Approximate Reasoning using Fuzzy Set Theory
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Lecture - 02
Fuzzy Sets - The Necessity

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Hello and welcome to the second of the lectures in this course titled Approximate Reasoning using Fuzzy Set Theory, a course offered through the NPTEL platform. In today's lecture we will look at how the need arose for Fuzzy Sets.

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The need for Fuzzy Sets can be traced back to both theory and the utility in practical applications. Allow me to give you a theoretical perspective of how the need arose for Fuzzy Sets.

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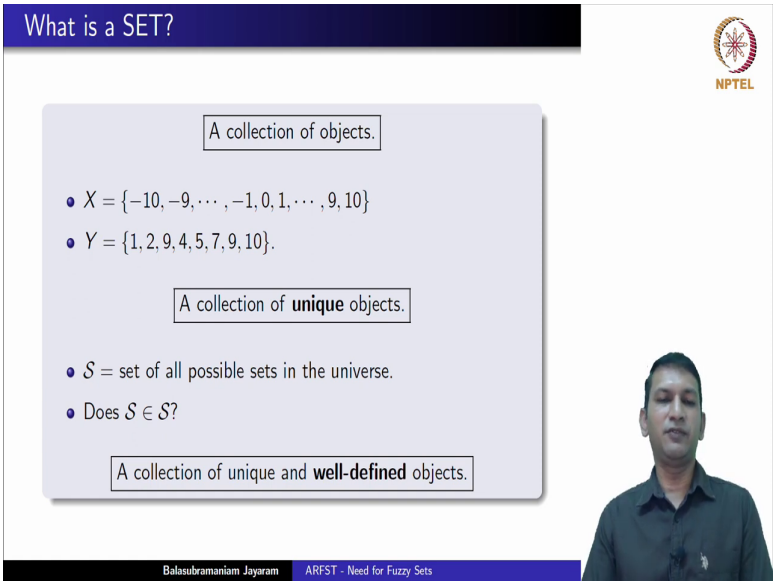

If we were to ask what is a set the perhaps the first answer that we would get is that it is a collection of objects. This definition is not far from what we have in mind of a set, for instance if you list out the elements of X like this yes it is a collection of numbers and it could

be called a set. However, when we think of this collection of numbers and call it Y we do not call it as set Y it is merely a collection of some numbers.

This is because the element 9 repeats itself; one might ask is there an issue? Well some properties of a set that we would like it to have may not be possible to be defined uniquely, for instance we may not be able to define the cardinality of a set if we allowed repetition of elements. Hence the definition itself was refined as a collection of unique objects. Now consider the set S which is the set of all possible sets in the universe.

Now, the question immediately arises does S belong to itself? Now towards avoiding such paradoxical questions the definition was further refined as a collection of unique and well defined objects.

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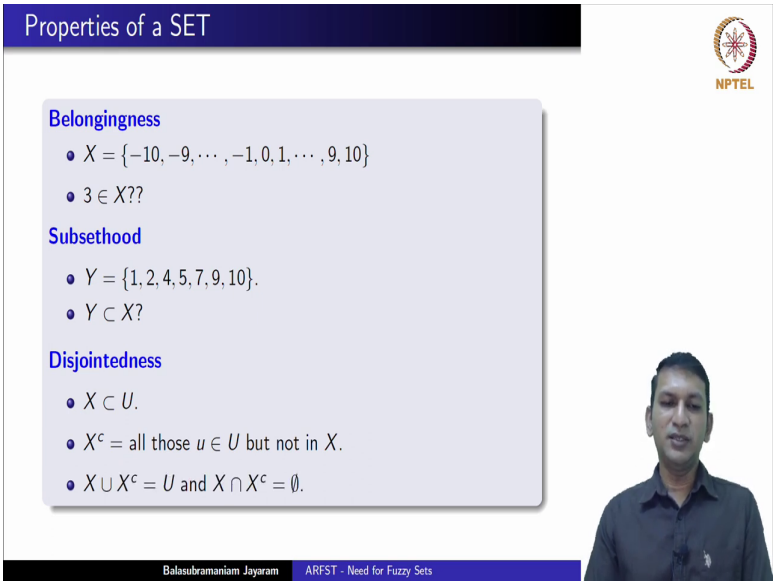

Now to be able to decide on the belongingness of an element to a set is very very essential. For instance if you are given this set X we can ask the question does 3 belong to X or not? We will get a unique answer.

Once we are able to answer this question at the level of an element we could also do this on a collection of elements. So, is Y a subset of X yes that is also answerable? Now based on this we could also create properties of sets, for instance if we know that X is a proper subset of another set U, then we could easily pick all those elements which belong to U, but not in X.

This is what we call the complement of X and the interesting property between X and X complement are these together they form the entire U and they are non overlapping.

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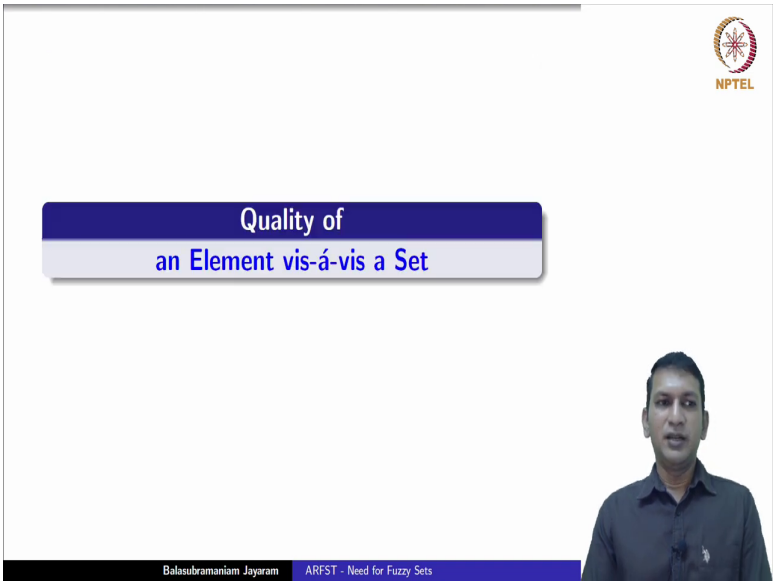

Let us look at some qualities of an Element of a set vis-a-vis itself. Allow me to present some scenarios let Q denote the set of all rational numbers. So, the Q complement those numbers which are not rational what we call as irrational numbers.

