

Approximate Reasoning using Fuzzy Set Theory
Prof. Balasubramaniam Jayaram
Department of Mathematics
Indian Institute of Technology, Hyderabad

Lecture - 02
Fuzzy Sets - The Necessity

(Refer Slide Time: 00:16)

Approximate Reasoning using Fuzzy Set Theory

Balasubramaniam Jayaram

Fuzzy Sets - The Necessity

"As complexity rises, precise statements lose meaning and meaningful statements lose precision."
- Lotfi A. Zadeh

NPTEL

Indian Institute of Technology Hyderabad

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

Hello and welcome to the second of the lectures in this course titled Approximate Reasoning using Fuzzy Set Theory, a course offered through the NPTEL platform. In today's lecture we will look at how the need arose for Fuzzy Sets.

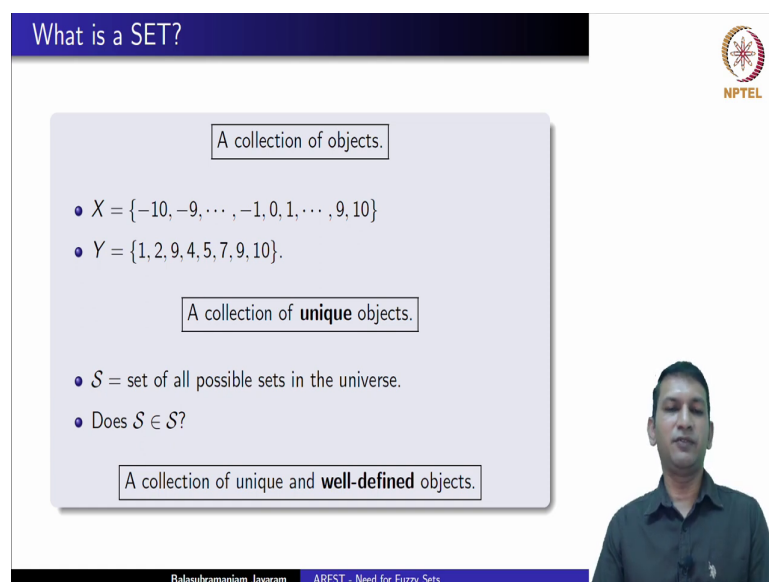
(Refer Slide Time: 00:37)



The slide features a dark blue header with the text "Need for Fuzzy Sets" in white, and a light blue sub-header below it with "A Theoretical Perspective" in dark blue. In the top right corner, there is a circular NPTEL logo. A video overlay of a man in a dark shirt is positioned in the bottom right corner. At the bottom of the slide, a dark blue bar contains the text "Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets" in white.

The need for Fuzzy Sets can be traced back to both theory and the utility in practical applications. Allow me to give you a theoretical perspective of how the need arose for Fuzzy Sets.

(Refer Slide Time: 00:51)



The slide has a dark blue header with the text "What is a SET?" in white. In the top right corner, there is a circular NPTEL logo. A video overlay of a man in a dark shirt is positioned in the bottom right corner. The main content area is a light grey box containing three definitions of a set, each in a white-bordered box: "A collection of objects.", "A collection of **unique** objects.", and "A collection of unique and **well-defined** objects.". Below the first definition, there are two bullet points: $X = \{-10, -9, \dots, -1, 0, 1, \dots, 9, 10\}$ and $Y = \{1, 2, 9, 4, 5, 7, 9, 10\}$. Below the second definition, there are two bullet points: \mathcal{S} = set of all possible sets in the universe. and Does $\mathcal{S} \in \mathcal{S}$? At the bottom of the slide, a dark blue bar contains the text "Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets" in white.

If we were to ask what is a set the perhaps the first answer that we would get is that it is a collection of objects. This definition is not far from what we have in mind of a set, for instance if you list out the elements of X like this yes it is a collection of numbers and it could

be called a set. However, when we think of this collection of numbers and call it Y we do not call it as set Y it is merely a collection of some numbers.

