Mathematics Optimization

Prof. Debjani Chakraborty
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture - 38

Dynamic Programming Problem (Contd.)

Today, we are dealing with the dynamic programming technique, and Dynamic Programming Problem can also be named as the multi staged decision making process or the recursive optimization process. The whole dynamic programming problem, the basic philosophy behind is that, any complex optimization problem where the number of decision variables are very high. Not only that if we need to take the decisions at different stages, we decompose the whole problem into several parts, and which have been solve sequentially.

And starting from one stage, we will move to the other stages and we will move further, and these are all interrelated, individual optimization problems and those are solve with optimization technique.

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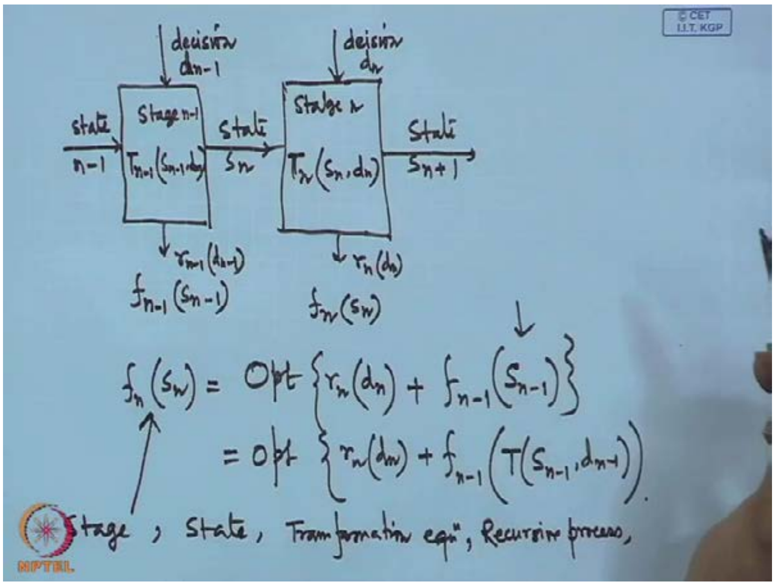

That is why with the diagram we can just explain in this way, as I explain the dynamic programming technique could be the forward recursive optimization technique we can adopt, or we can adopt the backward recursive process. If we just discuss the forward recursive process that means, I am in stage n minus 1, now here the inputs are the for every stage, as we know there are several states. That is why, if I consider the n minus 1 is state and the corresponding decision is d n minus 1, then that state transformation equation is there, which is the function of S n minus 1 and d n minus 1.

And for from this stage, if the state is n minus 1 there are several stages and then, we can have the immediate return that is r n minus 1 d n minus 1. Actually dynamic programming technique, this is the essential feature of dynamic programming technique that we have dividing into different stages. Now, for every stage there are number of state variables, and state variables are responsible to explain the stage, the corresponding stage completely.

And state means for every state there is a list of decisions decision is there, and we will incorporated the state as well as the corresponding decision; and the state transformation equation t n minus 1 is responsible to convert this state to the next state n. So, that we can move to the next stage n, that is the basic idea of the multi stage decision making process. Now, whenever we have considering state n minus 1 here, we need to calculate this is an individual optimization problem.

That is why we need to calculate the corresponding return, that optimal return I am gaining from the stage f n minus 1 s n minus 1 and this value is important, if I am moving to the next stage n. Because, it is not important for stage n, how we have reach to this value only this value is important, thus in stage n as well we have the state transformation equation T n, that is the function of S n and the decision d n.

Because, in stage n there is another possible decision are there, at the state transformation equation T n is again converting this state to the next state n plus 1, that is the idea for multi stage decision making process. And for every stage, there are alternative decisions are there not only that, for every recession how much return I am gaining that information we need. And we need we have to calculate the optimal return from the stage, and we are following the recursive relation for this.

And this recursive relation as we know this is equal to f n S n is equal to optimal over S n, and we are considering the immediate return of this. And the previous optimal return from the previous stage that is f n minus 1, that is important for the forward recursive relation. In other way also we can write it, optimum r n d n plus f n minus 1 and this S n minus 1 we got it from the previous stage, that is why there is a state transformation equation, that is T s n minus 1 d n minus 1.

That is the basic idea for considering the optimal solution of the current state, this is the we have consider the forward recursive relation, I am calculating the optimal value for the n th stage, which is depending up on the optimal value of the previous stage. That is the principal of optimality as I explain before, this principal of optimality says that whenever we are in the current state, that is only depended on the detail about the current state.

That how many state variables are there, how many decision alternative decision we can take in that stage and not only that, we need to depend on the information about the optimal solution of the previous stage. We do not have to have the information how we have reach to that point, what is the path up to coming up to that point that is not needed, that is the beauty of the decision dynamic programming problem. Once we are breaking into several parts, we can concentrated on the individual part, and we have to take decision upward that part only.

Now, here the things we need to understand, we need to discuss that how to due decompose the problem into different stages, that is more important for us, this is first question, first key feature of the essential feature of the this dynamic programming problem. Once is state is define, next come how to define the state variables that is more important, because for each stage there are number of states, and the number of states are being mathematically representative the state variables, that is why this is very important.

But, how to select the state variables there is no hart and first rule, this is from the experience only we have to formulate the state variables. This is the next, the other thing we need to know what is the transformation equation, I have already discussed in the last class, transformation equation. And we have to formulate recursive process, we have to decide whether this is forward recursive process, or backward recursive process, accordingly we have to formulate the recursive relation, that is the mathematical equation we need to formulate.

Now, here one thing is very important, whenever we are taking the decision just look at the variable d n, d n can be discretion, d n can be continues even that means, d n there is a domain of d n. In the domain of d n is discrete, how to handle that, how to construct the stages, how to construct the states, I have already discuss before. Now, coming to the next when the decisions space is continues in nature, how to handle that kind of problem, that part now I am going to discuss, but with some example I will explain to you, so that it would be clear.

Now, once we are taking the decisions space as continues, then we are saying that individual stages are the individual optimization problem, thus the class since it is continues. Classical optimization techniques are very much applicable in individual stages, because the spaces the functions are continues in that, and continues the differentiable as well, that is why let me check the simple model first, that is a model 1.

