Numerical Linear Algebra
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Lecture - 43
Singular value decomposition of a matrix- II

Hello, friends welcome to this lecture, if you recall in previous lecture we have discussed the singular value decomposition of any real matrix of size m cross n where the number of rows is bigger than number of columns, and we have seen certain remarks on singular value decomposition. Now here in this lecture we continue our discussion on singular value decomposition and in fact, we start with finding the singular value decomposition of a given matrix. So, let us start with an example here, so as an illustration of the proof of the theorem.

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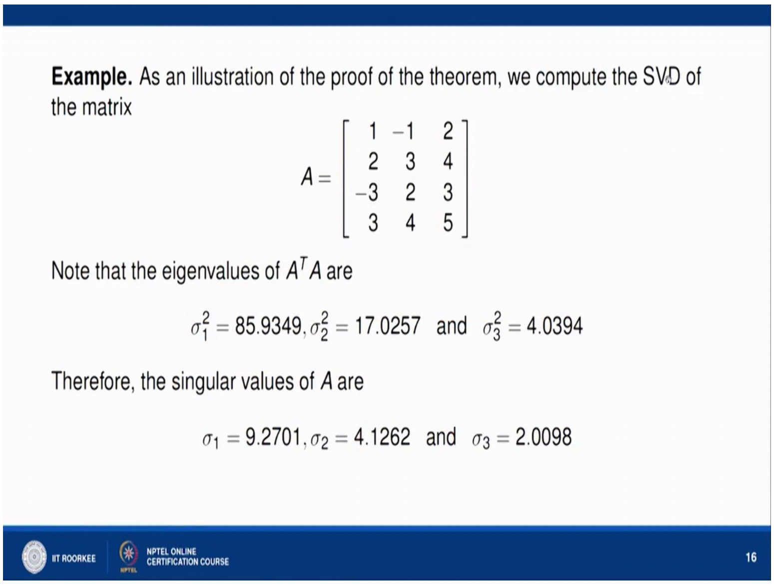

We compute the singular value decomposition of the matrix A, where matrix A is given as 1 minus 1 2 2 3 4 minus 3 2 3 and 3 4 5. And first thing we wanted to find is the eigenvalues of A transverse A, which are given as sigma 1 square as this quantity 85.9349 sigma 2 square as this and sigma 3 square is this, and we can calculate the singular values of a as the square root of these 3 value. So, we can say that sigma 1 is 9.2701, sigma 2 is this and sigma 3 is equal to 2.0098. Now, corresponding to these eigenvalues we can also calculate the eigenvectors of A transverse A.

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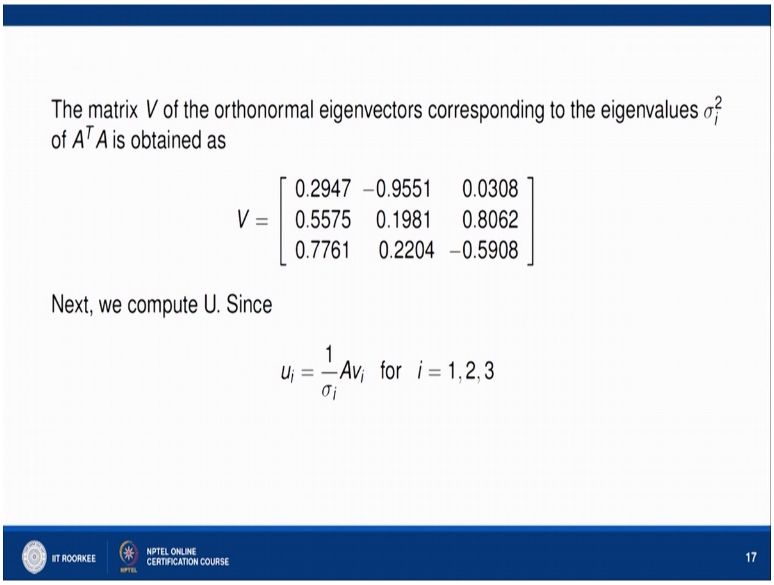

And we say that the corresponding to sigma 1 our Eigen vectors are given as 0.2947, 0.5575, 0.7761. So, this column first column of this matrix V is the eigenvector corresponding to sigma 1, second is corresponding to sigma 2, and third 1 is corresponding to sigma 3. So, we can say that these are what these are orthonormal eigenvectors corresponding to the eigenvalues sigma i square of A transverse A.

So, these are the singular vectors of A or we can say that these are the eigenvectors of A transverse A corresponding to sigma 1 square. So, now with the help of this we try to calculate our U. So, if you remember U i is given as 1 upon sigma i A of v i here, now V 1 V know V 2 know V 3 know. So, we can calculate U 1, U 2, U 3, so if you calculate U 1, U 2, U 3.