This is because the element 9 repeats itself; one might ask is there an issue? Well some properties of a set that we would like it to have may not be possible to be defined uniquely, for instance we may not be able to define the cardinality of a set if we allowed repetition of elements. Hence the definition itself was refined as a collection of unique objects. Now consider the set S which is the set of all possible sets in the universe.

Now, the question immediately arises does S belong to itself? Now towards avoiding such paradoxical questions the definition was further refined as a collection of unique and well defined objects.

(Refer Slide Time: 02:16)

The slide is titled "Properties of a SET" and features the NPTEL logo in the top right corner. It lists three categories of set properties:

- Belongingness**
 - $X = \{-10, -9, \dots, -1, 0, 1, \dots, 9, 10\}$
 - $3 \in X??$
- Subsethood**
 - $Y = \{1, 2, 4, 5, 7, 9, 10\}$.
 - $Y \subset X?$
- Disjointedness**
 - $X \subset U$.
 - $X^c =$ all those $u \in U$ but not in X .
 - $X \cup X^c = U$ and $X \cap X^c = \emptyset$.


A video inset in the bottom right shows a man speaking. At the bottom of the slide, the text "Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets" is visible.

Now to be able to decide on the belongingness of an element to a set is very very essential. For instance if you are given this set X we can ask the question does 3 belong to X or not? We will get a unique answer.


Once we are able to answer this question at the level of an element we could also do this on a collection of elements. So, is Y a subset of X yes that is also answerable? Now based on this we could also create properties of sets, for instance if we know that X is a proper subset of another set U, then we could easily pick all those elements which belong to U, but not in X.

This is what we call the complement of X and the interesting property between X and X complement are these together they form the entire U and they are non overlapping.

(Refer Slide Time: 03:14)



Quality of an Element vis-à-vis a Set




Balsubramaniam Jayaram ARFST - Need for Fuzzy Sets

Let us look at some qualities of an Element of a set vis-a-vis itself. Allow me to present some scenarios let Q denote the set of all rational numbers. So, the Q complement those numbers which are not rational what we call as irrational numbers.


(Refer Slide Time: 03:25)

Well-definedness vs Belongingness



- Q = set of all rational numbers.
- Q^c = set of all irrational numbers.
- Does $\sqrt{2} \in Q$?
- Does $\sqrt{2}^{\sqrt{2}} \in Q^c$?

Set is well-defined - Element's quality is not known.



Balsubramaniam Jayaram ARFST - Need for Fuzzy Sets

It is clear that the union of these two gives us the entire real line. Now if we were to ask this question does $\sqrt{2}$ belong to \mathbb{Q} ? Of course we know the answer $\sqrt{2}$ is irrational, so it will not belong to \mathbb{Q} . However, if you ask this question does $\sqrt{2}^{\sqrt{2}}$ belong to the set of irrational numbers, well of course there is a clear answer because it has to either belong to \mathbb{Q} or \mathbb{Q} complement.

But for the moment if we do not know the answer then we are not in a position to give an clear answer to this. Now if you look at this scenario the set itself is very well defined, but what we do not know is the quality of an element whether it is a rational or an irrational.

(Refer Slide Time: 04:24)

The slide is titled "Well-definedness vs Belongingness" and features the NPTEL logo in the top right corner. It contains a central collage of various birds, including a parrot, a duck, a seagull, a penguin, a hummingbird, and a flamingo. Below the collage, the text asks "Set of birds?". To the right of the collage, there is a smaller image of a penguin with the text "Set of animals with wings?". Below this, it asks "Is the Set well-defined? - Can be refined!". At the bottom of the slide, the name "Balasubramaniam Jayaram" and the text "ARFST - Need for Fuzzy Sets" are visible. A video inset of a man is shown in the bottom right corner of the slide area.

Now consider this picture which is a collage of smaller pictures, can we call this a collection of images is it a collection of unique images yes perhaps. Because the objects in the foreground they appear distinct.

So is this a set yes, but set of what perhaps we could say this is a set of birds. Now if we agree to this consider this image, of course immediately we recognize there is a penguin in it. But the question is if you were to ask does this penguin belong to the set of birds? No, this is an often debated question.

