SCIENTIFIC COMPUTING USING PYTHON
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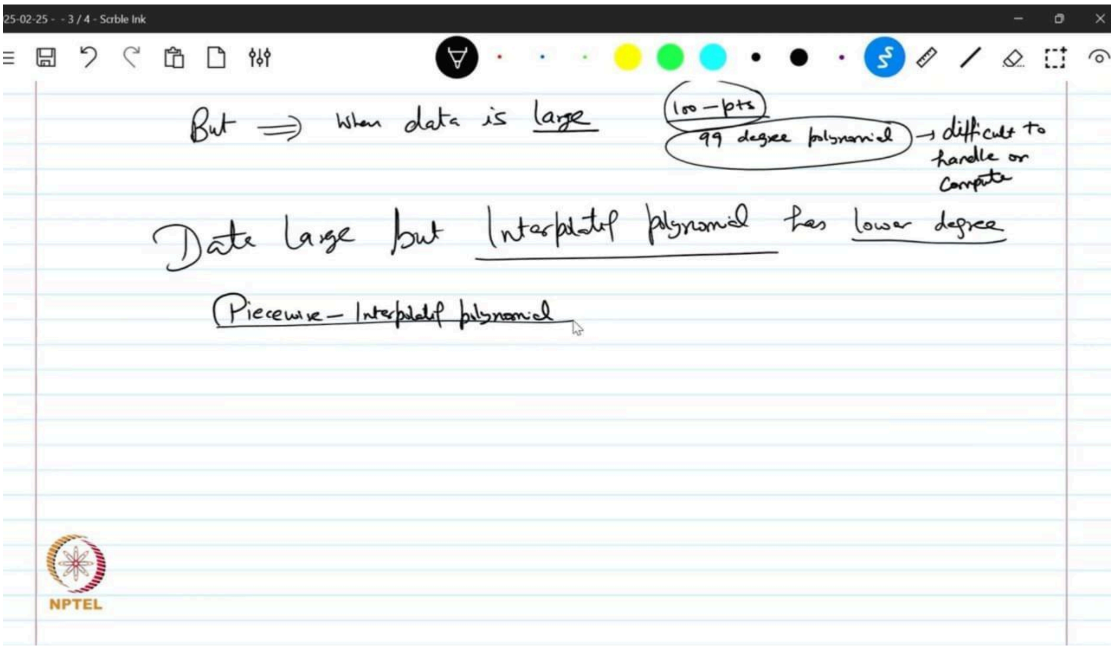
Lecture No. 24

Welcome all of you to Scientific Computing Python. So today we are going to start a new topic that is piece wise interpolating Polynomial. How can we find out interpolating polynomial in pcs. So let's get started. In the previous lectures, we saw that we have interpolating polynomial, we found it with Newton, found the Lagrange's from it and did Newton divided difference, after that did it with Hermite, so all these interpolating polynomial, it gives good values, it gives good approximation, but when will the problems arise when the data is large, the data is large, so for example, suppose we have just taken one data of 100 points, so if we have data of 100 points, then we know that we will get a polynomial of 99 degree, if we do it by Lagrange's, if we do it with Newton divided, any of the things that we have discussed, we will do it If we look at Hermite, then all the polynomials that we discussed and we will get will be of 99 degrees and from Hermite we will get even higher ones. So what is the problem in this case that the degree will increase a lot and what will happen if it increases a lot is that it will be difficult to handle. First of all, there will be a lot of problem in computing and handling. It will be a tedious process. Now we will find out 99 degrees and then find out the fit of it. So, this will be a very difficult process and even in a computer, if we try to find out a polynomial of 99 degrees, then the computer will also take a lot of time. And our data is nothing but points. Our data has thousands of points. There can be 10 thousand, 100 lakh such number of points in it. The data can be very large. So, in that case, all the methods that we have used now will fail. So if we have to deal with this large data and our purpose is also that our data should be It is large but the interpolating polynomial, whichever our interpolating polynomial is of lower degree, is very low, so this type of interpolating polynomial is called piece wise piece wise interpolating polynomial. In this, piece wise interpolating polynomial has to be defined, it has to be done in pieces, right?



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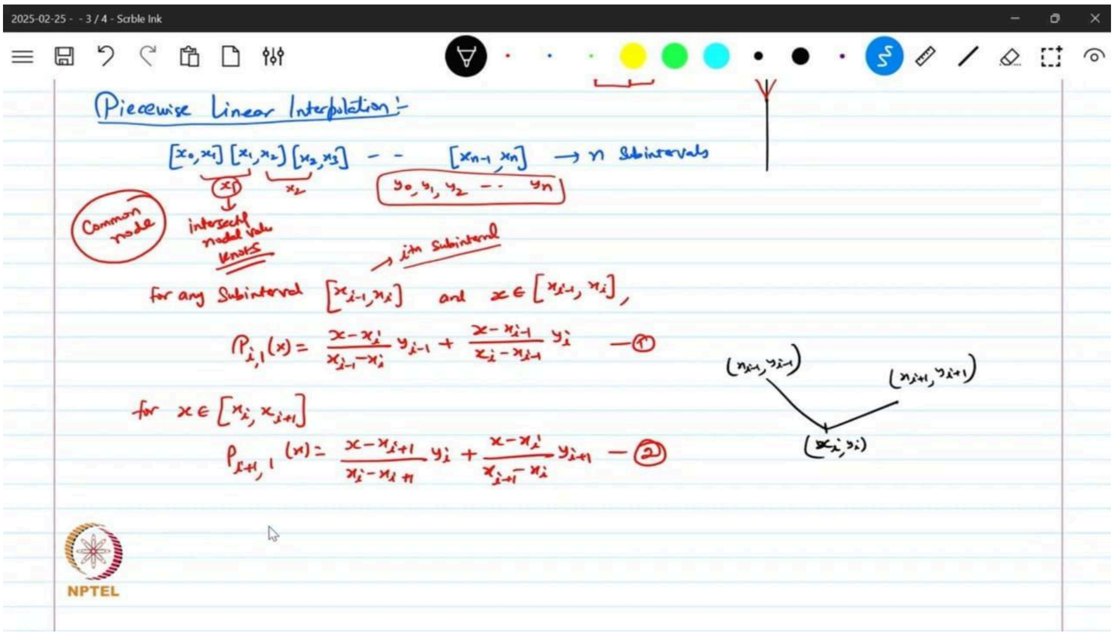
So, this is our main motivation that why do we have to do it piece wise, because the polynomial of high order degree is very difficult to handle and there is also fluctuation in it, as the degree increases, the fluctuation starts increasing, right? So now what is piece wise, as the name suggests it will be in pieces, so what is in pieces, so let me start, suppose we have data points, a lot of data points, the first data point is this, then the second one is this, one is this, okay, one is this, suppose we have a lot of data points, so what will we do with it, I can either do it like this, I have taken a linear line from here to here between two points In the middle, then I take a line from here to here, then I take a line from here to here, then I take a line from here to here, okay, then I take a line from here to here, then I take it from here, so you will see what we are doing in this, the linear interpolation that came between the lines which are two points, we took that, we took linear interpolation in the next one as well and here it will be coincide, it means that the continuity will remain in the next one, okay, here we took a linear interpolation between the two points, then took it in the next one, then took it in the next one, what did we do like this, all the data points were there, so if we have those data points, then they are x0, x1, x2, x n, okay, so these are n plus 1 nodal points and what happens in this, if we see, x0 and x n are boundaries, boundary nodal points and the ones that come in between, we call them these are interior nodal points. Okay, now if we get the nodal points, then suppose we have to do some linear interpolation. So, let's start with the first thing we do. I will name it piece wise because now we have to do it piece wise. So, what have we done? Let's start with linear interpolation. Now see what will happen in this. Now we have data points given, so what will we do? We will divide all the data points into sub intervals. Sub intervals are like these points. So, I can write it like this. One interval will be from x0 to x1. The second interval will be from x1 to x2. The third interval will be from x2 to x3. Like this, we have x n minus 1 to x n. Okay, so see, 1, 2, 3, we have these n sub intervals. Okay, so what have we done? We have defined these n sub intervals. Now if we take these two intervals, then there is a common value in them x1. Okay, what is the common value in them? x2. It is connected with each other, where is the