Introduction to Probability & Statistics
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Week - 3
Lecture -10
Independent Events

Toh ek aur example dekhte hain jisme hum independence ka istemaal karke probability calculate karte hain. Pehle wale example me humne coin toss ka experiment dekha tha, lekin ab hum consider karte hain ek aisa coin jisme probability of head p hai (fair coin zaroori nahi). Toh tail ki probability hogi $1-p$. Ab experiment yeh hai: coin ko baar-baar uchhalo jab tak pehli baar head na aa jaaye. Is experiment ka sample space hai: H, TH, TTH, TTTH, … yani ke sabhi sequences jisme last toss H hai aur usse pehle sab T. Ye countably infinite sample space hai. Independence ka istemaal karte hue har outcome ki probability nikal sakte hain, kyunki har toss dusre toss se swatantra hota hai. udaharan ke liye, T,H ka matlab hai pehle toss me tail aur doosre me head uski probability hogi $(1-p)\times p$. Isi tarah T,T,H ki probability hogi $(1-p)^2\times p$. General term: agar $n-1$ tails aur phir head aata hai, toh probability hogi $p(1-p)^{(n-1)}$. Ab hum event define karte hain Ei = "exactly i tosses are needed." Matlab first $i-1$ tosses T aur i-th toss H. Is event ki probability hogi $p(1-p)^{(i-1)}$. Ab sawaal hai: "probability that at least 3 tosses are needed." Iska seedha matlab hai Ei for $i \geq 3$ ka union. Direct sum infinite ho jaata hai, isliye hum complement event dekhte hain — i.e., at least 3 tosses needed nahi honge. Iska matlab hai H ya TH. Toh $P(E1) = p$, $P(E2) = (1-p)p$.

Toh required probability hogi $1 - (p + (1-p)p) = (1-p)^2$. Yani pehle 2 tosses me T,T aana zaroori hai. Independence ki madad se yeh simple form mil jaata hai. Ab ek aur example dekhte hain — ek electrical system jisme current left se right tak flow karta hai. System me 3 identical components hain (1, 2, 3). Har component failure probability 0.1 rakhta hai, aur sab completely independent kaam karte hain. System tab fail hota hai jab upper circuit aur lower circuit dono fail ho jaayein. Diagram ke hisaab se upper circuit fail tab hota hai jab component 1 fail ho jaaye. Lower circuit fail tab hota hai jab component 2 ya component 3 fail ho jaaye. Yani system fail event $F = A1 \cap (A2 \cup A3)$. Distributive property se $F = (A1\cap A2) \cup (A1\cap A3)$. Dono events mutually exclusive nahi hain, isliye union formula use karna padega. Let $B = A1\cap A2$ aur $C = A1\cap A3$. Toh $P(B)=0.1\times 0.1=0.01$ aur $P(C)=0.01$. Ab $B\cap C = A1\cap A2\cap A3$, jiska probability hai $0.1\times 0.1\times 0.1=0.001$. Toh $P(F) = P(B)+P(C)-P(B\cap C) = 0.01+0.01-0.001 = 0.019$. Matlab system fail hone ki probability 0.019 hai. Isliye system work hone ki probability hai $S = 1-0.019 = 0.981$. Yani lagbhag 98.1% chance hai ki system theek chalega. Is tarah independence aur probability properties ka istemaal karke hum complex systems ki reliability nikal sakte hain.