If you think of a bird what characterizes or defines the quality of being a bird is it? It is morphological feature of having wings or it is its essential quality to be able to fly. If we redefine the set as set of all animals with wings, then perhaps penguin will belong to this set.

Thus here the set originally was not well characterized, but it is redefinable and once it is redefined then the belongingness of another element in this case that of penguin to the set can be discussed and answered.

(Refer Slide Time: 05:59)

Well-definedness vs Belongingness

The Barber's Paradox

- In a village there is a lone male barber.
- M = the set of all the men in a village^a.
- S = set of men in the village who shave themselves.
- B = set of men in the village who are shaven by the barber.
- Clearly $M = S \cup B$.

$$\Rightarrow B = S^c$$

$$\Rightarrow B \cap S = \emptyset$$

Barber $\in B$ or Barber $\in S$?

Is $X \cap X^c = \emptyset$?

^aAll men are clean shaven. Barber is from this village.

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

Let us look at this famous Paradox which you may have heard. So, in a village there is a lone male barber. Let M denotes the set of all the men in a village S is a set of men in the village who shave themselves and B is a set of all men in the village who are shaven by the barber. So, clearly M is made up of the union of the sets S and B .

In other words you can look at B as the complement of S and in that sense B intersection S should be empty; that means, B and S should not overlap. However, if you ask the question the barber where does he belong does he belong to the set B or to the set S ? You see that we actually fall into a very paradoxical situation it appears that the barber probably belongs to both B and S . So now, it has questioned the wisdom of one particular property of insisting on this property which is X intersection X^c should it really be empty.



Of course note that we have made 2 important assumptions that all men in this village are clean shaven and the barber himself is from this village. Now so far we may have seen that the properties of the set which we thought were invaluable in some situations perhaps there is a need there is a ccause for relaxation.

(Refer Slide Time: 07:37)

Well-definedness vs Belongingness

Name	Age	Occupation	Name	Age	Occupation
Smriti	21	Student	Madan	45	Professor
Zuzanna	18	Student	Bharath	41	Professor
Andrea	17	Student	Shanthi	40	Professor
Shruti	23	Student	Victoria	46	Professor
Meena	33	Student	Fathima	35	Professor
Simon	35	Student	Jacob	43	Professor
Raja	24	Student	Akbar	54	Professor
Julia	14	Student	Anil	13	Student
Maya	14	Student	Anna	51	Professor
Kshama	21	Student	Gabriela	33	Professor
Adam	27	Student	Naveen	57	Professor

$T =$ set of all teenagers.



Balazubramaniam Jayaram ARFST - Need for Fuzzy Sets

Allow me to present to you another example consider this table in which you have a few names the ages of the corresponding people and their occupation. I have chosen only the occupation to be the student or professor. Now on given this data we can ask some questions.



So, if I were to ask you to pick up the set T of all teenagers I think it is easy to do or you will do is pick all those people whose ages fall between 13 and 19. So, in this particular list that we have these are the teenagers that we have. Now, we will slightly change the question and ask instead of picking the set of all teenagers, let us ask us ask ourselves to pick the set of all young people.

(Refer Slide Time: 08:29)

Well-definedness vs Belongingness

Name	Age	Occupation	Name	Age	Occupation
Smriti	21	Student	Madan	45	Professor
Zuzanna	18	Student	Bharath	41	Professor
Andrea	17	Student	Shanthi	40	Professor
Shruti	23	Student	Victoria	46	Professor
Meena	33	Student	Fathima	35	Professor
Simon	35	Student	Jacob	43	Professor
Raja	24	Student	Akbar	54	Professor
Julia	14	Student	Anil	13	Student
Maya	14	Student	Anna	51	Professor
Kshama	21	Student	Gabriela	33	Professor
Adam	27	Student	Naveen	57	Professor

$Y = \text{set of all young people.}$



Balazubramaniam Jayaram ARFST - Need for Fuzzy Sets

Now who is young? Let us for a moment assume that all those whose ages have not touched 35 are young. So, we put a hard threshold here. So, strictly less than the age strictly less than 35 we would call them young for the moment.