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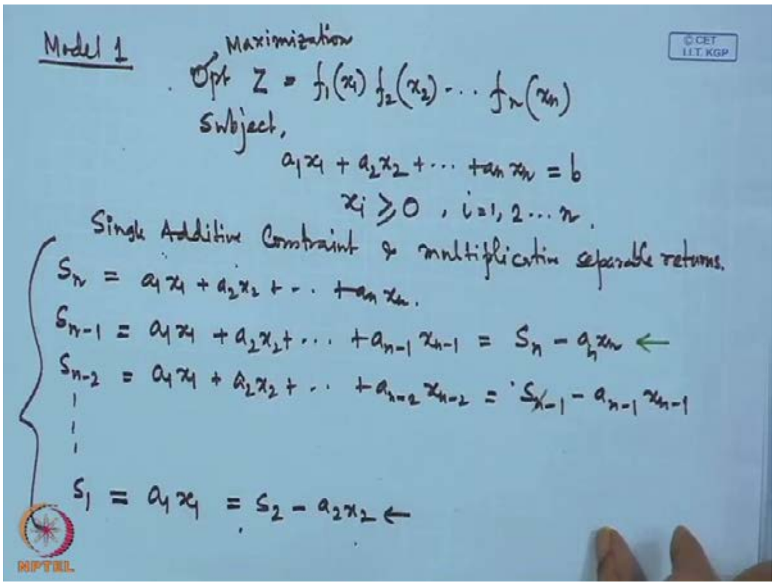

Where we are considering the objective function as f 2 x 2 up to f n x n, where x 1, x 2, x n these are the variables involved in the decision process. Now, this is a multi varied optimization problem, subject to say and it is also given that x i greater than equal to 0, where I is equal to 1, 2 to n this is our problem. Now, we can divide the whole problem into n stages, because n decision variables are involved, and by looking at the pattern of the model, we can divide into n parts.

And for individual stage we can define the state variable and we will solve it, let us see how we can do it, now we have taken, single additive constraints and multiplicative separable returns. One thing you have to keep in your mind that, if the objective function as well as the constraints are not, we are not able to break it properly, then better not to use the dynamic programming technique. Because, decomposition is very important at the decomposition, we have to keep in our mind, that for every state we should declare the corresponding state variable. If it is possible then only apply the dynamic programming technique.

Now, let us consider there are n stages and the first stage when n is equal to n, and the corresponding state variable, we can declare as S n as a x 1 plus a 2 x 2 up to a n x n. Let us move to the next step S n minus 1 and let us consider in this way a 1 x 1 plus a 2 x 2 plus a n minus 1 x n minus 1. Then, we can write this form as S n minus a n x n that means, if we are in the n stage we can reach to the n minus 1 state with this relation, thus this relation is the state transformation equation; and this is depending on the value of S n as well as a n x n.

Let us move to the next, S n minus 2 this is equal to a 1 x 1 plus a 2 x 2 up to a n minus 2 x n minus 2, certainly again the state transformation equation is S n minus 1 a n minus 1 x n minus 1. Similarly, if I just proceed further they will get S 1 is equal to a 1 x 1 and this is certainly dependent on the previous state, that is S 2 minus a 2 x 2 and this is the state transformation equation. If we consider the state variables like this, and if we consider there are n stages, now we will look at the objective function.

Let the objective function optimality we wanted to achieve as a maximization problem, it could be minimization as well the same process is there, same process means only the difference will be in the consideration of the objective function. But, the state variable or the stages definition you may consider as same as this one, then up to this once again I am just saying that, this is the problem of n variables, we are trying to decompose the problem into n stages.

For individual stage there are state variables and S n is the state variable, and you could see that S n is moving in the continues space, because x i’s are continues x i can be positive, it could take any value, that is why s n moves in the positive. Whatever alternative decisions, we are taking d n these are all the continues, and the positive values; similarly the this is the state variable in the next state that is n minus 1, this is also continue, these are all continues.

Now, we have form the stages, we have formulated the state variables, we have define the state transformation equation for individuals states. This transformation is responsible to move to the next state that means, it is depending on the previous state in this way.

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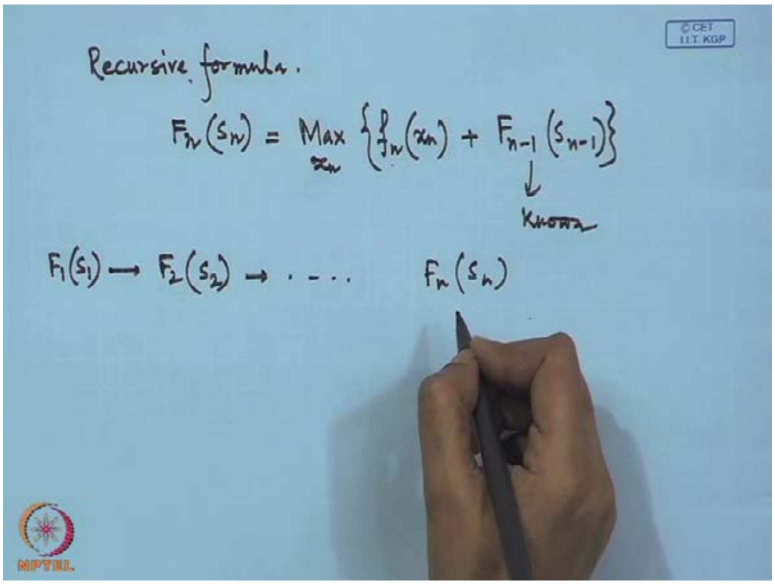

Then, we can formulate the recursive formula in this fashion that means, return from the n th state is equal to maximization of F n x n plus F n minus 1 S n minus 1, that is optimal solution, this is known to us, that is why these are only variable for us. And we can construct very easily, starting from F 1 S 1, F 1 S 1 is very easy to select, because it contains only one variable F 1 S 1 once we will it we know it, we will move to F 2 S 2. That means, we are doing the forward recursive process F 2 S 2 again would be the maximum of F 2 x 2 plus F 1 S 1.

In this way we will move to F n S n, and this is the final result, whatever optimal we are achieving here that is the optimal solution for the whole problem. That means, in each time the optimization occurs in the single variable, how it is coming as a single variable; let me take one simple numerical example then it would be clear to you.