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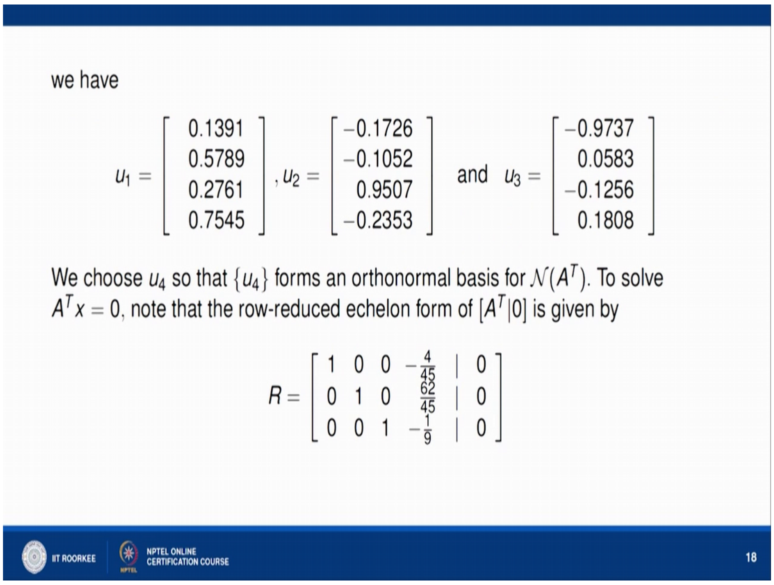

We have this U 1 as this column vector U 2 as this and U 2 as this we want to calculate the U 4 because what is the size of U, size of U is going to be m cross m now here m is what here m is 4. So, it means that U is going to be 4 cross 4 matrix and v is going to be 3 cross 3 matrix. So, we have already find out U 1 U 2 U 3 and we want to find out U 4, now if you remember what is U 4 U 4 is the orthonormal basis for null space of A transverse.

So, to find out orthonormal basis for null space of A transverse, we try to solve for A transverse x equal to 0. So, to solve A transverse x equal to 0 we look at the augmented matrix A transverse A and we try to find out the solution of A transverse x equal to 0. So, for that we U 0 row reduced echelon form of a this augmented matrix A transverse 0 which is given by this matrix R, and here R is 1 0 0 minus 4 by 45 0 and so on. So, this R matrix is known to us now with the help of this, we try to find out the solution for A transverse x equal to 0.

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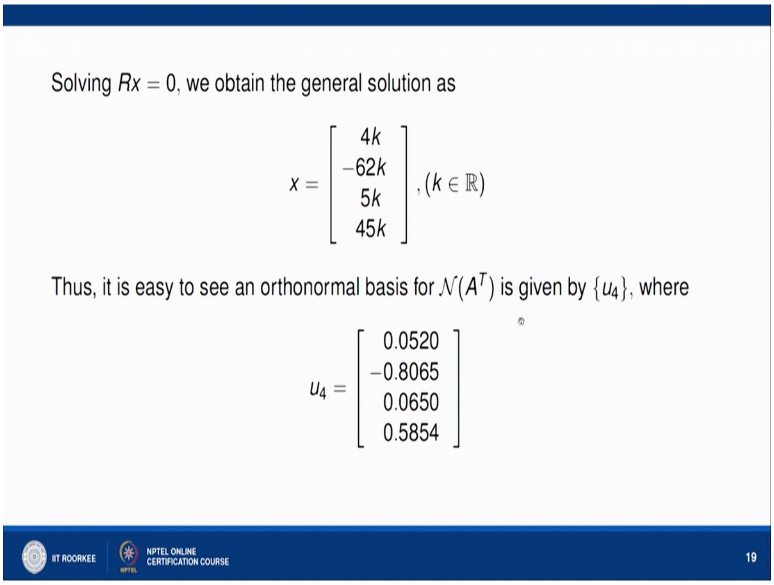

And if you find out the solution the solution of R x equal to 0 is given by x equal to 4 k minus 62 k 5 k 45 k, here k is any real value of real value. So, now since we know that this U 4 is going to be a vector which is orthogonal to U 1 U 2 U 3 and having norm 1. So, we divide x by the norm of x and we can say that in this way we can find out our vector U 4 as 0.0520, and minus 0.8065 and so on, so once we have U 4 we can.

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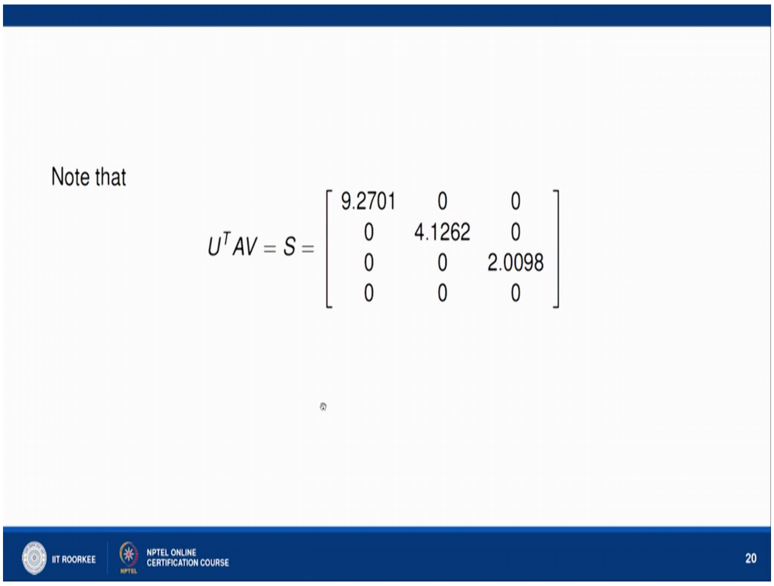

Find out our matrix U and we can say that U transverse A V is given as S, where S is this diagonal, diagonal rectangular matrix. So, here we try to show the same calculation with the help of with the help of mat lab. So, let us do the same thing with the help of mat lab. So, that we, we can say that all these calculation you can also do with the help of mat lab, so look at the mat lab session.