connection happening on the x1, so it means that their boundary points are the same and overlapping, right? So, we can say that in this case the x1 is the intersecting nodal, nodal value or nodes, also called nodes x1 x2 x3, such points which are connecting two sub intervals, we will call them intersect nodal points or intersect nodes, right? So, what is happening in these points, now like this was mine, suppose this is my x0, here somewhere my x1 is there, so see x1 is also going in this interval and suppose here we have our x2, so in x2 also, this one, this one is a sub interval and this one is a sub interval, in both, the x1 is the intersecting point, connecting both, right? So, what will we do in this way, pay attention to each intersecting point that this continuity of ours should remain, it will happen automatically because it is an interpolating polynomial, right? So now What we have to do is to calculate this, so what will we do to calculate it, so we will write it down mathematically, so see, we have a sub interval, so now I have taken any sub interval, so then any sub interval, now I have taken the sub interval, suppose let be x i minus 1 x i in between it, that means see, in the first one, it is going from x0 to x1, so this is the first sub interval, second sub interval, so we will call it i th sub interval, okay, so suppose we take this sub interval and any x of ours, suppose it is lying in it, so what will we do, now we have to interpolate it with a linear polynomial, n interpolation, we know, we can define it, p 1 , that means first degree polynomial x, how will I write it, now see, it is coming in it, so x minus x i divided by x i minus 1 minus x i and here we have the value given, what is the value y0, y1, y2, so we know, we have the values which are data values and given at this point, so this If the value of the function is, then this will come to our y i minus 1 because this is plus x minus x i minus 1 divided by x i x i minus 1 y i this comes to us so this linear polynomial is in this i th sub interval now let's see in the next one so what will happen in the next one okay in the next if we see 4 x belongs to now see x i is coming here it is intersecting this will give x i plus 1, okay this common node x i will come in both this one and this one as well okay so intersecting node or we can also call it common node okay so we can also call it common node so the common node which is in this interval and in this interval which is x i so in this case if we have x then I will write this polynomial as p i plus 1 because this is i interval and this is i sub interval or i plus 1 sub interval has at x so what will this be it will be x minus x i plus1 over x i minus x i plus y i plus x minus x i x i plus 1 minus x i y i plus 1. Suppose we take our one if I take this two, then what did we do, if we see, we took two points, any two points, one this, sorry we had two points, one this was, suppose one this was, so one I wrote this x i here any value, let's say this point is x i y i, this point is x i plus 1 y i plus 1, I wrote its coordinates, x i minus 1 y i minus 1, like this we drew two lines which are linear, like this we found it out.



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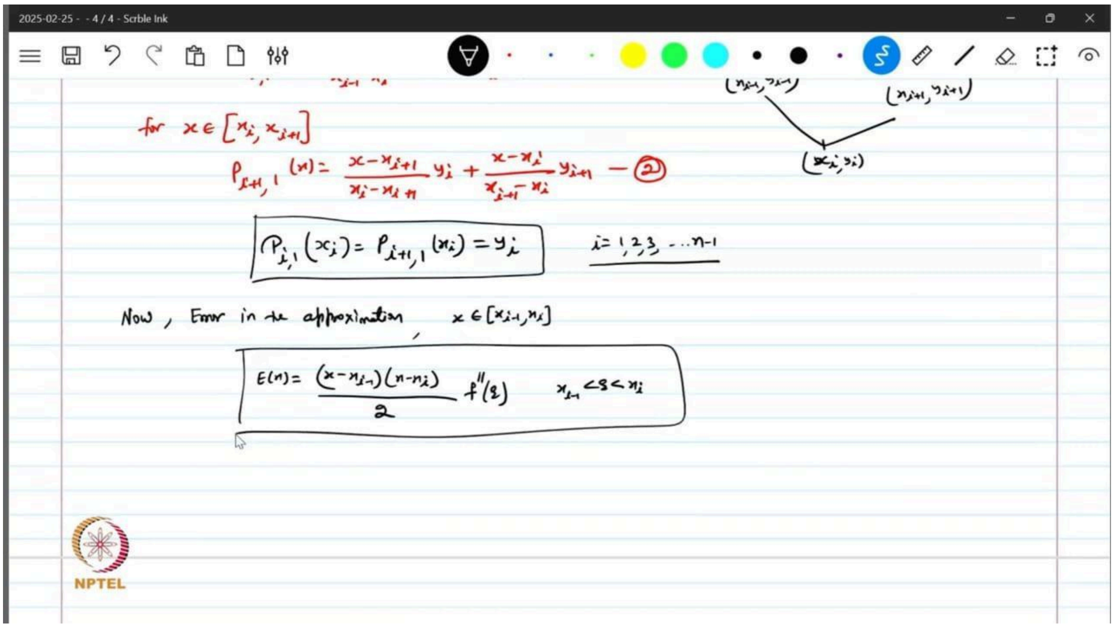
So, we know one thing for sure, we know what is it that p i,1 at x i is same as p i plus 1 at x i, okay, right, and both of these will be the same as y i because this is an interpolating polynomial, so from here we can see that if x is x i minus 1 then from here we will get y i minus 1, if x is equal to x i, from here we will get y i, the same case will be in this, so this condition is always it will be satisfied, okay, so if we satisfy this, then now we can define all the values that we have, for i from 1, 2, 3, up to n, okay, so now the values that we have are i, if we take two values, then if we see in this case, if I want to define this, then I will go up to i minus 1, okay, if I take two together because I am taking i minus 1 x i and from x i x i plus 1 so it will go in this way. So, now what can we do, now what we have, now the polynomials that we will get will come like this, linear polynomials will come and in every sub interval we will get a polynomial, now what will be the error, now if we want to make the error in the approximation, okay, so if suppose our x belongs to some x i minus 1 x i, then the error that we have will be x minus x i minus 1 x minus x i by 2 Factorial f xi now xi which will be now if we are going into x, then our xi will belong to from where to where it will belong x i minus 1 to x i, okay so in this way we will get these so in every interval we can find out the error like this depending on if x is belonging it then we will define it like this otherwise we can define it for all the values, so if we get an error like this.



