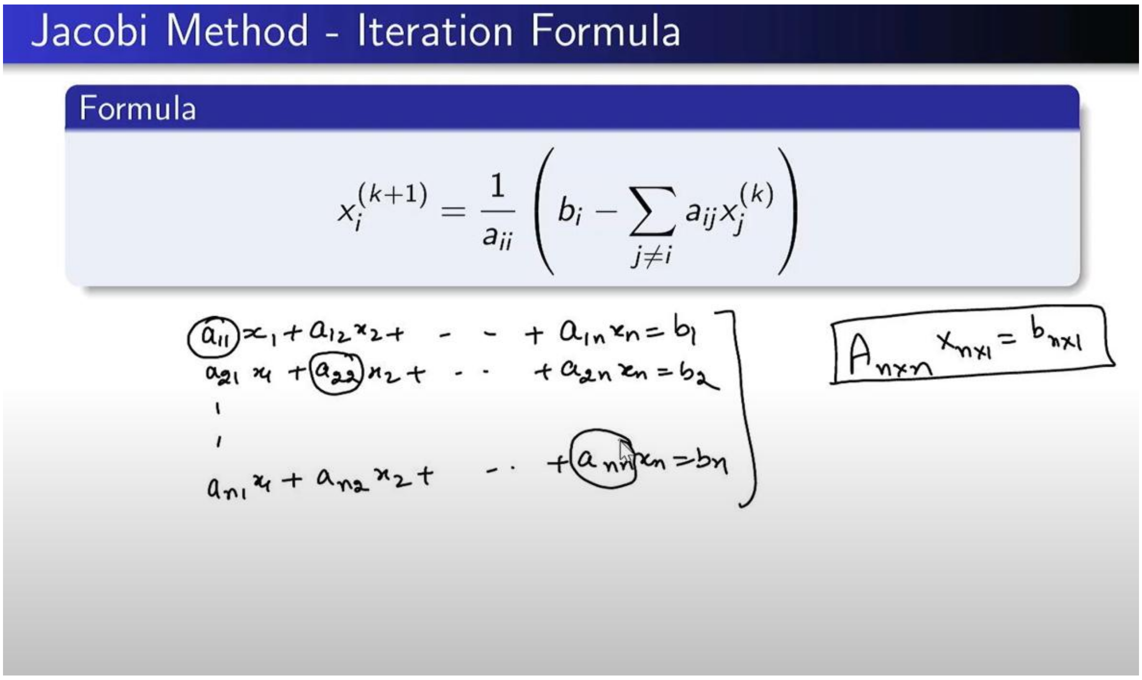

Now, what have we done? We have to solve this system. So, what will we do? Now, look, the first thing we have to do for this is, we will write this system like this. Look at the first equation. How do I write the first equation? I will write x1 like this, and I will bring everything b1 minus here. It will become a12 x2, a1n xn, okay I have taken this all this side and divide it by a11. So a11 as it is diagonal dominant so there is no problem. The a11 cannot be zero in any condition, so diagonal elements cannot be zero, even if these are small values then also there will be problem by loss in significant digits. So, diagonal dominance is required. So, look at this. How can I write it in short form? b1 minus — now here comes the summation — aij x. The x that is x, we will write j here, and I will divide it by a11. So, what happens here — 1 2 so x2 , 1 3 so x3 — so the i — let me make this i one.

Now, in this equation, it will become one a1j, okay? And j will go from 2 to n. So, here we have the first equation. Similarly, we have x2 ext. What will I do? I will keep it intact here, and all the other things — if I take the terms there — then it will become b2 minus — now here it will come a21 x1 plus a23 x3 a2n xn — has come till here. And after that, we divided it by a22.

Now, what did I do? I divided it by a22. What does it mean? That here also, our a22 cannot be zero. So, in dominance, the a22 here will be the largest element. So, I wrote it like this, and I wrote it here: b2 minus — like this. I did the summation, okay, a2j. Okay, and here I wrote xj, and here I can write j — that j is not equal 2. And after that, my a22 comes, so it cannot be 21 x1, cannot be two two, 23 x3, 24 x4 — will go on like this.

So, in this way, we will convert the whole system like this. And in the last line — xn — what will we be left with is bn minus summation anj xj n, j not equal to n divided by nn. Okay, so here we have such a system.

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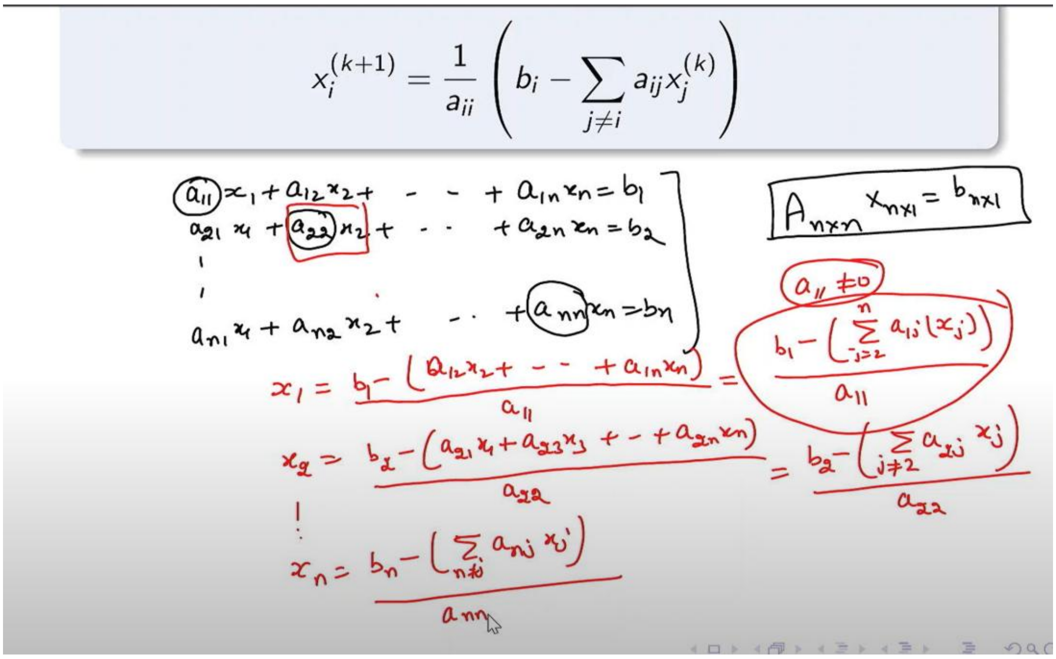

Now, after creating this system, we have to see how to solve it, because the main question will be how to solve it. So, for example, this is a system — how will we solve it? So, now we will convert what we did above into an iterative form.

So, we know what is iterative form and how to write a form. So, once we have got this, what is iteration? What does iterative form mean? Now, look, this is our system. I will write the entire system in this form. I will write it here. Let me write it on the next page. So, we can easily see it from here.

So, look, I will write xi, i are coming. This is equal to 1 over aii . Okay, and bi minus summation j, which is j is not equal to i a i j x j. I have written the entire system here.

Where i is 1, 2 up to n, we have a complete system. This is what we have just done. Now we have to make it iterative. So, what happens in iteration is that we give input from here. So, suppose we gave input at the k step and we got a new solution at the k + 1 step. So, this system will keep going on like this. For how long will it keep going on? Till we have x k + 1 - xk, where x is a vector. So, if it is a vector, then we have to take the distance between these two. So, we have to take the norm. So, it should keep going on till it becomes less than tolerance. So, what does less than tolerance mean? That I will keep running it till it becomes 0.5 in 10 to the power n. So, what does it mean? That the n that we get is a number of digits. The accuracy that will come, so accurate up to the end digit. So, depending upon how much accuracy we need, we will give tolerance accordingly and this iteration will keep going on.

