SCIENTIFIC COMPUTING USING PYTHON

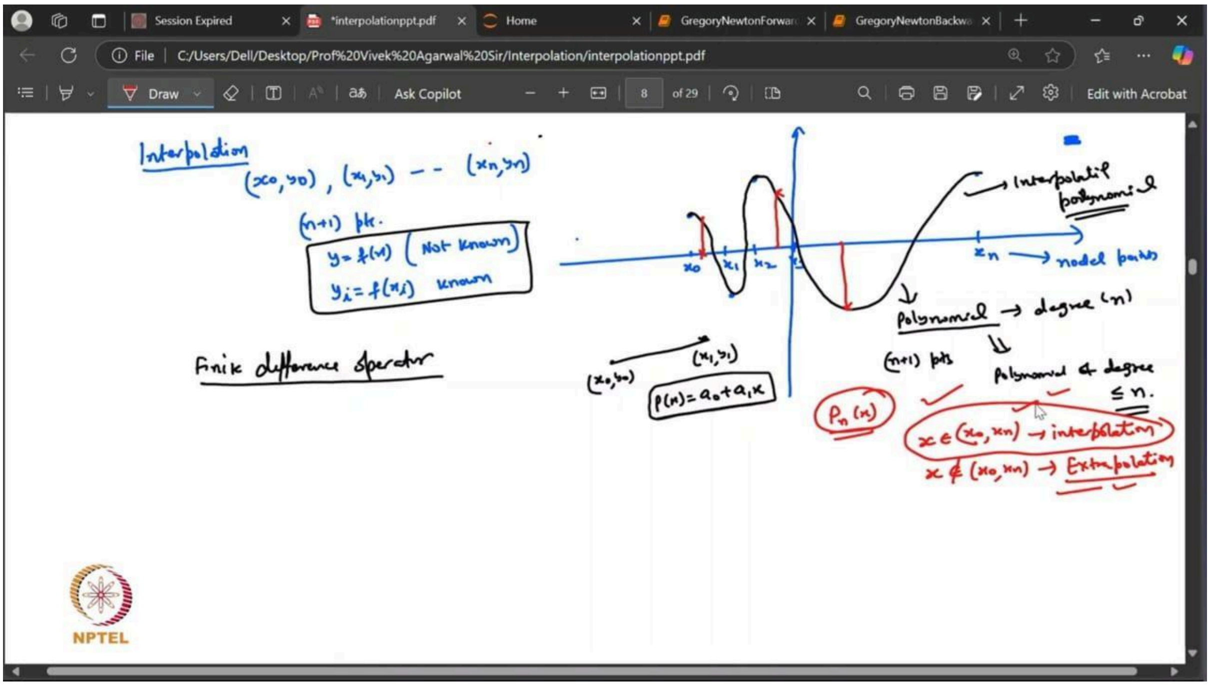
Professor Vivek Kumar Aggarwal & Professor Mani Mehra