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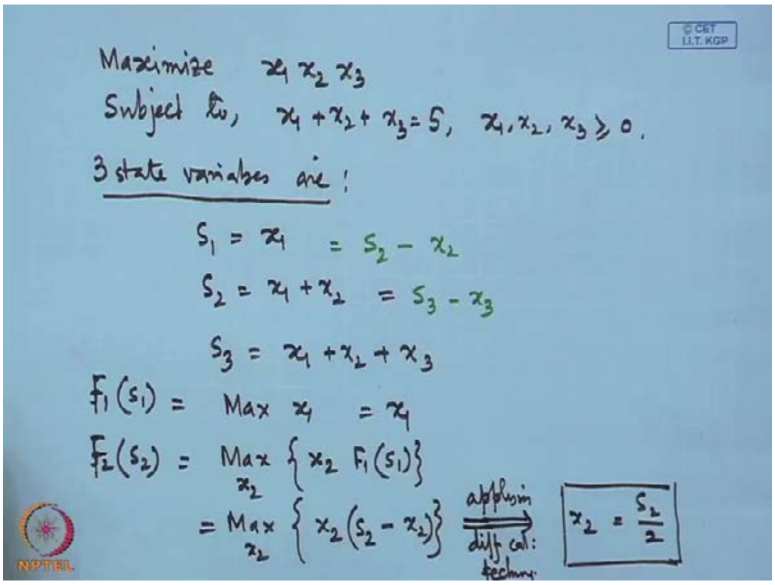

Corresponding to this model, where we are considering the single additive constraint and multiplicative separable returns, there are three decision variables are there, x 1, x 2 and x 3. We wanted to maximize x 1, x 2, x 3 subject to x 1 plus x 2 plus x 3 is equal to 5 and x 1, x 2, x 3 these are all positive values. Now, we need to solve it, dynamic programming technique is very nice, technique to solve this problem by confronting this problem into several three stages.

We will start from the first stage considering x 1 only the variable x 1, we will move to the next stage that is x 2 in this way we will proceed, for that thing we need to consider the state variables. Therefore, let us define three state variables, as S 1 is equal to x 1, S 2 is equal to x 1 plus x 2, S 3 is equal to x 1 plus x 2 plus x 3, these are the variables we are considering; as we did for the previous general model, the same model it is. If this is so we can write it as just see, these are the state transformation equation S 3 minus x 3, because x 1 plus x 2 comes from here.

And if I just move to the next, this is equal to S 2 minus x 2 this state transformation equations the very important for doing the optimal solution. Now, our first optimization problem f 1 S 1 is equal to maximization of x 1 only, x 1 is positive that is why we can consider x 1 and this is related with this relation. Let us come to the next stage F 2 S 2, I should consider capital, because in the model we have considered capital, F 2 S 2, if we consider this is equal to max only x 2 is varying x 1, and this is x 2 F 1 S 1.

And what is the value of F 1 S 1 this is the value is x 1, thus we can write it as S 2 minus x 2, now the problem of single variable, now we have to maximize over x 2, that is why we can apply the differential calculus method. Differential calculus technique tells us that, we have to differentiate this function with respect to x 2 once, and we will equate to 0 and we will get the corresponding optimal solution. And we will go for the second order derivative, in the second order derivative whatever not optimal, whatever point we got, stationary point we got we need to check whether this is maximum or not.

For if the second order derivative is coming negative, then it is maximum if it is positive you can minimum, that is why if we apply the differential calculus technique. We can do it we will get x 2 is equal to S 2 by 2, that is very easy to say, because past with respect to x 2 it is S 2 second is the 2 x 2 equal to 0; from here we are getting the x 2 value as S 2 by 2, that is the optimal in the second stage. Now, we will move to the next stage and was we know the principal optimality it says that.

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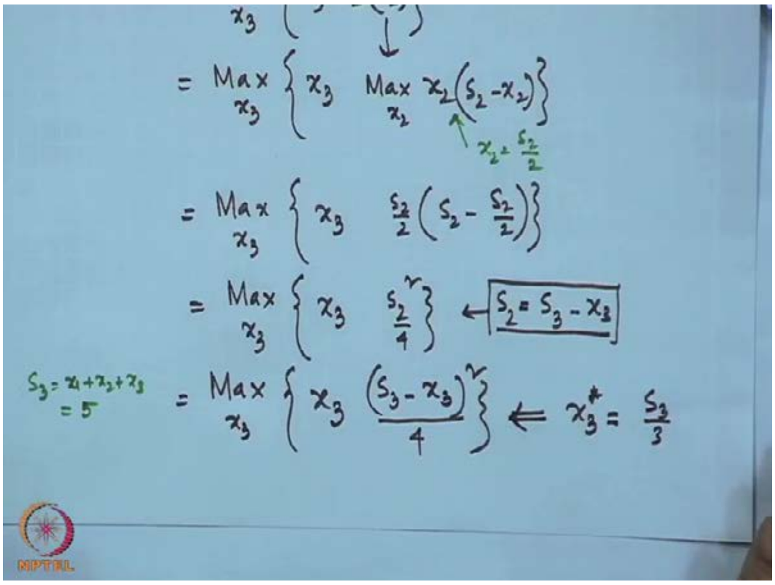

If I move to the next stage that is F 3 S 3, then the recursive process is that maximization over x 3, x 3 F 2 S 2 and F 2 S 2 just now we got this value, that is why here also we can write down the same. And we will just convert this optimization problem with the single variable, how just see what is F 2 x 2, F 2 x 2 was the maximization over x 2, this part I am just writing, this is equal to x 2 S 2 minus x 2. Just now we have seen that maximum occurs when x 2 is equal to S 2 by 2, that is why let us substitute here this value, then we will get maximization over x 3 x 3 and this is S 2 by 2 S 2 minus S 2 by 2.

This is the now, it is the problem of single variable and it is coming maximization of x 3 S 2 square by 2, and this is S 2 square by 4 that means, S 2 square by 4 we are getting. And just now we got the relation that S 2 is equal to S 3 minus x 3 we will substitute this fact here, then we will get maximization what you could see here, that the optimization problem is stage three, it is converting to the optimization problem of single variable x 3 only. And what is your S 3, S 3 is equal to x 1 plus x 2 plus x 3, this must be is equal to 5, because that is a constraint we are having, that is why we have reach to the final stage.

And here this is the problem of single variable, again the same technique differential calculus method, we will apply here. And we will get the relation here by applying the differential calculus technique, we will get x 3 is equal to that is star, that is optimal equal to S 3 by 3.