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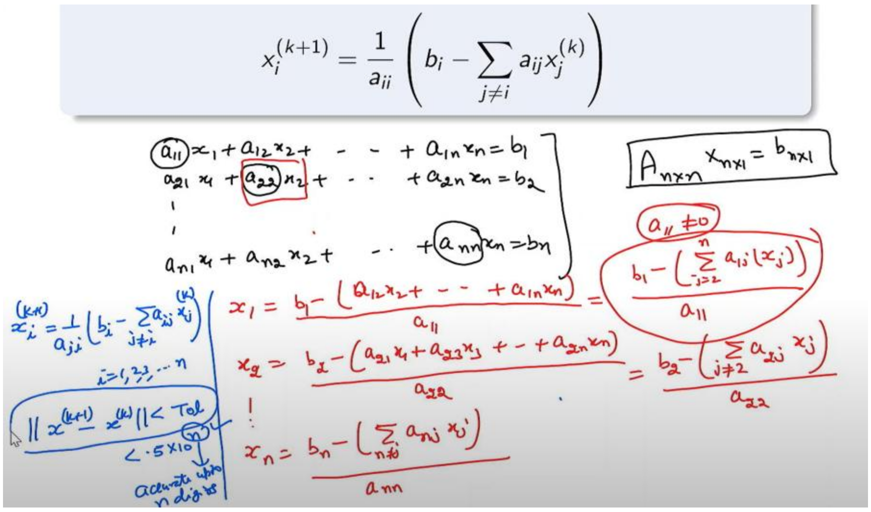

And this iteration may diverge. So, if then we have diagonal dominance, if it is a matrix, then in that case, it will definitely converge. That is why we call it a sufficient condition. Now, if this is a system, then what will I do? In this, the x1 becomes 27 minus. Okay, let me first write it down. I will write this system in matrix form. So, what did we do first? I wrote it in matrix form like this: 10, 2 , -1, -3, -6, 2, 1, 1, 5. This is what we have. And from here, x1, x2, x3 comes and the vector on the right side becomes 27, -61, -21.

Now see, we have a matrix. So, now we have to check it. So, you will see that this element is strictly greater than 2 plus 1, 3. This 6 greater than strictly 3 plus 2, 5. This is strictly greater than 2. So, it means that our matrix A is strictly diagonal dominant. Okay, so from here, we came to know that if we apply method on it Jacobi, it will definitely give us convergence. So, from here, let us try to write it in its form, that how can we make iterative.

So look, x1 is our 27 minus 2x2 - x3 divided by 10. So, from here the first equation is formed. If I make the second equation, I will write x2. How will x2 come? - 61 - now all these values have gone there, so minus 3 x1 plus 2 x3, okay, and we divided it by minus 6, so we got x2. And the x3 that comes out is minus 21 minus x2 x1. We got x1 plus x2 divided by 5. Okay, so we have a system.

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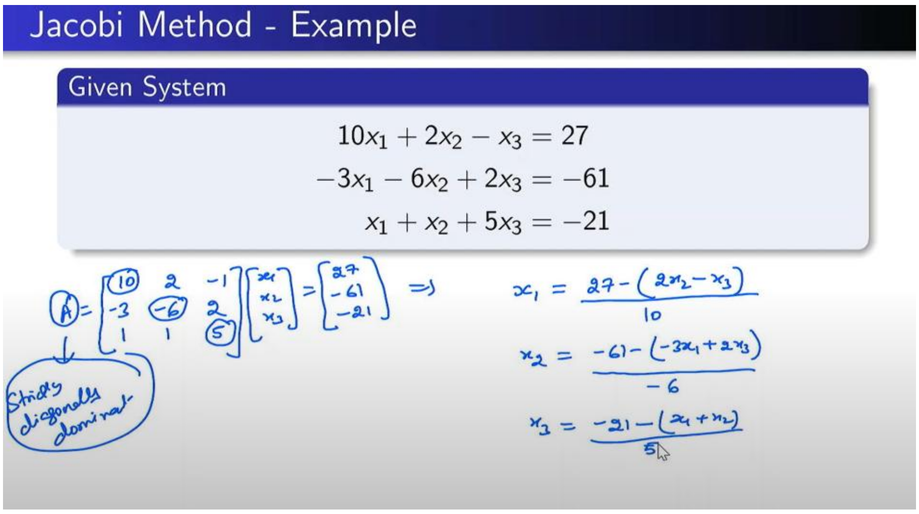

Now what do we have to do with it? We have to convert it into iterative form. So, what will I do to convert iterative form? k k k+1 k k k k k+1 k+1. And from here we can say that k is 0, 1, 2, 3 and so on depending upon how many iterations it will take to solve it. So now that we have a starting point, I can do this: that my x is this and this is basically x1, x2, x3. So I took x0, suppose I took the value, suppose I took 1, 1, 1. Okay, right? We have to start from 1, 1, 1 only, so it will be a little easier to start with this. Okay, so we started from 1, 1, 1. Otherwise you can start from any of them because this is the initial solution.

After that, if we put from here, then we will get x1 from here, then x2 will come. Now how do we stop? So I will see here that the name of x1 - x0, we are taking it. Okay, so this norm, is this norm. We know that we have to measure the distance between two vectors. So, what do we have to do? For this, we can also take the euclidean norm. I can also take its one norm. I can also take its two norm. I can also take infinity norm. So, I can take any norm. So, generally what we do is we take two norms. From that we come to know that as soon as we take y two norms, till it is not less than tolerance, this iteration loop, our loop, while loop will keep running. And as soon as this becomes less than tolerance, it will stop immediately. And our solution, we will see that the distance between the two solutions has become very less. So, what does it mean? That the values of x, our solution has come.

So, in this way, our Gauss Seidel works. Gauss Jacobi, sorry, works. So, this is our system that we solved in the first iteration, right? So, here we took the initial condition 0, 0, 0. So, like I took zero, my x1 came to 2.7, x2 came to 10 and ex3 came to minus 4.2. So, now see, the value that we have got, the value of x1 that we have got is 2.7, 10.16, minus 4.2. Now this is not a solution, right? After just one iteration, a new solution is obtained. So, what will you do with this? If I use x1 here, then I will get x2, right? I will get x2, then I will get x3. In this way, I will keep getting the solutions that I have, and I will keep checking its tolerance, that how much is the difference between the two. So, if the difference that we have between the two is less than the tolerance, then the program will go there and we will get our solution. So, we can solve this with the help of it.

So this is the Gauss Jacobi method. Now it is the same way. Gauss Seidel method is there. There is a slight difference between the two. What happens in Gauss Seidel is that it uses updated value immediately. And what will be its advantage? Its advantage is this — that fast convergence compares to the Jacobi. This is an advantage. Okay, now what have we done? We have solved this problem. See, in this, on the right side, k is coming everywhere. Okay, here all the k are coming. We have made this. See here, on the right side, there is k and on the left side, there is k+1. This is called explicit form. So what is happening in this — that we have done the Gauss Jacobi. But what will happen in the Gauss Seidel? See, we will get the k+1 of x1 from the first equation. So if I put this updated x1 here, then my x2 will come updated. Then if I put the updated x1, x2 here, then x3 will come. Okay, so what is happening in this is that the updated value is being used immediately. So this method is called the Gauss Seidel method.