So, if you were to apply this criterion then these are the people who could be called young? But notice that there are 2 people here Meena and Gabriela who are 33 years of age and are considered as young people, but there are 2 other people Simon and Fathima who are 35 years of age and they are not considered young anymore ok. Let us redefine this set little further instead of asking for the set of all young people.

(Refer Slide Time: 09:36)

Well-definedness vs Belongingness

Name	Age	Occupation	Name	Age	Occupation
Smriti	21	Student	Madan	45	Professor
Zuzanna	18	Student	Bharath	41	Professor
Andrea	17	Student	Shanthi	40	Professor
Shruti	23	Student	Victoria	46	Professor
Meena	33	Student	Fathima	35	Professor
Simon	35	Student	Jacob	43	Professor
Raja	24	Student	Akbar	54	Professor
Julia	14	Student	Anil	13	Student
Maya	14	Student	Anna	51	Professor
Kshama	21	Student	Gabriela	33	Professor
Adam	27	Student	Naveen	57	Professor

$Y = \text{set of all young students.}$



Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

Let us ask for the set of all young students, clearly Gabriela would lose it out because she is a professor not a student anymore. But what is interesting is that even Meena would not be considered a young student in the natural sense of people who joined school or college. Of course, it is contextual we will come to that in a later lecture.



So, the very age of 33 which allowed Meena and Gabriela to be called young in the sense of young persons, they do not allow them to be called young students. Now let us refine this or modify this a little further.

(Refer Slide Time: 10:28)

Well-definedness vs Belongingness

Name	Age	Occupation	Name	Age	Occupation
Smriti	21	Student	Madan	45	Professor
Zuzanna	18	Student	Bharath	41	Professor
Andrea	17	Student	Shanthi	40	Professor
Shruti	23	Student	Victoria	46	Professor
Meena	33	Student	Fathima	35	Professor
Simon	35	Student	Jacob	43	Professor
Raja	24	Student	Akbar	54	Professor
Julia	14	Student	Anil	13	Student
Maya	14	Student	Anna	51	Professor
Kshama	21	Student	Gabriela	33	Professor
Adam	27	Student	Naveen	57	Professor

$Y = \text{set of all young professors.}$



Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

Consider the set of all young professors; obviously, many of them would lose out because they are students. Who should be considered a young professor?

Let us say somebody who is 40 or below 40 of years of age and a professor is considered a young professor. In that sense these are the people whom you would call as young professor those whose ages are marked in blue, but there is also one person Bharath who is just 41 years of age who is not considered a young professor; whereas, Shanti who is 40 years of age is considered a young professor.



You see that to be able to put a threshold a hard cut off and distinguishing people is becoming a little unwieldy here.

(Refer Slide Time: 11:17)

Well-definedness vs Belongingness

Name	Age	Occupation	Name	Age	Occupation
Smriti	21	Student	Madan	45	Professor
Zuzanna	18	Student	Bharath	41	Professor
Andrea	17	Student	Shanthy	40	Professor
Shruti	23	Student	Victoria	46	Professor
Meena	33	Student	Fathima	35	Professor
Simon	35	Student	Jacob	43	Professor
Raja	24	Student	Akbar	54	Professor
Julia	14	Student	Anil	13	Student
Maya	14	Student	Anna	51	Professor
Kshama	21	Student	Gabriela	33	Professor
Adam	27	Student	Naveen	57	Professor

- Element details - well-known.
- Character of the set? Redefinable? **Definable?**



Balazubramaniam Jayaram ARFST - Need for Fuzzy Sets

You should also note here that all the details of the elements which are necessary for us are known. However, what we are not able to decide on is who is a young person, who is a young professor and who is a young student? So, it is the character of the set that we are not able to clearly define. Is it redefinable in fact the question is about is it definable at all or not.