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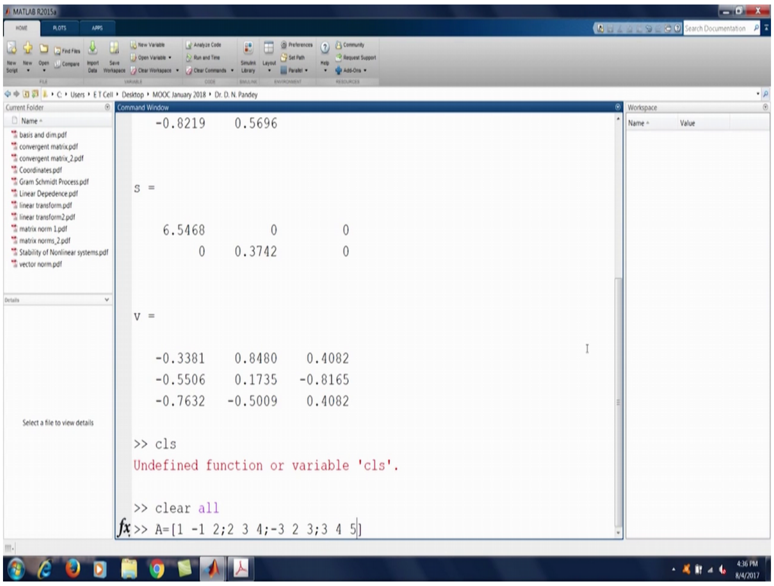

So, here first of all we need to define what is A here, so if look at you give this a as so here we have a can be given as a matrix. So, here A has column first row is 1 minus 1 and 2 and the second row is 2 3 and 4 third row is given as minus 3 2 and 3 last row is 3 4 and 5.

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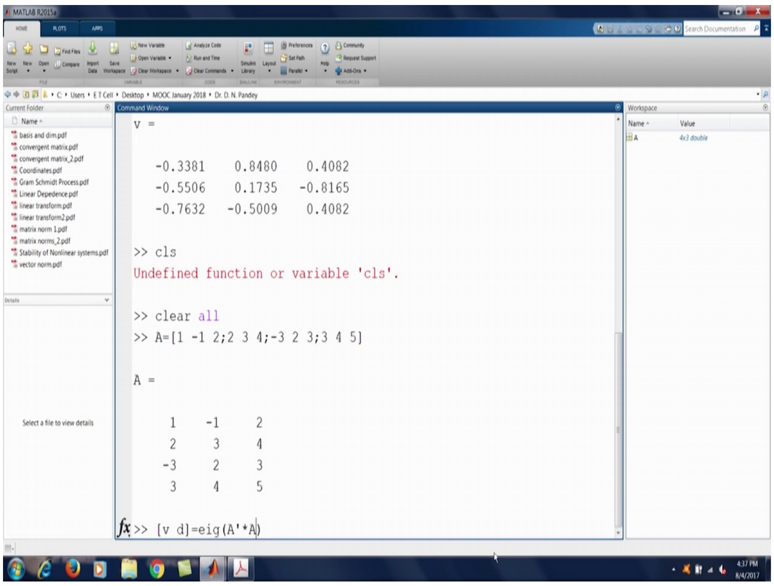

So, if you give this then our matrix A is given as these things, now we want to find out say eigenvalue and eigenvectors of A transverse A. So, for that we try to find out some eigenvalue and eigenvector for that we give v and d and will give as eigenvalue of A transverse A. So, for that we use this command A transverse A and which give you.

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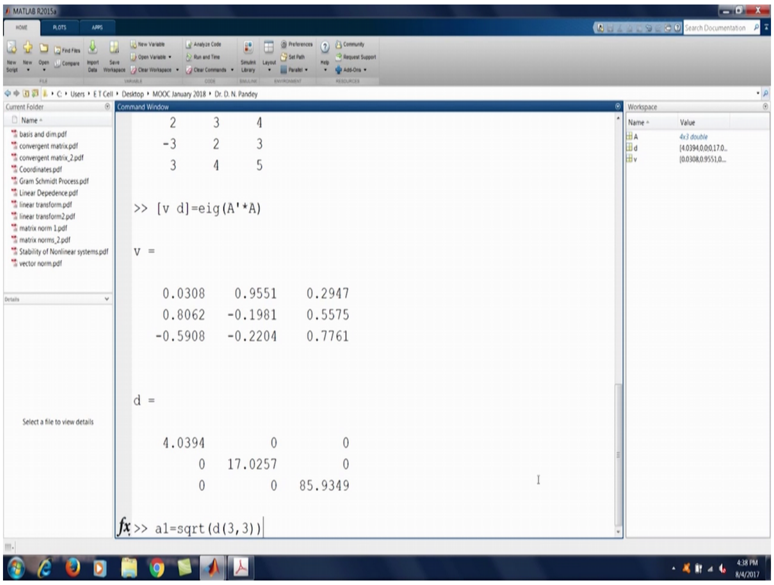

The eigenvalue and eigenvector, now here if you look at this is the eigenvector now here eigenvalues are this so, 4.0394, 17 point it is sigma 1 square, sigma 2 square, sigma 3 square, because we have denoted sigma 1 square as the largest singular value of A transverse A.

So, largest singular value is 85 85.9349, and this smallest is 4.0394. So, it is sigma 1 and this is sigma 2 sigma 1 square is 85 this sigma 2 square is 17.0257 and sigma 3 square is this. So, with the help of this you can find out what is a 1 let us denote a 1 as sigma 1. So, we write sigma 1 as a 1 and this is nothing, but square root of we can call this as d 3 comma 3, so here it is 3 comma 3. So, this will give you the entry square root of d 3 3.