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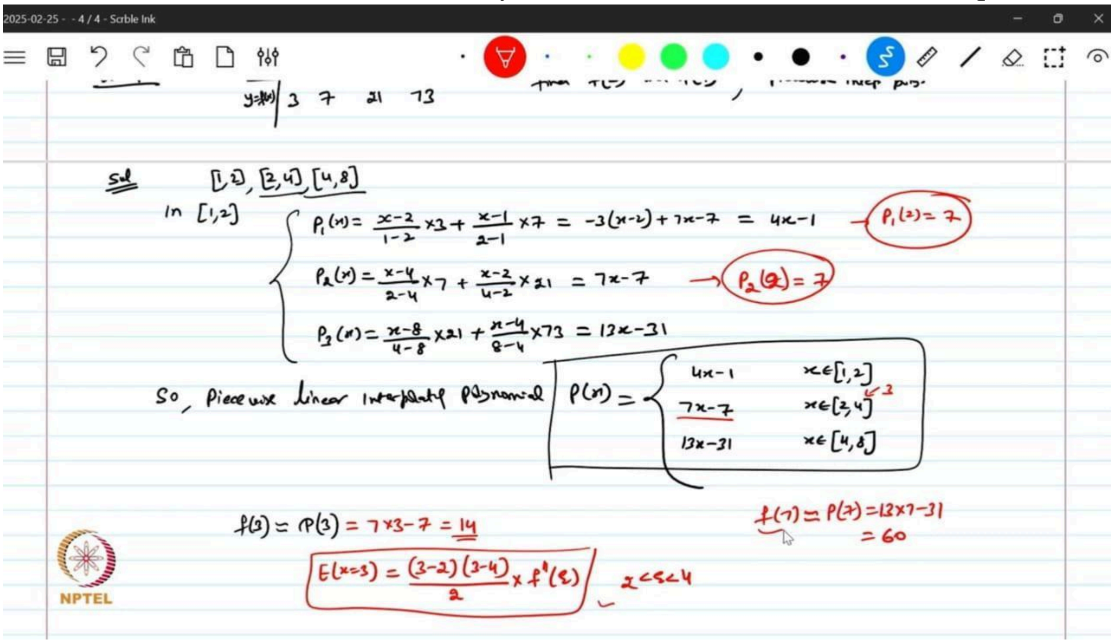
Let's take an example, suppose I have data like this given this is x and this is given y we have some values, so suppose the value of x is one, two, 4, 8 and this is 3, 7, 21, 73 this is given to us, okay so first of all what we need to do is find f3 and f7, we need to find these so first of all in this there are four data points, so we can do it with the interpolating polynomial that we have found out earlier but what we need to do in this is the estimation has to be done piece wise. We have to calculate the piece wise interpolating polynomial, so what will we do for this. Now look, we have three sub intervals, one interval is here, 1 to 2, 2 to 4 and 4 to 8. Okay, we have three sub intervals, and two and four are common nodes in these. Okay, so what will we do now? Now suppose I first go into this, in the sub interval 1 to two, so what will happen in this case if I take p, then I am defining p1 x, okay, so what will be my p1 x in this case, what will be the first one, x minus 2 1 minus 2, its value is 3 into 3 plus x minus 1, 2 minus 1 and its value is 7 into 7. From these, we have calculated it, so from here we get minus 3 x minus 2 plus 7 x minus 7, so from here minus 3 then 4 x and minus 1, so this linear polynomial that we have come in the first sub interval, like this I have defined the second sub interval We took out the second interval. What we would calculate in the sub-interval would be, so this has come in it, okay, x minus 4, 2 minus 4 into 7 plus x minus 2, 4 minus 2 into we have to see at 4 this comes 21, so this is what we have. So, we have seen the second one, so if we calculate its value, we will get minus 2 2 and minus 2 will come, so if we see this value, it will become 7x minus 7. Now see, p1 x at 2, but how much is it? If we substitute here, if I want to check here, then how much will be p2 at 0 and if I check here, how much will be p2 at because the common note is this, right? So, 14 minus 0. So, it means that our condition is getting satisfied. Now, in this way, we will get the p3. So, we will calculate p3, that is, in this one, it will come x minus 8 4 minus 8 into 21 plus x minus 4 8 minus 4 into 73. So, if we calculate this, we will get 13 x 3 minus and the common node is 4 in this. So, if I four put here, okay, then it will come to 21 21. And if I four put here, then how much will I get? So, 13 x minus 31. So, if I four put, then how much will I get? 52 minus 31 21. So, if both are the same, then we have three which we have, polynomial. So now if we are told that brother, what was the linear piece wise linear interpolation in this interval, so piece wise linear interpolating

Polynomial p x, what do we have? So, what we have is this, we have 4 x minus 1 when x that belongs to this 1 to 2 and this is 7 x minus 7 that x is from 2 to 4 and this is 13 x minus 31 x belongs to 4 to 8. So, this is our polynomial, this is our interpolating polynomial, so now see what happens in this, for all the data points we keep calculating it piece wise and we get our interpolating polynomial, it is linear, easily we can calculate it, so from there we get this, now we have been said to approximate f 3 and f7 and suppose we have to find f3, what is f3, this will approximate to p3 and what is p3, where will 3 come, it will come here, so what does it mean, I will take this polynomial, so its value will be 7 into 3 minus 7, so this comes to 14, so our three is 14 and the error is and if I have to find the error, then what will be the error on x is equal to 3, now I have to take this interval, so what is x 3, so 3 minus 2 3 minus 4 divided by 2 factorial into xi, xi goes between two and four, so if the value of the function, if we If someone gives it to us then what will we do, we will find its second derivative, from there we will find out its value, we will find out the maximum value and from there we will get the maximum error that our error is maximum, so we can find out the error according to this, now we have been told to find f7, so the approximate value of f7 is coming from here, so what will be that, which is p7, so how will we calculate, from here 13 into 7 minus 31, okay, it will come 31 91 60, so the approximate value of f7 that has come, we have 60, so in this way, on whichever x we have to interpolate, wherever we need the interpolating value, we will calculate it like this, okay, so this work is done for linear interpolation.