Department of Mathematics

Indian Institute of Technology, Delhi

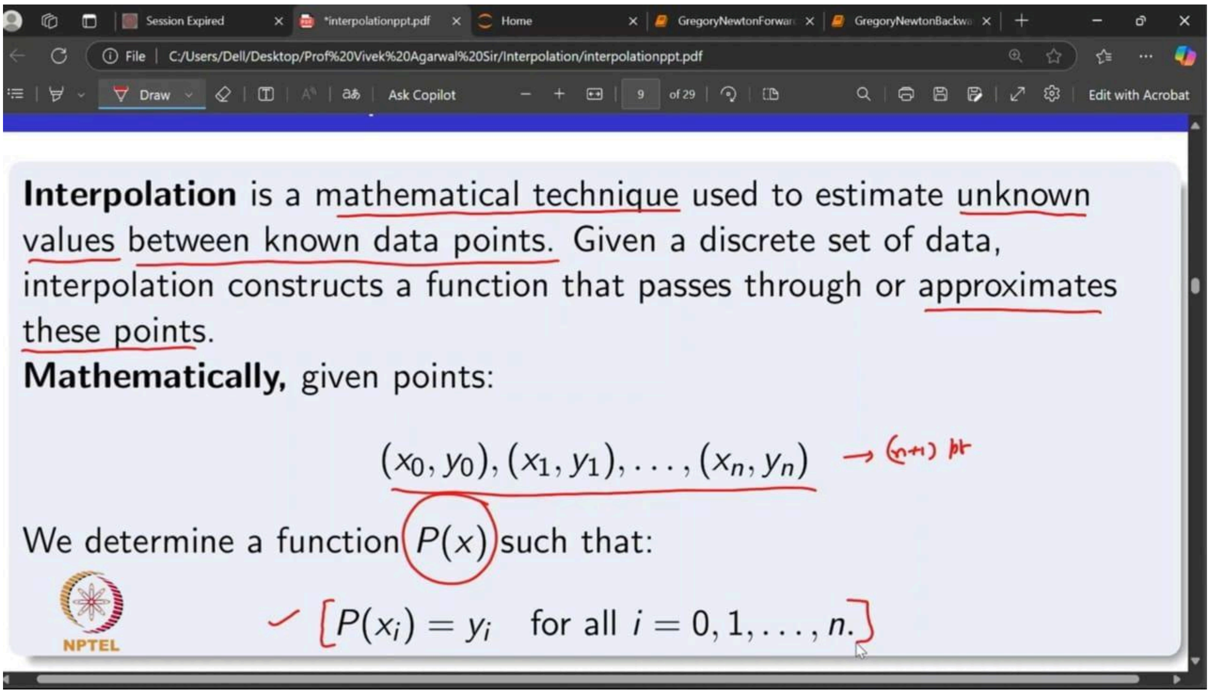
Lecture No. 20

Welcome everyone to Scientific Computing Using Python. So in the last lecture, we discussed the difference operator. So today we will use it to find out how we can use interpolation. If we have to define a Polynomial, then how can we use it. So let's get started. Today we will actually use the difference operator that we discussed to do the interpolation. So what is in interpolation, suppose we have data points. We are given data points in this form, x0,y0, x one, y one xn,yn . So these are its coordinates and this is the total n plus 1 points. Now what will we have? Sometimes, if our function is y is equal to f x, we don't know, okay, not known. So if we don't know this function, we just know that the values of this function, these nodal points, are known. So, the x i's x zero, this is my x one, this is my x two, this is x 3 up to x n. If x is a, then these points which are given, we call them nodal points. The nodal points which we will have will be given now what do we have to do, now suppose we have given some values on this, some value on y, I have given a value here, suppose it is given here, okay, given on y, so now we know the values of the function only for these nodal values and we do not know any other value, so what will we do, we will try to construct such a polynomial that this polynomial, all these points, their points, should be brought on this polynomial, so we need such a polynomial now what is a polynomial, we have a n plus 1 points, so it will be a polynomial, its degree will be maximum degree will be n, so what do we get, in this a polynomial of degree less equal to n, okay, there are n plus 1 points and a polynomial of degree n, why, like we have two points, these are any two points, so this x zero y zero is one, point one x one and y one, so we know So we can draw a line in it and the line is a polynomial, we can write it like this, a0 plus a1 x, there are two points and the polynomial that we can draw, in this the maximum is a linear polynomial, so we can define a quadratic if three points, so if there are n plus 1 points, then the maximum polynomial of n th degree, we can make such a polynomial that passes through all these points, so this is called interpolating polynomial. So in this we do not know the function but we know the value because we know that if we know the function, then we can write that function as in terms of the polynomial by Taylor expansion. So what do we have, there are two types of data, that is, we have data, so what is in this, the function is unknown, so we do not know the values, we do not know the function, we know the values of the function, so what do we have to do in it, let's see how can we fit this polynomial to a polynomial that passes through all these points, this is the condition, so now what have we done Till now we had defined the final difference operator, so if we have defined the final difference operator, then first we use it to find out how we can interpolate, interpolation interpolating polynomial and after that what will be the advantage of interpolation that if we have such a polynomial, I have named it p n x. Now if someone tells us that the values here, the first value was here, the second value was here, then if someone tells us that we have to calculate the value at this point or predict the value at this point or predict the value at this point, then we can predict these values with its help, the value of x, which will be ours, from where will it take x from x zero to x n, so if this value of x is given from x zero to x n and we will find x somewhere in this interval, then we call it interpolation and if x goes outside it, okay, it does not belong to this, then we call it Extrapolation: So what do we have to do? Here we have to do interpolation. It means that we need the value of x somewhere and if we have to find out the value of that x, then we will need a polynomial, i.e. interpolate polynomial and with its help we can find it out. So this is the work that we do because nowadays we have data and if we have to interpret the data and use it to make predictions, then we can do this type of interpolation, then we can also do extrapolation and after that we can also make predictions. What would be the meaning of prediction? Extrapolation: On the basis of the data that we have been given, suppose we have been given the data till yesterday, we can see whether it is going to rain today or not, how will the oil prices fluctuate today, what will they be tomorrow, so all these predictions will come, so we can use it in that. If we have any data and there are values missing somewhere in that data, then if we approximate the missing data, then if we have to take the definition of it, then we should do the definition.



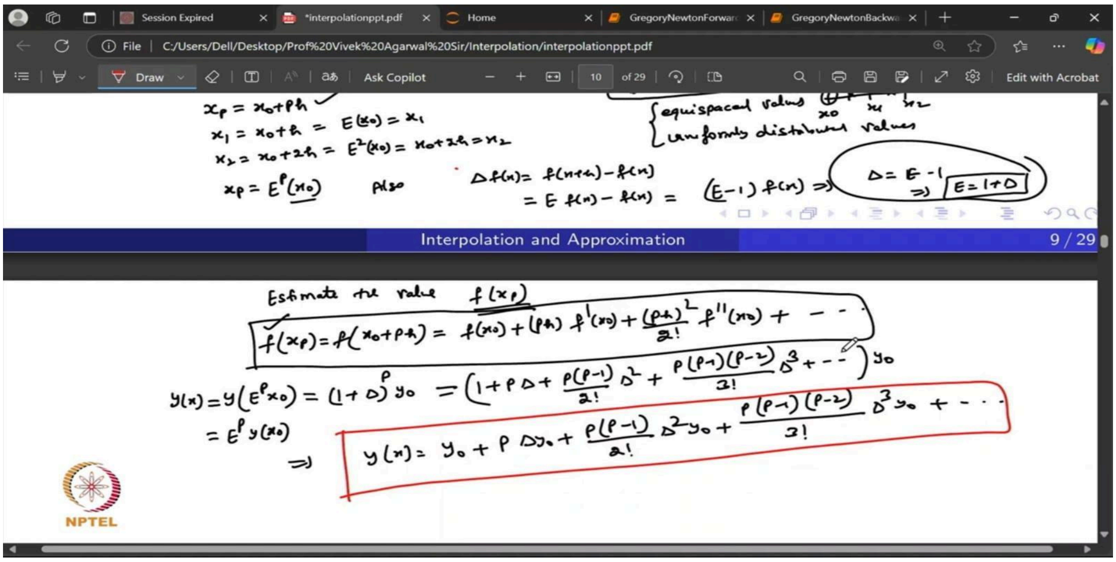
(refer slide time:8:21)

So this is the definition of interpolation. So interpolation is a mathematical technique used to estimate unknown values between the known-data points. So known-data points were our x0, y0, x1, y1, and so the total is n plus 1 points given a discrete set of data or our data is a discrete set. So, the interpolation construct function passes through and approximates these points. So, we should pass it through it. This is the point and this condition should be there here. If we are representing it by a polynomial, approximating it, then the condition of the polynomial should come here. So, if we do this, then we will say that we are going to find the polynomial as interpolating polynomial and from that we are going to find the values of x between these values. So, we will name it interpolation. So, now we have to see how to do it because in what way we can do it. Okay, and how much accuracy will we get in doing this? How accurate will our predictions be?



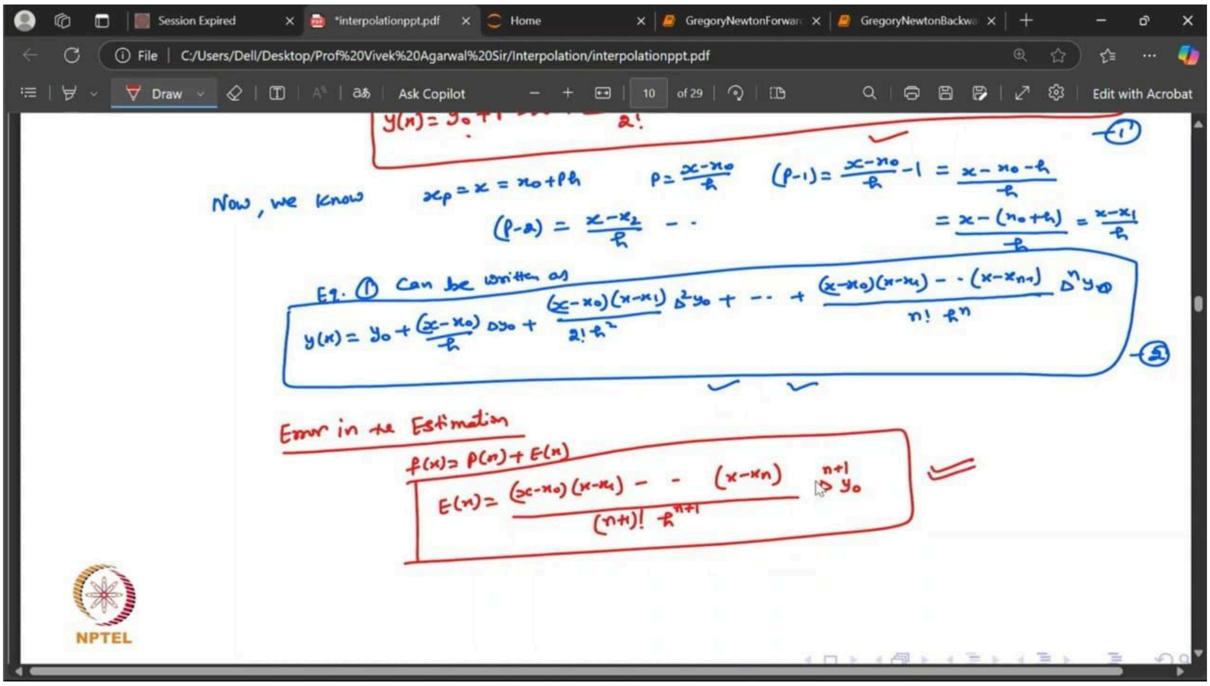
(refer slide time: 9:41)