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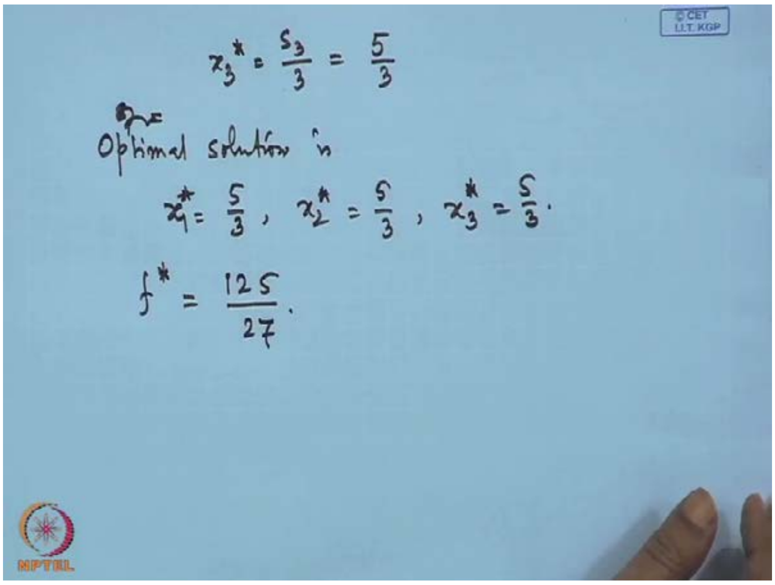

Now just now we have said that, x 3 means x 1 plus x 2 plus x 3 that the value is equal to 5, because that is the given constraint for us, thus we can say that this is equal to 5 by 3. One we have risk to this point S 3 is equal to 5 by 3, we have the relations with us that is S 1 is equal to I need to write again, just let me show you ((Refer Time: 25:14)) just now we have a S 1 is equal to x 1, S 2 x 2 this relation and from this relation by substituting the value of x 3, as we got this is equal to 5 by 3, that is 5 minus 5 by 3, we will get S 2.

Once we are getting S 2 after substitution, we will get the value for S 1, in this way we can say that the optimal solution is x 1 is equal to 5 by 3, x 2 is equal to let me put star here, because these are optimal values 5 by 3. And x 3 star is equal to 5 by 3 with objective function that is f star it is equal to these value, because value is x 1, x 2 and x 3, that is all about the model I wanted to save for the problem of the single additive constraint, and multiplicative return.

And we have applied the dynamic programming technique, thus formally we can say that, we can summarize the process, the dynamic programming process as this one.

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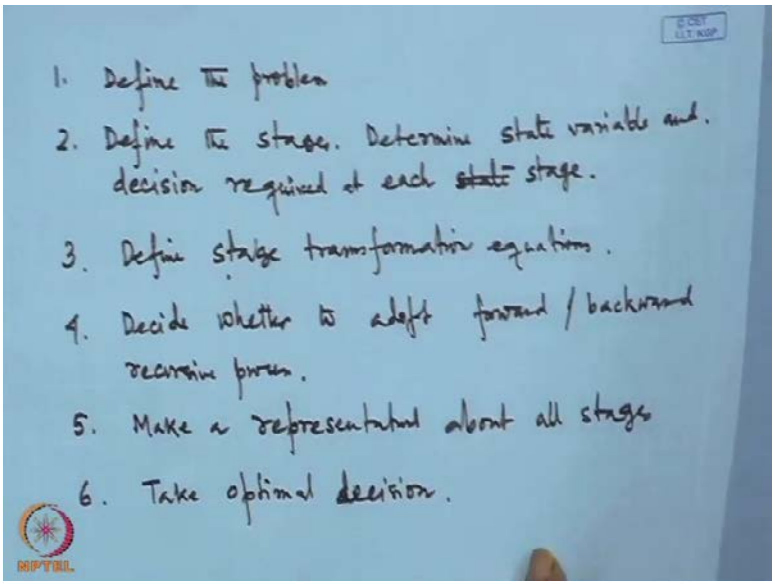

The first step is the define the problem means, specify the objective function and the constraint that is our problem, then decompose the whole problem into several stages. One stage is I have fixed, we need to determine state variables and we have to fix about the decision required at each stage, rather states we will move to the third, we will specify the relation that how one stage can be move to the other stage rather. One state of one stage can be move to the other state of the other stage, that is why we need to define the state transformation equations.

Once we are forming the state variables, from there we can formulate the state transformation equations as we did there, because we have the state transformation equation in this case, that S 1 is equal to S 2 minus x 2, S 2 is equal to S 3 minus x 3, except the these are the state transformation equations. And then, we have to decide whether we will adopt the forward recursive relation or the forward recursive process, that is why decide whether to implement forward or backward recursive process.

Then we have to reach to the last stage, and after that we have to take the decision about the whole problem that is it has been said that, after reaching to the last end, make a representation about all stages, rather the optimal solutions of every stages. And then, take optimal decision these are the steps, now that much I wanted convey regarding the continues, no I will take some other models as well, but before to that depending on this model, let me consider one problem here, which contains uncertainty as well.

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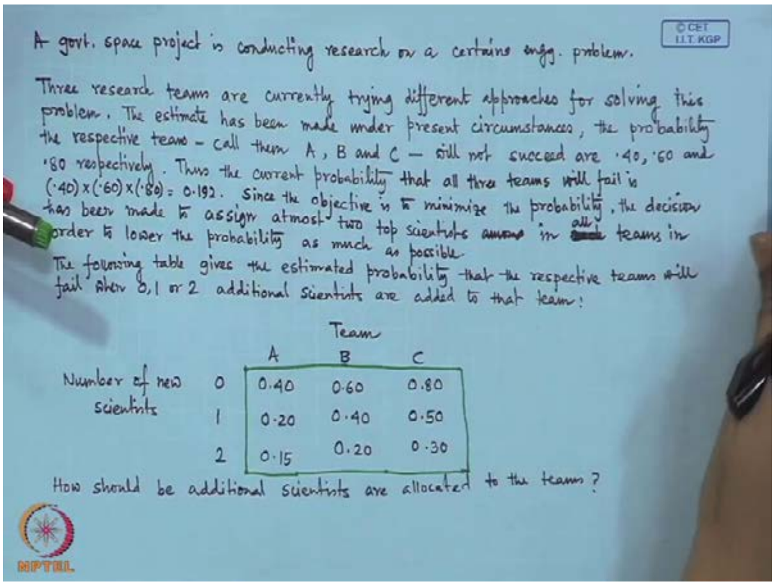

But, the problem is of similar type we have the single additive constraint, and multiplicative severable returns, this is the problem for us. And here the probability factors are involved in the process, just look at the problem, a government space project is conducting research on a certain engineering problem. Now, three research teams are currently involve for solving the research problem, now the estimates are in the present circumstances, actually the estimates regarding their failure of the respective teams are given.

If we consider the teams as A, B and C respectively then, the we have the chance figure, so that the corresponding team will not succeed are 0.4 0.6 and 0.8 respectively. Once we have this information that means, at the current state if the organization wanted to involve this teams, the same teams in the project. Then there is a probability of failure that is the multiplication of three failures together and it is coming 0.192, now the organization wants to reduce these value, thus they wanted to introduce few new scientist in this process.

