Numerical Linear Algebra

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Lecture - 14
Eigen values & Eigenvector – II

Hello friends, I welcome you to my lecture on Eigen values and Eigen vectors, the second lecture on this topic. So, in the in this lecture the first theorem which we are going to do is, the Cayley Hamilton Theorem. This theorem is very important in the discussion on Eigen values and Eigen vectors; it says that every square matrix satisfies it is on characteristic equation.

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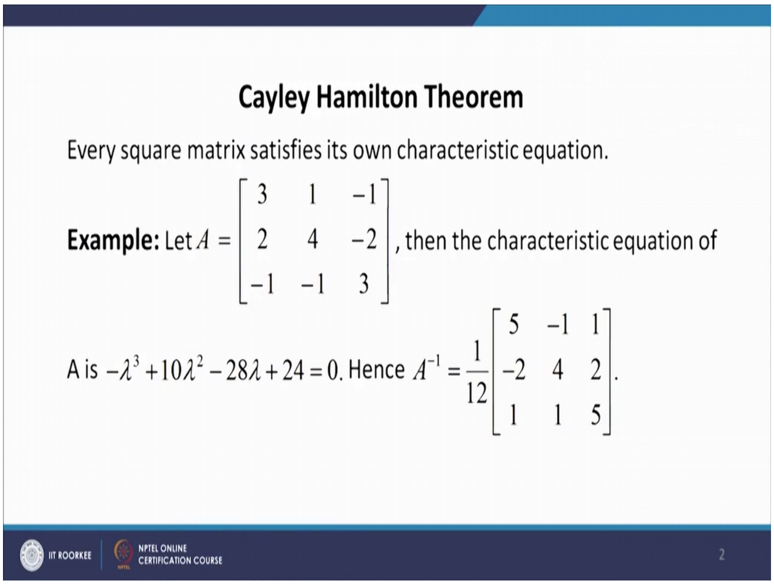

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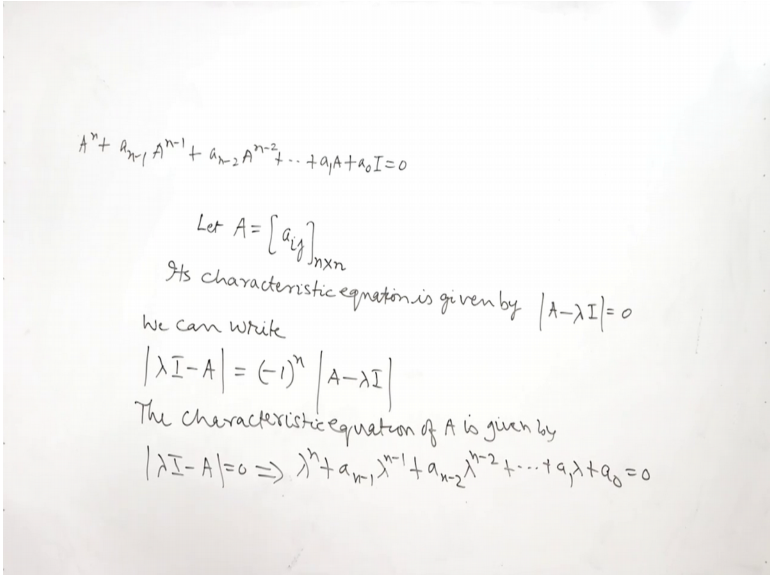

So, if you have a square matrix, let us say A equal to a i j of order n. Then we know that its characteristic equation is given by determinant of A minus lambda I equal to 0. Now when you explain this determinant you get a polynomial equation in lambda of degree n. So, the theorem says that if you replace lambda by the square matrix A in the equation the constant; of course, will be then presented by the constant times, the identity matrix of order n.

So, then that equation, when the equation, characteristic equation among its satisfied for the matrix A. So, suppose the characteristic equation we can write we can write for convenience. I will write lambda I minus A this is equal to minus 1 to the power n determinant of this, I am doing to make the coefficient of the lambda to the power n equal to 1, because in the in the, when you explain the determinant mod of A minus I mean determinant A minus lambda I, then the coefficient of the lambda to the power n is minus 1.

So, in order to prove the Cayley Hamilton Theorem, we will like to have the coefficient of lambda to the power n equal to 1 for a convenience. So, we write a determinant of the matrix lambda I minus A which is nothing, but minus of A minus lambda I. So, minus 1 will be the, when you take the determinants is minus 1 to the power n will be there. So, what we say, is that when you explain with this determinants let us. So, let us focus on, so lambda I minus A, the characteristic equation lambda A minus lambda I equal to 0 will give determinant of lambda I minus A equal to 0.

So, I can take the characteristic equation of as, which implies when you expand it lambda to the power n plus a n minus 1 lambda to the power n minus 1 plus n minus 2 lambda to the power n minus 2 and so on a 1 lambda plus the constant a naught. This is equal to 0. So, this is the characteristic equation. The theorem says that what we have is the following. The theorem says that A to the power n replace lambda by a. So, a n minus 1 A to the power n minus 1 a n minus 2 A to the power n minus 2 and so on a 1 lambda a 1 A 1 A plus a naught.

Now we cannot add a scalar these are m by n matrices. So, we multiply a naught I identity matrix of order n. So, a naught I this is equal to 0. We are going to prove that this equation puts, and therefore, from here we can say that every square matrix satisfied this characteristic question, this is what we are going to prove. So, if we can prove this, we will have to prove the Cayley Hamilton Theorem; that is every matrix satisfies this characteristic questions. So, let us begin from here. We shall be considering the characteristic question determinant lambda I minus A equal to 0 with gives us this equation.