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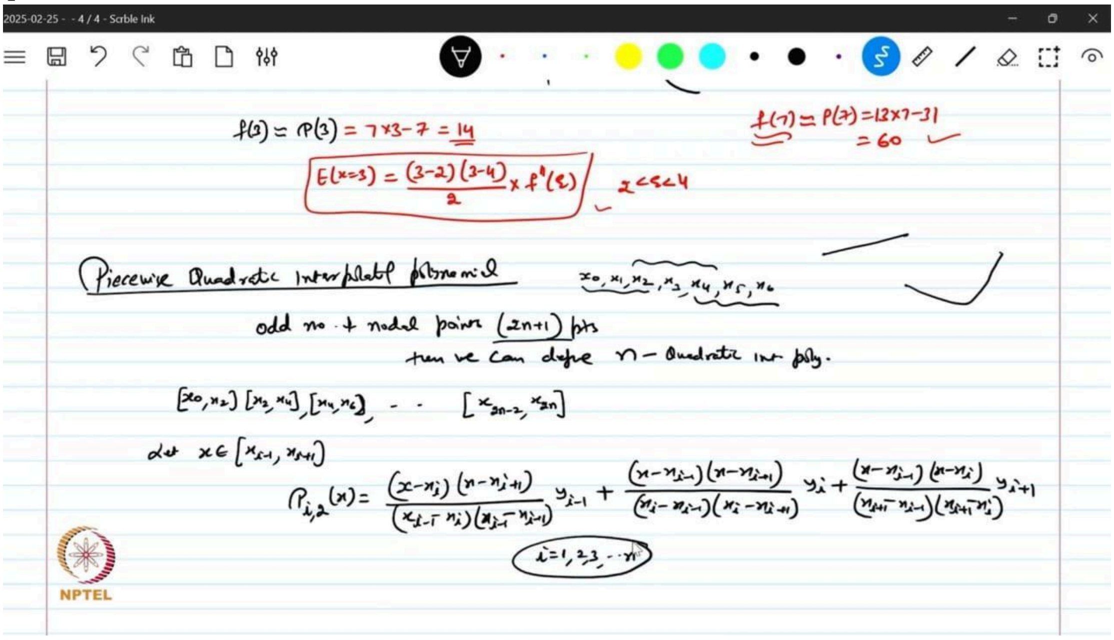
Like this we do the second one, piece wise quadratic, not interpolating polynomial, you do piece wise quadratic not linear, so quadratic we know that if we have to make a quadratic, then we need at least three points are required. Right? If we have two points, then we can draw a line. But if we have three points, one point is this, one is this, one is this, then I can draw a quadratic from here. So, to draw a quadratic, we need a minimum of three points. So, what does it mean that if I have to write a quadratic piecewise, then it should be like this: x0, x1, x2, these are the three points then x3, x4. So, what will happen? First these are the three points. Then these are the three points. Because there will be a common point as well. Both of them should be connected to each other. So, their common point is x3. Then the next one will be x5 x6. So, what does it mean that if we have to get one polynomial or two polynomials or 3 polynomials, then we should have seven points. So, if we see like this, then the first condition that should be there should be an odd number of nodal points. So, if we have 2 n plus 1 point, then we can define n piecewise, so we can find out quadratic and we can calculate the interpolating polynomial. So, the first thing we have to do is that we should have odd number of points. So, if we have odd number of points, then look how the substitution will happen now, from x0 to x2, the first interval is the sub interval, the second sub interval is the x2 to x4, x4 to x6. We have split it and after splitting, look, we will get x2 n minus to x2 n, so there will be a total of 2n plus 1 point. Now what we have to do is now we have to define it like this, so let x belongs to someone, if I go between x minus 1 and x plus 1, I take i th. So, i th will be i minus 1 and i plus 1 x i will come somewhere in between this. So, if I have to define it in this, then I define the polynomial p, the i th polynomial of degree two will be x minus x i x minus x i plus 1 here. So now we have x i minus 1 minus x, x i minus 1 x 1 plus 1 and the value that will come here will be y i minus 1 plus x minus x i minus 1, x minus x i plus 1 from here x i minus x i minus 1 and from here x i minus x i plus 1 will come our y i plus x minus x i minus 1, x minus x i, x i plus 1 minus x i minus 1 and x i plus 1 minus x i y i plus 1 and this is to i plus 1. So, we will keep on defining all the i and whatever i we have, we will keep on taking 1 2 3 up to n. So, the n numbers that we have are the polynomial that we will get. All of them will be quadratic.



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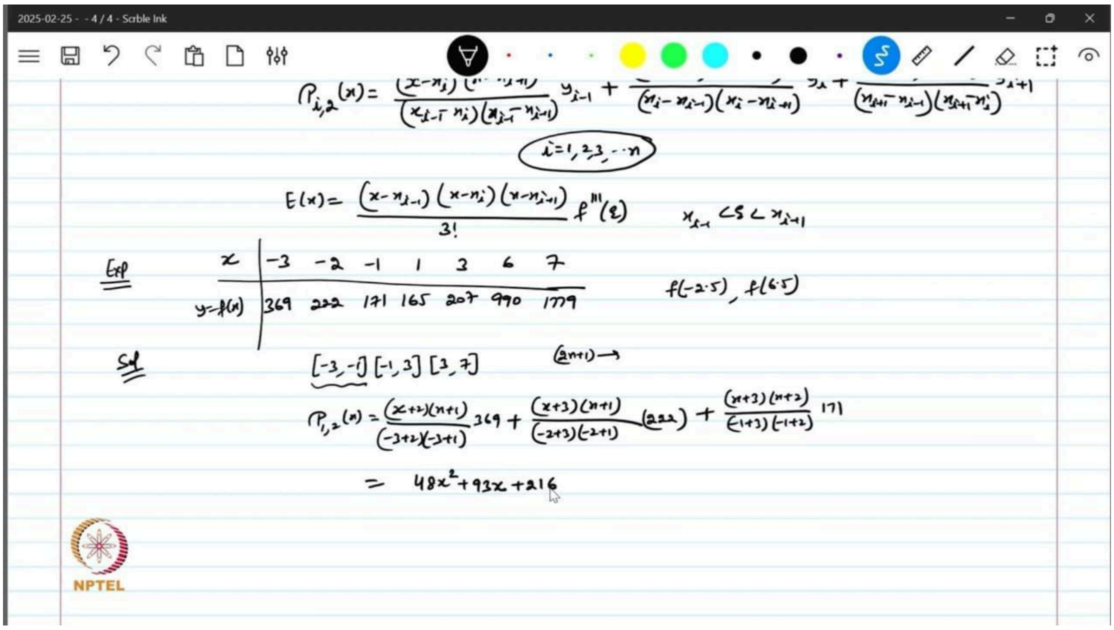
And as we found the error last time, the error that will be there if we take any x and i th is in the sub interval, then it will be x minus x i minus 1, x minus x i, x minus x i plus 1 by 3 factorial and the third derivative of i is and xi is lying somewhere between x i minus 1 and x i plus 1. So, from here we can remove the error. So, with its help we are calculating it. So, if we suppose we have any these values. Now like in the example that we took above we have four number of points. So, we cannot apply it in that. Why can't we apply it because we have been told that the minimum value should be either 3 or 5. So according to this we will calculate it. So now let me take another example and have taken this. Now suppose we have some value of y is equal to f x and this value is this, then this value will come to us minus 3, minus 2, minus 1, 1, 1, 3, 6, 7. And if we have this value, then this value is 369, 222, 171, 165, 207, 990, 1779. This value is given to us and we have been told to find piecewise quadratic interpolating polynomial and approximate at 2.5 and 6.5. What will we have in this case, then we have to form sub intervals, now see what will be the subintervals the first one becomes minus 3 to minus 1, second to minus 1 to 3 and third 3 to 7. So we have these three sub intervals, so the value that we had should have 2 n plus 1 points, so if I take n as 3 then it will become 7, so we have this, so now what will I do, that piecewise, I will define the quadratic polynomial, so in the first one we will do, so suppose I am taking p1 2 x, 2 means it is of second order and one means in the first one, so this will come, x minus 2 is coming, so x plus 2, x plus 1, minus3 plus 2, minus 3 plus 1, into 369 the first one came like this x plus 3, x plus 1, minus 2 plus 3, minus 2 plus 1 and the second value came, our double triple two, okay plus I will calculate it, so it comes x plus 3, x plus 2 and it will become minus 1 plus 3 and minus 1 plus 2 and 171 so if we calculate this we will do it because we know how to calculate the quantity. So, this should come out to be 48 x square plus 93 x plus 216. We will also see it with the help of code. We will try to verify whether it is coming out the same or not.