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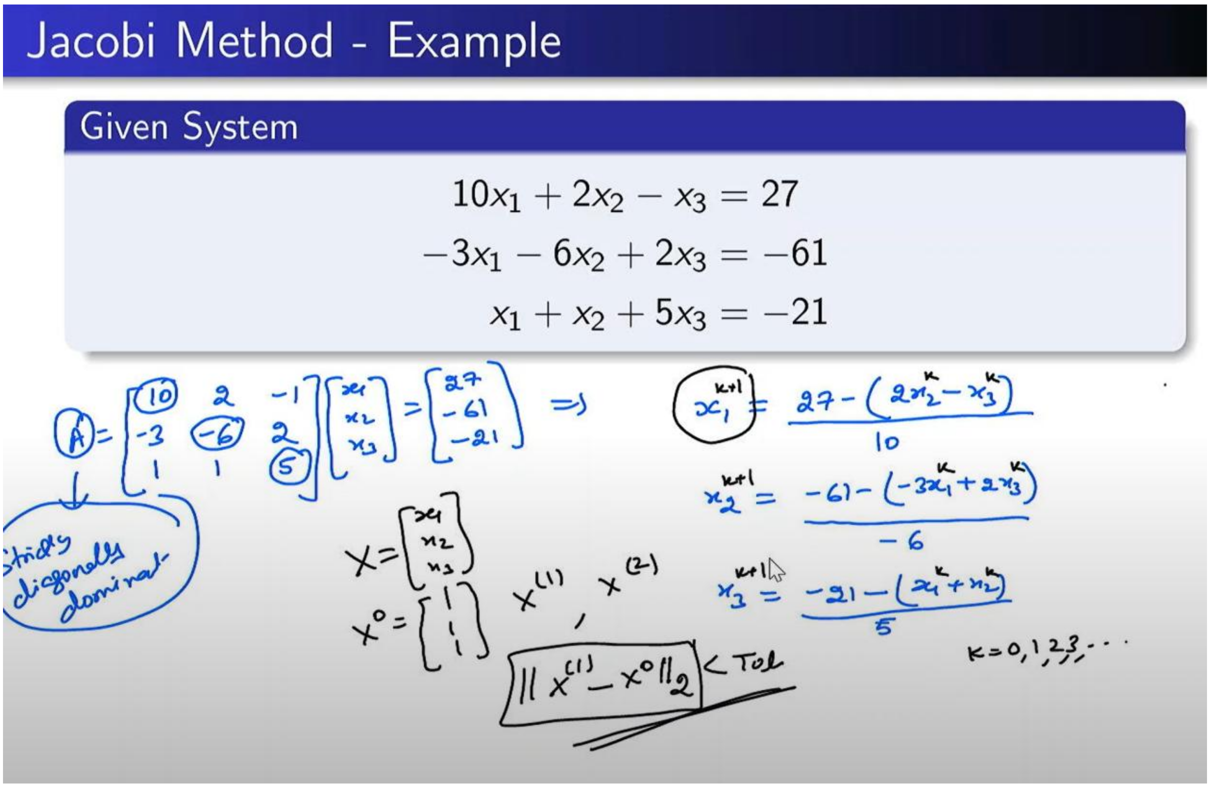

So this is the difference between the two. So see what is happening in the Gauss Seidel method — the k+1 values are also being used in the Gauss Seidel method, and the xk is also being used to find out the updated value. So see, the updated value is being used here, and it is also being used here. So from here, we are finding out and using it here. So if we solve this method, what will we get?

Like my previous equation was this one. So what is happening in this? So what is happening in this is that now look, we had found out the x1 first. And what was the b1 minus summation j is not equal to 1 a, a1j xj. So here we have our x1. So this is our x1. Now what am I doing to find out the x2, and what will come out? We have to divide it by a11.

Now our x2 is here. So what was written in it — b2 minus. Now see, what have we done here? I will use it in a21 — with that, x1 will come. Plus, now the a22 is already there. Now I will do the summation — j is not equal to 2 and j is not equal to even one — a2j xj is here. Now what I am doing, I want to make it iterative. So here, x will come, but here I am taking x+1 — the value which I got from here, I used it here.

After that, I mean x1, k+1 used here, xk left here. Now I did x3. So what happened in x3 — b3 and all this — I divided by a22. Now I did b3 in the next. After that, see what happened — a31 x1, k+1 we already have, a32 x2 k+1 — this too we already have. And the remaining one we do not have v. So here a3j xj k. Okay, and here we will take that j is not equal to — j is now. So I write not or j — we will make it greater than 3. Okay, so at less than this has come. So 31 and two elements. Now see, those which are below the diagonal element have been used here. The values above have come here, and we have divided it by a33. So we will keep doing it like this. There is no problem in the end. In the end, we have nk + 1 left. So what will become of bn minus — now all these terms are new to us. So we will write j is not equal to n anj xj. And here you have k+1. And we have divided it completely by the diagonal element ann. So this is our system. So this system is using our updated value immediately. So we call this system the Gauss Seidel method.

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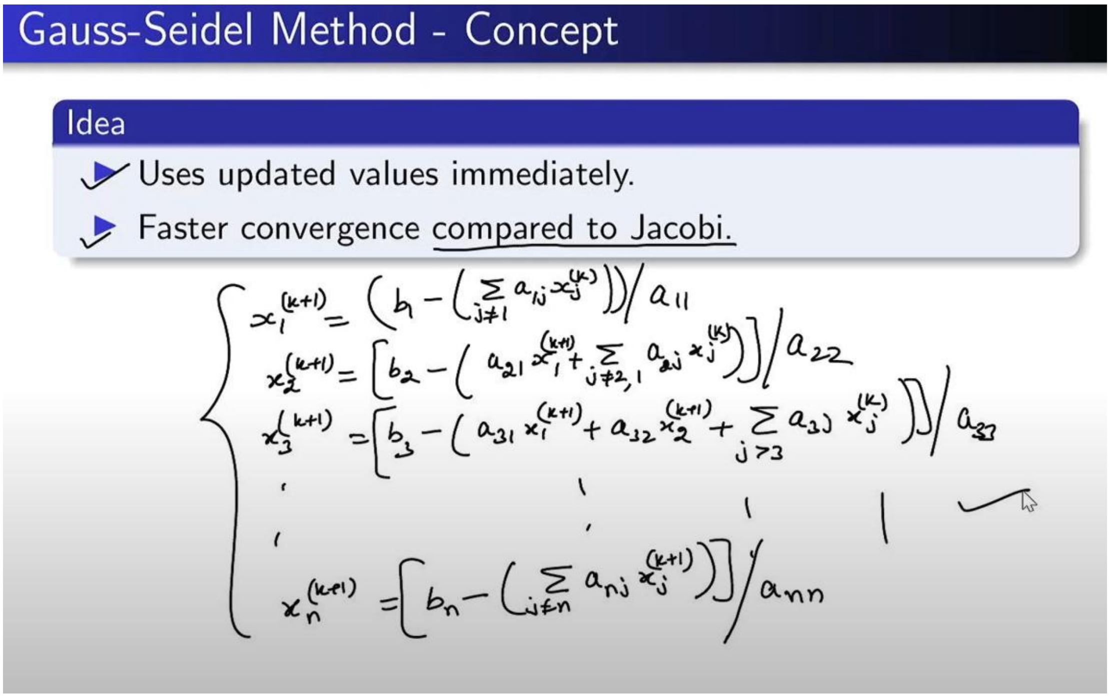

So there is a little difference between the two. So see, this j is less than i. So for that, this value is the updated value. These are the updated values. And this is the value of the previous step. We used the values at kth iteration and steps, and we got a new value. So in this way, we will do the Gauss Seidel method.

Now, as this was the question — how to solve this — now we will do it with the help of Gauss seidel. So, with the help of Gauss Seidel, see what happened — x1 will be the same 2.7. What happened in x2 — we used this 2.7 here. So our values have changed. 8.95 came in x3. In this, we used 2.7 and 8.95. So, here we have used the updated value.