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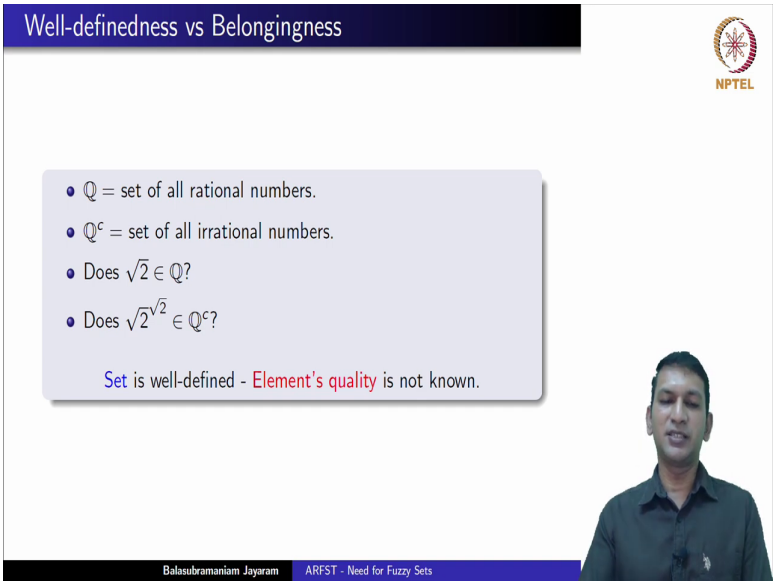

It is clear that the union of these two gives us the entire real line. Now if we were to ask this question does root 2 belong to Q? Of course we know the answer root 2 is irrational, so it will not belong to Q. However, if you ask this question does root 2 power root 2 belong to the set of a irrational numbers, well of course there is a clear answer because it has to either belong to Q or Q complement.

But for the moment if we do not know the answer then we are not in a position to give an clear answer to this. Now if you look at this scenario the set itself is very well defined, but what we do not know is the quality of an element whether it is a rational or an irrational.

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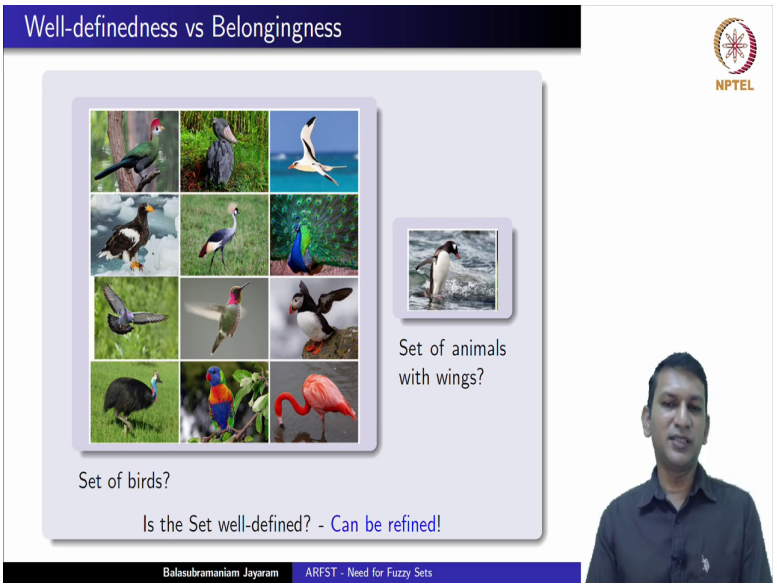

Now consider this picture which is a collage of smaller pictures, can we call this a collection of images is it a collection of unique images yes perhaps. Because the objects in the foreground they appear distinct.

So is this a set yes, but set of what perhaps we could say this is a set of birds. Now if we agree to this consider this image, of course immediately we recognize there is a penguin in it. But the question is if you were to ask does this penguin belong to the set of birds? No, this is an often debated question.

If you think of a bird what characterizes or defines the quality of being a bird is it? It is morphological feature of having wings or it is its essential quality to be able to fly. If we redefine the set as set of all animals with wings, then perhaps penguin will belong to this set.

Thus here the set originally was not well characterized, but it is redefinable and once it is redefined then the belongingness of another element in this case that of penguin to the set can be discussed and answered.