(Refer Slide Time: 11:43)

The slide features a central image of a white square with a jagged, star-like border. Inside this square, there are six blue circles of varying shades of blue. Below the image, the text reads "Set of blue balls?". The slide is part of a video lecture, with the NPTEL logo in the top right corner and the speaker's name, Balasubramaniam Jayaram, and the course title, ARFST - Need for Fuzzy Sets, at the bottom.

Let us look at this picture. So, if I were to ask what does it consist of? Once again if you say that it is a set of blue balls it consists of blue balls yes we would not be far away from the truth. So, let us say that it is a set of blue balls, now the question is I have another ball here.

(Refer Slide Time: 12:02)

This slide is similar to the previous one, but with a seventh blue circle placed outside the jagged boundary, at the bottom right corner. Below the image, the text reads "How blue is blue?". The slide is part of a video lecture, with the NPTEL logo in the top right corner and the speaker's name, Balasubramaniam Jayaram, and the course title, ARFST - Need for Fuzzy Sets, at the bottom.

Now, ask the question does this ball at the right bottom corner belong to the set of blue balls? Now if you take a moment part of you wants to say yes, part of you wants to say no. Because you are asking the question how blue should something be to be called blue.

(Refer Slide Time: 12:26)

Well-definedness vs Belongingness

NPTEL

Blue is a **concept!**

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

The slide features a central diagram with a white star-shaped boundary on a light blue background. Inside the star, there are six blue circles. Outside the star, there is one blue circle. Below the diagram, the text reads "Blue is a concept!". The slide is part of a video lecture, with a small inset of the speaker in the bottom right corner.

Because blue itself is a concept we talk about darkness, we talk about pitch darkness; so it is a Concept.

(Refer Slide Time: 12:35)

What is a FUZZY set?

NPTEL

A Concept!

- X = set of teenagers.
- \tilde{X} = set of **young** people.
- \tilde{Y} = set of **blue** balls.
- E = ϵ -nbd around 0.
- \tilde{E} = points **close to** 0.

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

The slide is titled "What is a FUZZY set?". It contains a list of five items under the heading "A Concept!". The items are: X = set of teenagers, \tilde{X} = set of **young** people, \tilde{Y} = set of **blue** balls, E = ϵ -nbd around 0, and \tilde{E} = points **close to** 0. The slide is part of a video lecture, with a small inset of the speaker in the bottom right corner.

So, if you were to ask what is a Fuzzy set? Perhaps one of the best ways to explain it is it is a mathematical construct which captures the concept for instance consider this sets that we have discussed. So, far if x is set of teenagers it is easy to discuss whether somebody belongs to the set or not, but the moment you relax and call it a set of young people, then we know that the concept young applies differently it is varying, it is subjective, it is contextual.

Similarly, if when we talk about set of blue balls blue becomes a concept. If E is just the epsilon neighbourhood around 0, the moment you fix epsilon then we are in a position to say whether an element belongs to the set or not. However, if you modify this and say that let E tilde represent all those points that are close to 0, then this closeness this proximity it is a concept it is contextual and subjective.

(Refer Slide Time: 13:46)

The image shows a video lecture slide. At the top right, there is the NPTEL logo. The main content is a blue and white box with the text "Need for Fuzzy Sets" and "An Application Perspective". In the bottom right corner, there is a small video feed of a man in a dark shirt speaking. At the bottom of the slide, there is a black bar with the text "Balasubramaniam Jayaram" and "ARFST - Need for Fuzzy Sets".

So, far we have seen the need for generalizing classical sets themselves and we have reached a point where we would like to abstract the concept and Fuzzy sets give you a way of doing it. How we do it is a will be covered in the next lecture, but now let us look at the need for Fuzzy sets how they were motivated from an application perspective.

(Refer Slide Time: 14:14)

The slide features a dark blue header with the text "Need for Fuzziness and Fuzzy Inference Systems". In the top right corner is the NPTEL logo. The main content area has a light blue background and contains a smaller box with a dark blue header "How does a human being think?". This box includes a small image of the "The Thinker" statue and a bulleted list of questions. Below this box is a light blue button with the text "Capturing Knowledge". At the bottom of the slide, there is a dark blue footer with the text "Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets". A small video inset of a man in a dark shirt is visible in the bottom right corner of the slide.