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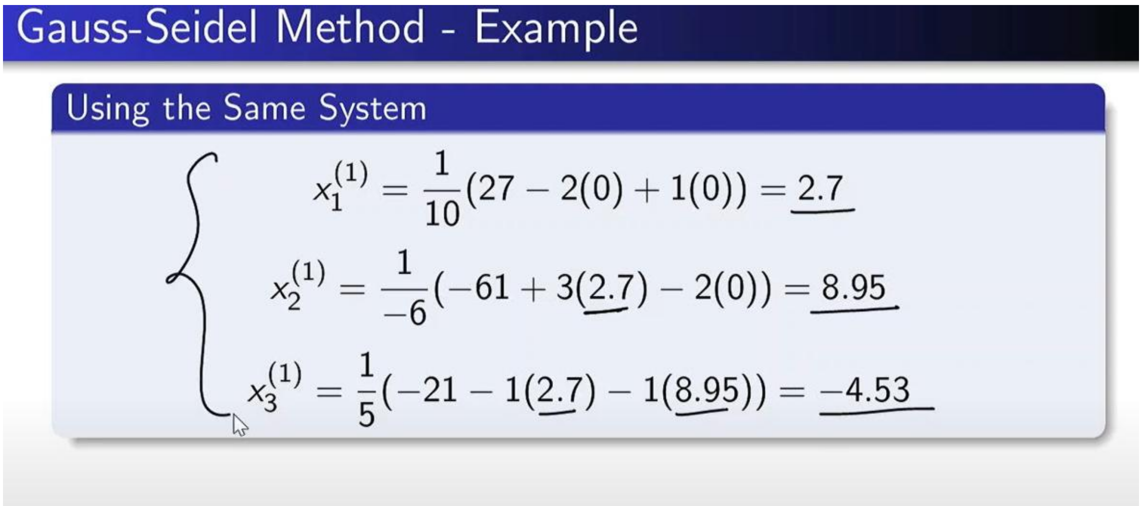

So, now we can solve this thing in one step or two steps, but if we have to solve this with more iterations, then we will have to make a computer program for that, which we will do in Python. So, this is our Gauss Seidel. Now look, we have seen this method. So, we have to see what is the convergence of this—meaning is it giving fast convergence or slow convergence, or how much convergence is it giving? So, we have to check.

So, we saw that the convergence of this method depends on the convergence matrix, its spectrum radius. If we see that if its spectrum radius is less than one, then we can say that this method converges. So, we have to check what is the convergence. For this, we will have to look into the analysis a little bit. So, let's see what it means.

See, I am doing this—the convergence analysis of Gauss-Jacobi method (also called Gauss-Jacobi). So, let's see what to do with it. Now see, we had a system A x =. We had to solve this. The matrix A was diagonally dominant. Now see, any matrix A can always be written like this: L plus D plus this, L would be—take all the elements as zero—and here a21, a31, a32; in the last an1, an2 will go on like this D.

Okay, what would D be? Only diagonal elements, a11, a22and all these zero, and U is upper and lower—zero everywhere, zero below as well. What will happen here, a12, a13, a1n, here a23, a2n. In this way, the non-zero elements that we have will come here, which is above the main diagonal. So, always in this matrix, if I sum all three of them, then we will have matrix A.

So, what did we do? I converted this matrix. So, this matrix became like this: L plus D plus U, X is equal to b. This is what we have. Now see, what is happening in Gauss Jacobi? These diagonal elements are being used a lot because division is happening through this. So, what did we do? I kept D here, and I took it and all of them here—L plus U, okay, minus, okay. So, what did I do? This is x, sorry, this is also to be written as minus (L+ U)X+b. The minus L plus U was picked up by me and brought to the right side and plus b.

Now D is our diagonal dominant matrix. It means that the diagonal element is there, and there is no zero. So, if there is no zero, then the D inverse will exist, then the D inverse exists. So, I can write X like this: minus D inverse, L+U, X plus D inverse b. This becomes ours. Okay, so I wrote it like this. Now I will name this equation as one. So, this x will be its solution, and it will be formed in this way. Now what have I done? I have made this equation a little like this.

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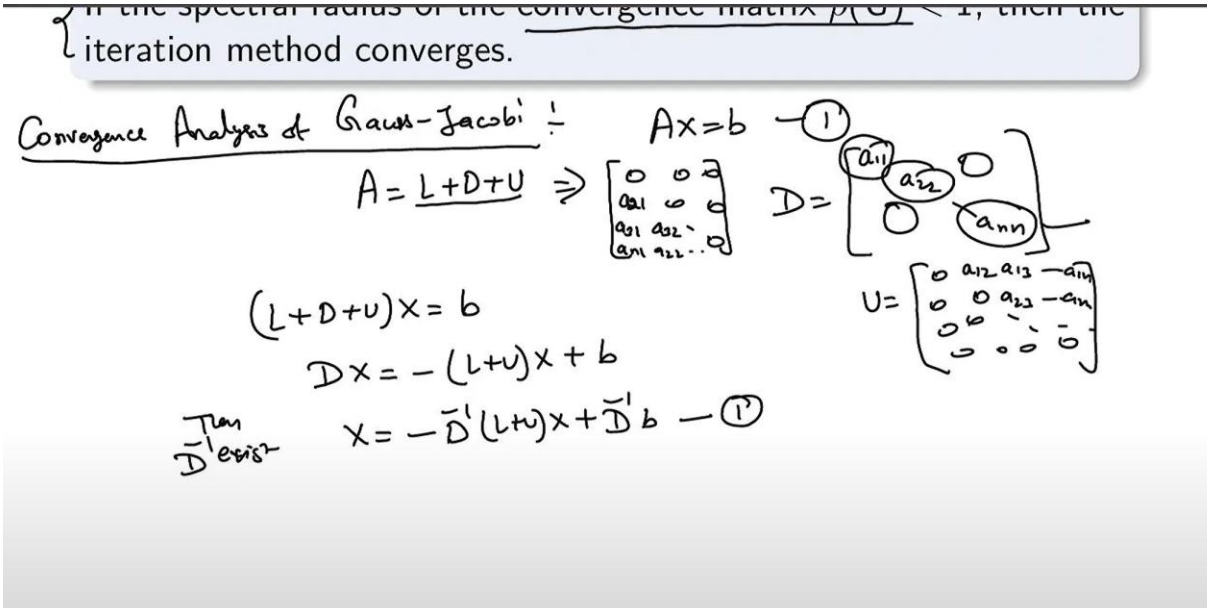

Now look, if we want to convert it into iterative form, then I will convert it into iterative form. Let's take x n+1`or k+1, take k no problem, because it is same, so we can take it. x k + 1 = - D inverse L + U x k plus D inverse b. Now what should we do? Subtract one minus two. What does this become? x - x k + 1 is equal to—now this will become common L+U. Now here this comes x minus x k, and this is the same. So, this cancels out. So, this thing is left with us.

Okay, so we have written this thing like this. Now what are we doing? If we see, what is this thing? We are minusing the value at approximants value from the solution at k+1 step. So, if we see, we have got an error at the k+1 iteration. We have got an error at the kth iteration. Okay, so this error that has come—we have got an error—it can be positive or negative, because we have not taken the modulus yet.

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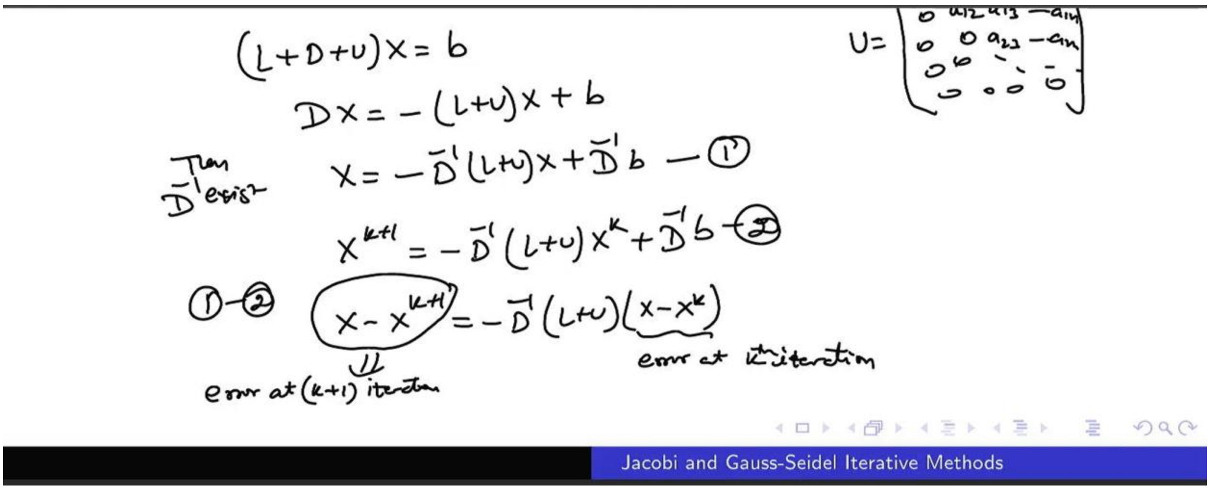

Okay, now if there is an error in this, then what is happening in this? See, there is a matrix in this because these are vectors. So, if there are vectors, then a matrix is coming here. Minus D Inverse L plus U. Okay, so I name this matrix G. Okay, so if I name it, then I have written it like this. Now what do I have to do? I will use this. So, see in which form am I writing it? I have written it like this, e k+1 is equal to G e k. Okay, now I am e k, starting from zero—we can write it like this: G square, e k-1 can go on like this also. So, if you see e k, I can write it like this also with the same iteration, right? Because look, whatever will come from it—e1 will come G e0, e2 will come G e1 that will come G square e0, right? In this way, the square, cube will keep on going like this, so we will get this. So, I have e k+1 becomes G power k e0 like this.