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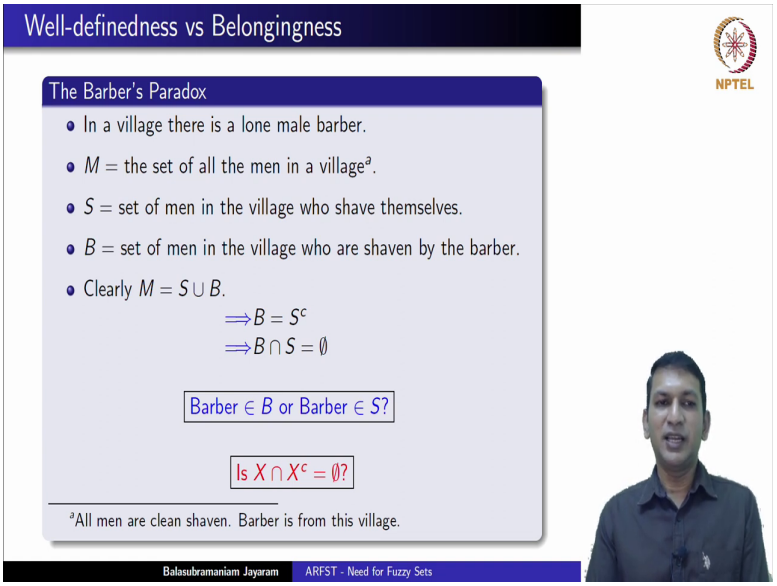

Let us look at this famous Paradox which you may have heard. So, in a village there is a lone male barber. Let M denotes the set of all the men in a village S is a set of men in the village who shave themselves and B is a set of all men in the village who are shaven by the barber. So, clearly M is made up of the union of the sets S and B.

In other words you can look at B as the complement of S and in that sense B intersection S should be empty; that means, B and S should not overlap. However, if you ask the question the barber where does he belong does he belong to the set B or to the set S? You see that we actually fall into a very paradoxical situation it appears that the barber probably belongs to both B and S. So now, it has questioned the wisdom of one particular property of insisting on this property which is X intersection X^c should it really be empty.

Of course note that we have made 2 important assumptions that all men in this village are clean shaven and the barber himself is from this village. Now so far we may have seen that the properties of the set which we thought were invaluable in some situations perhaps there is a need there is a ccause for relaxation.

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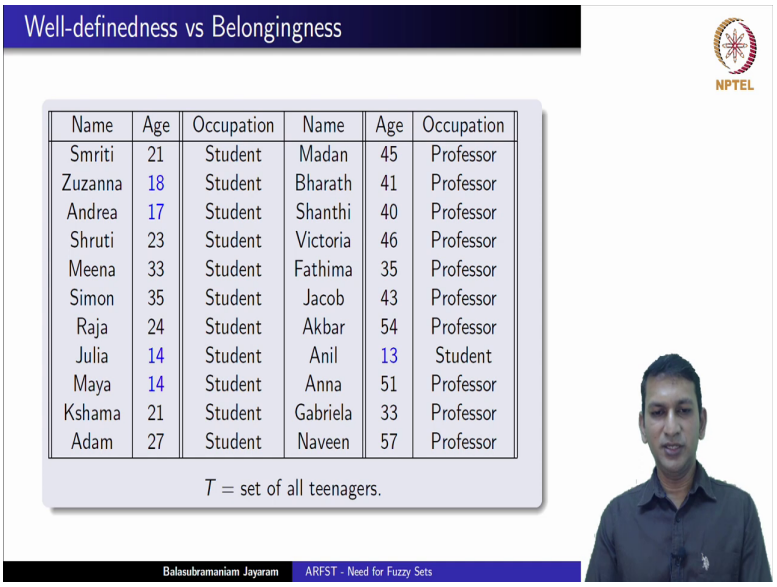

Allow me to present to you another example consider this table in which you have a few names the ages of the corresponding people and their occupation. I have chosen only the occupation to be the student or professor. Now on given this data we can ask some questions.

So, if I were to ask you to pick up the set T of all teenagers I think it is easy to do or you will do is pick all those people whose ages fall between 13 and 19. So, in this particular list that we have these are the teenagers that we have. Now, we will slightly change the question and ask instead of picking the set of all teenagers, let us ask us ask ourselves to pick the set of all young people.

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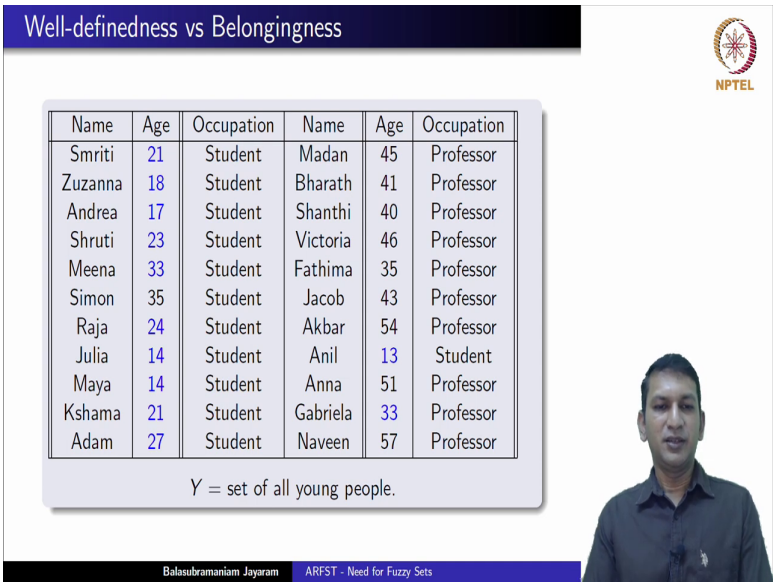

Now who is young? Let us for a moment assume that all those whose ages have not touched 35 are young. So, we put a hard threshold here. So, strictly less than the age strictly less than 35 we would call them young for the moment.

So, if you were to apply this criterion then these are the people who could be called young? But notice that there are 2 people here Meena and Gabriela who are 33 years of age and are considered as young people, but there are 2 other people Simon and Fathima who are 35 years of age and they are not considered young anymore ok. Let us redefine this set little further instead of asking for the set of all young people.