And at the most they can add two scientists in three teams, and if they want to introduce three scientists here, what are the probability rest for estimates that given here also, it is says that. Since, the objective is to minimize the probability the decision has been made you assign at most two top scientists in all teams, in order to lower the probability as much as possible. The following table gives the estimated probability that the respective team will fail, when 0 scientists are being added that means, the current situation 0.4 0.6 0.8.

If one scientist is added to team one, then the failure will reduce to 0.2, if one scientist is added to team B 0.4 reduce to 0.4, 0.8 is reduce to 0.5. Similarly, if two additional scientists are added, then team A the situation is little better, the probability of failure is coming down 0.15 0.20 0.30, but the situation is such that, we wanted to reduce the probability of failure. But, total number of scientist are two, therefore which scientist to be allocated, the additional scientist to be allocated in which team that is the question for us.

Thus this is the objective of the problem, how should be the additional scientists are allocated to the teams, so that the failure rate will decrease, that is the whole problem. If we consider this problem, then again this problem can be solve with the dynamic programming technique, and dynamic programming technique will convert the problem into several stages.

Here the stages are being considered by considering one team at a time, first it has been considered say team C. And we will analyze the situation, we will get the optimal decision if we consider team C, then we have move to team B, then we have move to team A, in that way the stages are being constructed thus.

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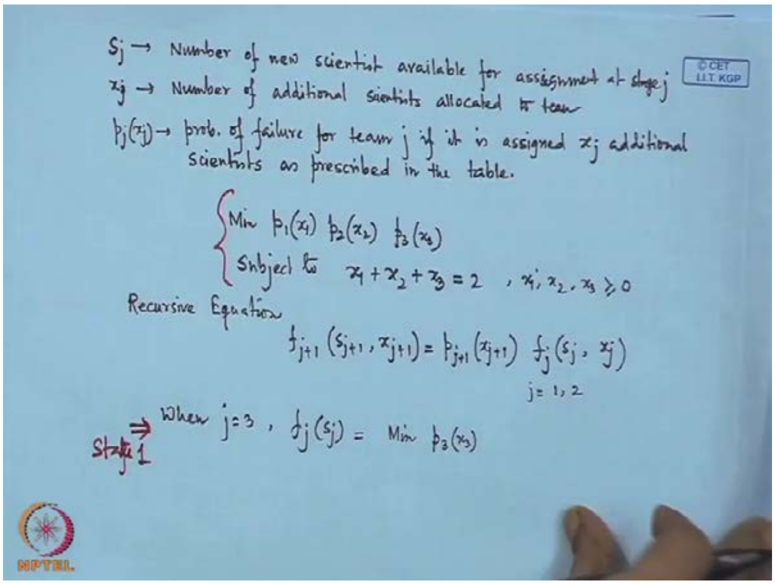

We can say that, this is the problem for us, that is minimization of the total probability of failure that is coming, because these are all independent evens, that is why the total probability of failure will be multiplication of all events. And p 1 x 1 is the probability of failure for team one, if it is assigned x j number x 1 number of scientist, x 1 can be adding from 0 1 2. Here x 2 is the probability of failure for team two, if x 2 number of scientist are being allotted, and x 2 can again take the value 0 1 2, similarly p 3 x 3.

And there is a constraint the total number of scientists must be two, that is why x 1 plus x 2 plus x 3 equal to 2, x 1, x 2, x 3 these are all positive values certainly and there is another constraint that these are all the integer values, that is why this is the integer programming problem. And we can solve it with the dynamic programming technique very easily, now you see we have consider the states S j as the state variables x j at the corresponding decisions. Thus we can have the recursive equation that j plus 1 th optimal decision for j plus 1 th state is depending on the j th stage.

If we start from j is equal to 3, that is if we consider the last team that is team C as the first stage, because we wanted to move backwardly, in the backward recursive process. Therefore, f j S j is equal to minimum of p 3 x 3, here only one variable is involve that is x 3, these are the decision variables and S j are the state variables for this. Now, this is the problem for us, now this is the first, this is stage 1, let us stage 1.

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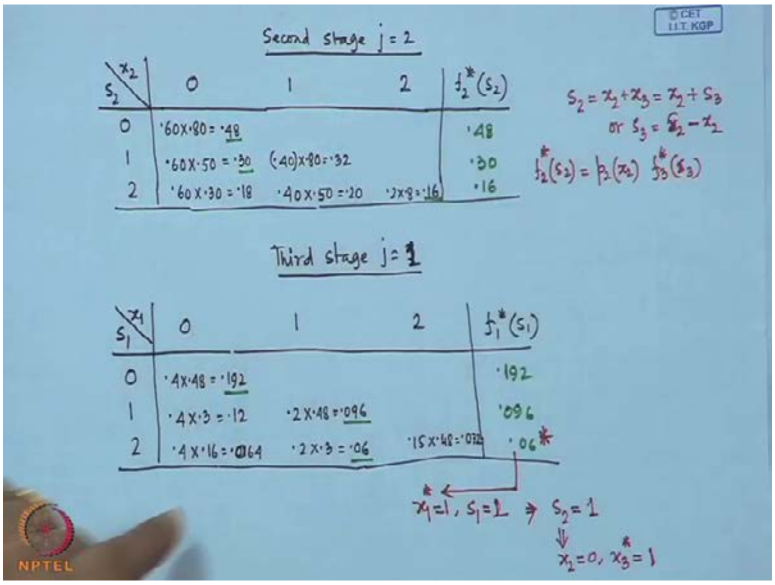

Let us move to the next stage, that is stage 2 and look at the problem, we are adopting the backward recursive process, that is why j equal to 3 we consider that was stage 3, that was not stage 1. Now, coming to the next stage that is second stage j is equal to 2, these are the state variables 0 1 2 and x 2 is the decisions we can consider either it could be 0, it could be 1, and it could be 2.

Now, if x 2 is equal to 0 that means, we are not including any new scientist in second team, but S 2 is equal to 0, how S 2 is been constructed S 2 is equal to x 2 plus x 3 that means, S 3 that is a state transformation equation, S 3 is equal to S 2 minus x 2, as we did for the continues case. The same problem occurs here, then we can calculate the probability value, when x 2 equal to 0, S 2 equal to 0 that means, no team is getting team B or C they are not getting any new scientist.

Then the probability would be the multiplication of the corresponding probability 0.6 and 0.8, for 0 addition there is coming 0.48. Now, for the next when x 2 is 0, but S 2 is 1 that means, team C got one scientist, but team B did not get any scientist. That is why by considering the probability value, we have to calculate the probability that is coming 0.6, that is team B is not getting any scientist, but team C is getting one scientist that is 0.5.