Now we want that the initial one that we have is like this. So, in this we want that the error at kth step and the error k+1 step should be less than that. If it is less than that, then G should have some value that is less than that. Now G is a matrix. So, what should we do with the matrix so that we get something that we can know that our convergence is happening and what will be its contribution to G, right?

So, for this, we have to check some of these things. So, if we see, we can write it like this also. So now we have to do it a little mathematically. So, what will we do mathematically? Now you see that we have this system. Okay, so we wrote it like this or we converted it and wrote it like this. Now, G is a matrix, and we saw that minus D inverse L+ U is this. So, the matrix will be a well-defined invertible matrix—an invertible matrix—and G is basically a non-singular matrix.

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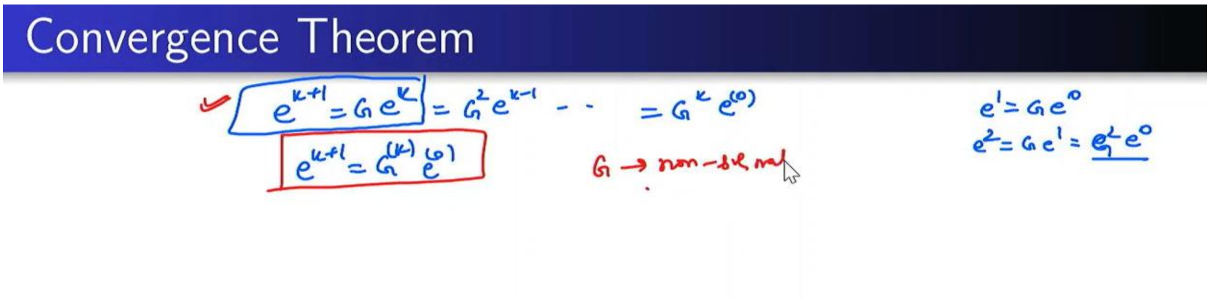

So, if we have a non-singular matrix, then we assume that it has n eigenvalues, okay, and this means that the eigenvectors are linearly independent. So, we assume that G is having n eigenvalues and correspondingly n linearly independent eigenvectors. And these eigenvectors, if we look at it, will be in the vector space Rn, okay.

So now, see, what is e_0? It is a vector. I can write it as a linear combination, or I can write it directly like this. Then, if there are eigenvectors, then I take their eigenvectors—suppose v1, v2, vn , I have taken eigenvectors like this—these ones are okay. So now these are eigenvectors. These become basis of Rn. So if I pick any vector e_0 from that Rn, I can write it as a linear combination uniquely, cn vn, okay.

So that means from here we will have n eigenvectors which will be linearly independent. They will become basis of Rn, which we know. So, if it is a basis, then if we pick any element from it—then what is e_0? That too is an n-dimensional vector. We can write it in a way—we can write it in linear combination. So I wrote it like this.

Now I wrote here G, I pre-multiplied it, so it came out c1 Gv1, c2 Gv2, cn Gvncame out. Now see, we can write it like this c1, now what is this, what is G1v1 is lambda1 v1 because this is eigenvector corresponding to any eigenvalue, so let's name it λ1 . So, I can write this λ1 v1 plus c2 λ v2, cn λn vn, I wrote it like this, okay. Now we have it. Now what did I do? I pre-multiplied it one more time. So what will it become? $c_1\lambda_1^2v_1$, $c_2\lambda_2^2v_2$, $c_a\lambda_a^2v_a$—it will become like this. So, we will keep going like this.

So you know what we will have with G k e0—let me do the summation. It will become $\sum c_i\lambda_i^kv_i$ from 1 to n. This will become what we have, okay. So now these values have come. Now what we have to do is that if we want, then I just write it like this, okay. So I write $\lambda_i^kc_iv_i$, okay.

Now see, now what I want is that if we have $k \to \infty$, then what has it become with us? If we see, this is e k, so as $k \to \infty$, which is e k should be going to zero—the error is zero. If it goes towards zero, only then we will say that it is converging.

So if this—because we said that our system is diagonally dominant—it is a sufficient condition. It is a sufficient condition, and it is diagonally dominant. Then, for sure, it will converge. So we applied the method from here, and we came to know that if we make $A \to \infty$, then e k will tend to zero.

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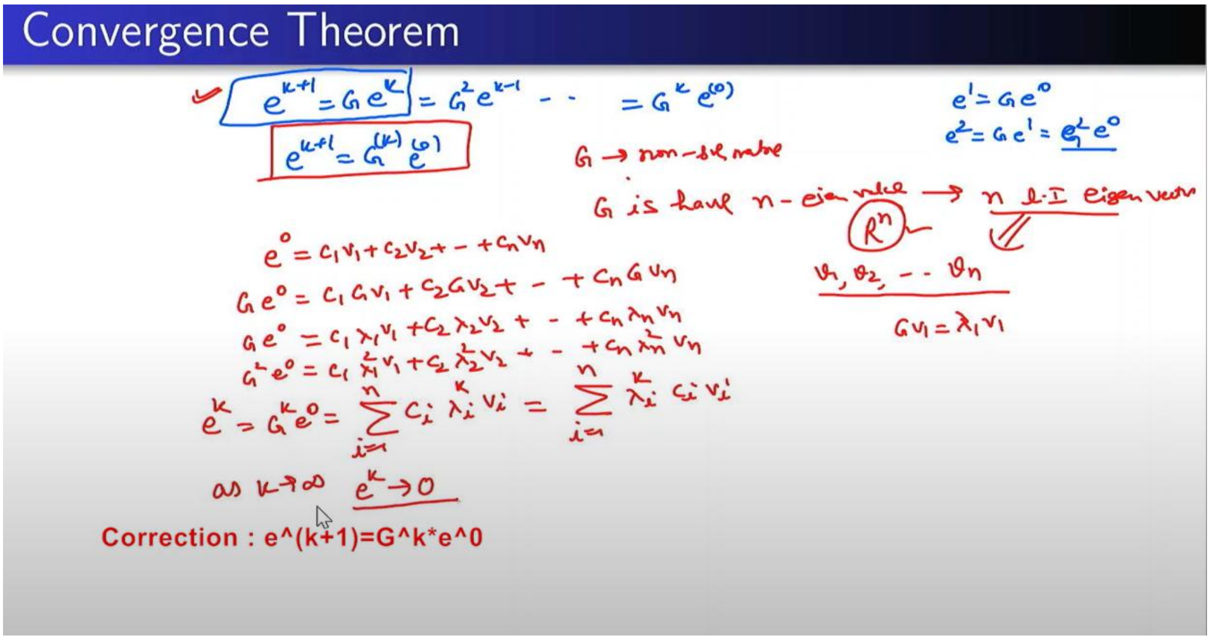

If e k is tending to zero, then what would be happening here is that it would be going towards zero. So in this, if we assume that the eigenvalues—its values are very small, less than one—so if I say here that if all the values, their modulus value is less than one, suppose, then what will happen? I will take their power, 1, 2, 3, it will become smaller and it will converge towards zero.

So, if I say from here that this is its largest eigenvalue—we know how to write the largest eigenvalue, rho of G, spectral radius—if this is the largest eigenvalue, okay—if its value is also less than one, then it means all the eigenvalues will be less than one in magnitude. So, what will happen is that, if it is converging from zero, then this must have definitely happened. This is what will happen—only then it will converge towards the solution, okay.