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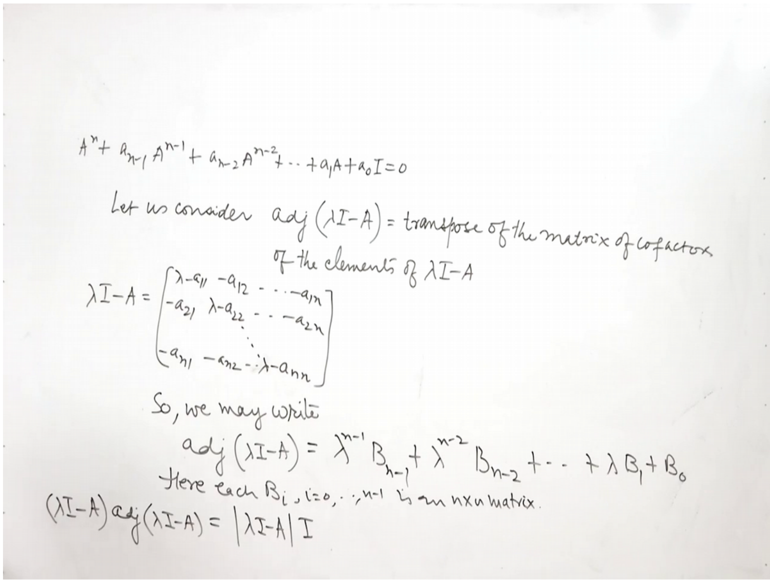

Now, what we do is, let us consider the adjoint of the matrix lambda I minus A, the adjoint of the matrix lambda I minus A is the matrix transpose of the matrix of cofactors of the elements of lambda I minus A. It is the transpose of lambda I minus A. Now let us look at the matrix lambda I minus A, lambda I minus A is looks like this from the identity matrix after multiplying by lambda you are subtracting A. So, diagonal elements will all be subtracted from lambda.

So, we get lambda minus a 1 1 lambda minus a 2 2 and so on, lambda minus a n n. And here we get minus a 1 2 minus a 1 n minus a 2 1 and so on minus a 2 n, and here we get minus a n 1 minus a n 2 and so on, lambda minus a n n, this is lambda I minus A. Now if you take the mat cofactor of any element here what you get is an n y n matrix, whose determinant will give you a polynomial in lambda of degrees n minus 1.

So, what we can say is the following. So, we can write adjoint of lambda I minus A adjoint of lambda I minus A is the transport of the matrix of cofactors of the elements of A, each cofactor here gives you a polynomial in lambda of degree n minus 1. So, adjoint of lambda I minus A, we can write as B n minus 1 B n. Let me write matrix here lambda to the power n scalar first, let us letters write a scalar first. So, lambda to the power n minus 1 B n minus 1 lambda to the power n minus 2 B n minus B n minus.

sorry not like this B n minus 1 like this, B n minus 1 and then B n minus 2 and so on lambda B 1 plus B naught, because the cofactors are polynomials in lambda of degree n minus 1. Now here each B I is an n by n matrix, is an n by n matrix. Now we know that if you mat multiply A matrix by its adjoint, but do that is determinant of the matrix into identity matrix. So, let us use that, so adjoint of B lambda I minus A into lambda I minus A when you multiply lambda I minus A by adjoint of lambda minus A, what is that is determinant of lambda I minus A into identity matrix. So, let us write this.

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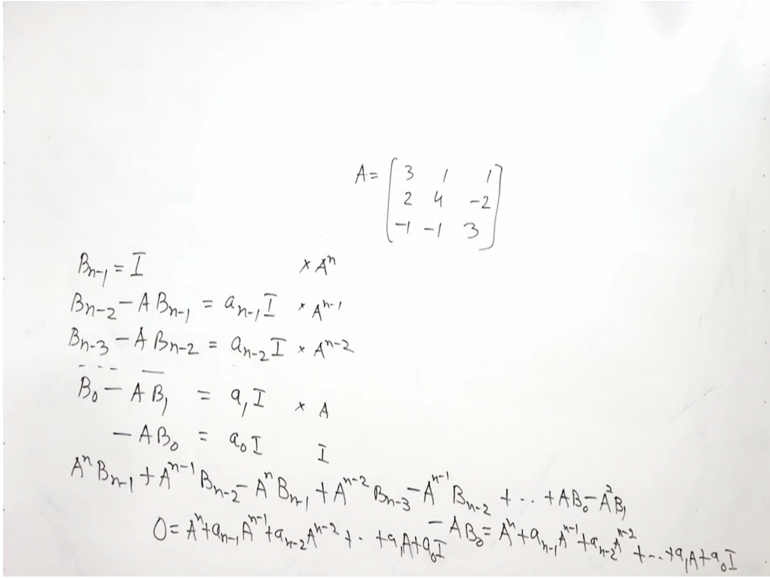

Now lambda I minus A times adjoint of lambda I minus A is lambda to the power n minus 1 B n minus 1 lambda to the power n minus 2 B n minus 2 and so on, lambda B 1 plus B naught, and this is equal to determinant lambda I minus A, which is ah, which we wrote, which we have written as lambda to the power n plus n minus 1 lambda to the power n minus 1 n minus 2 lambda to the power n minus 2 and so on a 1 lambda plus a naught multiplied by identity matrix. Now let us equate the corresponding powers lambda on both sides.

So, if we do they that, the coefficient of lambda to the power n here is I into B n minus 1. So, we get B n minus 1 equal to the coefficient of lambda to the power n here is I. So, we have coefficient of lambda to the power n, here is I, and then we get lambda to the power n is a I, ya. Now the coefficient of lambda to the power n minus 1, the coefficient of lambda to the power n minus 1 is how much, when you multiply lambda I to this, you get lambda to the power n minus 1.

So, B n minus 2 minus A B n minus 1, this equal to a n minus 1 I. Now we coefficient of lambda to the power n minus 2 if you find, then what will get lambda to the power n minus 3 into B n minus 3 multiplied by this. So, B n minus 3 minus, then A B n minus 2, and this equal to a n minus 2 I. So, this may be go on. Then we shall have, when you multiply lambda I by lambda B 1, will get the coefficient of lambda I square. We can write the coefficient of lambda. So, coefficient of lambda will be minus A B 1 and then B naught.