But, in the next, if we consider the team C is getting two scientist, team B is not getting any scientist in the probability would be 0.6 into 0.3, that we have calculated, just see 0.6 into 0.3. Come to the next x 2 is 1 S 2 is 0, not possible, because S 2 is equal to x 1 plus x 2, x 2 is 1 S 2 is 1 that means, team B got 1, team C did not get anything, that is why 0.4 into 0.8 that is equal to 0.32. In this way we completed, they completed the whole, similarly for x 2 is equal to 2, S 2 equal to 0, S 2 equal to 1 is not possible S 2 equal to 2, is only possibility team B is getting two scientists, but team one is not getting any scientist.

That is why up to this whatever analysis we have done, we have analyze the situation for team C in the first stage, then team B and C together rather, in the next stage. And we can get the corresponding optimal solution, this is the state transformation equation f 2 S 2 is equal to p 2 x 2, certainly this would be minimization, minimization of p 2 x 2. And p 2 x 2 and f 3 star S 3 star and in the previous stage no alternative was there, that is why we did not need not to take any decision bit here, if we consider the minimum values that the we are getting these values.

Now, we are moving to the next stage that is stage 1, stage 1 means we are considering the teams A, B and C together, and here the stage transformation equation is S 1 is equal to x 1 plus x 2 plus x 3. Just now we have seen x 2 plus x 3 is equal to S 2, that is why this is equal to x 1 plus S 2, in other way we can say that S 2 is equal to S 1 minus x 1, this is the state transformation equation for us. And here the same logic we are adopting team one is getting 0 scientist, and altogether there is no inclusion that is why the probability is coming 0.192.

That was given previously already, that if no team is getting any new scientist and this is the probability for failure. Now, considering any randomly any value that x 1 equal to 2 that means, team A is getting two scientist, team B and team C they are not getting anything. That is why the probability is coming from here for 0, S 2 equal to 0 the optimal value is 0.48, we have just taken 0.48 and 0.15 is the probability of failure, if team A is getting two scientist at a time.

Therefore, the total probability is coming 0.072, in that way we have just completed the table, these are the infeasible options for us. If this is so, then we can see for S 1 equal to 0 this is optimal solution, S 1 equal to 1 this is the optimal solution and S 1 equal to 2 this is optimal solution. And from the least if I pick up the minimum one, because I have the recursive optimization equation f 1 S 1 is equal to minimization of p 1 x 1 and f 2 x 2.

Then this is the minimum value point 0.6 is a minimum value and how we got 0.06 that is important for us, just look back the table, here we have considered that S 1 equal to 2, S 1 equal to 2 means and x 1 is equal to 1 that means, S 2 equal to 1. Here go here then is S 2 is equal to 1 we are getting 0.3, that is why we have just picked up the optimal value in this stage, thus this is the very nice principal for optimality. Whenever you are in the current stage, that is only depend on the previous state optimal value, it is not depended on anything else.

The whole information about the current stage and the optimal values for the previous stage, that is why from here we are getting 0.06, we are getting x 1 star is equal to 1 that is the optimal and we got S 1 equal to 2. How we can get S 1 equal to 2 and with this, then we can get the value for S 2 is equal to 1 as I said, and this S 2 equal to 1 how we got 0.3 look back to the previous table, that is why that is the last stage of the dynamic programming technique is that, you have to trace back the whole optimal solutions.

So, that from here I will come here, I will move 0.3 has been used come here, 0.3 where is 0.3, 0.3 is here what is the option, then only you just get the x 2 is equal to 0 and S 2 is equal to 1. What is S 2, S 2 is just now we got from S 2, if we know S 2 and x 2 we will get S 3 that means, we are getting S 3 is equal to 1 that means, the conclusion says that that add, one scientist in team A and one scientist in team C no inclusion in team B. Then, the probability value that is a probability of failure, joint probability of failure is reduced from 0.192 to 0.06, and that is the beauty of dynamic programming technique.

Even we have the situation where the variables are moving in the discrete space, and the alternatives are dependent on the probability values, but still we can calculate the solution for it. Now, let me discuss the next model, because once we will do different models together you can have certain idea, that how to construct the stages, how to determine the state variables, that is why that experience one need together.

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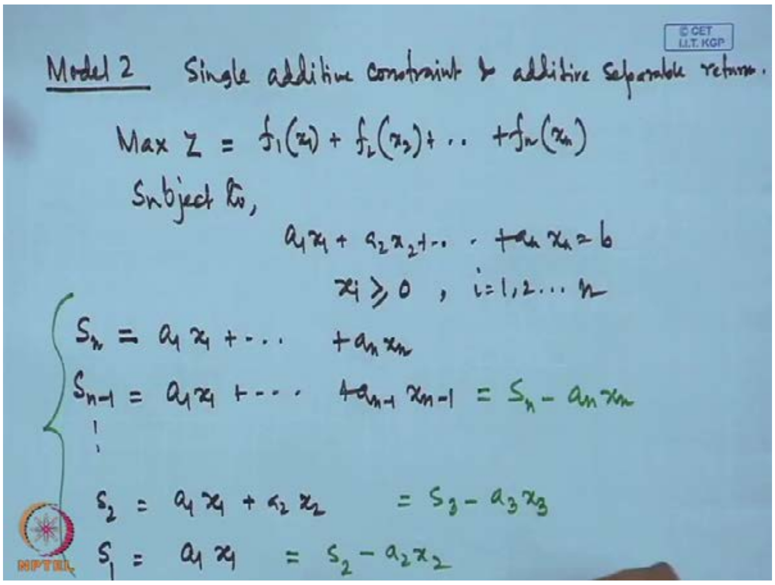

That is why I am coming to the next model, model 2 considers single additive constraint and additively separable return, thus the optimization problem can be considered as max Z equal to f 1 x 1 plus f 2 x 2 plus f n x n, subject to a 1 x 1 plus a 2 x 2 a n x n equal to b. And x i these are all positive there is a non negativity constraints, for i equal to 1 to n, if we look at this model, this model can be solved with the classical optimization technique by forming the Lagrange function, this is one way for solving it.

Another way is that this objective function is the separable objective function, as well as the constraint is also separable, we can adopt the separable programming technique for solving this. But, we can also apply the dynamic programming technique that is the defining stages, define state variable etcetera, etcetera further, that is why let us start working on it. Let us first defenses n variables are there from your experience, you must have realize that we will considering n stages, and corresponding n state variables, we will consider as this one.