So, from here we can say that if this method is converging, then it is necessary that the rho of G is the less then 1, right? Rho means the spectral radius—there is no need to take its modulus. It become a less than one, then we can say that, if this method is converging, then it is a necessary condition that the convergence matrix—okay—its eigenvalue will be the largest eigenvalue of this G. Its largest eigenvalue—its magnitude—will also be less than one, okay.

So, if this happens, then we can say that, as we keep taking its power, that value will keep on decreasing and trending towards zero. So, this condition is a necessary condition. What was the sufficient condition? Diagonally dominant matrix. And this is our necessary condition. If we see that, if it is less than one, then it will also converge in it. So, in this, we say that it is necessary as well as sufficient.

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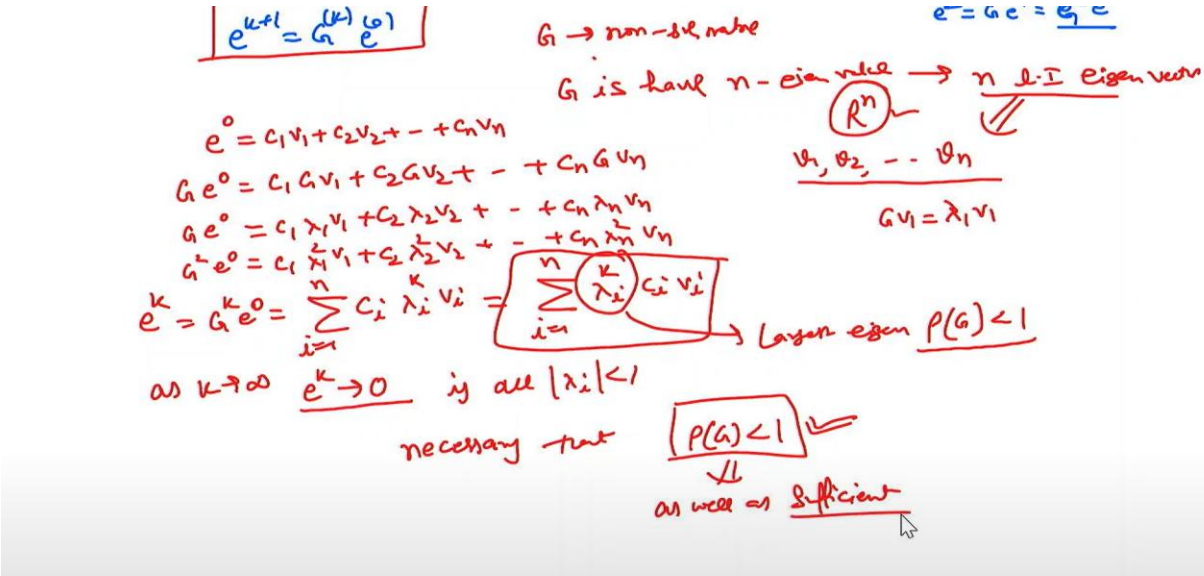

Now we have to see how fast it will be. So now, the method that we have—Gauss-Seidel, Gauss-Jacobi—the rate of convergence of these, we had told you above. So, the rate of convergence is minus log rho of G, okay. Rho of G means the largest eigenvalue, that is less than one. So, if we take the log of a number less than one, it comes in negative, so we took the negative. So, this is the rate of convergence, okay.

And how much more will this rate of convergence be? If the eigenvalues are very small, if they are close to one, then these values will go close to one. If they are less than, if it will be very small, then these values will become very high values. So, the rate of convergence will increase a lot. So, this matrix is ours in this case—we have the convergence matrix, okay. So, we have to remember the matrix because its role is very important.

In this case, we have the convergence matrix in Gauss-Seidel. Now, we can do it in the same way. Now, what will be the convergence matrix in Gauss-Seidel? Okay, so what will be the convergence matrix in Gauss-Seidel?

Now, see, in Gauss-Seidel and Gauss-Jacobi, we know in which way we can say that if you see between the two in Gauss-Seidel, then this was happening. So, it means that k+1 values, this j is less than i came will remain with us on the left, okay. So now, let's see what happens in this—what will happen in Gauss-Seidel?

So, if you see in Gauss-Seidel, then our iterative method will be formed—this is for sure. We have L plus D x equals b. This is the system. Now, we have seen that k+1 is acting on L and D and not on U. So, we will do this as L plus D minus this, okay. And I can write it like this: b minus Ux.

Now, L plus D has diagonal elements which are non-zero, so this matrix will be invertible because it has become a lower triangular matrix—the diagonal elements are non-zero—so this L plus D become invertible. So, if this becomes invertible, then I can write it as x as L plus D whole inverse b minus L plus D inverse Ux, okay.

So, in this case, let's take this as equation number one. Now, I have made it in k+1 form, so this is L plus D inverse b minus the inverse of L plus D , Ux k. This is what we have.

Now, I will do the same thing. I will do 1 minus 2, okay. So, just as I did 1 minus 2, this is what we have—e k+1 is equals two. Now this cancels out. So, it will become minus the inverse of (L plus D) and this will become e to the power k.

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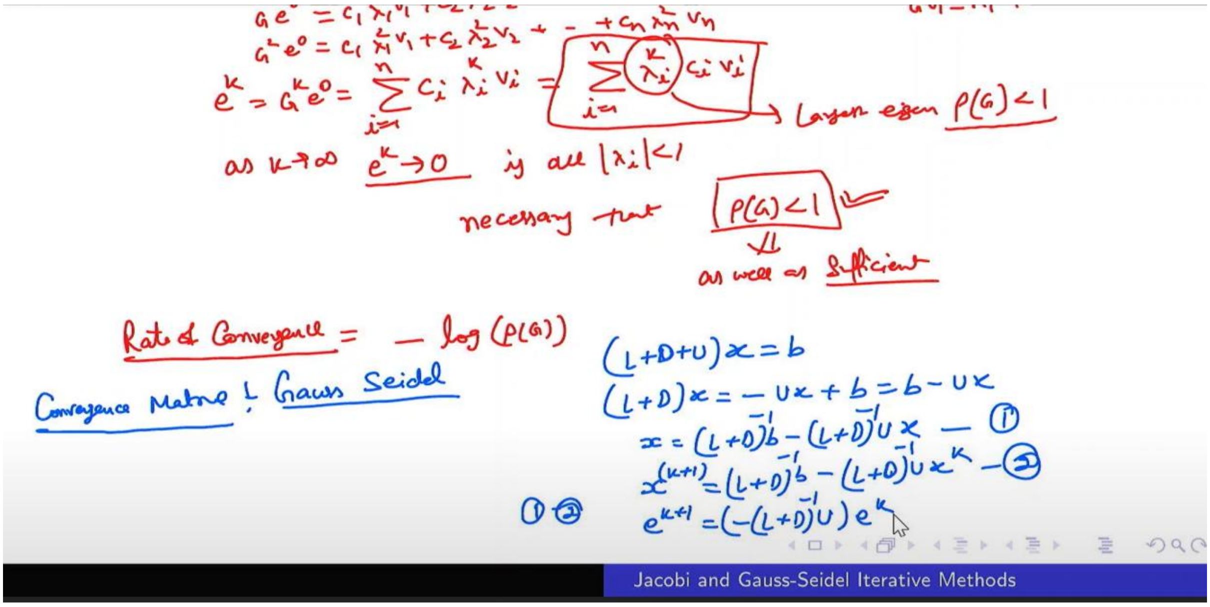

So, if we look at this, our convergence matrix—this convergence matrix—let's name it H—so it has changed, right? This convergence matrix of ours has become this in this case. This is called the convergence matrix. And all the other things will be same, if we find its spectral radius, that is less than one.

And in this case, we will say that the solution will exist and converge, and the rate of convergence will come out like this—the convergence matrix is related. So, this means that now let's see how to find out this matrix. Let's see it—I will try it.