Need for Fuzziness and Fuzzy Inference Systems

NPTEL

How does a human being think?

- We think in words ...
 - Linguistic terms.
- How do we describe someone?
 - Tall, slim, fashionably-dressed.
- More qualitative than quantitative.

Capturing Knowledge

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

Now this course is about approximate reasoning. So, for reasoning you need both knowledge and intelligence as we have seen before. But let us ask ourselves a question a different question perhaps how does a human being think? Thinking is very much a part of reason if you were to ask this question without much effort we would realize that we actually think in words in terms of linguistic terms.

Imagine you are sharing a room with your roommate and a visitor drops by, but unfortunately your roommate is not available at that time and even before you could find out the name of the visitor he leaves. Now when the when your roommate friend comes back to the room you would like to describe this person to him. How would you describe him? Clearly you would not have had the time or even the inclination to measure his height.

So, that you can tell your friend that he was 5 feet 7 and a half inches in height and perhaps 63 kgs in weight, no that is not how we describe somebody. Your description perhaps would be as he was tall, slim and fashionably dressed perhaps. Now the question is what is tall what is slim and how should one be dressed to be called fashionable. Now let us look at it.

How tall should one be to be called tall? What should his height be? For instance in India you might say that somebody who is around 5 foot 9 inches can be considered tall; however, if you travel to northern European countries perhaps it is more or less an average height of anybody there. On the other hand what you would consider as an average height for an Indian could well be somebody who is tall in Japan.

So, tall as you see is not only subjective, but also contextual. Similar is the thing with being slim and now coming to the last word fashionably dressed. We will know that this is a very time varying concept too not just subjective, but contextual and time varying too so all of this point out to this. That we perhaps think more qualitatively than quantitatively. Why is this important?

Because we want to be able to capture knowledge, knowledge comes from human beings and if you want to capture them you need a way of capturing this into a construct. A question might arise as to why we need to capture this knowledge? These days we talk about machine learning perhaps a little bit of going back in history might give us a good perspective.

There were times when automated systems were being used to support human decision makers, these so called expert systems used probabilistic tools to arrive at a decision. However, people wanted machines to think to think like experts, if you want the machine to think like a human being then it was important to understand how human beings thought and as is clear largely we think in terms of linguistic terms.

So, it is essential that we capture knowledge into a mathematical construct and give it a representation that is both appropriate and amenable for further processing.

(Refer Slide Time: 18:05)

The slide is titled "Need for Fuzziness?" and features the NPTEL logo in the top right corner. A central box asks "Are we measuring precisely?" and lists three types of errors: "Zero-error", "Parallax error", and "Subjective error". To the left of this list is a photograph of a long-haired dog. Below the error list, a blue button contains the text "Imprecision in Measurement". In the bottom right corner, there is a video feed of a male presenter. The footer of the slide identifies the speaker as "Balasubramaniam Jayaram" and the course as "ARFST - Need for Fuzzy Sets".

Now consider this question when we measure something we know that there are lots of possibilities for errors to creep. In there are different kinds of errors all of us would have done some amount of experimentation either in chemistry lab or physics lab.

There are these errors that are associated with the instrument itself, for instance 0 error then there are these errors that human beings introduce like the parallax error, but then there are also this subjective errors. For instance if you look at the picture on the left if I were to ask you is it a cat or a dog each of us would have his own answer for it. Of course, it is clear it has to be either a dog or a cat.

But looking at it I think our subjectivity of how we are conditioned by the morphological features of a cat or a dog would lead us to answer accordingly. So, imprecision in measurement also necessitated the need for fuzziness. Many of these things we will see in more detail as the lecture series progresses.

(Refer Slide Time: 19:21)

The slide is titled "Need for Fuzziness and Fuzzy Inference Systems" and features the NPTEL logo in the top right corner. The main content is a box titled "Non-linearity is ubiquitous" which contains a graph of a non-linear signal and a bulleted list of characteristics. Below this box is a button labeled "Model-Free Estimators". At the bottom of the slide, there is a small video inset of a man and a footer with the name "Balasubramaniam Jayaram" and the text "ARFST - Need for Fuzzy Sets".