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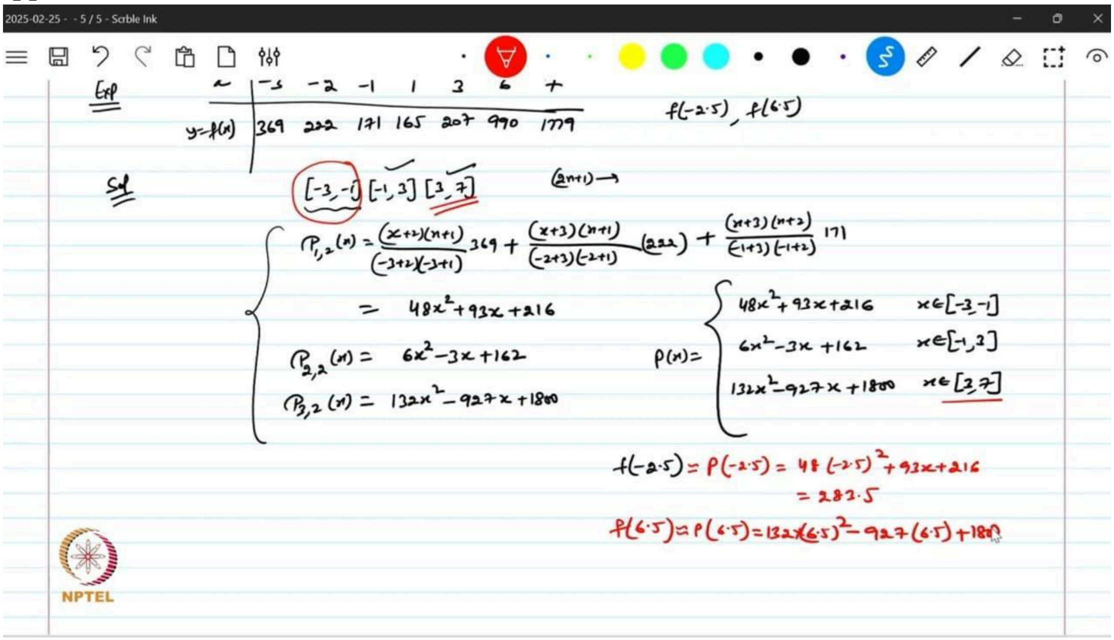
Similarly, I will find out p2, 2 the second. Okay, so what will we do in the second, we will calculate in this one. If I calculate in this one, then like this we will do our Lagrange's interpolating polynomial. I will take the points minus 1, 1 and 3. Okay, so in this case I can calculate it and I saw that the value is 6 x square minus 3 x plus 162. So, you can also calculate it yourself and it came out to be 32 x in the third one between three and 7. We calculated between 3 and 7. So, this is coming out to be 132 x square minus 927x plus 1800. So, we have three interpolating polynomials second degree and i.e. okay so if someone tells us that f which is an interpolating polynomial of second degree and if we have to write it, then how will we write it, so this will come to us, one which will come, if we write it like this, then p x will come here 48 x square plus 93 x plus 216 when x is in minus 3 to minus 1, 6 x square minus 3 x plus 162 when x is between minus 1 and 3 and 132 x square minus 927 x plus 1800 when x is between 3 and 7, so if we calculate it, then I hope that this will come, we will also verify it, okay so now this is our quadratic, so now if we calculate it, then we were told to calculate f at minus 2.5, so minus 2.5 will come in it in the first one, this one, so what will I do in this, I will approximate it, then this will come p at minus 2.5 is 48 into minus 2.5 square plus 93 x plus 216 this value will come so whatever value comes from here I will calculate it If I calculate it, then the value that is coming out is 283.5. Okay, I will calculate it like this f 6.5 6.5 will come in this. So, what will I do for it? Its approximate value p at 6.5 will go in this one, so it will come 132 into 6.5 whole square minus 927 into 6.5 plus 1800. So, this value that has come out, the approximate value that has come out is 1351.5.