So, B naught minus A B 1 and coefficient of lambda where is a 1 I. Then we can equate the constant both sides ah; that is con not, not constant, let me say the term which is free from lambda. So, minus A B naught is equal to a naught I. So, we have these equations, they are n plus 1 equations. Now what we do? We multiply the first equation by A to the power n second equation by A to the power n minus 1. So, we multiply by A to the power n A to the power n minus 1 A to the power n minus 2 and so on. This is by A and this by I, and that is we can do pre multiplication.

So, A to the power n B n minus 1, A to the power n B n minus 1 ah, and then right hand side will be A to the power n. Here left hand side we are writing first, so A n minus 1 when we multiply here A n minus 2, we have n minus, sorry A n minus 1 A n minus 1 B n minus 2 and then we get minus A to the power n B n minus 1. So, you can see becomes will go on cancelling, and then you will pre multiplied by A to the power n minus 2. So, A to the power n minus 2 B n minus 3, and then we get minus A to the power n minus 1 B n minus 2, and we can go on like this, and then let us multiply the last, but 1 equation. So, plus A B naught A B naught minus A square B 1, then we have minus A B naught.

And right hand side will give you A to the power n into I. So, A to the power n plus a n minus 1 a to the power n minus 1, and then a n minus 2 a to the power n minus 2 and so on a 1 a plus a naught I. Now we can see A to the power n B n minus 1 will cancel with this, A to the power n minus 2 B n minus 2 will cancel with A to the power n minus 1 B n minus 2 with this and so on. So, the terms will go on cancelling A last B last in the end, at the end A B naught will cancel with A B naught, so left hand side becomes 0.

So, 0 equal to A to the power n plus a n minus 1 A to the power n minus 1 a n minus 2 A to the power n minus 2 and so on, a 1 lambda a 1 A plus a naught I. So, we have this equation. So, that is why we say that every matrix satisfies its characteristic equations. Now let us take an example on this theorem. If the size of the matrix is not very large, we can cal calculate the inverse of the matrix by this theorem, say let us take a 3 by 3 matrix; A equal to 3 1 1, A equal to 3 1 1 and then the second row 2 4 minus 2, and then we have minus 1 minus 1 3.

Then what we will do is, we will find the characteristic equation for this matrix, and then use Cayley Hamilton theorem, to determine the inverse of the matrix A. So, here we are assuming the matrix A whose determinant is non zero, so that its inverse exits. So, by Cayley Hamilton Theorem we will find the inverse.

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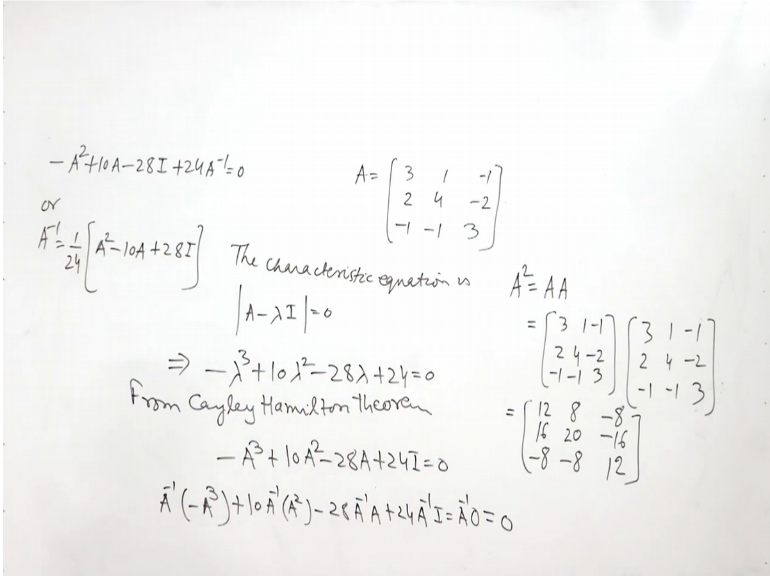

So, if you, the characteristic equation is determinant of A minus lambda I equal to 0. So, from the diagonal elements of the matrix A we subtract lambda, and then find the determinant of the resulting matrix. So, you can check check that, this will give you the question minus lambda cube plus 10 lambda square and then minus 28 lambda plus 24 equal to 0. So, this is the characteristic question. Now you can see we have a 3 by 3 questions, so its characteristic question is a polynomial in lambda of degree 3. Now every matrix satisfy its characteristic questions. So, from Cayley Hamilton Theorem we have, we can replace lambda by A cube, so by A.

So, minus A cube plus 10 A square minus 28 A plus 24, we multiply by I equal to 0. This right hand side is a 0 matrix. Here this 0 is a 0 scalar. Now what we will do in order to determine the inverse of A, let us pre multiply this equation by A inverse. So, A inverse minus A cube plus 10 times A inverse A square minus 28 A inverse A plus 24 A inverse I equal to A inverse 0. Now any matrix multiplied by 0 matrix is 0 matrix, so we get 0 matrix here. Now this is, this A cube is, can be regarded as A into A square.

So, A inverse into A is identity matrix identity, identity into A square will give A square and minus 1 is A scalar, we can write outside this matrix, so minus A square we get, and then we get similarly 10 A, A inverse A is identity, identity into A is A. So, then minus 28 I minus 28 I, this identity matrix, and then plus 24 A inverse equal to 0, A inverse I is A inverse, or we can say a inverse is equal to, we can transfer the terms with the site, so 1 by 24 A square minus 10 A plus 28 I. So, big for the given matrix A we can find A square. So, A square is equal to A into a. So, I can write it as 3 1 1 2 4 minus 2, then minus 1 minus 1 3, and then here 3 1 1, then 2 4 minus 2, and then we have minus 1 minus 1 3 yes.