Need for Fuzziness and Fuzzy Inference Systems

NPTEL

Non-linearity is ubiquitous

- Data from non-linear processes
- Underlying model is difficult to abstract
- Usually approximated with linear models
 - Differential Eqns, Difference Eqns, etc.

Model-Free Estimators

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

Finally we talk about data analysis. Data normally are collected from processes which are typically non-linear; however, it is very difficult to abstract the underlying model. Often as engineers you would readily understand that we resort to approximations and often these approximations are with linear models in the time Fuzzy set theory was introduced in the in mid-sixties the controllers that were used or the models that were applicable or applied typically used differential equations or difference equations or integral equations.

At that time they felt a need for model free estimators and Fuzzy Inference Systems were one way of building model free estimators.

(Refer Slide Time: 20:28)

A quick look - back and ahead

NPTEL

What have we seen so far?

- Set boundaries can be vague.
- Need a mechanism to handle it.
- Theoretical and Practical motivation.
- A fuzzy set can be thought of as capturing a **concept**.

Next Lecture:

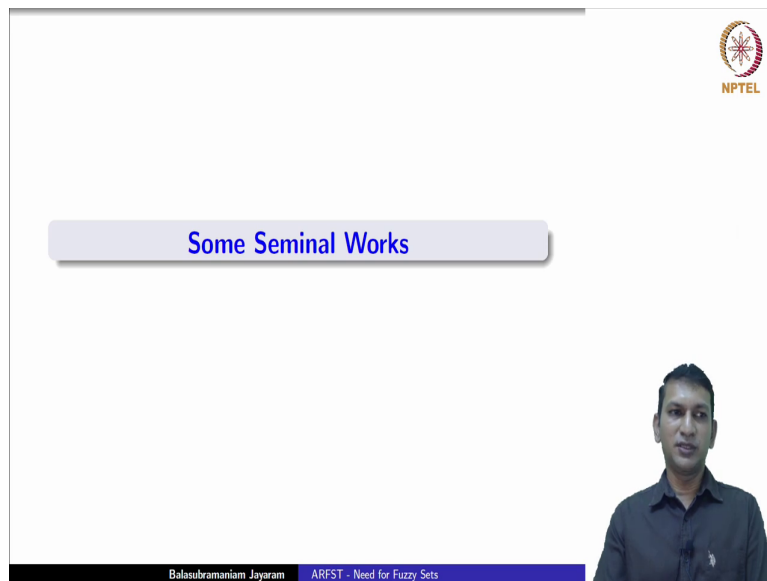
Representations of fuzzy sets.

Balsubramaniam Jayaram ARFST - Need for Fuzzy Sets

A quick look at what we have seen in this lecture so far, from a theory perspective we have seen that set boundaries can be vague. We need a mechanism to handle it and there were clear theoretical and practical motivations for these.

And if you were to be asked what is a Fuzzy set it can be thought of as capturing a concept. As we saw a few moments before the representation took that is given to the captured knowledge is very important and if you want to capture it through this concept of fuzzy set a useful appropriate and amenable representation is very very important and this is what we will see in the next lecture.

(Refer Slide Time: 21:25)