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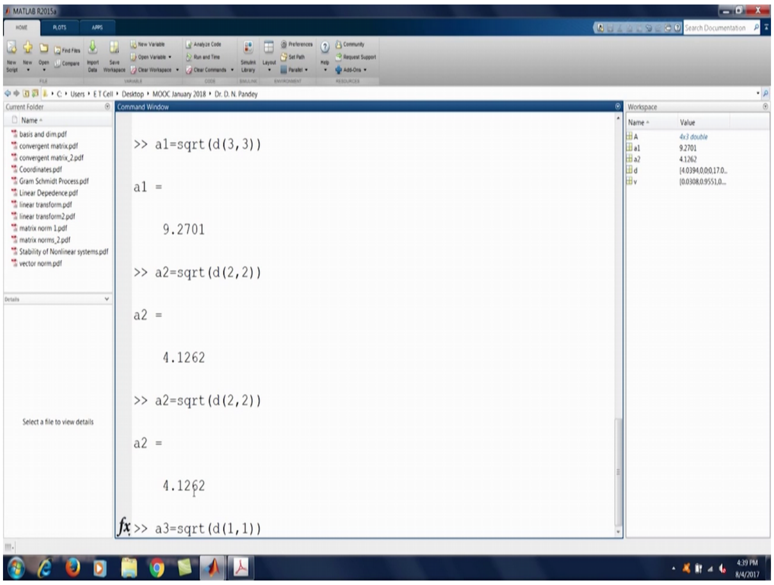

And which is coming out to be 9.2701, which we call this as the first singular value of a, similarly we can find out a 2 the second singular value of a which is given as S q r t and here we have d and 2 comma 2. So, this will give you the second similar value of a similarly we can have a 3. So, here we can say a 3 is going to be S q this is the command for square root of we need to find out d 1 1, so d 1 1 means here.

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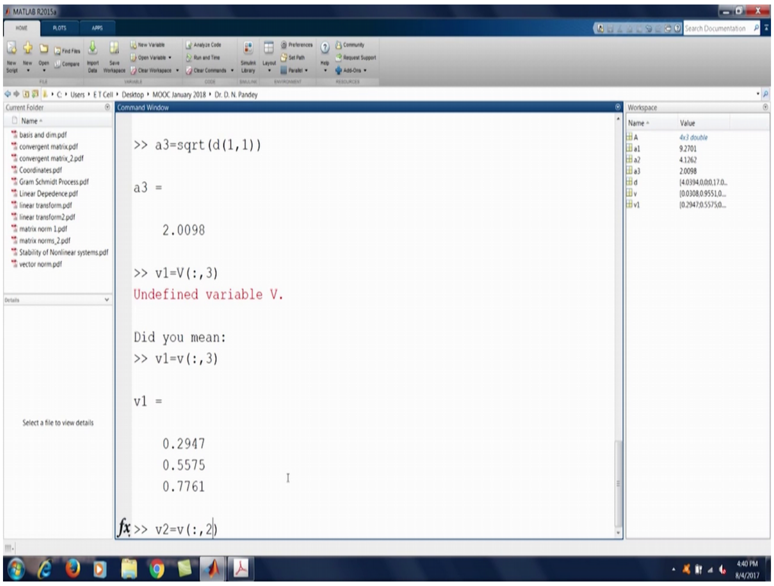

So, this a 3 will give you the smallest singular values corresponding a. So, here your a 1 is basically what if you a 1 represent the sigma 1, 9.2701 if you look at it is matching with this. So, sigma 1 is 9.2701 sigma 2 is 4.1262, which is look at here sigma 2 is going to be 4.1262 similarly here. So, so here we have sigma 1 as this sigma 2 is and it is denoted as a 1 a 2 a 3.

Now, corresponding to this we also want to find out your v 1 v 2 v 3. So, what is v 1 here? So, v 1 is going to be now if you remember when you calculate your eigenvalue vector here. So, v 1 means the eigenvector corresponding to sigma 1 square, now sigma 1 square is this quantity. So, eigenvector we want to consider as this, so here your small v 1, I am calling as this is v I am writing this as third column.

So, third column is corresponding to the largest eigenvector corresponding to the largest eigenvalue and which is defined as here just a minute yeah it is given by. So, it is 2947 0.5575, 0.7761, 6 1 which is given here. So, here what is v 1 v 1 is this 2947 0.5575, 0.7761. So, this is the corresponding eigenvector corresponding to the largest eigenvector similarly ok. So, here we have defined v 1 similarly we can find out v 2 as v and it is.

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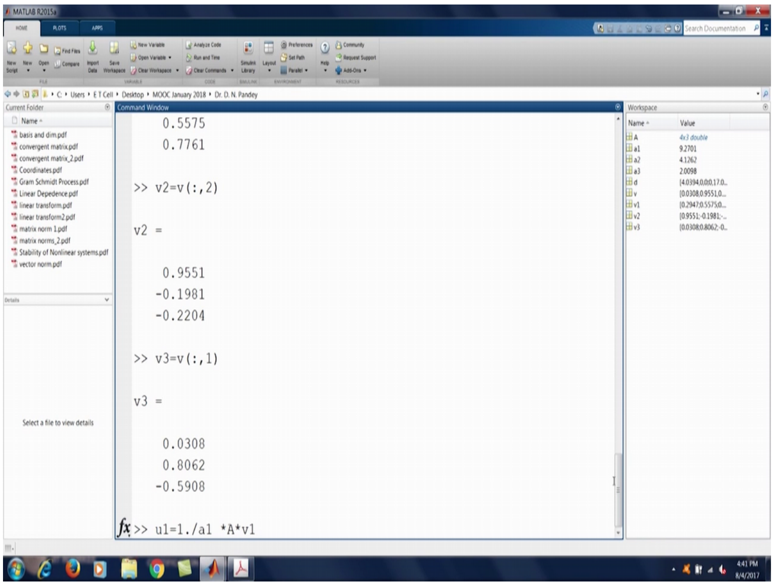