So, we have to discuss all these things. So, in this case, the first thing that we do is we use the method that is Newton Gregory's forward difference formula. So, using this means what are we doing? We will have to use this somewhere. So, this formula that comes, this formula, basically if we see, what do we have in this case, the values that we have are x zero,y zero, x one, y one such values given or this forward difference operator of ours is useful only when suppose we have to interpolate the value of x, so if our x is lying somewhere here, it means that this is x zero, this is x one, this is x two, it is lying somewhere here, it is lying somewhere here, x is lying somewhere here, it is lying somewhere in the beginning, so, to apply this, we apply forward difference. We need to have a condition that if I look at the space in any nodal point, whatever be the x i minus x i minus 1, it should always be constant. So, this type of values we have are called equispaced values or uniformly distributed values or nodal points. So, we can use it for this. For example, we have data and the data is of suppose temperature, so we have fixed in it that our temperature throughout the day and after every 15 minutes or after 30 minutes after half an hour and we will record the observation, so what will happen at that time in which the observation is being recorded, the spacing between the two observations will be 30 minutes and it should be constant, so that type of data we will call that we have equispaced data. So, our Newton Gregory formula is specially for that. When we have data, it will be equispaced now, see, we have to use it, so how to use it? So the difference table that we had created, let us first write the formula that suppose we need to find out the values of interpolate at some of these points which are in some starting value itself, so for this we have a formula, so what do we need to do, we need to find out the value of x p at some point, okay, so x p, I have written x zero plus p h, so if it is p one then it will come x zero plus one h, so it will become x one, if it is p 2 then it will come x zero plus 2 h x 2 ,okay, so what does it mean? This is shifting us, we have put the value of p in x p, so we have been shifted here, so it has been shifted here, so it means that I can write this from here that I did e off one at x not , so I got x one, here what did I do e square x, so I got two shifting plus 2 h, so I got x 2, so if I write this, then I can write my x p as e to the power p of x not Now see, x not is being used, it means the starting value is being used and we had seen in the finite difference table that the first value y not, its difference was being used repeatedly in first order, second order, third order, etc. in the forward difference table. So we can take it like this. Now we know that we found out another relation also. Now the value of p can be anything. And we also know that we had done that day that if the forward operator is f x, then f of x plus h minus f of x was defined like this. Then I had also written it like this that this e of f x minus f x, right, can we write e minus 1 f x like this and from there we came to know that we can write it like this. So from here we came to know that the delta is one minus sorry e minus 1 and what did we do from here, we have to calculate it, so the value of e that will come to us is one plus this. So now we have the value given to us. Someone tells us to find estimate. The value of f at x is calculated when we don't know it. The value which we know is only known at these points. So if we find out , then what will we do? What do I want to do? I want to find out x p. Look at x. What can I write? x not plus p h. Okay, there is a function which we don't know right now. If we approximate its value, Suppose our function is continuous and its derivative exists it means suppose I can write this function as a Taylor series. So I will write it like this. So I will write it like this f of x not plus p h f dash x not plus p h whole square by factorial two double dash f x not. Like this, we will calculate it. Okay, this is ours. In this way, now we know that this is possible only when we have the values of the function. The function is known. So if we don't have the function known, then I will use this and see what I can write. So now see, we don't know the value of the function. We only know the values. x not x one x two in this so I will use this now so look what I have to do is find out e of p this is suppose y of x so I can write it like this what is e it is 1 plus delta to the power of p okay so I will find out on this okay so now what we have to do is I will write x not okay because x is not so now what we have to do is I will expand this by using the binomial expansion so what will happen after doing binomial expansion 1 plus p delta plus p p minus one by two factorial delta square plus p p minus 1 p minus two by 3 factorial delta cube and it will come out y not our values, that is, delta will be applied on what because we need this right this because x was this I need y and from here this y e p x not okay and what will that be so if I correct it a little bit so it will become my plus to the power of p y okay so we wrote e p after that We put the value and what will this value be, it will depend on the y not. So we can use it in this way. If this is a linear operator, then I can write it like this, e p y not. So if we wrote it as, then we expanded it and we got this. Now from here we get to know that our this, we can write it y like this, y not plus p delta y not plus p into p minus one delta square y not by 2 factorial plus p into p minus one p minus two by three factorial delta cube y not and so on. Alright and so it will go on like this because it is a binary expansion. So we get a series.



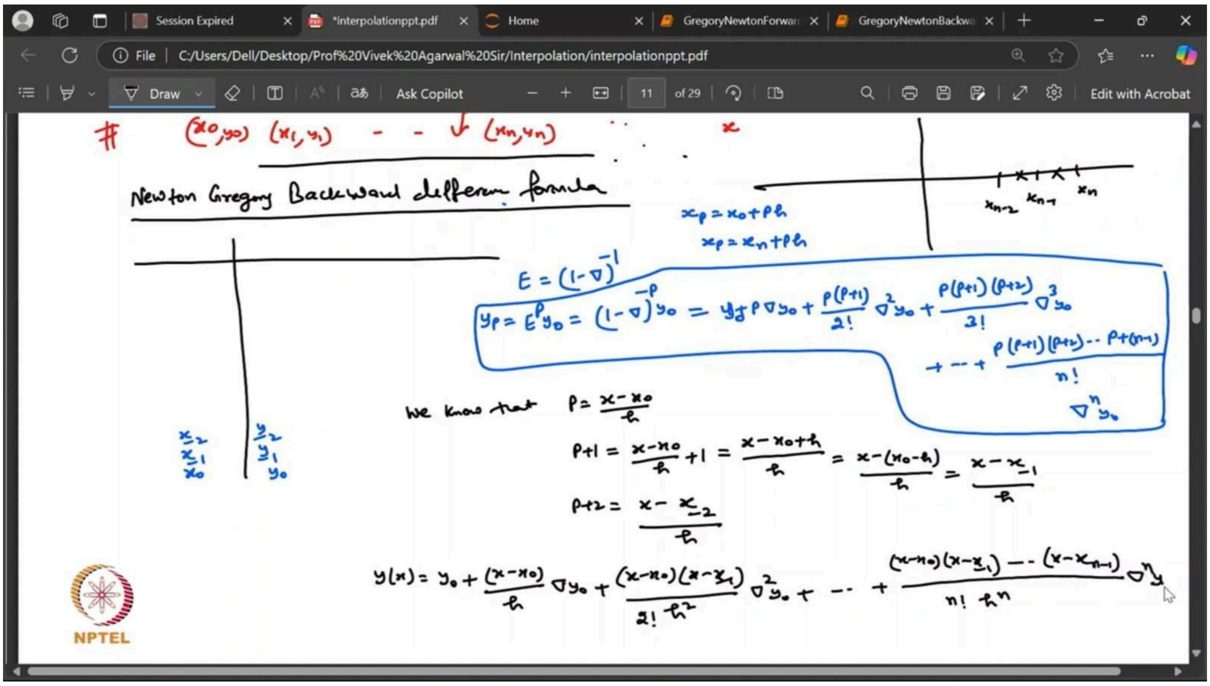
(refer slide time:21:48)