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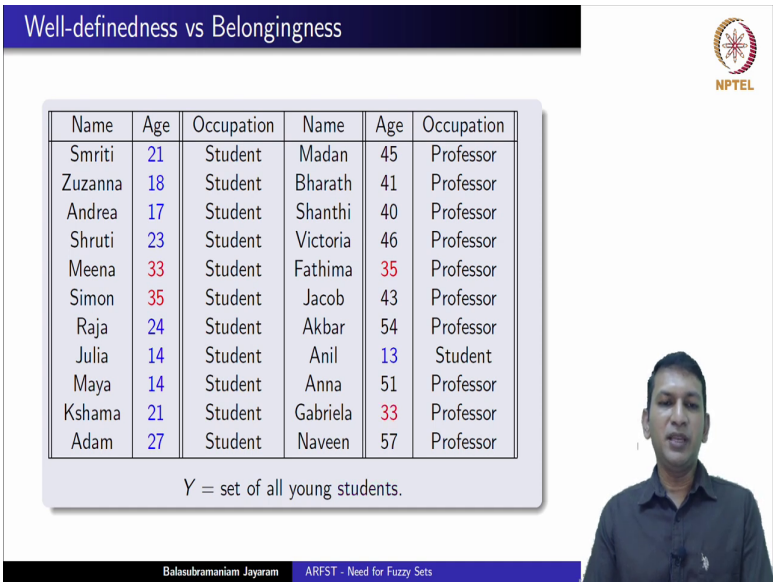

Let us ask for the set of all young students, clearly Gabriela would lose it out because she is a professor not a student anymore. But what is interesting is that even Meena would not be considered a young student in the natural sense of people who joined school or college. Of course, it is contextual we will come to that in a later lecture.

So, the very age of 33 which allowed Meena and Gabriela to be called young in the sense of young persons, they do not allow them to be called young students. Now let us refine this or modify this a little further.

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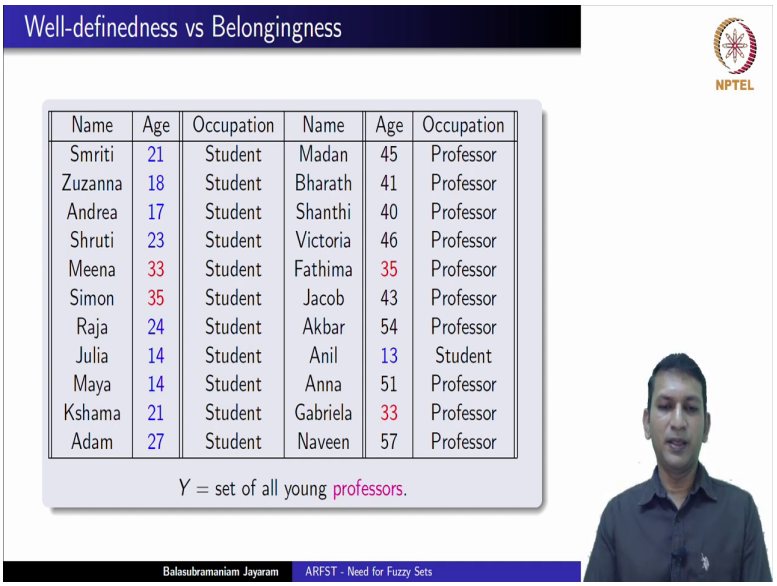

Consider the set of all young professors; obviously, many of them would lose out because they are students. Who should be considered a young professor?

Let us say somebody who is 40 or below 40 of years of age and a professor is considered a young professor. In that sense these are the people whom you would call as young professor those whose ages are marked in blue, but there is also one person Bharath who is just 41 years of age who is not considered a young professor; whereas, Shanti who is 40 years of age is considered a young professor.

You see that to be able to put a threshold a hard cut off and distinguishing people is becoming a little unwieldy here.

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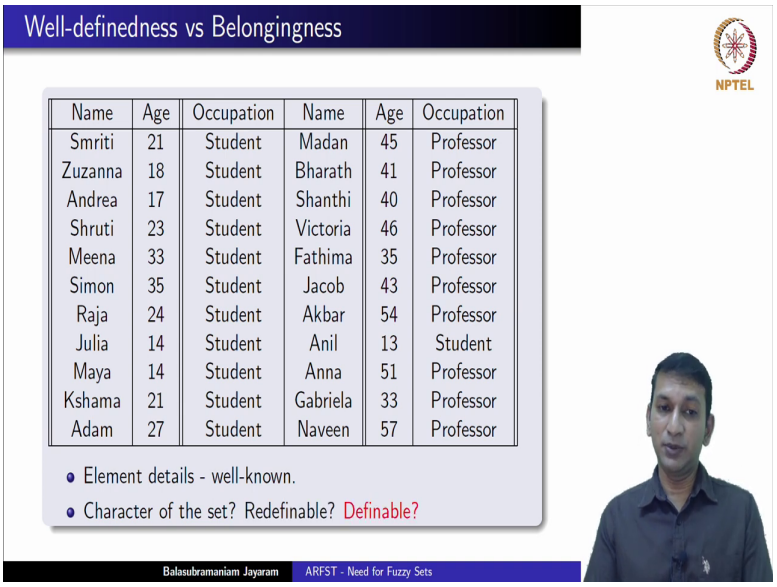

You should also note here that all the details of the elements which are necessary for us are known. However, what we are not able to decide on is who is a young person, who is a young professor and who is a young student? So, it is the character of the set that we are not able to clearly define. Is it redefinable in fact the question is about is it definable at all or not.