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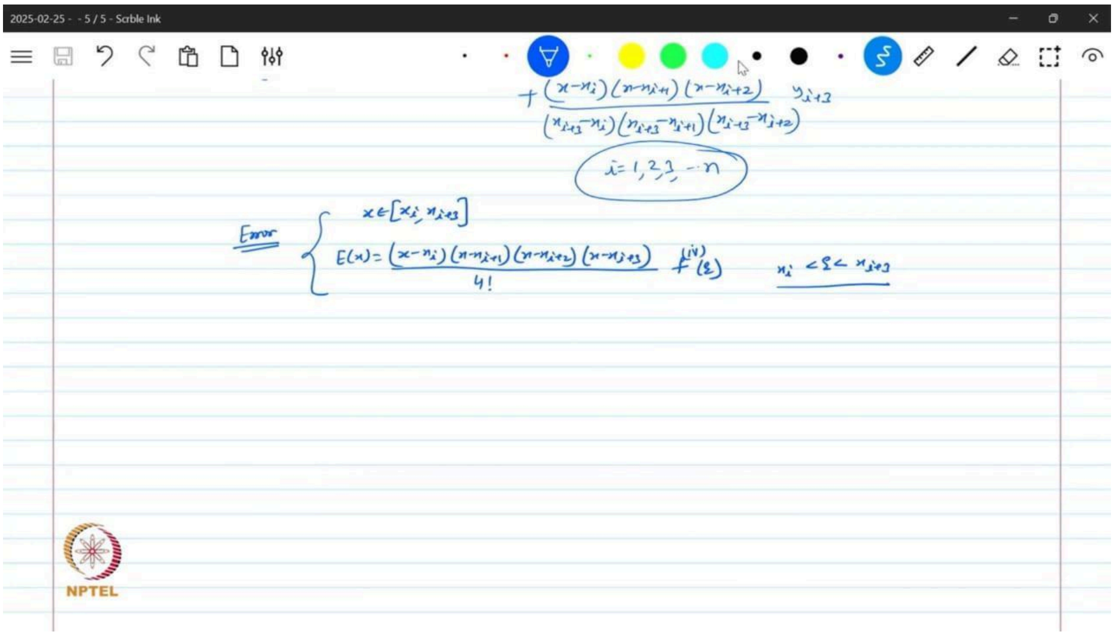
So, this is our interpolating value that we have found out, see, we did it from our 7 data point, but from the second-degree polynomial, we have approximated it as interpolating polynomial. In this case, we have polynomials quadratic, piecewise, what will happen if we go to piece wise cubic interpolating polynomial. Now see what to do in this. Now we need cubic. So for cubic, we know that the minimum that we have should be three points, there should be four points, so ours will come out like this from this. While doing this, okay, so what will we do now, the values that we have, x0, x1, x2, the values that are given, how much should they be? If I have these values, x0, x1, x2, x3, then this much value should be there. Then if I go to the next one, then x4, x5, x6, so how much will this be for the next one and x3 will be common, then the next x6 will be common, what will happen in this, x6, x7, x8, x9 will come out from x6, so what does it mean that we should either have four points, okay, or 7 points, or 10 points, so from this we should have points, so what do we have, I will say that if we look at it, I can say that if I have points here, then I can write it like this, x0, x1, x2 up to x3n, then times 3n will go, then if n is 1 then 1st, n is 2 then 2nd, n is 3 then 3rd, so it will be created like this, okay, now that we have these points, then we have a total of 3n plus 1, which are nodal points. If we have to apply cubic interpolating, piecewise, we should have points in this manner. So, if we have n and suppose I take 2, then how many points should there be? 6 plus 1 7 points from the piecewise, then what would be the points? x0, x1, x2, x3, x4, x5, x6 So in the first one, this will come, in the second one. So, on this, we get our two piecewise cubic interpolating polynomials. So, I can write the first one like this, x0 to x3, the sub interval will come in between this. The second one will come; the sub interval will come between x3 to x6 and x3 is the common node. Okay, so if we have to write it piece wise, then see how we can write it. So, if we keep writing in this way, then if I keep doing it like this, then I can write like this, any x i and x i plus 3. If this is any i sub interval, then I can write, we can write p i 3 x. So, this will become x minus x i plus 1, x minus x i plus 2, x minus x i plus 3, divided by x i minus x i plus 1, x i minus x i plus 2, x i minus x i plus 3, now this is done, okay plus the next one that will come is x minus x i, x minus x i plus 2, x minus x i plus 3 divided by now going to i plus 1 so we will take x plus 1 minus x i we will take minus x i plus 1 minus x i plus 2, okay and we will take x i plus 1 minus x i plus 3 and here y i plus 1 will come, okay so this is how we have this one next one that will come is x minus x now here i + 2 will go so i x minus x i plus 1 x minus x i plus 3 this will become divided by now i plus 2 plus x this will go to x i plus 2 minus x i plus 2 minus x i plus 1, x i plus 2 minus x i plus 3 and this is done and this will come y i plus 2, okay and the last one that will remain is x minus x i x minus x i plus 1 x minus x i plus 2 and this becomes our x i plus 3 minus x i, x i plus 3 minus x i plus 1 and x i plus 3 minus x i plus 2 and y i plus 3, okay so this is like this we will have to write a lot and the i in this case will come 1, 2, 3, up to n, so like this after writing all the values we will get polynomials okay and there will be polynomials which will be cubics and the nodes in between them we will know that the node value means our polynomial so depending upon where our x from where we have to do interpolation is coming to, okay and the error that will occur in this case we know that if we are going in this, going in the interval, that means x belongs to x i and i plus 3 goes up to 3 then what will happen is that we will have x minus x i, i plus 1, i plus two, x minus i plus 3, I divided it by 4 factorial and our derivative is the fourth derivative at some xi. It is okay and it depends on the interval. So if we now suppose that our xi will lie between x i and x i plus 3, then the error that we will get is this. So if we want to take an example of this thing, then we can calculate it by example also. And we just know that the number of points will be 3n plus 1 point, so it should be multiple of 3n. Only then we can calculate it. So now we have done this work.



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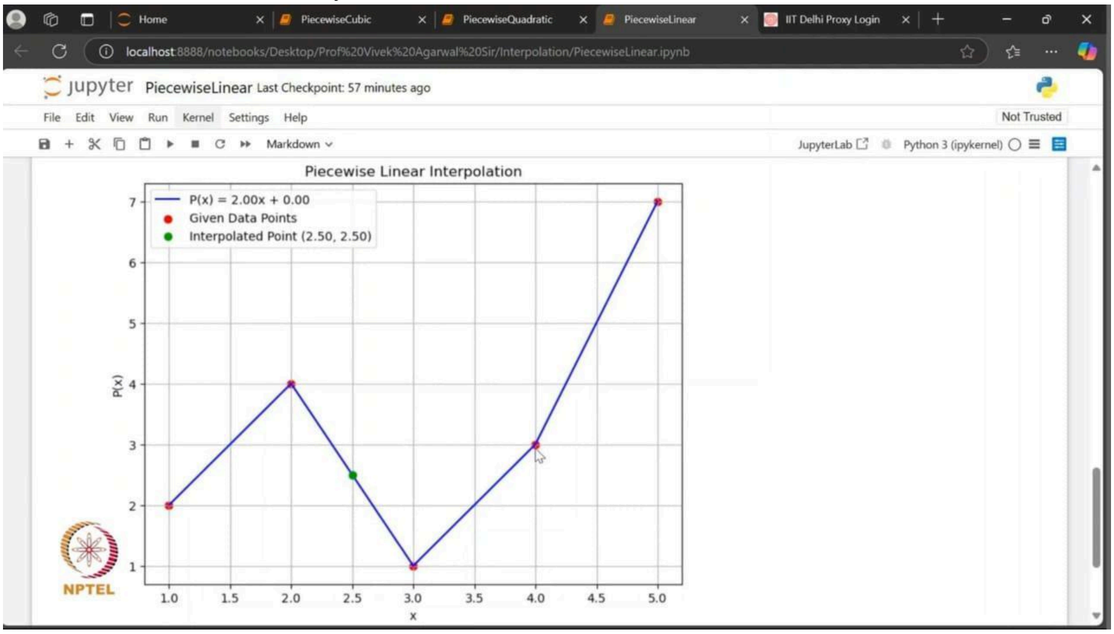
Let us see with the help of code what will happen if we write the code. So now we will check on what basis the polynomial that we have created, let us calculate it. So now I have named this code of ours, we have given piece wise linear interpolating polynomial, okay because it will be given in piece, so what we have to do is we have to give the x data and y data in this code and we have to give the point where we have to do the interpolation, okay so x data and this data should be sorted, there should not be any mixture, it should be in increasing order, okay so we have given the x that should be sorted like the x axis and corresponding y, okay so what have we done, we will define it from here, so in this we will define the polynomial in each sub interval, linear interpolation, we already know how to do it, we have already done it with the Lagrange's, okay so we will calculate it and plot it and we will also write the polynomial that we have, okay so now the values that we have, like we had taken an example, so let me take an example, let me first take this example, here our example is here So see, the number of points we had was 5, so four, the piece wise linear interpolation that we had came in the first one, i.e. 2 plus 2 x in the first one, i.e. minus 3x plus 10 in the second one, then 2 x minus 5 and then 4 x minus 13 and the sub intervals that we created were 1,2; 2, 3; 3, 4; 4, 5. So we plotted it like this, so you see what is happening in the plotting, first from here, in the first 2 sub interval, this polynomial is the interpolating polynomial, then in the second one, then in the third one and then in the fourth one and our 2.5 is being drawn in the second one, so the value there will be 2.5 only.