Student: Why is last set is minus 1.

[FL] sorry, this is, this is minus 1 here. So, what we have here minus 1. So, when we multiply the two matrices what we get is. So, first column be multiplied to the all the rows of the A matrix to get the first column. So, 3 into 3 9, 9 plus 2 11, 11 plus 1 12, then 3 into 2 is 6, and then 4 into 2 8. So, 6 plus 8 14, 14 plus 2 16, then we have minus 3, we have minus 2, so minus 5, minus 3 is minus 8. And then, so this is first column. Now go to the second column.

So, 3 into 1 3, 3 plus 4 7, 7 plus 1 8, and then 2 plus 16 18, 18 plus 2 20, and then we have minus 1 minus 4, so minus 5, minus 3 means minus 8. And then we have third column, so minus 3 minus 2 minus 5, minus 5 minus 3 is minus 8. Then we have minus 2 minus 8 minus 10, minus 6 minus 16, then we have plus 1 plus 2, so 3, and then we have 3 3 9, 9 plus 3 12. So, this is A square ok. From is square A square, we subtract 10 times a matrix. So, 10 times A means, you multiply all the entries of a matrix by 10. So, let us find A inverse here. So, so this is A square right.

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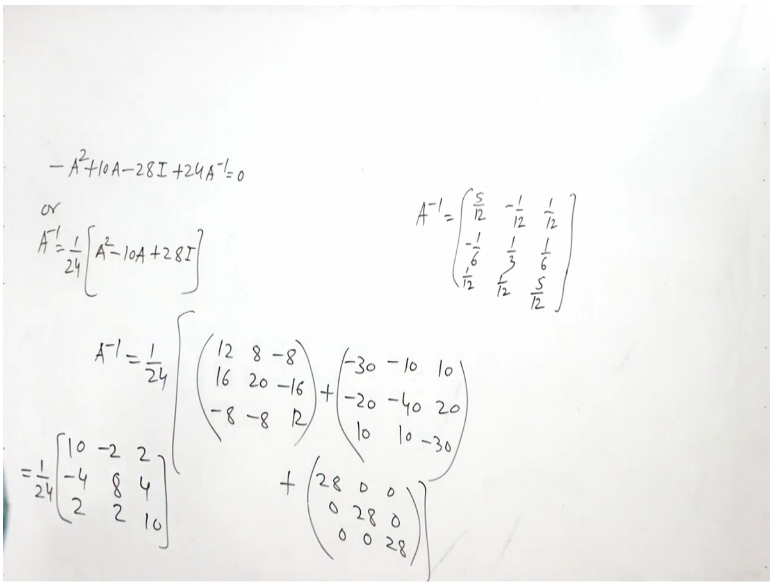

So, A inverse equal to 1 by 24, then I write this matrix. Let me write this way. So, 12, 8, minus 8, then 16, 20, minus 16, minus 8 minus 8, 12 and then minus 10, minus 10 be multiply, so I put minus, plus sign here. So, minus 30, minus 10 multiplying, so minus 10 plus 10, minus 20, minus 40, and then we get 20 here, then we get 10, 10 minus 30, I multiplied by minus 10, and then 28 times identity matrix. So, identity matrix of order 3 multiplied by 28. So, 28 0 0 0 28 0 0 0 28. Now let us add the 3 matrices and divided by 1 divided by 24.

So, 12 minus 30 minus 18 minus 18 plus 28, so we get 10, then we have 16 minus 20, so minus 4 minus 4 plus 0, so minus 4. Then we have minus 8 plus 10 we get 2, then we have 8 minus 10 minus 2, then we have 20 minus 40, so minus 20 plus 28, so we get 8, and then we have minus 8 plus 10 we get 2, and then we have a minus 8 plus 10. So, we get 2 and we have minus 16 plus 20, so 4 and then we have 20 12 minus 30. So, minus 18 plus 28, so we get 10. So, this is the bracketed expression this multiplied by 1 by 24.

So, you multiply each element of the matrix by 1 by 24 to get A inverse. So, finally, A inverse comes out to be, so 10 by 24 5 by 12, then we get minus 1 by 12, then we get 1 by 12, we have minus 1 by 6, we have 1 by 3 then 4 by 24. So, 1 by 6 then 2 by 24, so 1 by 12, then 1 by 12 again and then 10 by 12 24, we get 5 by 12. So, this is how we get the A inverse of the given matrix by using k Cayley Hamilton Theorem, but I, let me again remind you that this theorem can be used only for matrix, those matrices where the value of n is not large; that is the order of the matrix is not large, because we have to carry out the multiplication of the matrix with itself A square, if you did A to the matrix A to be of order 4, then you will have to find A cube and so on.

So, that is not an easy job. So, then if you want to find the inverse of a given matrix, then we use elementary row operations to reduce it to an identity matrix. So, whatever value elementary operations we do on the given matrix to reduce it to identity matrix, same elementary operations be, when we do on the identity matrix, we get the inverse of the matrix. So, we use that method. Now let us move to a result which is very important in the study of Eigen values and Eigen vectors, we have the following yeah. We will first discuss the properties of Eigen values of a special matrices we are different earlier discuss the a Hermitian matrix A, matrix is called Hermitian A is Hermitian if A is equal to a conjugate transpose.