Now the series that we have, we will have to terminate this series somewhere because this is a series. It depends on how far our finite difference operator is going. So in case, if we have to see that we know that it depends on our number of values. So in the number of values, we had said that if we had three So if we have three values then we said that it can go up to the second order finite difference operator. So if this is n plus 1 value then it can go up to a finite difference. So in this case we will say that then a n plus 1 data points which will be our series will truncate and values will be y x is equal to y not plus p del y not plus p into p minus 1 by two factorial del square y not and last terms will p p minus 1 p minus 2 and so on p minus n minus 1 over n factorial del n y not. So ultimately our series which is our polynomial will give us this. So with the help of this polynomial, we will calculate it. So we have this polynomial. So we can name it 1. Now these terms which have come are these terms of p . Now see, if we have to write this polynomial then we write polynomial in the form of x but here everything is in the form of p. So this thing is useful when we substitute the value of p and we will get the interpolation value for that x. So we will get its value on that x. So this is one. Only then it will be Is good when we want the exact value now see what I can do here I have p so we calculated so we have p and this is what we have we calculated so we had this now we know that x p or x whatever it is let me write x or let me write x okay what was that x 0 plus p h this was okay so now look let me write it in this form now from here if I want to find out h then what will come x minus x not by p this is what we have come okay so this is our values this is what we have come or else let me calculate h and write p so p is h now because p is coming here I want to see p minus 1 what will come so this will come x minus x not by h minus 1 so this will come x this will come x minus x not minus h by h and I can write it like this x minus x not plus h by h and what is x not plus h is x one so this becomes we have x minus x one by h similarly p minus 2 what can I write, so we will see that we are x minus x two by h so if we keep writing like this then the equation that we have written above one equation so now equation one can be written as so this is our polynomial now whatever values we want to tell we can write polynomial in place of it so how will we write polynomial y not plus x minus x not by h into delta y not x minus plus x minus x not x minus x1 by 2 factorial and h will come from here and one h from here h will become h square and this will come delta square y not so if we see then what will we get in the end x minus x not x minus x1 x minus x n okay because in p minus it was going to one x will go to a x n minus one over n factorial h to the power n this sorry okay so we will calculate this value so in this case if we see then what we have This expression is our polynomial. So, this will give us the polynomial and it will give the exact value. As soon as we put the value of p, okay, so depending on what value we have asked for or the polynomial that we have asked for, we will use it. Now this is okay, we have done it, so now we have to see its error, how much will be the error. Now we are doing it in approximation, so we have to find out the error in the estimation. So, if we see, the error is calculated by Taylor series expansion, so what we have to do is find out the error, E x, so E x will be basically the value of the polynomial, p x plus sum. If we see, f x is somewhere, we will see it will be p x plus some error. So, we have to find out the error. So look at the error, our x zero, x one, x two up to x n , so the error was at those points. If it will be zero, then how can we write our error? x minus x not, or x minus x one, x minus x n by n factorial into h power of n, see, it is coming from here, so the next term will be n, and this one will become a n plus one y zero. So the error that we have found out is the error for the forward, which is the Newton forward operator, the forward formula's error, okay, and we will have to do that, we will have to write a n plus one, n plus 1 factorial over h to the power of n plus 1 because if it is up to n, then a n plus 1 will come, so this error will be involved, so this is the error for the Newton Gregory forward difference formula.



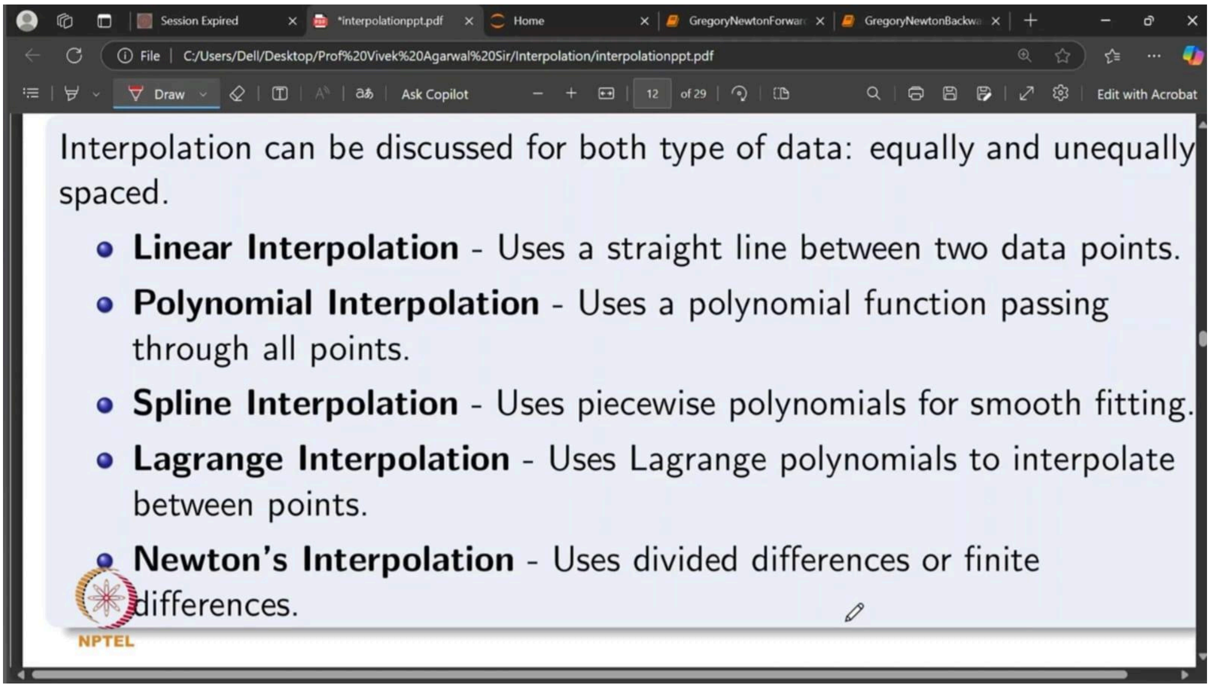
(refer slide time: 30:17)