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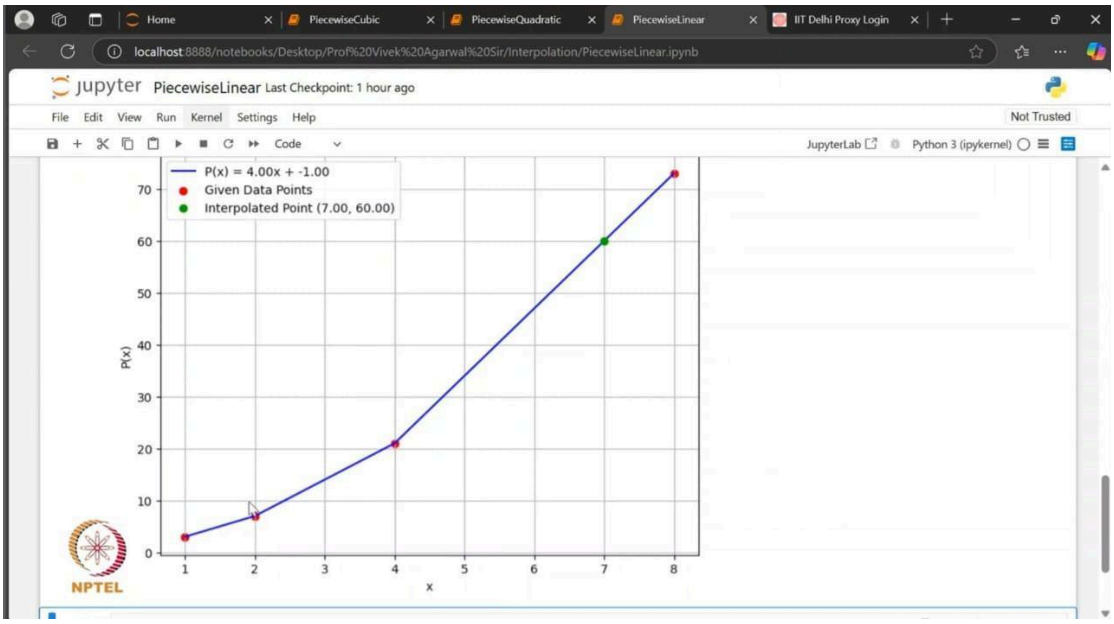
So, in the same way, suppose we have instead of 2.5, I make it 3.5. So, by making it 3.5, the polynomial will not change, this value of 3.5 came as 2, the value came, okay, so here, if we find the value at 3.5, then in this way, which we can do piecewise linear interpolation. So, the second example that I just took, if I copy this, Control V. Okay, this point, so let's say it's a point, we can use it anytime. So, we wrote in it 1, 2, 4, 8 we had defined 4 and 8. Okay. And the second one we had defined, its values were 3, 7, 21 and 73, okay. And the calculation that we have to do is x, suppose I take x as 3, I calculate it, then we will get this polynomial. And if you remember, we had calculated this, that 4x minus 1, 7x minus 7 and 13x minus 31. If you see, we had calculated this, where is it? So we had calculated this, okay, so the code that we have done has also given the same values and this is our piecewise interpolating polynomial. Now we have on 3.5 values were calculated, then 14 came and from there also 14 came, right? So, we calculated from three. Now the second value that we had calculated was for seven. So, I will write it as 7. So, we did 7 and ran it and the value on 7 60 came out. That was our value. So, if we see, this piece wise in every sub interval is linked with each other and the continuity is maintained here. So, this piece wise calculation of ours was done for linear interpolating polynomial.