Now, see, this is a 3x3 matrix—this is a matrix. So, A which is ours is 4, 1, 1, 1, 5, 2, 1, 2, 3—this matrix, x_1 x_2 x_3, 2, -6, -4. It is. Now, this matrix is diagonally dominant. It is diagonally dominant. We can see it from here. That this is greater than 2—this is greater than

3. This is greater than 3 is equal to here. So, this is diagonally dominant but not strictly because 3 is coming and equal to 3, okay but not strictly.

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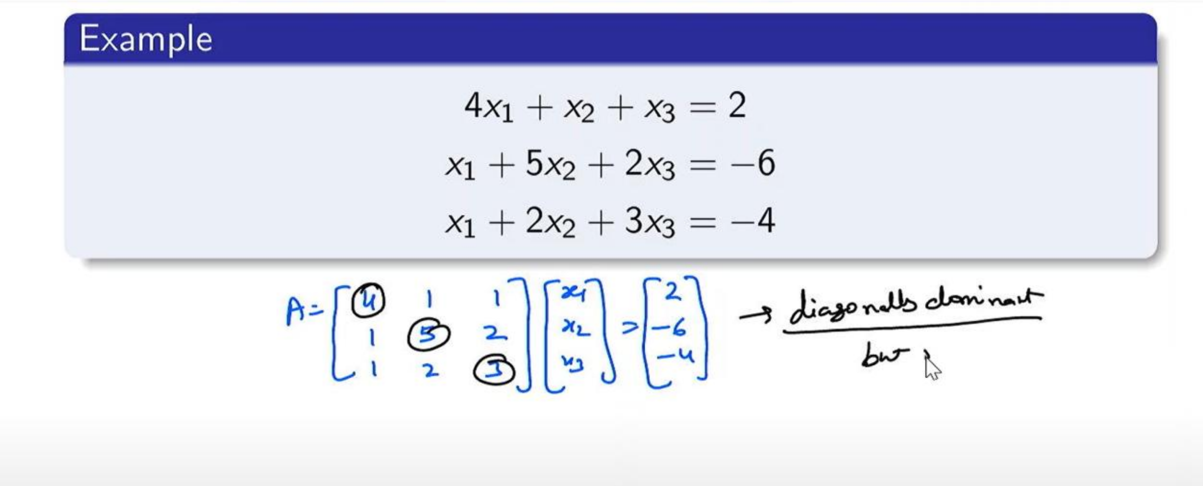

But here, it is strict. Here, it is strict, okay. So, it will converge. It is not coming zero either. But now I have made this matrix like this—see: 0, 0, 0, 1, 1, 2. This is our L which becomes D—ours becomes 4 ,5, 3, okay. I will put zeros everywhere. And U we have 0, 0, 0, 1, 1, 2, 0 0. This is what we have.

Okay, so now we have to see what will happen in Gauss-Jacobi. Now, the convergence matrix that we had there was minus D inverse, (L + U). This was. Now, look—D is the diagonal element. It is the inverse of the diagonal matrix—we know that. The inverse of the diagonal elements. So, this will become minus 1 by 4, 1/5, 1/3. And everything will remain zero only. The diagonal element matrix will remain diagonal, and we had taken this L+U on the right side. So we have 0, 0, 0, 1, 1, 2, 1, 1, 2. Okay, it is a symmetric matrix. So from here, this is what we have. Now what did we do? We will multiply these two with each other, okay, whatever matrix we have now. If I use this, then this is what we have: 1 by 5. Whatever we do with this minus, this is what we have.

See how much zero it will be. We did this, so this 1 by 5 came. I multiplied it with this, so 1 by 3 came. Then after that, I did this, so 1 by 4 came here. From here, I did 1, zero zero or zero came. Then I did this, so 2 by 3 came. Then I did this, so 1 by f came. I did this, so 2 by 5 came. And I will do this, then zero came. This is the matrix we have, okay. So this matrix is G matrix. So this matrix which is formed, now we will call it convergence matrix. Okay, so this is the convergence matrix now.

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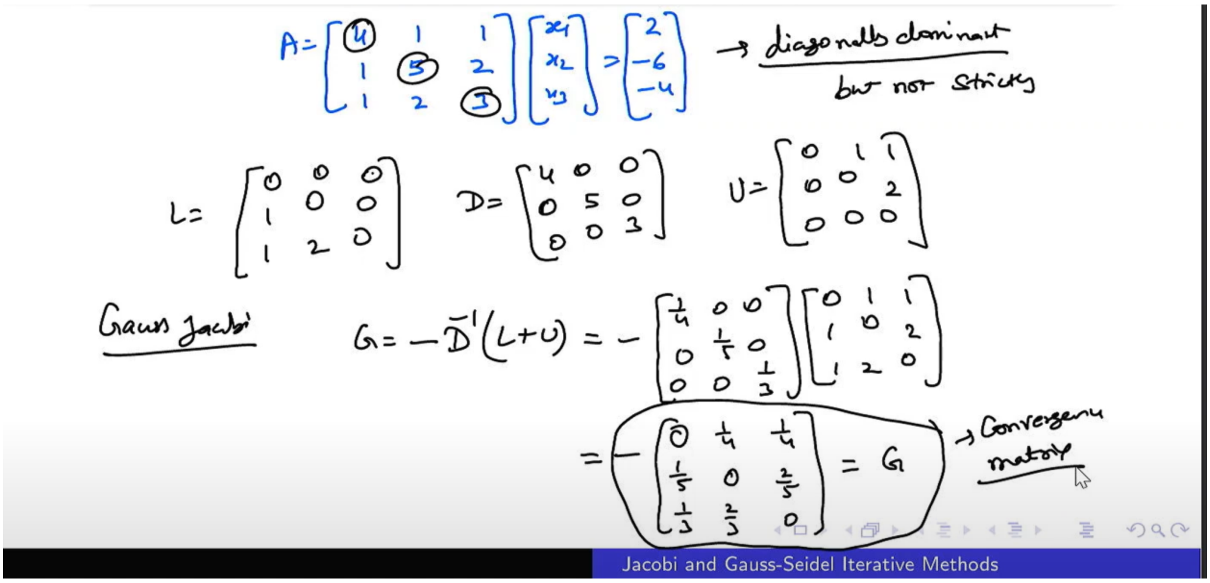

Convergence matrix, in this case, the Gauss Jacobi which was formed in Gauss Seidel is easy. Now in the same way, we will try to convert it into Gauss seidel. So look at Gauss Seidel, what is happening in it? What did I do in Gauss Seidel? I represented it with H. The inverse of P + L inverse this. So what will happen in this case? This will become minus 4, 5, 3. After that,1, 1, 2, 0, 0, 0. Its inverse — and 1, 1, 2 — comes. We will take its inverse, we will multiply it by it. So this matrix which will be formed will become our convergence matrix in this case. So the convergence matrix which is this, we have solved it and we will get the solution.