Now we can do the same thing for backward, okay, so what do we have to do for backward, so if we see, let us see the backward, now what do we have to do, suppose we have the values that we have, x zero y zero x one y one x x n y n this Now we have to find out the value of x, suppose it is bringing it somewhere here, bringing it from behind, so x is not going to the front but bringing it somewhere in the previous value, so suppose the x that we calculated, here we had x n, somewhere here x n is minus 1, somewhere here x minus 2, so suppose our value of x is somewhere here or somewhere here, then if we apply the forward method, then what will happen is that the error will be very high, so if we do not apply the forward operator and apply backward because we know that we need the value of the past, so here we are, then in that case our accuracy will increase, so our method for that is Newton Gregory backward difference formula or now what does backward difference formula mean that we have to find the value somewhere here, so for this, see what is the first thing to do, we had done the forward difference tables, in that we had seen that backward ones The last value is used in its first order difference, second order difference and third order difference. We saw this thing in the previous section. So what does it mean? What do we do in this? The table which is made with the values, we consider the last value as x note. Okay. And we consider it as x one. We consider it as x two. This way it will become our last x n. We reverse the x. Or if we don't want to do this, we will keep it like this x n x n minus one and x zero the formula that have to write will be in the form x zero or y zero here and y n here so what we do lets not change the ordering keep this in this order in some books Okay And if x is zero, then I have seen that in some books it is written like this x zero x minus 1 x minus 2. It will go on like this. So we are going backwards. We can represent it by x. In some books, we can represent it by x. But our main purpose is to use this last term. Now let's see how we have to use it. Okay. Now look at our values. Now we know one more thing that x not plus p h. Okay. Now it depends on what our x not is. What is that? Either this is x note, if I consider it as x not, then it will become or it will become x p, which will become p in this case will be negative, right? So p will be negative, so we will go backwards, okay, so everything depends on what we are assuming, so let's do it like this, for the time being, let's assume it as x zero x one x two backward, so this becomes x zero or it becomes x minus one , this becomes x minus 2, so what did we do by not using this terminology, we defined it like this, first we defined x zero, okay, so after that we have to calculate the value of y p, so what will be the power of e p y not, we are going backwards, okay, x is given here, now see how will we write it, I will write it as 1 minus minus p, y not has come to me, I will apply the binomial expansion to it, so it will become 1 plus p y not plus now the value of p in this is p plus one It will go, okay, it was -1, right? And this will become 2 factorial. And this will become second order. Now this will become p p plus 1 p plus 2 by 3 factorial. Backward third order difference operator, so it will go on like this and at the last it will come p p plus one p plus 2 p plus . Now till where will it go, in three, two is coming, so till n minus one it will go by n factorial and n th difference. So this formula will become ours. Now, we know that this formula is used when we need to find the direct value. But if we don't need to find the direct value, then what will we do? For this, we will change it in the same way. So, if we change it, what will come out? We know that our p was x minus x not by h . Okay, so now this is what we have. Now, see what will be p plus one, so x minus x not by h plus one So how much has it come x minus x not plus h by h this will come and how much is x minus I can write it like this x not minus h by h so the value that will become will become x minus now if we have to go backwards from x not then we should tell it x minus 1 so it has come okay so here what do we have in this case x minus 1 y minus 1 okay this is x minus 2 y minus 2 so in this way we have values given let's say that the value that we have is like this then what will be p plus two it will become x minus x minus 2 so if it goes backwards then we will substitute like this so if we substitute this then the values that we will get which are polynomial and we will get this then what will become y of x we will also get here it will become y not okay so the value that we will have y not plus x minus x not by h backward first order x minus x not x minus x minus 1 by h square 2 factorial second one The operator will come and it will go on like this, okay and in the end we will get this x minus x not x minus x minus one up to x minus x n minus one to the power of n factorial h n this is our formula if we get the polynomial word now if we see in this backward then this value is coming, okay so if we want to calculate this then suppose I want to find out the error, so how will we get the error, we will get it in this way, so the error that will be there, I can look at it and write it from here, so what will be x minus x zero x minus 1 x minus n to now the next thing that will come will come , okay minus and here the minus will also come sign this is minus p because going with minus minus a minus the next thing that will come will come minus a divided by now see a was then a plus will come to the power of factorial a a plus and the backward difference which will be n plus 1 th plus order or comes, this is ours that will be the error involved In this, if we go backward, then okay, then we use Newton Gregory forward and backward.



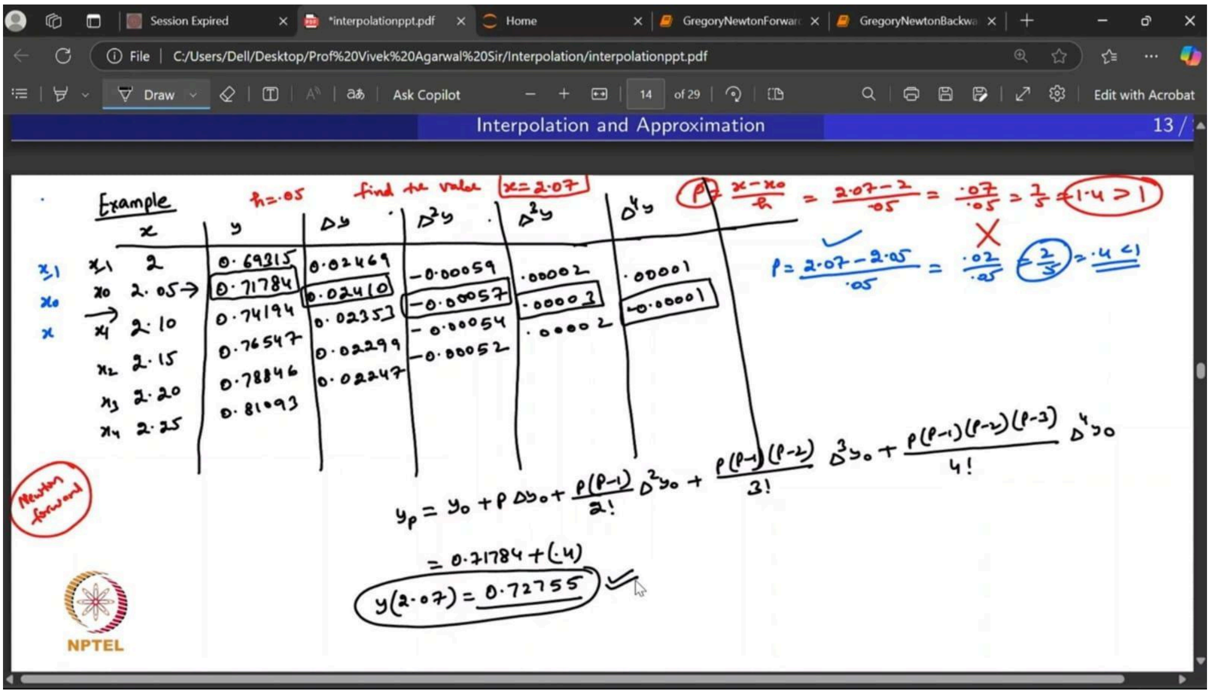
(refer slide time: 39:30)

Now we have to see that it is not that these two are used only. If we see, these two interpolating are used only forward and backward values. So if we say that this interpolation is an interpolating Polynomial, it is using only some front values and in backward it is using the last values. So the interpolation is like I discussed that it is passing the value from it, so there can be many interpolations, linear interpolation, Polynomial, spline, Lagrange, Newton. We will do all this later. So now we have done this Newton forward, so this is our formula, we got it. So we have calculated it and from here we have found out.