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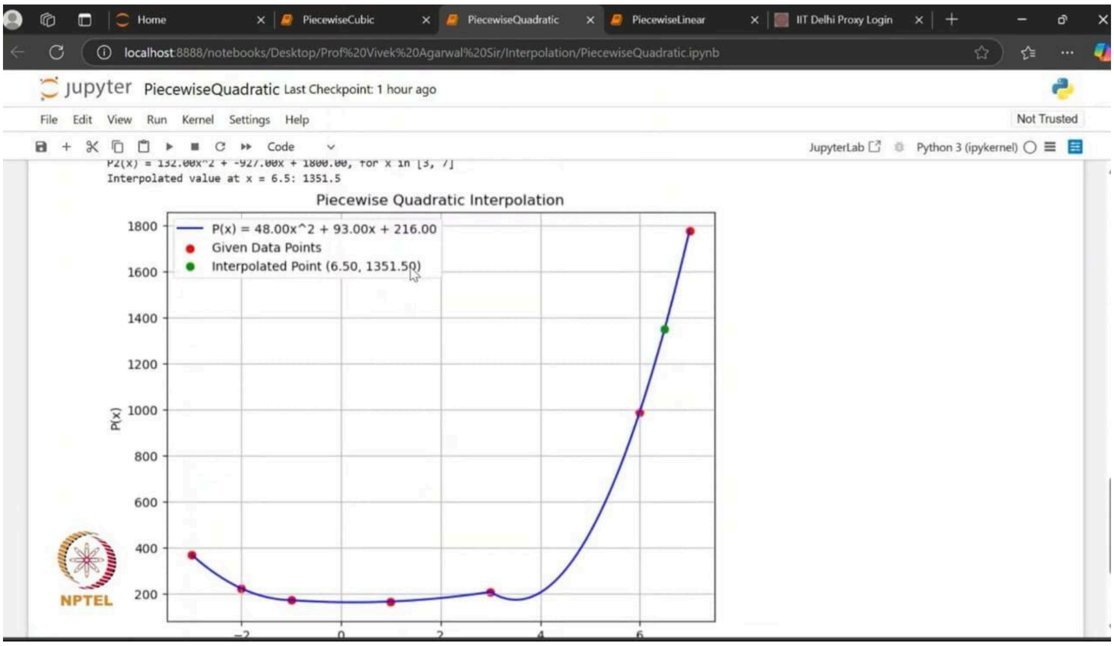
So, in quadratic we know that the calculation is a little difficult, but we did quadratic in the same way for quadratic and we took points in quadratic. So, we know that we have to take odd number of points. So, we took odd number of points and ran it. We saw that there are five points, so two will come to us. So, these two were piece wise. We have the quadratic polynomial and 2.5 brings it in the first one, so the value came in the first one and this came to us. Okay now see, the common node was 3, so the quadratic that came from the first on three and the quadratic formed in the second, they are showing continuity here with each other, but if seen, the function is not differentiable on three in this case, but we mean we have not done anything that we should get differentiable, okay so what will happen in this case is that the function is maintaining continuity and we have the polynomial that we have, so let me try this with some data that we had done and what we had defined was minus 3, minus 3, minus 2, minus 1 and then one, then three, then six and from there we had taken these data points, okay and corresponding values 369, 222, 171, 165, 165, 207, 990 and 1779. We took this, okay and after that we calculated minus 2.5. Now let's see, let's run it. The polynomial that we have is this. So, if we see, 48x square 93x plus 216 is the first one correct. The second one is coming 6x square plus 3x plus 162, this is also correct. And the third one that came was 132 x square minus 927x pus 1800. So, if we look at it, this is the value that we got. We had calculated it. So here we have this quadratic polynomial, and at minus 2.5 we will get 283.5. And we calculated it with the help of this, so we got minus 283.5. So, look at this, we have a quadratic polynomial, first this one is going till here, then from here to here, then from here to here, then from here it went to here and from here to here, and from here to here, like this we have a quadratic polynomial and the minus 2.5 will come in the first sub interval. Now I will change it to 6.5. So, if we see 6.5, it comes here. So, if I calculate it, it comes out to be 1351.5, and this is what we should get.



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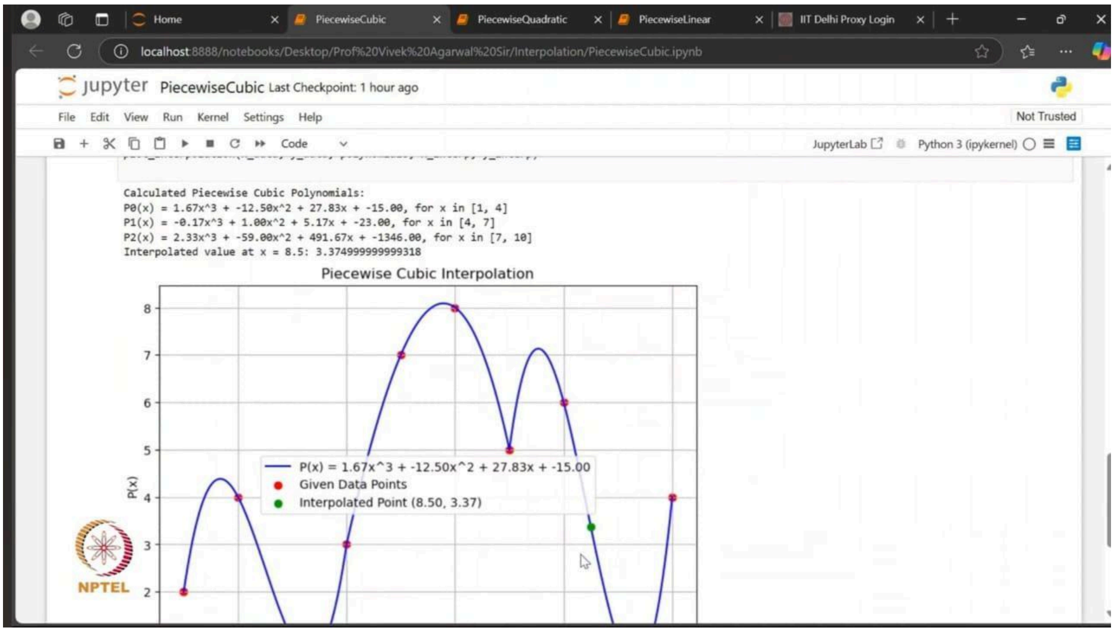
If we see according to this, we have got the piece wise three quadratic polynomial. We have calculated the value that we have. So, it has become quadratic. Now we have it for cubic. So, in cubic we have seen that the number of points is three times. So, for this, if I take the values that we have, see, if we take 2 plus 2 4 5 6, 7 then we have 7 points. So, it means 3 n plus 1. So, it should be like this. So, if I take n, it should become 7. So, we took this value and, in this approximation, we have two. In this, we have an interpolating polynomial. The polynomial will come cubic, this is here, the first one is from here to here, so now look at this, we need four points till here, so from 1, 2, 3, 4 we need 4 points and in the four to the second one, this four here is a common node, so if you see on the common node, this continuity is maintained, after this we know that the function is differentiable, it is not at this point, okay, but the continuity is maintained here very well, so we saw the value at the interpreting point, it came 5.31 and I am bringing it to the second one, okay, similarly we have any values, now I will take this because the first sub interval will be from one to four, so if I take 2.5 in the first sub interval, then I will see in the first one, it came here, so it came 2.5 on 2.5 only, okay so this is our cubic interpolating polynomial will come between one to one to four and the second interpolating poly. Now between four and 7, I can increase it like this. I can make it three. So, to make three, in the space here, 8 9 10, we have put these points, so we have 10 points. So, I took the value of this. I took 6 from here. Okay, suppose here i took 1 I took it 4 here. Okay, so we also got the value of 10. Now when I calculated this, we got three interpolating polynomials. And there are three interpolating polynomials. So, the first one is here, then the second one is still here and the third one is here. So, from here we came to know that the 7 point is at the corner. Corner means that this function is not differentiable. Okay, but this polynomial, we have three polynomials. And we have three polynomials. So, we have 2.5, we had already seen that the value of 2.5 is coming for 2.5. But this is that I can also calculate further, so I saw at 8.5, the value came to 3.37; here it is lying here and it came to 3.37, so with the help of this we can find out the cube, so we have come to know that we can use the linear piece wise, we can use quadratic piece wise and we can do cubic.



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So today in today's lecture we saw that if the data points are too many and we do not need a high order polynomial of degree, the interpolating polynomial, then one of its alternatives is that we can find out a polynomial in piece wise piece and the polynomial that we will have can be linear, it can be quadratic, it can be cubic, and we have also seen from the python code that what we had defined, what we had calculated, is coming out, so with the help of this we can define piece wise interpolating polynomial of any degree. If you can, I hope you understood today's lecture and liked it, and thank you for watching it. Hello