So, here we have v 2 similarly we can define v 3 and v 3 is again v and so, here we have v 3 as this, so here v 1 v 2 v 3 is given here. So, we can say that this is matching with this. So, v 1 is this v 2 is this and v 3 is this v v 3 is 0.0308, 0.8062 and minus 0.5908.

So, that is what we written here v 3, so now, you have v 1 v 2 v 3 and a 1 a 2 a 3 as singular values of a now we calculate u 1, so if you look at what is u 1. So, u 1 is basically 1 upon 1 upon for that we use this symbol a 1 and a of so here we find out this is A and v 1, so here we have v 1.

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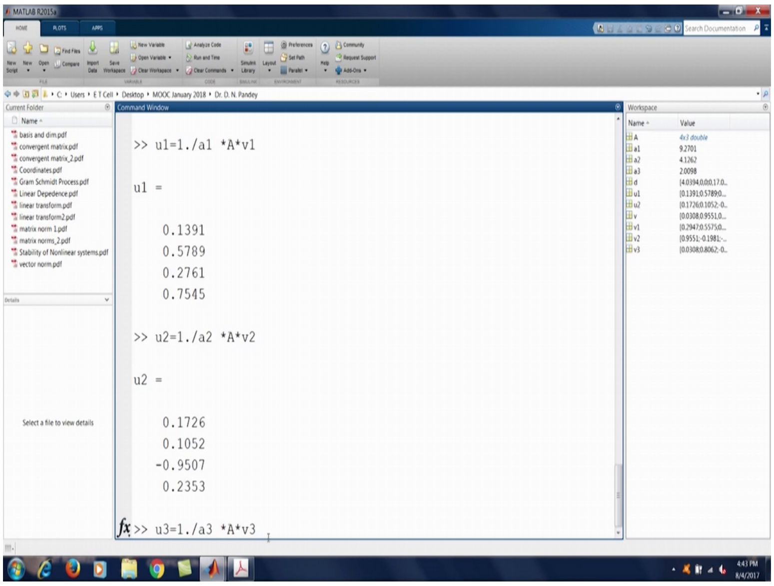

And if you look at this is coming out to be 0.1391, 0.5789, 0.2761, 0.7545, if you look at our u 1 is going to be this thing 0.1391, 0.5789, 0.2761, and 0.7545. So, this is our U 1 which is listed here similarly we can find out u 2. So, to calculate the u 2, u 2 is basically 1 upon second Eigen second singular value of a into a v 2.

And if you calculate it is coming out to be u 2 as 0.1726, 0.1052 and that we can verify that it is given here, here u 2 is point minus 0.1726 and. So, on which is given here, now to calculate u 3 we again use the similar expression u 3 is given as 1 upon sigma 3 and a v 3.

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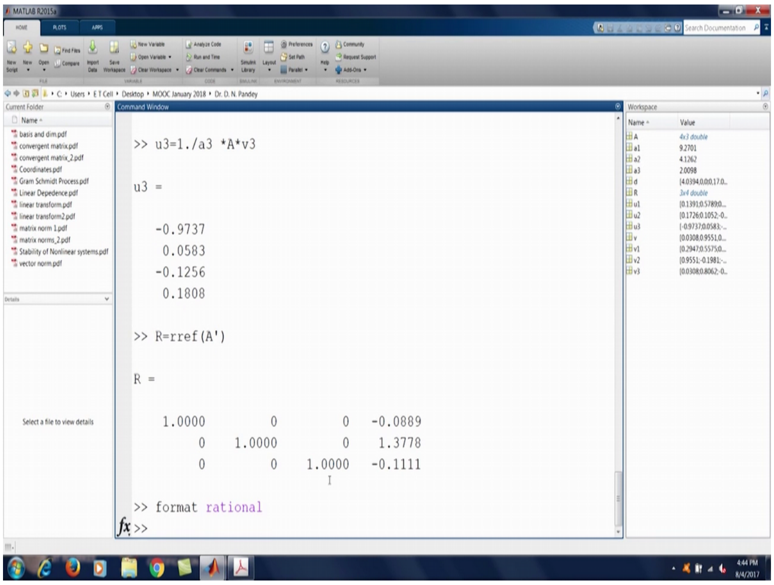

So, it is given here, so U 1, U 2, U 3, U can calculate by this expression. So, it is given as this now to calculate U 4 you need to solve for A transverse 0. So, for that we need to find out this row reduced echelon form of A transverse 0. So, for that we look at we use a notation R as row reduced echelon form, here this r r e f gives you the row reduced echelon form and now here we want to define for A transverse.

So, if you look at, but here we are getting in some kind of decimal. So, to, but here we are using in rational, so let us change the format here. So, if we use the format rational then again doing the same thing.