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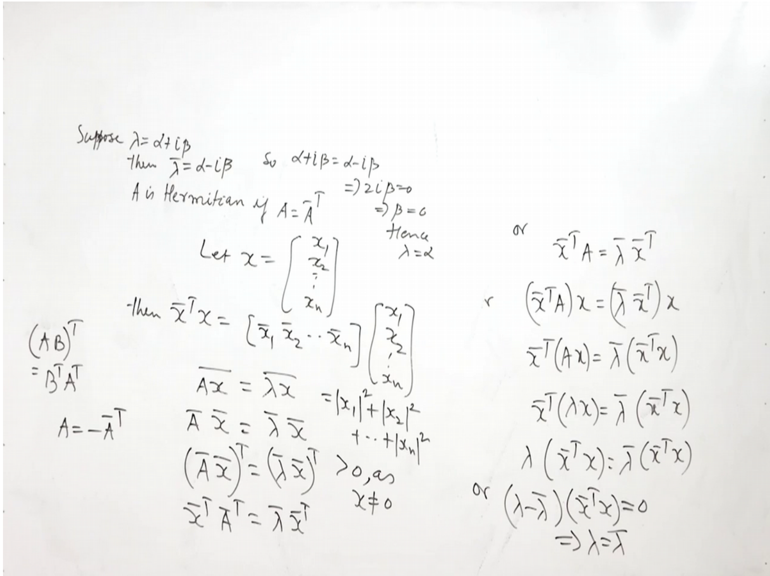

So, let us prove that every Eigen value of a Hermitian matrix is a real number. So, let us say, let lambda be an Eigen value of A and x the corresponding Eigenvector. Then we have the matrix equation A x equal to lambda x, then we have this matrix equation A x equal to lambda x. Now we have to make use of the definition of a Hermitian matrix; that is A is equal to a conjugate transpose. So, what we can do is, let us take the conjugate on both sides. So, conjugate on both side when we do, we have this. Now consecutive x means consecutive A and conjugate of x, then here lambda conjugate x conjugate. Now let us say transpose on both sides. So, A conjugate x conjugate transpose equal to lambda conjugate x conjugate transpose.

Now we know that when we have a product of 2 matrices A into B, the transpose of A B is equal to B transpose A transpose. So, let us apply this property. So, we will have x conjugate transpose A conjugate transpose equal to, lambda is a scalar, so we will have, so it will not be affected. So, lambda conjugate by transpose it is not affected, and then we have x conjugate transpose. Now A conjugate transpose is equal to A. So, we have or x conjugate transpose A equal to lambda conjugate x conjugate transpose. Now, we post multiply this equation by x. So, x conjugate transpose A x equal to lambda conjugate x conjugate transpose x.

now we can write by, by the associativity of the matrix multiplication x conjugate transpose A x equal to lambda conjugate x conjugate transpose x. A x is equal to lambda x, so we can write this, lambda is a scalar I can write it like this, lambda times x conjugate transpose x, or lambda minus lambda conjugate into x conjugate transpose x is equal to 0. This is what we have. Now let us note the following. See x is an Eigenvector, let us say, let x be having component x 1 x 2 x n, where we are dealing with n by n matrix, so Eigenvector will have n components, then x conjugate transpose, x will be equal to, x conjugate means take conjugate of all the components, when we take transpose this column vector becomes row vector. So, x 1 conjugate x 2 conjugate and so on x n conjugate these x conjugate transpose into x, so we have x 1 x 2 x n.

When we carry out this multiplicand, this 1 by n matrix these n by 1 matrix will get 1 by 1 matrix, and the 1 by 1 matrix means one number. So, x 1 conjugate into x 1, if you multiply complex number by its conjugate, you get the absolute value of the complex number is square. So, mod of x 1 square, then mod of x 2 is square and so on. Mod of x n square, x 1 into x 1 conjugate is mod of x 1 square. Now x is non zero vector. So, at least one component is here is non 0, and therefore, this sum is non 0, because the sum of non negative quantity, which can be 0 only when each quantity is 0.

So, this is not 0, inside it is strictly positive, as x is not equal to 0. So, this quantity is not 0, it is positive therefore, lambda is equal to lambda conjugate. When a complex number equal its conjugate, it is always a real quantity, because suppose lambda is a complex number lambda equal to alpha plus I beta, then lambda conjugate is alpha minus I beta. So, alpha plus I beta equal to alpha minus I beta means, 2 I beta equal to 0 or beta equal to 0; hence lambda is equal to alpha. So, it is a real. So, each Eigen value of the Hermitian matrix is a real quantity.

Now, Eigen values of a skew Hermitian matrix are purely imaginary or 0. The only difference in the case of a skew Hermitian matrix is that, you have a negative sign A is equal to minus A conjugate transpose. So, in this proof, when I hear I replace A conjugate transpose by A, you replaced by minus A. When you replace by minus A, same proof will carry on, here we will have lambda plus lambda conjugate. So, lambda plus lambda conjugate will be 0, because x conjugate transpose x is not 0. And then lambda plus lambda conjugate is 0, lambda plus lambda conjugate will be 2 alpha, 2 alpha equal to 0 means alpha equal to 0. So, lambda will be equal to I beta; that means, either lambda is purely imaginary or it is 0.

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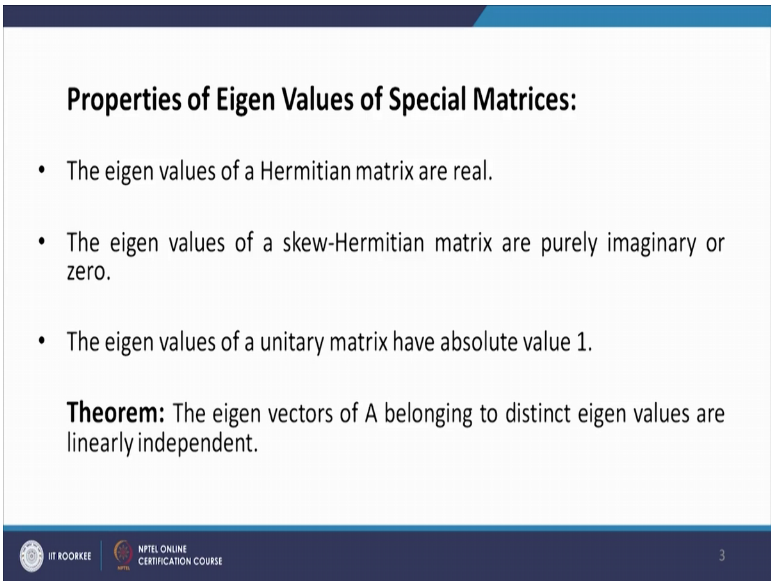

Now let us take the third property be Eigen values of A unitary matrix have absolute value one. So, let us say, let A be a unitary matrix.