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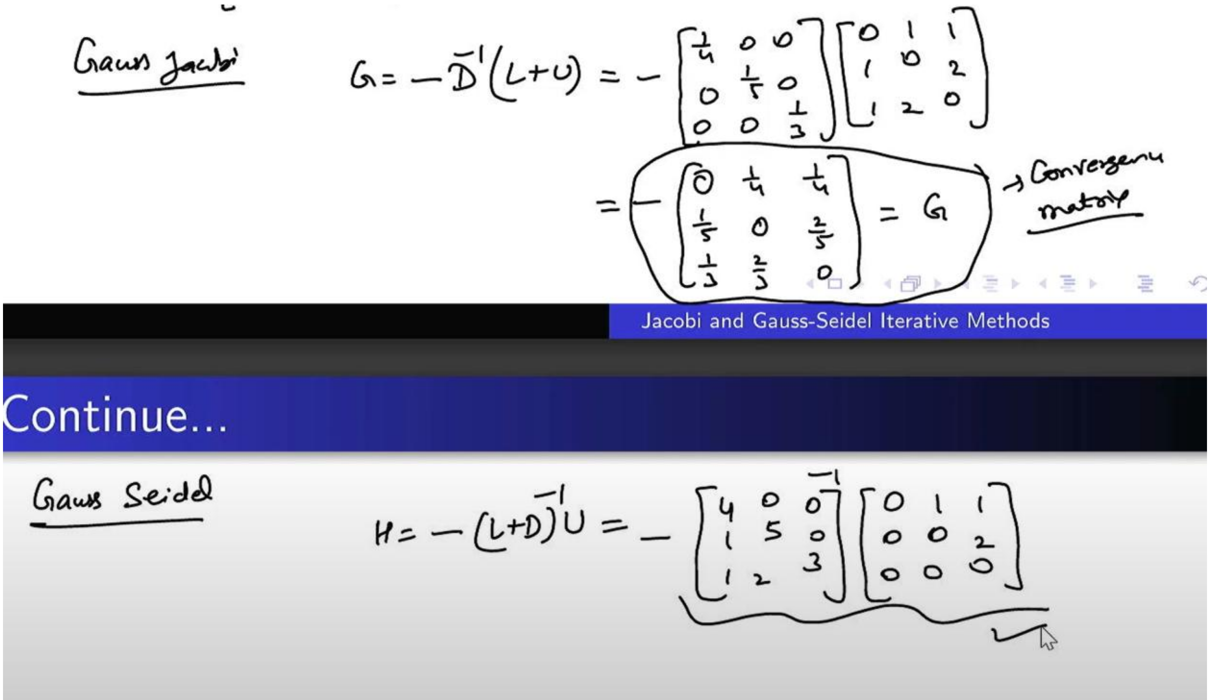

So in the same way, we can do it for many matrices. Now, like we have another example. I have taken this example. Now I will see. Let me change it a little. Now we will solve it with the program and we will see that — brother — this program, if we are doing this, then what convergence are we getting? Is there divergence?

Now what did we do? Let me take another example. So what did we do in this example? Let's take another example. Now suppose I took a system: minus 4 x1 plus x2 plus 10 x3, is equal to 21 y came out. Then we got: 5 x1 minus x2 plus x3, is equal 14. Then we got: 2 x1 plus 8 x2 minus x3 equals to minus 7 came out. See what I have done.

Now we have to solve this system with the help of Gauss Jacobi and Gauss Seidel. This is the work we have to do. So now, if we see in this case, the matrix that will be formed will be this: minus 4, 1, 10, 5, -1, 1, 2, 8, -1 — this became — and the right side vector is this. Now see, in this case, if we see, then there is a problem. This is not a diagonal dominant matrix because minus 4 is less than 11. Here also, the diagonal element is very small. It is also much smaller than the other sum. So this matrix is not diagonally dominant. So in this case, there is no guarantee that our method — Gauss Seidel or Gauss Jacobi — will converge or not. The condition is sufficient, and it is not necessary. So, sufficient means that if it converges, it will converge. If it does not, anything can happen. It may converge, or it may not. So now, in this case, what do we do? If we solve this, we will see whether it is converging or not. But what are we doing? I looked and tried to understand it a little. So I saw that this 10 here is greater than these. Here, 8 is greater than both of these, and 5 here is greater than both of these. So I did a little swapping. I did the permutation — of which? — permutation of equations.

So I wrote this system like this: 5 x1 – x2 + x3 = 14 , okay? And the second one I wrote: 2 x1 + 8 x2 - x3 = -7. And I wrote: -4 x1 + x2 + x3 = 21. I wrote this. Now after writing this, we saw that our matrix has become like this. See: 5, -1, 1, 2, 8, -1, -4, 1, 10 — this is what we have.

Now look — this matrix: 5 is greater than 2, 8, which is greater than 2+1, 3 , 10 is greater than 4+1, 5. So this has strictly diagonally dominant. Okay. So by manipulating the same system a little bit, we converted it into diagonal dominant. The solution will remain the same, right? Right. So in the solution, we know that if we are applying row operations, interchanging them, then there will be no problem. So the solution is always the same.

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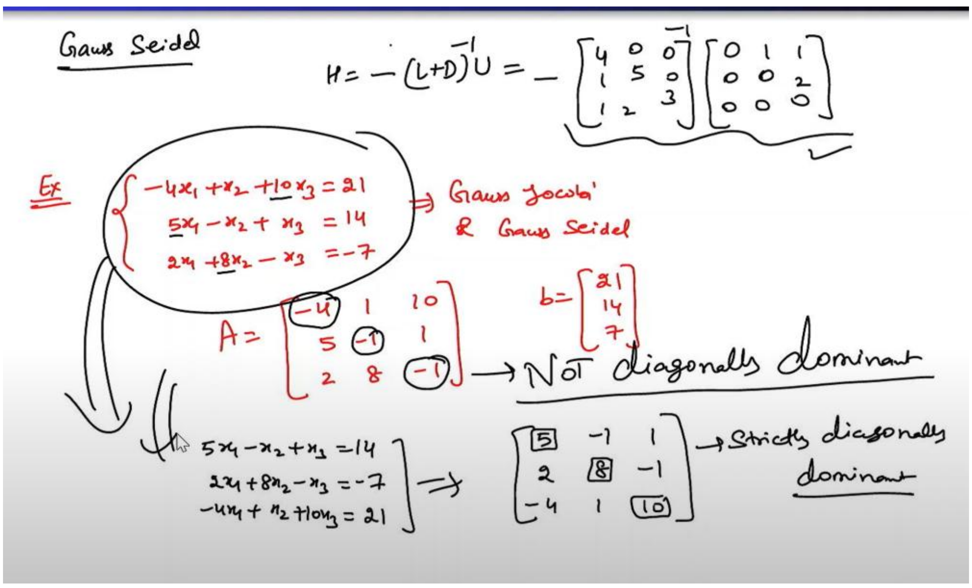

So now, if we had applied gauss jacobi earlier, then we would have known — it was not — that the solution is converging or not. But we applied our row operations on it, and we came to know that now this system has become strictly diagonal dominant, and now it will definitely converge. And we will see after solving both of these that it is converging. So what is the rate of convergence of this thing? We will also check that. The rate of convergence is a diagonal dominant matrix, and if it is not, then what effect is it having? We will pay attention to that. So our rate of convergence is this G or H — whatever it is — so the convergence matrix gives, where G is — we have written it here — we can take G. That is the convergence matrix.

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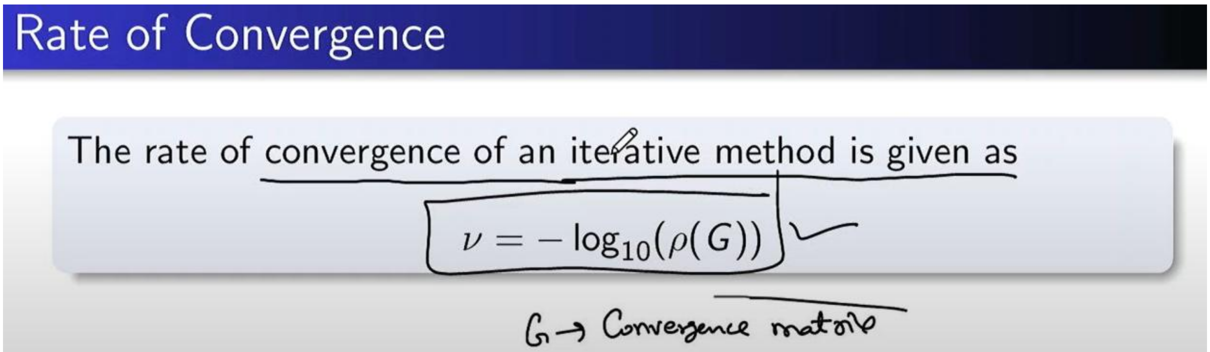

So with the help of the convergence matrix, we can see whether this is converging or not.In this way, we can make many solutions, many systems which we can solve with the help of Gauss Jacobi or Gauss Seidel, right? We can do all these things easily with the help of methods.

So today we discussed Gauss Jacobi and Gauss Seidel — what is their rate of convergence, and what is the convergence matrix. We discussed all that. And we will do the program related to this — a Python program — in the next lecture. So I hope you have understood this lecture. And we will do the remaining part of it in the next lecture. So thank you very much for watching this lecture.