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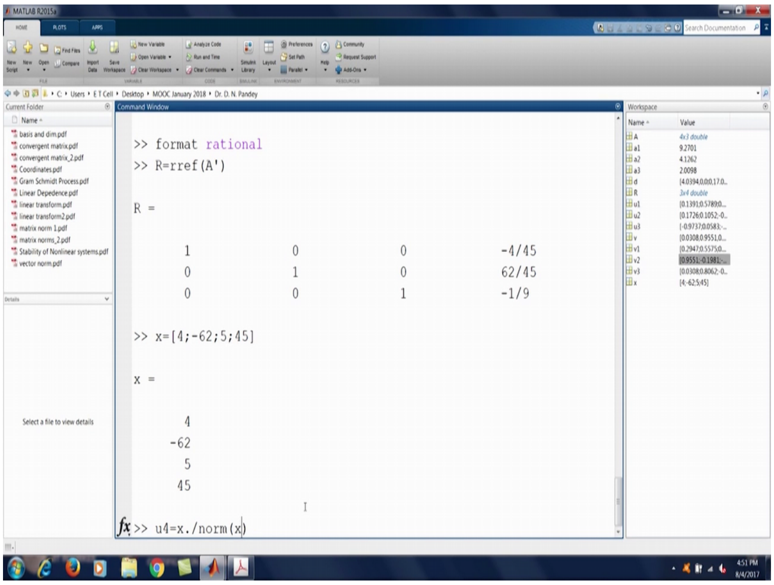

We can say that our R is coming out to be this and which is nothing, but you're R here. So, R is 1 0 0 minus 4 by 45 0. So, if you look at the R is going to B 1 0 0 minus 4 by 45 and so on. So, so we have equivalent form, so now, if you solve this A transverse x equal to 0 we have solution given as 4 k minus 62 k and so on.

So, to find out U 4 we have solve this equation R x equal to 0, where Rs is the row reduced echelon form of A transverse comma 0. And we are able to solve this equation as x as 4 k minus 62 k, 5 k 45 k, where k is any real number and with the help of this x we try to find out say orthonormal basis of null space of A transverse. So, we have say that the dimension of null space of A transverse is only 1 because we are getting only 1 linearly independent vector, which can explain this whole thing.

So, we simply find out say U 4 as normalized vector of x. So, we divide x by norm of x to find out U 4. So, for that let us consider this x here I am using k as 1, so x is given by 4 minus 62 5 by 5 and 45. So, with the help of this we can define U 4 as x divided by norm of x. So, we can say that x divided by norm of x here, so here this I hope that when we discuss the vector norm, we have seen that this norm of x represent the 2 norm of vector x.

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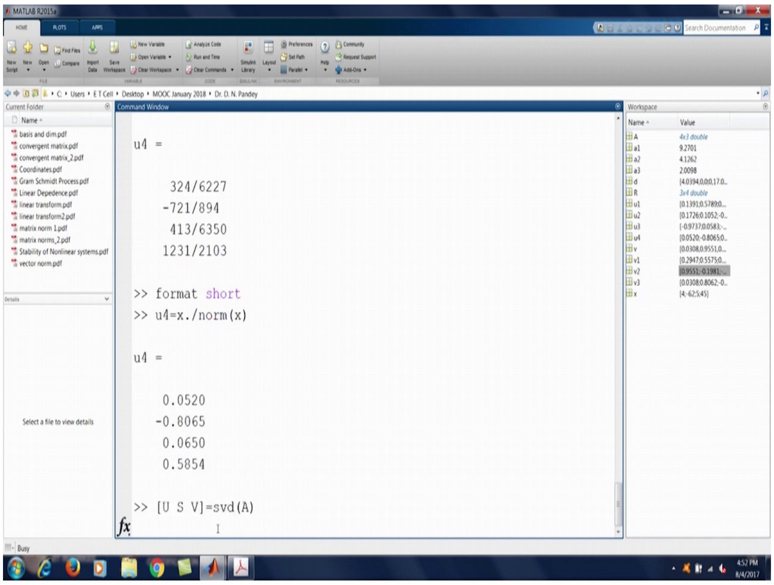

So, using this you can say that U 4 is coming out to be this, but again here we are using the decimal thing and here we are getting the fractional thing. So, here we change the format and we look at the format at short format, and then we can calculate U 4 and it is coming out to be this that 0.0520 minus 0.8065 and it is look at here we have this U 4 as this.

So, once we have U 1, U 2, U 3, U 4, and V 1, V 2, V 3, we can say that our matrix U is given to us, V is given to us, and a is already given to us and S is nothing, but diagonal matrix consisting the diagonal entries as your singular values of A. So, here we have seen that all these, calculations you can do on your, mat lab and, one very easy way to find out say this singular value decomposition to use this simple command that is USV, we can call this as USV, U comma S comma V is equal to here we have this command svd of A, so here if you use this command.

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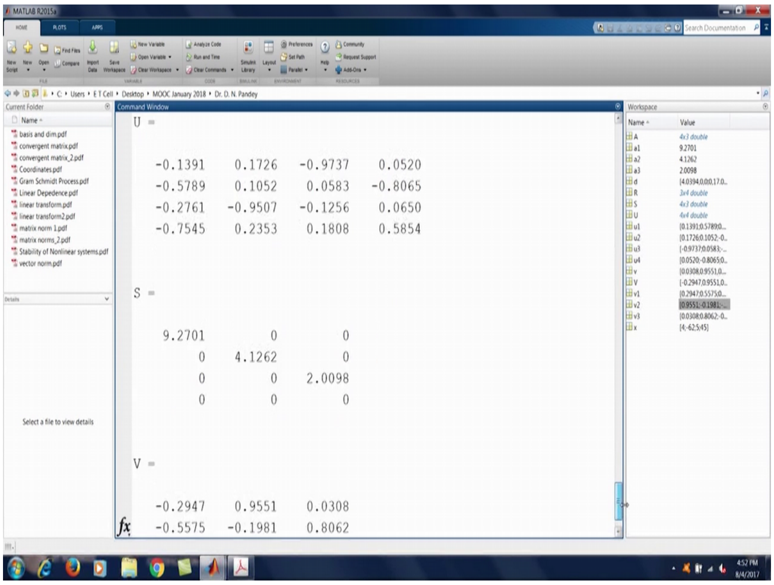

Then also you will get all the representation of U S and V. So, if you have we have this U S and V here.