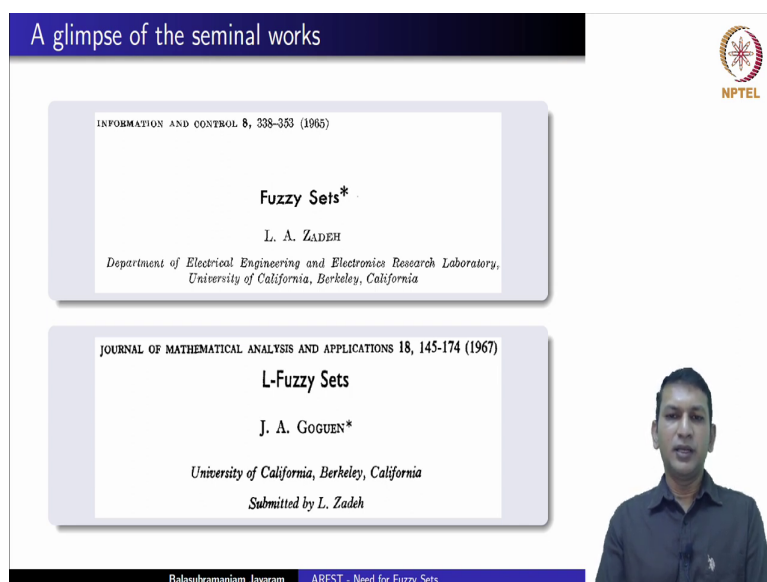


Some Seminal Works

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

However before we close this lecture allow me to flash at you a few seminal works, there have been quite a few perhaps my choice has been motivated by the content that we have covered in this lecture.

(Refer Slide Time: 21:40)



A glimpse of the seminal works

INFORMATION AND CONTROL 8, 338-353 (1965)

Fuzzy Sets*

L. A. ZADEH

*Department of Electrical Engineering and Electronics Research Laboratory,
University of California, Berkeley, California*

JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 18, 145-174 (1967)

L-Fuzzy Sets

J. A. GOGUEN*

University of California, Berkeley, California
Submitted by L. Zadeh

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

This was the original paper of Lotfi A Zadeh who is credited as the father of Fuzzy Set theory, his paper appeared in 1965 in the journal Information and Control.

Wherein he propounded his new theory of sets the generalization of classical sets to Fuzzy sets mind you professor Zadeh himself was an engineer an electrical engineer from UC Berkeley and he came to propound this concept of Fuzzy sets from a purely practitioners perspective. Soon enough mathematicians also got interested in this and within 2 years time there was a seminal paper outlining the theoretical underpinnings of fuzzy set theory in a very reputed mathematical journal by Goguen.

(Refer Slide Time: 22:38)

The slide is titled "A glimpse of the seminal works" and features the NPTEL logo in the top right corner. The central focus is the cover of the "JOURNAL OF SEMANTICS", an international journal for the interdisciplinary study of the semantics of natural language. The cover specifies the issue as "DECEMBER 1983, VOL. II - NO. 3/4" and highlights the article "A FUZZY-SET-THEORETIC APPROACH TO THE COMPOSITIONALITY OF MEANING: PROPOSITIONS, DISPOSITIONS AND CANONICAL FORMS*" by L.A. Zadeh. A video inset in the bottom right shows a man in a dark shirt, presumably the presenter. At the bottom of the slide, the text "Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets" is visible.

This is another paper of Zadeh himself which deals in a more semantic way on what a Fuzzy set is, this appeared maybe almost 15 years later 1983 allow me to flash at you just 2 more works which you could see as seminal.

(Refer Slide Time: 23:01)

A glimpse of the seminal works

ELSEVIER Fuzzy Sets and Systems 66 (1994) 207-221
FUZZY sets and systems
Fuzzy sets and vague environments
Frank Klawonn

ELSEVIER Fuzzy Sets and Systems 90 (1997) 141-150
FUZZY sets and systems
The three semantics of fuzzy sets
Didier Dubois*, Henri Prade

Balasubramaniam Jayaram ARFST - Need for Fuzzy Sets

The first of them was almost 25 years after the first paper on Fuzzy set theory by Zadeh by professor Frank Klawonn, where he discusses fuzzy sets and vague environments and this could probably resonate with you given the content that we have covered in this lecture. And finally, there is another very well known well cited paper the Dubois and Prade which deals with the different semantics that you could assign to fuzzy sets.

We encourage you to have a look at these papers too they are written very nicely and more in a story like form and we believe that these research papers are also at an accessible level to the audience of this course. So, with this we will come to the end of this lecture, in the next lecture as was flashed we will look at finding a useful appropriate and amenable representation for fuzzy sets.

Thank you.