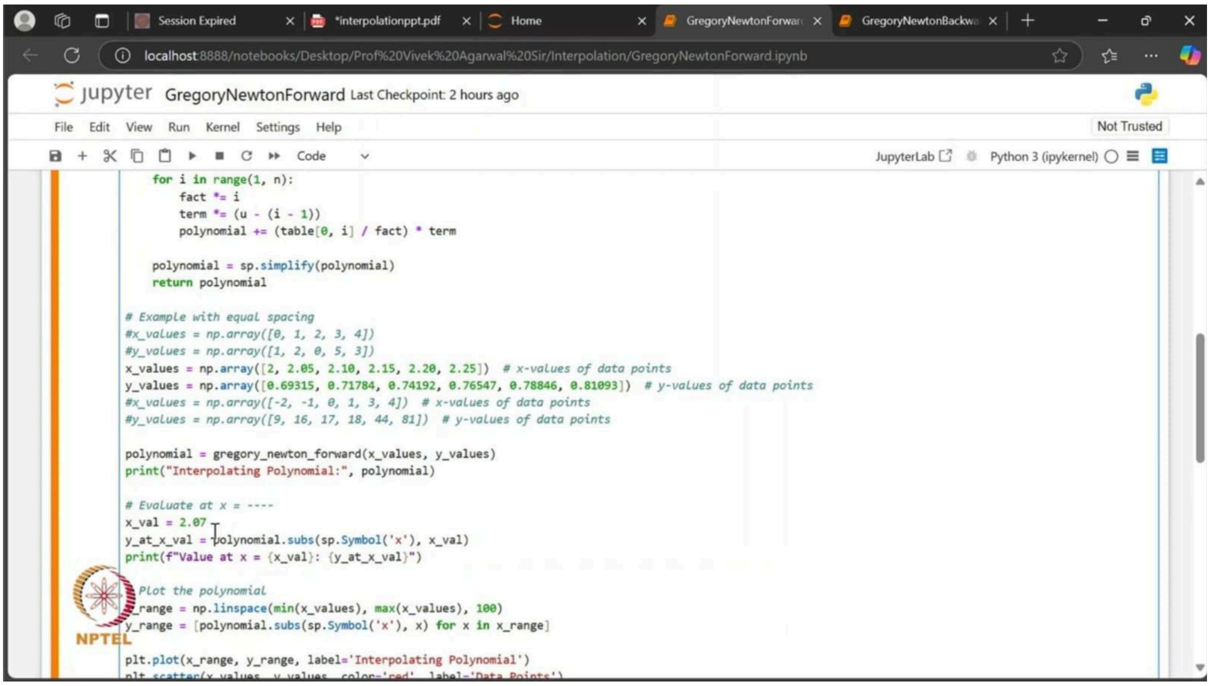
(refer slide time: 41:38)

So we can use this formula now to calculate various values, we have to calculate for its purpose. For this, I think some error will also come in this, but here h comes, here h square like this, now look, let's do an example and see how we can calculate, so let's take an example and see how we will apply forward and backward in this example, so like we have an example, we have been given some values, first we will write it in the table, this is x and 0.71784 this value is 0.74194 this is 0.76547 this is 0.78846 and this 0.81093, these values are given to us, now we do not know which formula we have to apply, newton forward, backward, what have we to do, there are even some central ones, but we are doing that forward and backward only, so we have been told that these values are given to us, find this, find the value for x is equal to 2.07, we have to calculate on this, this is said, so 2.07 see where it is coming, if it comes somewhere here then the above values are being used in y but okay so what will we do in this case, these values are there, so we will apply Newton forward in it, we have decided that Newton forward will be applied only, now we have to see that for applying Newton forward, now our value here, so the value of x is this, okay so now we have to see that this value is x, this is x not, this is x1, this is x2, so after x1 and x2 is coming in between x2 and x2 so what did I do, we have to calculate the value of p because we need the formula, so this was x minus x not by h h ours and how much is a .05 this much difference, so we calculated this, how much is x mine and minus 2 and it came out .05 how much did it come out, .07 divide by .05 came out 7 by 5 made 1.4 so 1.4 is greater than one so what do we do, the value of p is always less and should be less than one So what do we do in this case, the error that we get will increase a lot. So we cannot use this, so what do we do, I will shift the x not because that is what we have to use. So what did we do, I shifted the x not and what did I do, let's shift it like this, from here, I will consider it as x not and I will consider it as x minus one and I will consider it as x one, so I did it like this, let's see what came out, 2.07 minus 2.05 divided by .05 .02 by .05 2/5, so as soon as this 2/5 came out, we will say that ok .4 came out less than one, so we will calculate it, so our p should be less than one, so what did we do now, I will erase it and now this was two, so what happened now, I considered it as x zero, this is my x one, this is x two, this is x three, this is x 4 and this is x minus one, so what do we do now y not what happened y not mine this is done ok so now I will put the forward which is first forward first forward we know that this minus this so I will calculate this so if we calculate this then we are getting 0.02469 this is coming 0.02410 this is coming 0.02353 0.02299 and this is coming 0.02247 this value so we know that this value is minus and this is then we will go like this we will calculate it I calculated the second one so the second one came minus of minus 0.00059 this value is a minus of 0.00057 I am writing it between these two I wrote this value between these two I wrote this value between these two I will write this value between these two so it will come minus 0.00054 and this one came minus 0.00052 this is it so after this we have to use That is to do this value ok so now I will write the third one, in the third one we saw that this will come here point 002 and here point 003 is coming this value .0002 it means I will use this value now we have to go one more step to find the fourth difference, if we find the fourth difference then we will have this value point 1 and minus .00001 this value ok so it means we have to use this value we can go only till here we cannot go beyond that ok so we can go till here only now if we go from here then in the next we will not get this y not because we have to take the corresponding from it so it means now whatever value we have to substitute we will substitute this so the values that will come to us or the polynomial will come or the value will come then we will calculate so what do we do if we want the direct value then we will write y p what will that be y not plus p del y not plus p minus one by two factorial del square y not plus p p minus one p minus two by three factorial del cube y not it will come like this now where do we have to go So I will take it here, p p minus one p minus two p minus three, okay, 4 factorial and it comes four, so I can go till here. Now the value of p is point four, so I have substituted it, it comes 0.71784 plus point 4 into it. Let's calculate and substitute the value of p here, okay. Then I will substitute it here, I will substitute it here, so we substituted this value and the value that we will get from there will be 2.07, and if we solve this, then its value will come out to be 0.72755, okay. So in this we had said in the formula that if we are substituting and using this p formula, then we will get to know the direct value, so it means that y at 2.07 will bring it somewhere here, its estimated value, okay, so we have used this value, that means we will use it for our purpose. Estimating and we have estimated it, so look, whatever formula we had applied or we have used the same formula, this one is fine, so we have calculated the formula of P on Y, so all the values , we will substitute and we will get the answer, if we do not want to do this work, we need a polynomial, so I will put a polynomial here, y not we already know this value, we know h as well, we know everything, if you see, the value that we will get in this, the value of x minus x not, here, if we see, if we have to go up to four, then we should get a polynomial of fourth degree, so if we try to calculate it with the help of computer, then we will see that in this case, we will get a polynomial of fourth degree and we will get a polynomial of fourth degree, so with its help we will calculate it.