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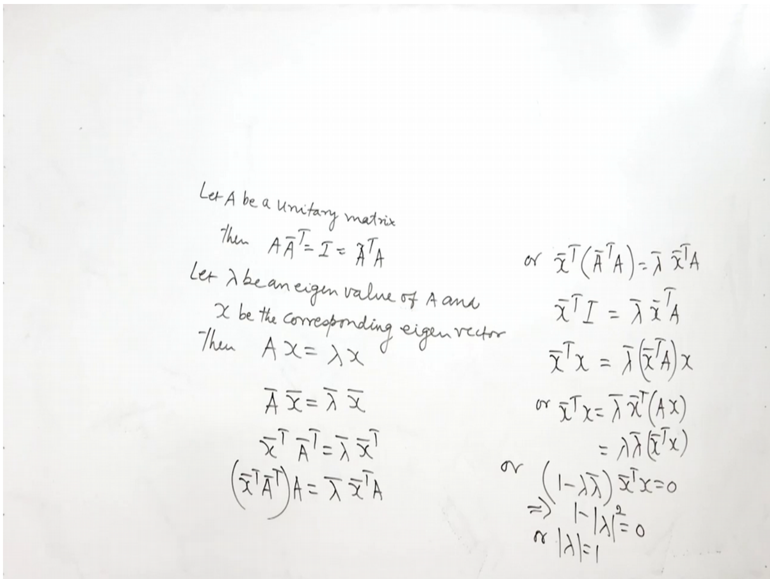

Then A A conjugate transpose is equal to identity matrix or you can say A inverse is equal to A conjugate transpose. Now let us say let lambda be an Eigen value of A and x be the corresponding Eigenvector, then we have these matrix equation A x equal to lambda x. So, again take conjugate transpose here, first we take conjugate, so A conjugate x conjugate equal to lambda conjugate x conjugate; like in the previous exercise. Now we take the transport on both sides x conjugate transpose A conjugate transpose, and then we have lambda conjugate x conjugate transpose, then we post multiply by A.

So, x conjugate transpose A conjugate transpose be post multiply by A, and then this will be equal to lambda conjugate x conjugate transpose A. Now this is we can write or x conjugate transpose A conjugate transpose A equal to lambda conjugate x conjugate transpose A, A conjugate transpose is equal to I, A conjugate transpose into A is equal to I when A A conjugate transpose is I. This is also A conjugate transpose A equal to I.

So, this identity matrix for x conjugate transpose into identity is equal to lambda conjugate text conjugate transpose A. Now this is matrix this multiply by I. So, it will give you x conjugate transpose. Now let us post multiply by x, let us post multiply by x. So, this is or A x equal to lambda x. So, we will have lambda, lambda conjugate x conjugate transpose or we can write 1 minus lambda lambda conjugate x conjugate transpose x equal to 0. We have shown that when x is an Eigenvector x conjugate transpose is never 0, it is positive.

So, 1 minus lambda lambda conjugate is 0. Now lambda into lambda conjugated mode of lambda square, so this is 0. So, this implies that mod of lambda is equal to 1. So, the Eigen values of a unitary matrix have unit models or there absolute value is 1. So, now, we know that a real unitary matrix A really unitary matrix is orthogonal matrix. So, in the case of a real matrix, the real orthogonal matrix, the Eigen values will be either 1 or minus 1 known as go to the Eigen vectors of a belonging to distinct Eigen values are linearly independent. So, suppose you are given an n by n matrix.

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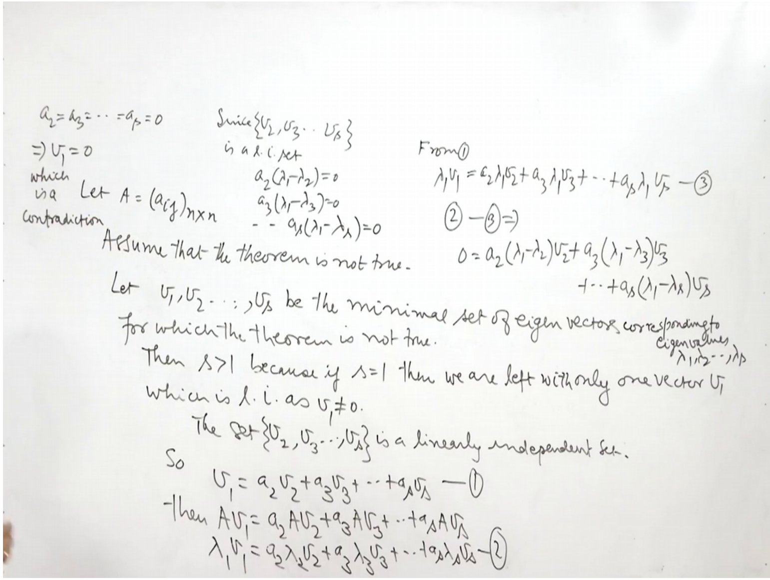

Let A be equal to a i j n by n. We want to show that the Eigen vectors of the matrix A which correspond to distinct Eigen values, they are linearly independent. So, let us assume that the theorem is not true. We are proving it to by contradiction. So, assume that, the theorem is not true. Then assume that the ah, then let us let v 1 v 2 and so on v s be the minimal set for which the term is not true, be the minimal set of Eigen vectors for which this theorem is not true. Then it is clear that s cannot be equal to 1, then s is strictly greater than 1, because if s is equal to 1.