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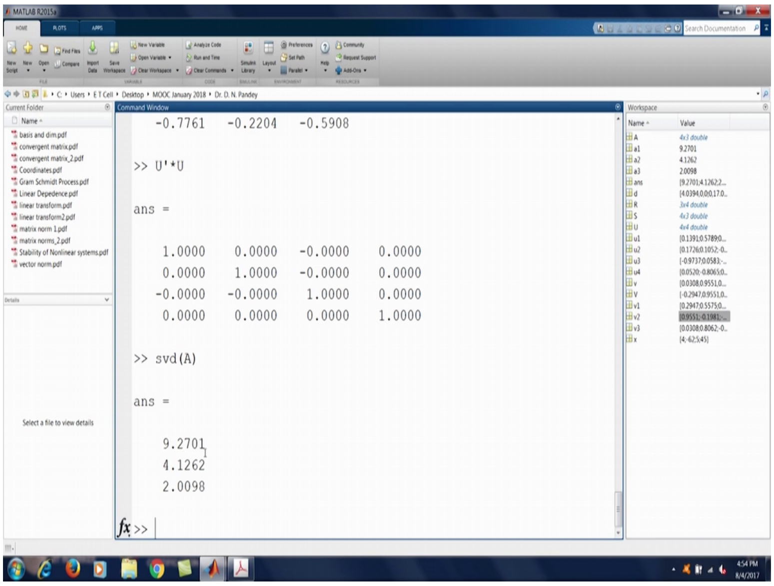

So, first of all whether see whether it will be coming out to be same or not. So, for that let us this look at this V, V is minus 0.2947, here we are getting 0.2947, please remember that if V 1 is an eigenvector corresponding to A transverse A, then minus of V 1 is also an ortho eigenvector corresponding to A transverse A.

So, the columns of v may differ by say minus sign, so here we can say that V is given by this V 1 this V 2, V 3 and this your U 1, U 2, U 3, U 4. So, that is just look at your U 4, U 4 is just now we have calculated as this. So, here you say that it is coming out to be same here right. So, we can say that if you don't want to use any kind of calculation then you can straight away calculate your singular value decomposition of a given matrix using this symbol USV, as svd A, where U represent the orthogonal matrix in off size m cross m and V is an orthogonal matrix of size V.

So, that also you can check by calculating U transverse U, so that you can easily check. So, here you're U and transverse U you can check there it is coming out to be identity matrix. So, it is if you look at the diagonal entries it is coming out to be 1, so here you can calculate your S singular value decomposition on mat lab by simple writing USV as svd A. Now, if you don't write anything then we simply write svd of A, then it will give you only the the diagonal entries of the S. So, here these are the entries of your singular values of a 9.2701, 4.1262, and 2.0098, these are the diagonal entries of S.

So, if you don't write anything if you simply write svd A then it will give you the singular values of A. But if you use in place of svd A, if you use this this command that is USV equal to svd A, then it will give you the matrix U, matrix S, and matrix V, such that USV is given as your a U transverse S USV transverse is given as A. Now so that is v have done it on mat lab, so now, moving on to.

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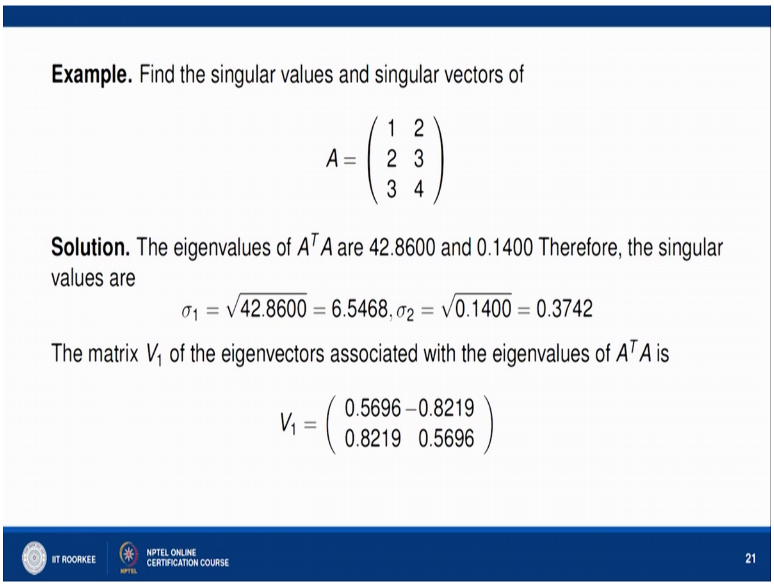

Second example here also we discuss the similar kind of problem. So, here find the singular values and singular vectors of matrix A, and with the help of that we try to find out the singular value decomposition of this matrix A.