Similarly we will move to S 2 that is a 1 x 1 plus a 2 x 2 and S 1 is equal to a 1 x 1. If this is, so then we can formulate state transformation equations very nicely here what are the state transformation equation, this is equal to can be written as S 2 minus a 2 x 2.

Similarly, this could can be written as S 3 minus a 3 x 3, similarly this can be written as a n x n, these are about a state variables. Once we have constructed the number of stages corresponding state variables, decision are moving in the continues space, because x i's are all move are moving in the continues decision space that is why we have to adopt the classical optimization technique for solving it.

And not only that we have formed the state transformation equation; that means, how one state is moving to the other state. Now, only thing is left that is the recursive formula, we can use the forward recursive equation we can use the backward recursive equation as well.

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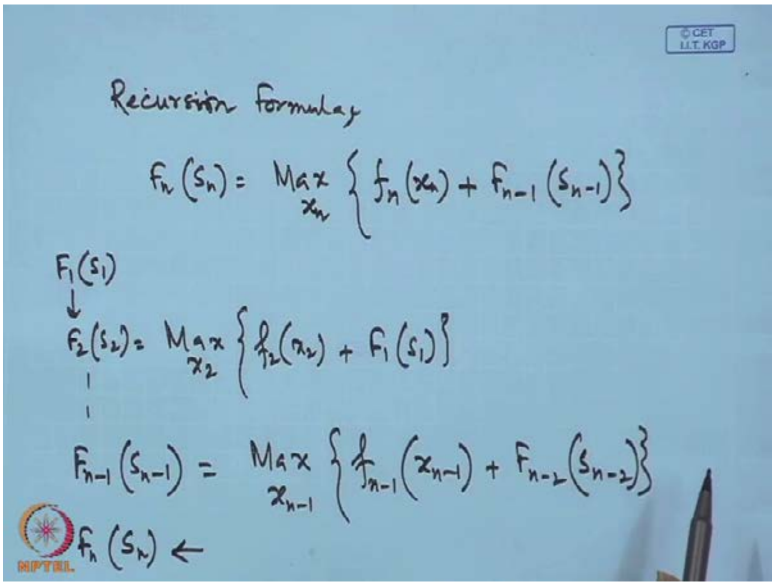

Now recursive formula can be written very nicely again F n S n is equal to by considering one variable at each stage then n'th stage would be F n x I am sorry, this is not F n x n this is the objective function F n x n plus F n minus 1 S n minus 1.

That means the n'th stage is depending on n minus 1 stage thus we have to move the process from F 1 S 1 then we will go to F 2 S 2 by considering max of x 2 F 2 x 2 plus F 1 s 1, f 1 is s 1 is the optimal value of the previous state. In this way if we just proceed we will move to f n minus 1 s n minus 1 this is equal to if I write again x n minus 1 this is the objective function in the n'th stage.

And then we will reach to the final stage that is the whole problem will be abreast once, this calculation will be based on this Richardson formula. Now, we can see that for each stage we are solving only one optimization problem at a time and you have experience the model one in the similar way we will handle let us consider some numerical example, so that you can understand this model in a better way.

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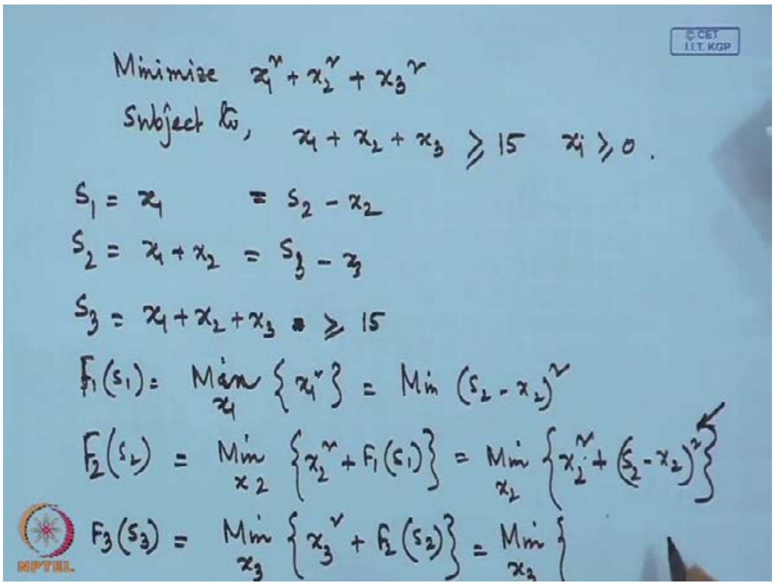

Greater than equal to 15 the value is there alright there is no equality constraint single additive constraints are only there and these are not integers these can be in a continues space it can take any real number x 1 x 2 x 3. Now, very quickly let me do the problem let us consider the first stage variable x 1, the second state, state variable not stage variable, then it is equal to x 1 plus S 2; that means, it is equal to…

Then the state transformation equations I will do later on just let me consider the we wanted to convert in one variable that is why the values are coming here S 2 minus x 2 am I right this is S 3 minus x 3 alright. Now, this is given from the constraint this is greater than equal to 15. Now, we have constructed the stages we have constructed the state variables as well as the state transformation equation, let us move to the problem.

Let us solve in each state individual optimization problem first problem is f 1 S 1 maximization over x minimization problem minimization of x 1 square alright. In other way we can write minimization of S 2 minus x 2 whole square let us move to the next that is f 2 s 2 let me consider the capital F because we have considered in this model small f as variable name if this is. So, this is equal to minimization of x 2 x 2 square plus F 1 S 1 this is F 1 S 1 for me that is why we can consider minimization of x 2 x 2 square plus S 2 minus x 2 whole square.

Now, once we are getting that the see second stage problem is a problem of single variable only x 2 we can apply with the optimization technique move to the next F 3 S 3 well this would be minimization x 3 x 3 square F 2 S 2; that means, minimization x 3 and F 2 S 2 again we have to convert it further.

That's why let us solve the problem, let us first solve the problem then only we can construct the we will get optimal solution then only we can construct this one; otherwise its not possible for us that is why out problem is coming here.