Then we will be having only one vector here which is v 1, and a single vector when we have which is an Eigenvector, it is linearly independent. Then we are left with only one vector v 1, which is linearly independent, because it is non zero vector, which is linearly independent as v 1 is not equal to 0 vector. Now, so, s is greater than 1. Now by the our assumption that v 1 v 2 v s is minimal an set of Eigen vectors for v term is not true. These set, if you take the set of vector v 2 v 3 so on v s, then it will be a linearly independent set. So, the set v 2 v 3 and so on v s is a linearly independent set and. So, v 1 can be written.

Now v 1 v 2 v s is linearly dependent, because for this the term is not true, v 1 v 2 v n is linearly dependent while v 2 v s v 3 v s is linearly independent. So, we can write v v 1 as a linear combination of v 2 v 3 and so on v s. So, a 2 v 2 a 3 v 3 and so on a s v s I can write. Now let us pre multiply this equation by A, the matrix A. So, then A V 1 is equal to A is a linear operation. So, will get A 2 A V 2 A 3 A V 3 and so on a s A V s. Now V 2 V

3 V s are Eigen vectors of the matrix A. Let us say they correspond to the Eigen values lambda 2 lambda 3 lambda s by v 1 corresponds to the Eigen value lambda 1. So, then we will have lambda 1 v 1 is equal to a 2 lambda to v 2 and a 3 lambda 3 v 3 and so on a s lambda s v s.

We are assuming that set of Eigen vectors corresponding to Eigen values lambda 1 lambda 2 and so on lambda s, which are distinct. Now we have this equations. Let me call it as 1 equation number and this has equation number 2. Now in the equation 1 we multiply by lambda 1. So, from equation 1, lambda 1 be 1 is equal to a 2 lambda 1 v 2 plus a 3 lambda 1 v 3 and so on a s lambda 1 v s. Let me call it as equation number 3. Then from equation 2 subtract equation 3. So, 2 minus 3 will give you 0 equal to a 2 times lambda 1 minus lambda 2 into v 2 a 3 times lambda 1 minus lambda 3 v 3 and so on a s lambda 1 minus lambda s v s.

Now, v 2 v 3 and so on v s form a linearly independent set. Therefore, a 2 lambda 1 minus lambda 2 is equal to 0, a 3 lambda 1 minus lambda 3 is equal to 0, a s lambda 1 minus lambda s equal to 0. So, what we get since a 2 since v 2 v s form a linearly independent set, is a linearly independent set. We have a 2 times lambda 1 minus lambda 2 equal to 0, a 3 times lambda 1 minus lambda 3 equal to 0 and so on, a s times lambda 1 minus lambda a s equal to 0. Now lambda 1 lambda 2 lambda s are district. So, lambda 1 minus lambda 2 is not 0, lambda 1 minus lambda 3 is not 0, lambda 1 minus lambda s is not 0.

So, we get since Eigen values are distinct, so we get a 2 a 3 and so on a s all equal to 0, but a 2 a 3 a s all equal to 0, means v 1 is equal to 0, but v 1 is a non zero Eigenvector. So, at least one a k here is non0, since v is v 1 is an Eigenvector, at least one a k here is non0, but what we get here, all a 2 a 3 a n a so on a s are 0. So, this implies that v 1 is 0 which is a contradiction. So, Eigen vectors of a square matrix belonging to distinct Eigen values are all linearly independent v and then we have result which says that if A is real symmetric matrix then corresponding to distinct Eigen values, the Eigen vectors of A are very, of A are orthogonal.

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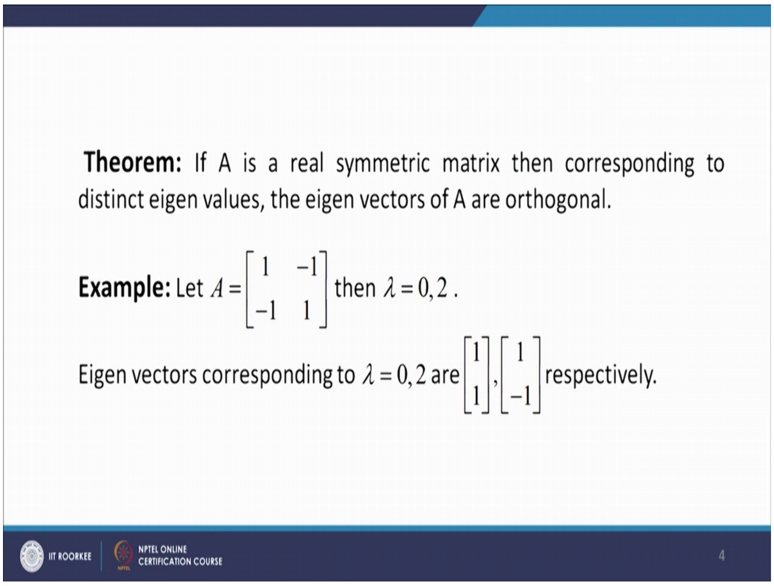

Now this is very important result which we will use in our next lecture when we discuss the diagonalization. So, let us prove this result, the prove is not difficult.