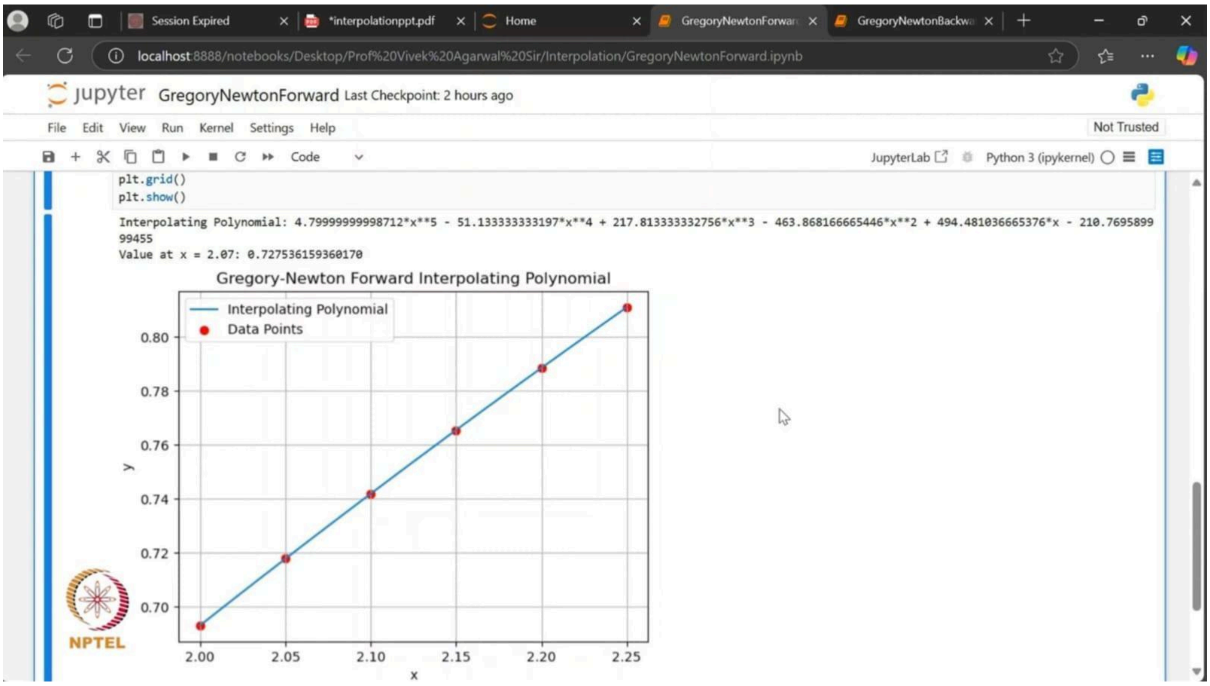
(refer slide time: 52:32)

so if I suppose I see a python code and check it whether we are getting this value or not, like I have given this code to ours Pass this of Newton Gregory Forward, okay, so in this code we are using numpy, we are using sympy and matplot is used for plotting, sympy is used for symbolic computation, numpy is used for numerical computation, okay, so we will create a forward difference table, after that we will use Gregory Newton Forward, which we have used here, okay, and the symbol means we will come in the form of x, so we have calculated it like this, now what I did is I tried to test it for different examples to check robustness, okay, so that we can see the examples, so what I did to see it, we took an example and took this x value, see 2 2.05 2.10 2.15 2.20 2.25 and these corresponding values i have taken and these values are to be checked if i have to check then have to check here 2.07 why because the values we had to see was at 2.07 so we have to find out its values



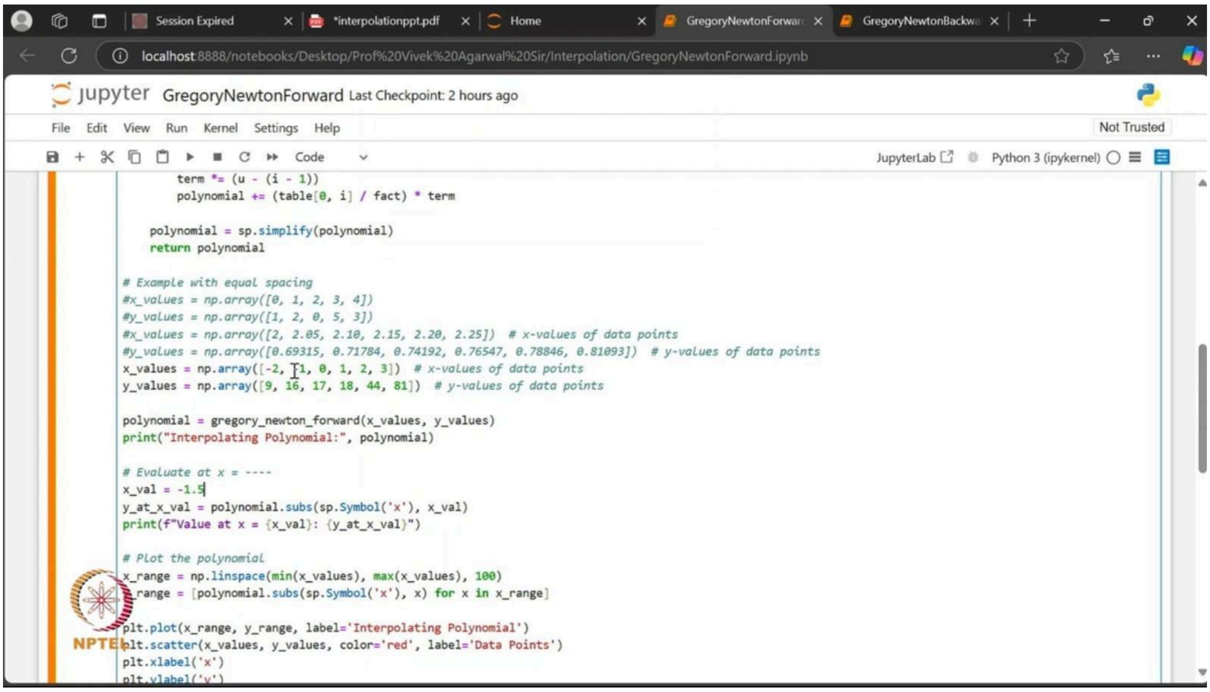
(refer slide time: 55:23)

so if i run so look, we have this one 4.9 x raise to power 5, okay 51, this is the Polynomial, this is the Polynomial i, which We have a polynomial in this, we have calculated it from the first element, so 5 has come, if we use the second element which we had written as x not, then a polynomial of the fourth degree will come, so x equal to 2.07 if we see, see, this value has come and the same value was coming there, 0.72753, see, it is coming here 0.72755 and it is coming here 0.72753, because it is taking the value in the beginning, it is starting the x zero. Okay, so from here we have these values and I have used it from here.



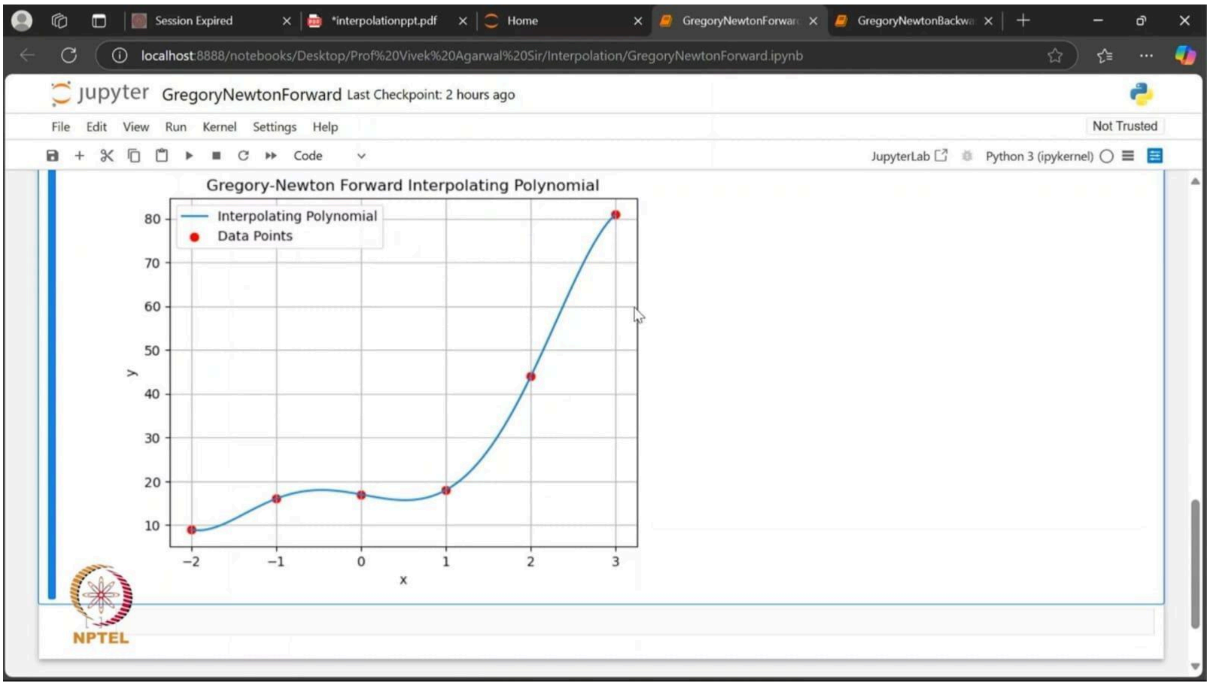
(refer slide time: 56:24)