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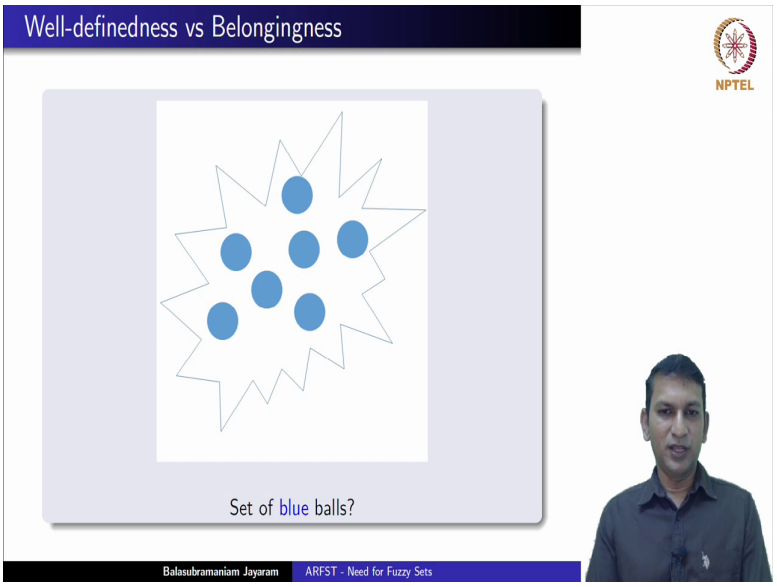

Let us look at this picture. So, if I were to ask what does it consist of? Once again if you say that it is a set of blue balls it consists of blue balls yes we would not be far away from the truth. So, let us say that it is a set of blue balls, now the question is I have another ball here.

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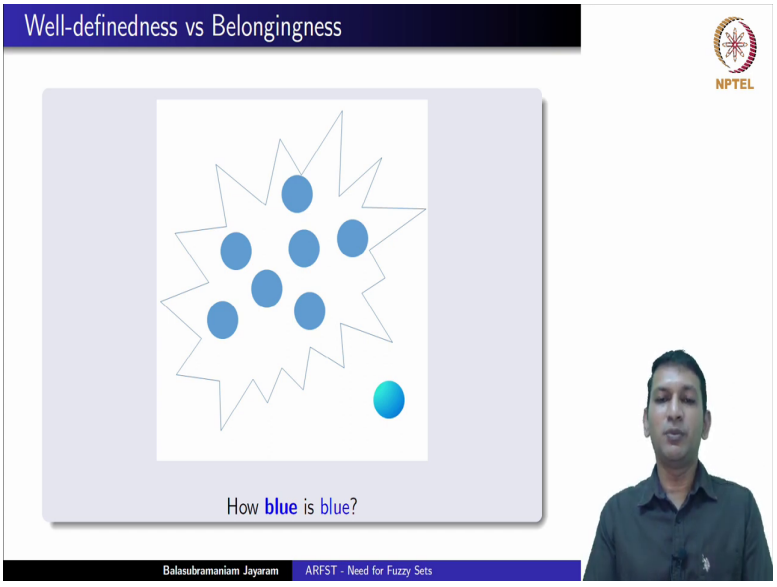

Now, ask the question does this ball at the right bottom corner belong to the set of blue balls? Now if you take a moment part of you wants to say yes, part of you wants to say no. Because you are asking the question how blue should something be to be called blue.

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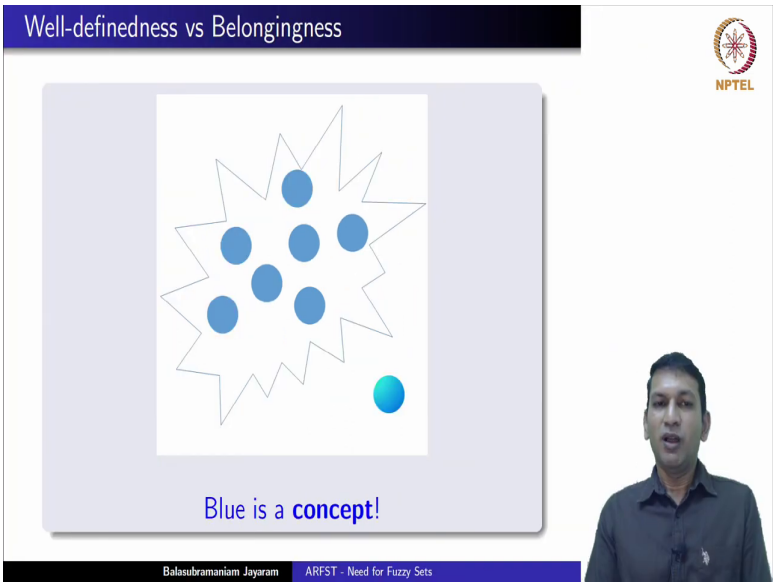

Because blue itself is a concept we talk about darkness, we talk about pitch darkness; so it is a Concept.

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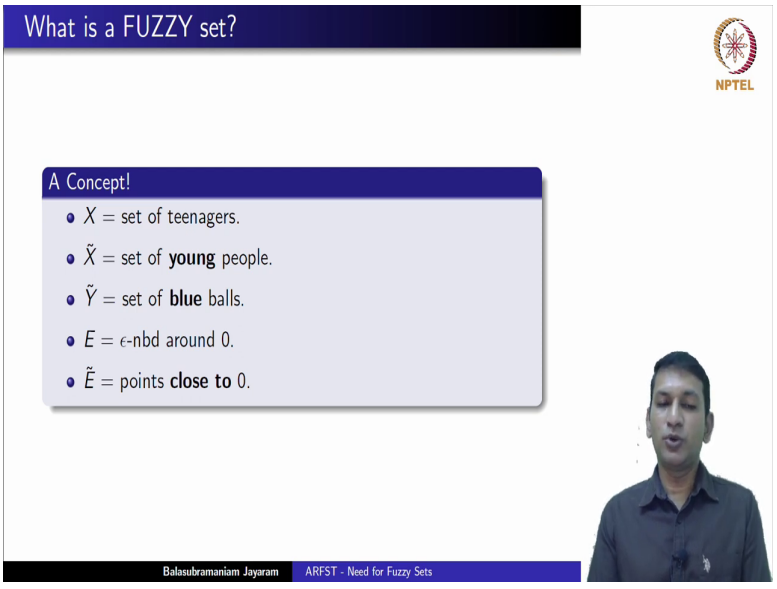