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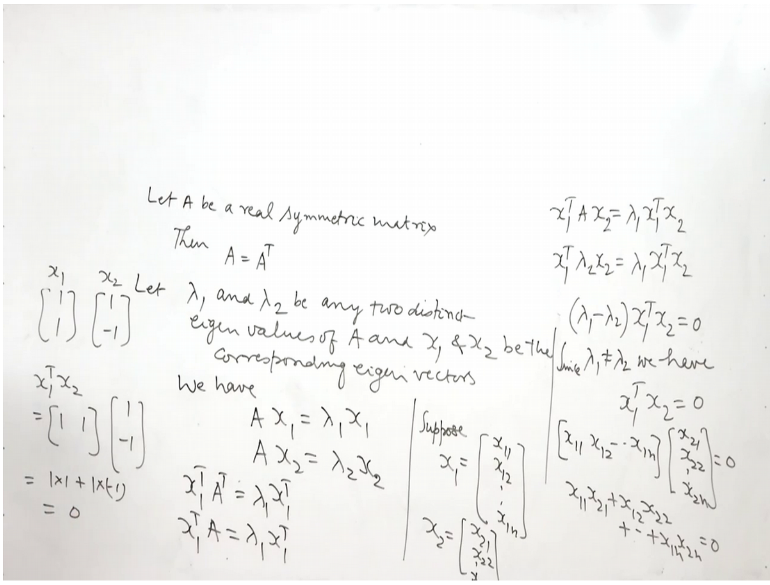

So, let A be a real symmetric matrix, then A is equal to a transpose. Now we want to prove that corresponding to distinct Eigen values, the Eigen vectors of a are orthogonal. So, let us say let lambda 1 and lambda 2 be any two distinct Eigen values of A and x 1 x 2 be the corresponding Eigen vectors. We want to prove that x 1 and x 2 are orthogonal to each other. So, we have A x 1 equal to lambda 1 x 1, because lambda within lambda 1 is an Eigen value of A an x 1 is the corresponding Eigenvector, and then we have second equation A x 2 equal to lambda 2 x 2.

we can pick up any equation in order to make use of the given condition on A; that it is real symmetric matrix A, A is equal to A transport, we have to take transpose of any one of these two equations. So, let us take the transport of the first equation. So, we get x 1 transpose A transpose equal to lambda 1, lambda 1 is a scalar, so it is not affected by transpose, so x 1 transpose. Then A transpose is equal to A. So, x 1 transpose A is equal to lambda 1 x 1 transpose. Now we post multiply by x, so x 1 transpose A x is equal to lambda 1 x 1 transpose x, A x is given to be, oh be A A x we are post multiply by x 2.

So, be post multiply by x 2. So, A x 2 is equal to lambda 2 x 2, so x 1 transpose lambda 2 x 2 is equal to lambda 1 lambda 2 x 2, oh no this is x 1 transpose x 2. So, we have x 1 transpose x 2. So, we will have, I can write it as lambda 1 minus lambda 2 x 1 transpose x 2. Now lambda 1 and lambda 2 are distinct. So, lambda 1 is not equal to at lambda 2, since lambda 1 is not equal to lambda 2, we have x 1 transpose x 2 equal to 0, which means that the vector x 1 and the vector x 2 are orthogonal. See how we get that suppose x 1 x 1 is equal to, we write a vector, Eigenvector as a column vector.

So, x 1 is suppose having components, say say let me call it A x 1 1 x 1 2 and so on x 1 n and x 2 has components. Let us say x 2 1 x 2 2 and so on, x 2 n then x 1 transpose x 2 will be what x 1 transpose, means row vector. So, x 1 1 x 1 2 and so on, x 1 n and we have x 2 x x 2 1 x 2 2 and so on, x 2 n equal to 0. So, we get x 1 1 into x 2 1 plus x 1 2 into x 2 2 and so on, x 1 n into x 2 n is equal to 0; that is the dot product of the 2 vectors x 1 x 2 is equal to 0. So, are a (Refer Time: 53:04) a scalar product is 0. So, the two vectors x 1 x 2 are orthogonal and here we have given example where A is real symmetric matrix you can see.

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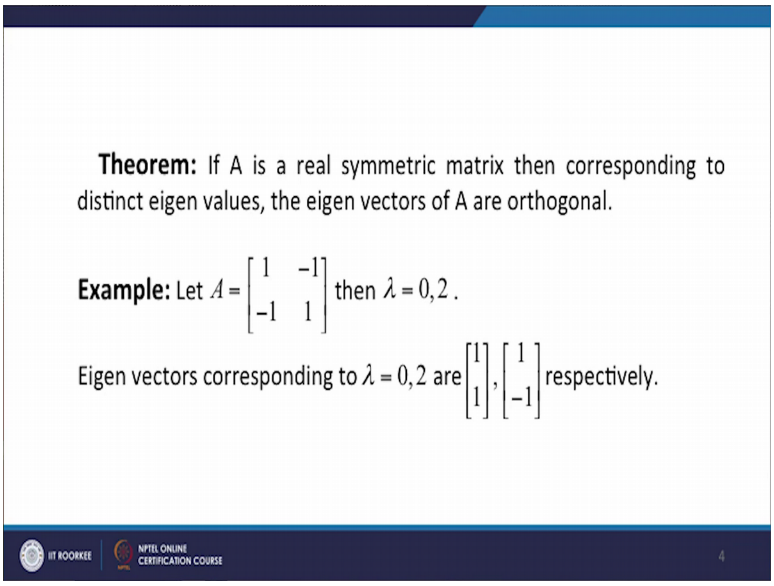

When you find the Eigen values of this, you get Eigen values s lambda equal to 0 and 2, so which are distinct Eigen values. And when you find the corresponding Eigen vectors like we have found earlier, the Eigen vectors corresponding to lambda equal to 0 is 1 1, the Eigenvector corresponding to lambda equal to 2 is 1 minus 1 and you can see if you take these dot product or scalar product of these two vectors, then 1 1 1 1 1 1 vector is 1 1, the other vector is 1 minus 1.

So, suppose this is our x 1 vector and this our x 2 vector, then x 1 transpose x 2 is equal to 1 1 and here we have 1 minus 1. When we take the matrix multiplication we do, so 1 into 1 and then 1 into minus 1, so which is equal to 0. So, the two vectors are orthogonal to each other. So, by this theorem, by this result we can verify the theorem, and with that we come, we will conclude this lecture.

Thank you very much.