Okay, so this is our forward, we can do this for different values. Now suppose I use this and suppose I try to calculate the value of the last, I will do 2.21. Let's do 2.21 now if we see, where will 2.21 come, if it comes somewhere in the last, then it should be applied backward, but we can also do it with forward. It's just that the error will increase, so the values of error which is 2.21 are 0.79299. So we will check how accurate these values are. We will also find out its accuracy. We will find out the error which is in it and we will get to know how much error is coming. So this is how we took out the value from the polynomial. Now suppose I take any other value. Suppose I took the value y. In this way I took any random number minus 2 minus 1 3 4 between the two. Minus 2 minus 1 zero from zero to one and 2 from 1 and 3 from 2. Okay, because the values should be equispaced. We have 9 16 17 18 44 and 81. So the first thing we have to do is that the values should be equispaced. So what did we do? We used these values. Now suppose we have to calculate here. I want to do the value at minus 2.03 or in between these and the extrapolation will be done so I want to do it on minus 1.5 but in between it's okay because minus 2 point will go out and there will be an extra position inside it taking minus 1.5 somewhere here I want to calculate it



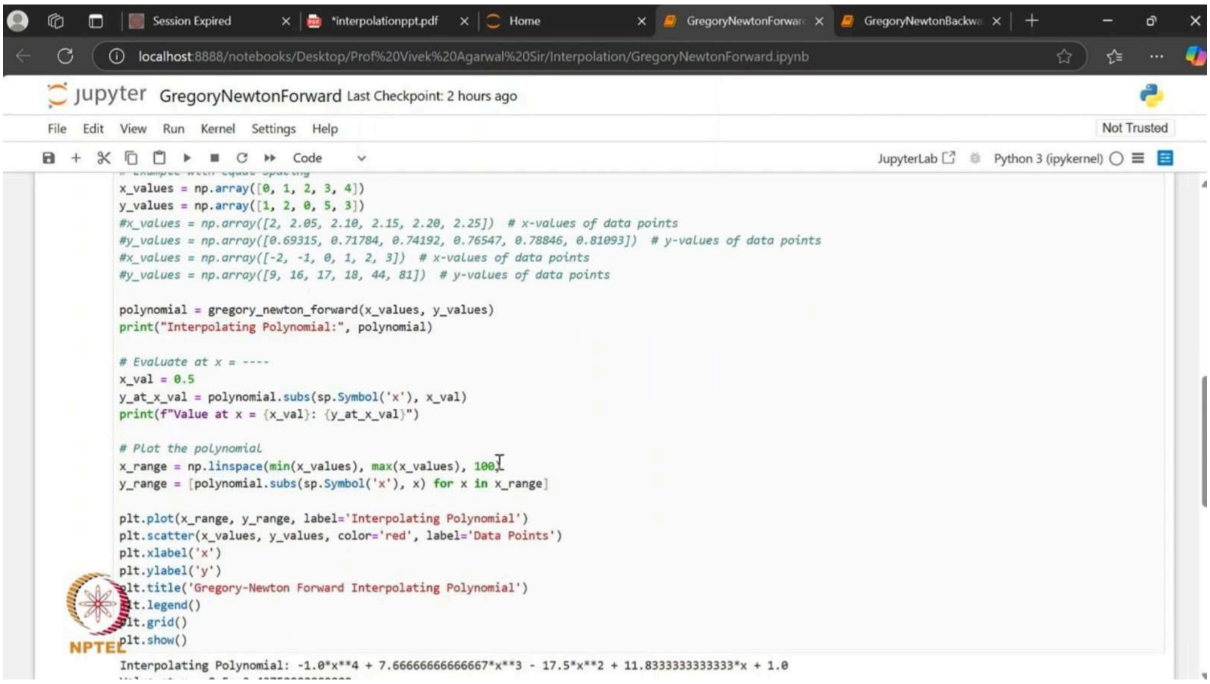
(refer slide time: 58:46)

so I ran it and see the polynomial of fifth degree came and the approximations of minus 1.5 we calculated that and what are we doing in this, I considered all the values as the starting values, okay and we calculated it and the value that we have is like this I want to do minus .5 in place of minus 1.5



(refer slide time: 59:33)

let's see that, okay the polynomial will remain the same, there will be no change in it, the value which is point -.5 18 came, so these values have become 18 here, okay so we have this depending upon how our polynomial is being calculated so this polynomial was, we have its The answer is here, this polynomial constant value is 17 and this is the coefficient of x square x cube x4 and x5 so on its basis we have got the interpolating polynomial and this is obtained with the help of forward mixed Newton forward now I do not use any data but use any other data, I use this one because once I have created my code, now keep changing the data value and our value goes, now I took 0 1 2 3 4 and took some values, so suppose I have to take it on point five so I calculated it, okay, we got a polynomial of fourth degree and the value on point five is 3.43 and this was our interpolating polynomial which we used from forward but there is only one problem in it that all of it should be in equispaced so if we see, equispaced data is on zero then on one then on two then on three then on four okay so this is interpolating polynomial It is passing through all the points, so we said that it is an interpolating polynomial, but in this, the forward values is being used, or the backward value is being used, but here we have to do interpolating polynomial, so what was this interpolating polynomial, which was passing through all the points, so it is passing through all the points, okay, so on its basis, we have now created the code and we keep increasing our data because we can have any data, this is small data, we may have some bigger data, if we have real data, then its file will be CSV file or Excel file, we will upload it here and with its help we will calculate the polynomial and after calculating the polynomial, we will put any value where we need interpolation and we will get the interpolation value, the only thing in this is that as the number of points increases, the degree of the polynomial that we have will also increase, okay, so the degree will increase, so we can calculate it like this We can do that and with the help of this one we have used it.



(refer slide time: 01:02:46)

So today, with the help of Newton Gregory Newton forward formula, we have seen that if we do not interpolate any values, which are in the starting values, then by this method, we will get a polynomial of degree n, and we can interpolate on the basis of that polynomial, and we can use those values if we want to do any work related to this data. So, today's lecture was based on the Newton Gregory forward difference formula, so I hope you liked and understood this discussion, and thanks for watching this lecture.