So, if you were to ask what is a Fuzzy set? Perhaps one of the best ways to explain it is it is a mathematical construct which captures the concept for instance consider this sets that we have discussed. So, far if x is set of teenagers it is easy to discuss whether somebody belongs to the set or not, but the moment you relax and call it a set of young people, then we know that the concept young applies differently it is varying, it is subjective, it is contextual.

Similarly, if when we talk about set of blue balls blue becomes a concept. If E is just the epsilon neighbourhood around 0, the moment you fix epsilon then we are in a position to say whether an element belongs to the set or not. However, if you modify this and say that let E tilde represent all those points that are close to 0, then this closeness this proximity it is a concept it is contextual and subjective.

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So, far we have seen the need for generalizing classical sets themselves and we have reached a point where we would like to abstract the concept and Fuzzy sets give you a way of doing it. How we do it is a will be covered in the next lecture, but now let us look at the need for Fuzzy sets how they were motivated from an application perspective.

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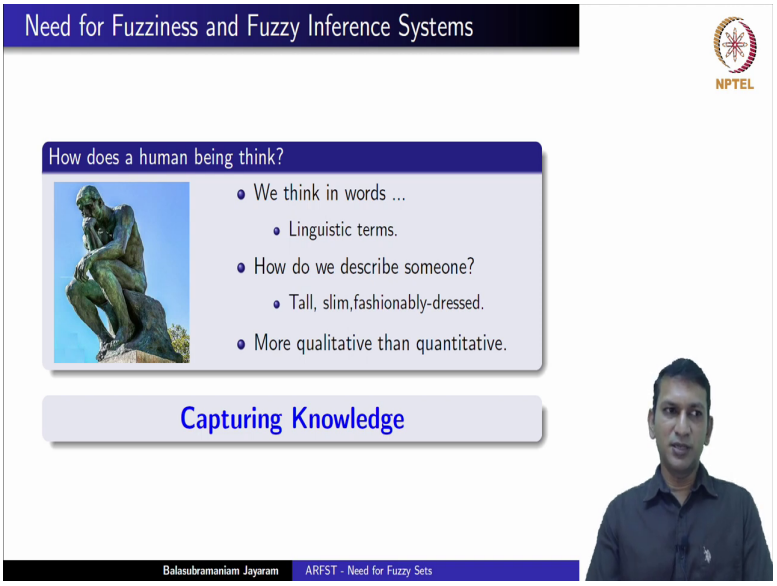

Now this course is about approximate reasoning. So, for reasoning you need both knowledge and intelligence as we have seen before. But let us ask ourselves a question a different question perhaps how does a human being think? Thinking is very much a part of reason if you were to ask this question without much effort we would realize that we actually think in words in terms of linguistic terms.

Imagine you are sharing a room with your roommate and a visitor drops by, but unfortunately your roommate is not available at that time and even before you could find out the name of the visitor he leaves. Now when the when your roommate friend comes back to the room you would like to describe this person to him. How would you describe him? Clearly you would not have had the time or even the inclination to measure his height.

So, that you can tell your friend that he was 5 feet 7 and a half inches in height and perhaps 63 kgs in weight, no that is not how we describe somebody. Your description perhaps would be as he was tall, slim and fashionably dressed perhaps. Now the question is what is tall what is slim and how should one be dressed to be called fashionable. Now let us look at it.

How tall should one be to be called tall? What should his height be? For instance in India you might say that somebody who is around 5 foot 9 inches can be considered tall; however, if you travel to northern European countries perhaps it is more or less an average height of anybody there. On the other hand what you would consider as an average height for an Indian could well be somebody who is tall in Japan.

So, tall as you see is not only subjective, but also contextual. Similar is the thing with being slim and now coming to the last word fashionably dressed. We will know that this is a very time varying concept too not just subjective, but contextual and time varying too so all of this point out to this. That we perhaps think more qualitatively than quantitatively. Why is this important?

Because we want to be able to capture knowledge, knowledge comes from human beings and if you want to capture them you need a way of capturing this into a construct. A question might arise as to why we need to capture this knowledge? These days we talk about machine learning perhaps a little bit of going back in history might give us a good perspective.

There were times when automated systems were being used to support human decision makers, these so called expert systems used probabilistic tools to arrive at a decision. However, people wanted machines to think to think like experts, if you want the machine to think like a human being then it was important to understand how human beings thought and as is clear largely we think in terms of linguistic terms.

So, it is essential that we capture knowledge into a mathematical construct and give it a representation that is both appropriate and amenable for further processing.

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Now consider this question when we measure something we know that there are lots of possibilities for errors to creep. In there are different kinds of errors all of us would have done some amount of experimentation either in chemistry lab or physics lab.

There are these errors that are associated with the instrument itself, for instance 0 error then there are these errors that human beings introduce like the parallax error, but then there are also this subjective errors. For instance if you look at the picture on the left if I were to ask you is it a cat or a dog each of us would have his own answer for it. Of course, it is clear it has to be either a dog or a cat.

But looking at it I think our subjectivity of how we are conditioned by the morphological features of a cat or a dog would lead us to answer accordingly. So, imprecision in measurement also necessitated the need for fuzziness. Many of these things we will see in more detail as the lecture series progresses.