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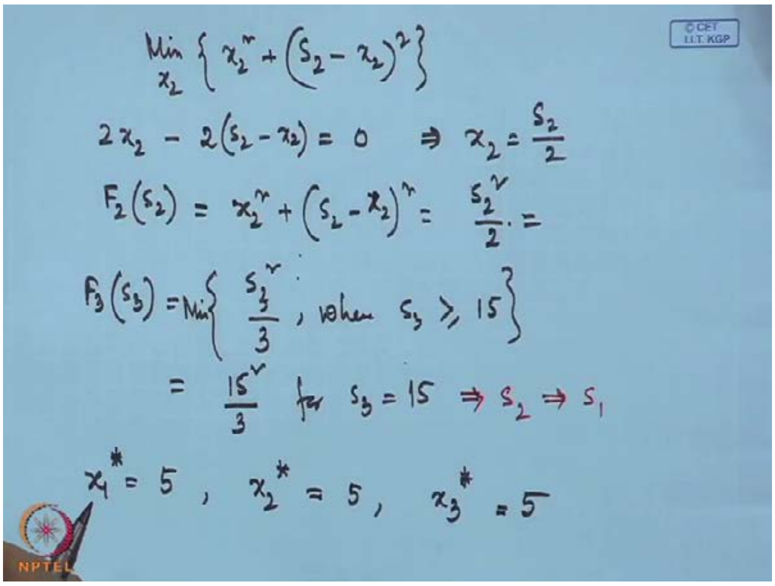

That minimization of x 2 square plus s 2 minus x 2 whole square now if we apply the differential calculus technique then by differentiating this objective function with respect to x 2, x 2 is only the variable for us; then it will be 2 x 2 minus 2 S 2 minus x 2 equal to 0 and this gives the value x 2 is equal to s 2 by 2. Once we have done, so then our F 2 S 2 can be nicely calculated x 2 square by S 2 minus x 2 whole square, after substitution of this we will get this value as s 2 square divided by 2.

Now, let us go for the next decision process that is again s 2 is equal to from here ((Refer Time: 54:34)) we got s 2 is equal to S 3 minus x 3 that is why we can consider this as the x 3 square plus S 2 square by 2. Let me put s 2 as s 3 minus x 3 that is why it is coming again the optimization problem of single variable alright that is the beauty after applying dynamic programming technique.

And once we are calculating this we will get F 3 S 3 is equal to and we have reach to the final state and here the values are given x 1 plus x 2 plus x 3 can be minimum value is 15, maximum value can be anything. Now, our problem is for minimization problem if we apply the differential calculus technique here ((Refer Time: 55:27)) whatever value for x 3 we will get that would be dependent on the value of s 3. Therefore, the minimum objective function will be obtained when x 3 is equal to 15 clear.

Thus we can say that minimum is obtaining as S 3 square divided by 3 and we need to consider and here there is a condition that s 3 greater than equal to 15 minimum of this, the certainly it would be 15 square divided by 3 that is a optimal value s 3 is equal to 15. Now, once we get s 3 is equal to 15 we can calculate s 2, we can calculate s 1.

In other way we can calculate on the values for x 1 stare that is 5 x 2 star is equal to 5 and x 3 star just now we have calculated is equal to 5, because s 3 is 5 plus 5 plus 5 clear. Let me repeat once more this process s 3 will give you the value for S 2 value for S 2 means this is the function of x 1 and x 2 and function of x 1 x 2 means again we will move to s 1. Then we can calculate x 1 x 2 to 2 equations to unknown from there very nicely we can calculate the value of this.

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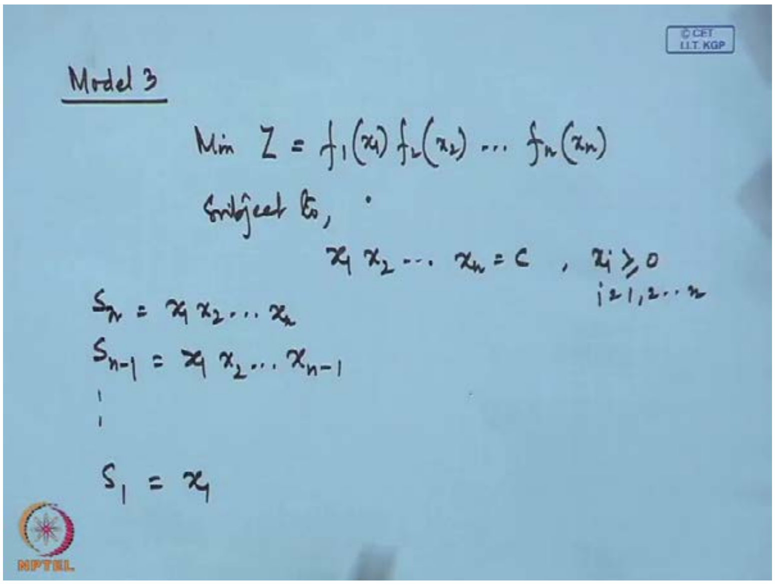

Now, let us move to the next model, model 3 where we are considering the problem minimization or maximization anything we can take where the objective function looks like this these are all in a multiplicative way. Now, here also we can solve it by dynamic programming technique and we need to declare the state variables in this way you will proceed further and we will get s 1 equal to x 1 alright. And from here we can calculate the state transformation equation as well, thus I can say that you can apply this for the example.

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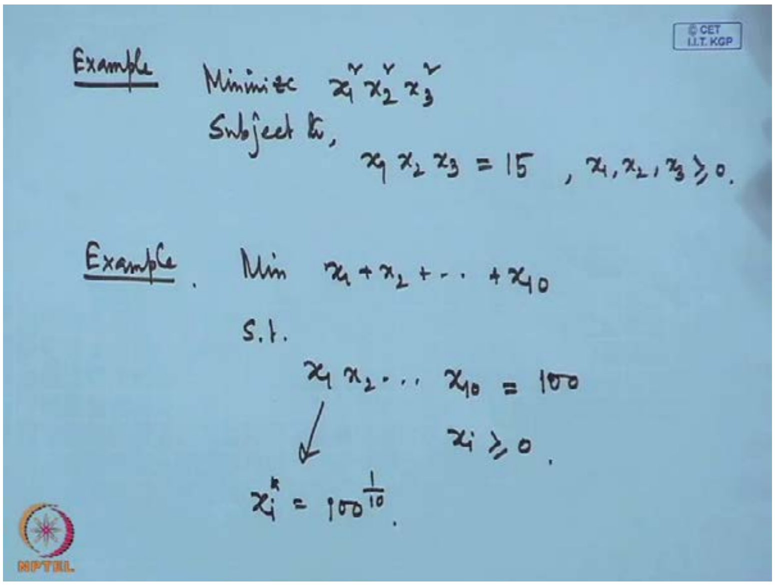

Minimization of no this is not the problem, equal to say 15 and some other problem I can also give to you for practice. Now, that is all about the problem and this problem solution I can say that x I star is equal to, that is the solution of this and that is all about the dynamic programming technique.

Thank you very much.