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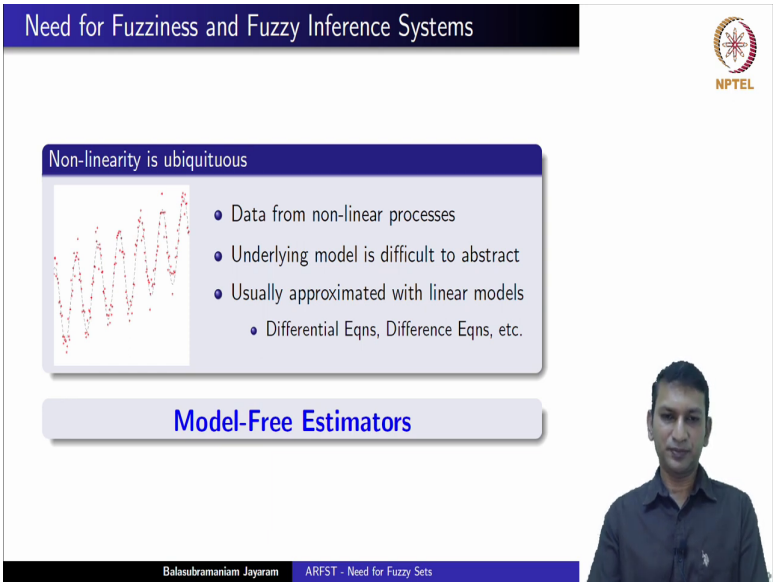

Finally we talk about data analysis. Data normally are collected from processes which are typically non-linear; however, it is very difficult to abstract the underlying model. Often as engineers you would readily understand that we resort to approximations and often these approximations are with linear models in the time Fuzzy set theory was introduced in the in mid-sixties the controllers that were used or the models that were applicable or applied typically used differential equations or difference equations or integral equations.

At that time they felt a need for model free estimators and Fuzzy Inference Systems were one way of building model free estimators.

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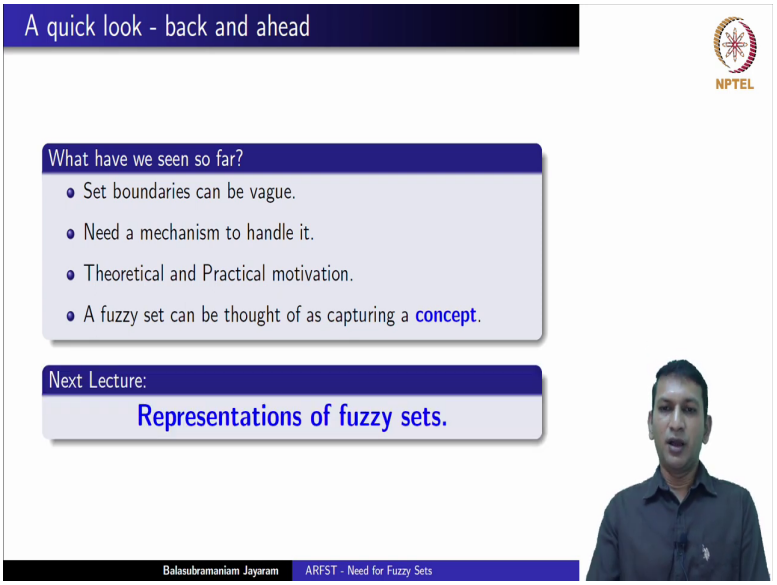

A quick look at what we have seen in this lecture so far, from a theory perspective we have seen that set boundaries can be vague. We need a mechanism to handle it and there were clear theoretical and practical motivations for these.

And if you were to be asked what is a Fuzzy set it can be thought of as capturing a concept. As we saw a few moments before the representation took that is given to the captured knowledge is very important and if you want to capture it through this concept of fuzzy set a useful appropriate and amenable representation is very very important and this is what we will see in the next lecture.

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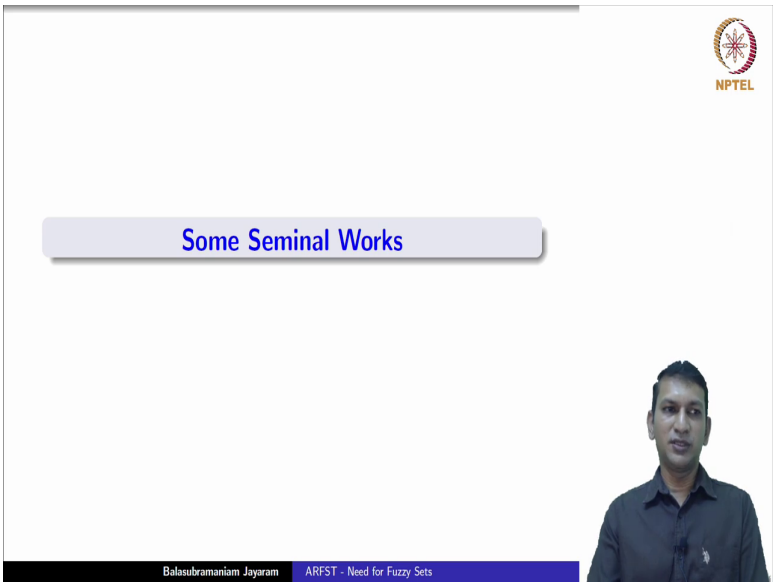

However before we close this lecture allow me to flash at you a few seminal works, there have been quite a few perhaps my choice has been motivated by the content that we have covered in this lecture.

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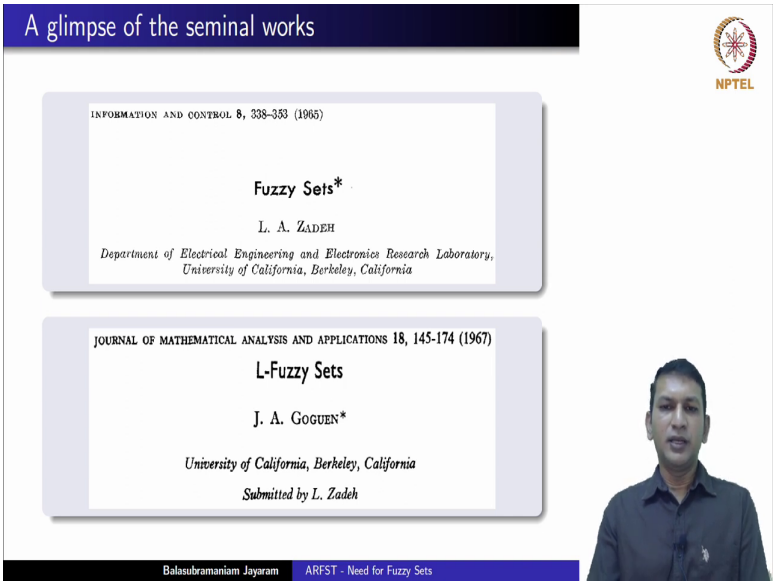

This was the original paper of Lotfi A Zadeh who is creditor as the father of Fuzzy Set theory, his paper appeared in 1965 in the journal Information and Control.

Wherein he propounded his new theory of sets the generalization of classical sets to Fuzzy sets mind you professor Zadeh himself was an engineer an electrical engineer from UC Berkeley and he came to propound this concept of Fuzzy sets from a purely practitioners perspective. Soon enough mathematicians also got interested in this and within 2 years time there was a seminal paper outlining the theoretical underpinnings of fuzzy set theory in a very reputed mathematical journal by Goguen.

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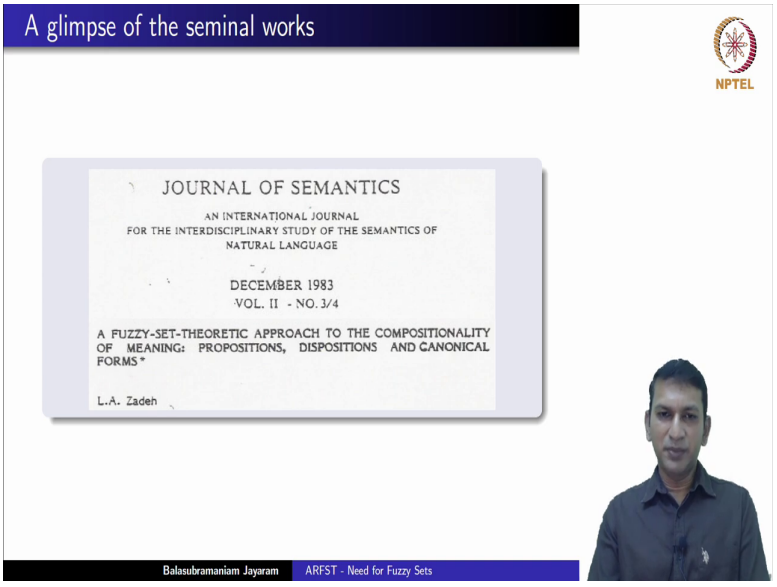

This is another paper of Zadeh himself which deals in a more semantic way on what a Fuzzy set is, this appeared maybe almost 15 years later 1983 allow me to flash at you just 2 more works which you could see as seminal.

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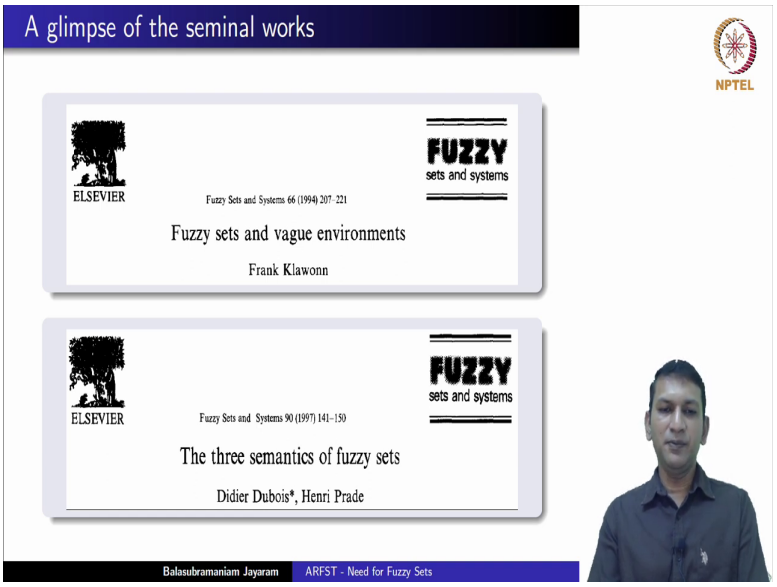

The first of them was almost 25 years after the first paper on Fuzzy set theory by Zadeh by professor Frank Klawonn, where he discusses fuzzy sets and vague environments and this could probably resonate with you given the content that we have covered in this lecture. And finally, there is another very well known well cited paper the Dubois and Prade which deals with the different semantics that you could assign to fuzzy sets.

We encourage you to have a look at these papers too they are written very nicely and more in a story like form and we believe that these research papers are also at an accessible level to the audience of this course. So, with this we will come to the end of this lecture, in the next lecture as was flashed we will look at finding a useful appropriate and amenable representation for fuzzy sets.

Thank you.