So, again in a same way we can calculate the eigenvalues of A transverse A as this and therefore, this singular values of matrix A is given as sigma 1 as 6.5468, and sigma 2 as this and we have both the Eigen singular values are nonzero. So, corresponding to this we can calculate the Eigen vectors of A transverse A as this. So, this is the Eigen vector corresponding to sigma 1, this is the Eigen vector corresponding to sigma 2.

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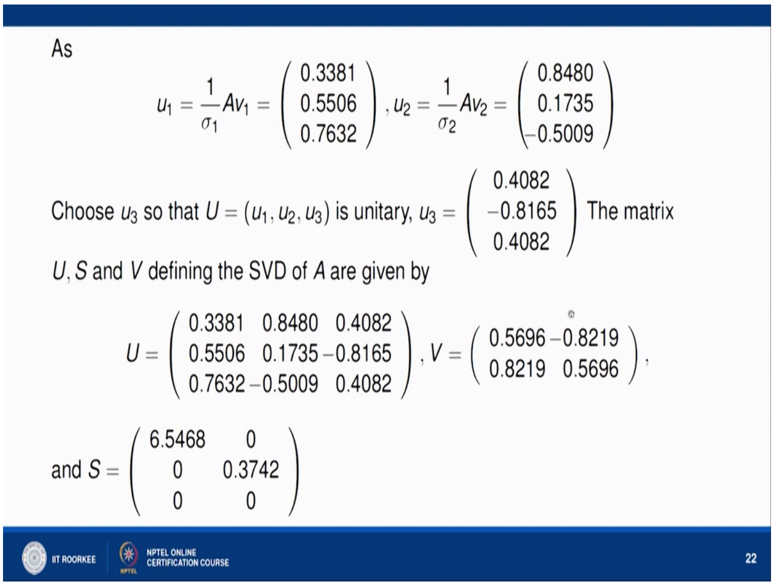

And with the help of this v 1 and v 2, we can calculate U 1 as 1 upon sigma 1 a v 1 and U 2 as 1 upon sigma 2 A of v 2, and to find out U 3 such that U 1, U 2, U 3, is orthogonal or say unitary real unitary matrices orthogonal matrix.

So, U 3 we can calculate as this 0.4082 minus 0.8165, and this to find out this we need to look at the orthonormal basis for null space of A transverse, for that again we have to solve the system A transverse x equal to 0. So, here we can find out U 3 using the row reduced echelon form of A transverse 0.

And we can say that the matrix U S and V, define defining the singular value decomposition of A are given by this is U 1 this is U 2 and U 3 we have calculated like, this and v we have already calculated as an Eigen vectors of A transverse A. So, V is this and S is given by this. So, basically if you remember this V can be easily calculated with the help of eigenvectors of A transverse A.

Now with the help of this v we can calculate U is the remaining U is can, can be calculated as as an orthonormal basis of null space of A transverse, and S is also calculated because S singular values of A is calculated. So, singular values A is basically sigma 1, sigma 2, those are the element here listed here. So, basically to calculate singular value decomposition the only thing is basically left is to find out this complete matrix U.

So, now moving on this we can say that there is a relation between V and U, and why we call the vectors of U and V are singular eigenvectors.

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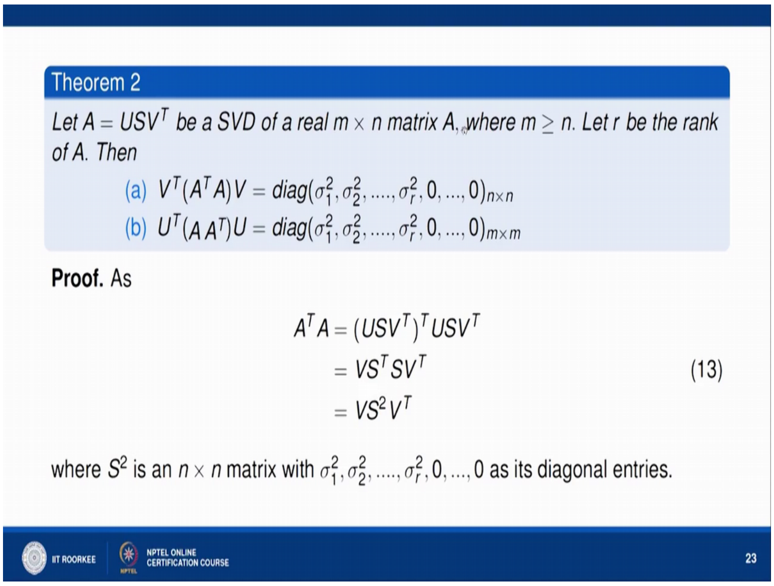

So, here let a is USV transverse the singular value decomposition of a real m cross n matrix a where m is greater than or equal to n. And let r be the rank of a it means in r is the number of nonzero, rows in row reducing echelon form of A then we can say that v transverse A transverse A equal to A V equal to diagonal matrix sigma 1, consisting the diagonal entries sigma 1 square, sigma 2 square, sigma r square.

And similarly, U transverse A transverse A U is equal to this thing. So, if we look at if you look at carefully then this will give you what it will give you A transverse A V equal to diagonal matrix V into diagonal matrix of this, and this simply clarify that the elements of, it means columns of V are known as right.

Singular vectors of A or we can say that columns of V are right singular eigenvectors of A transverse A. So, first of all to calculate this let us calculate this A transverse A, so A transverse A is basically A is U transverse. So, A transverse can be calculated as USV transverse transverse, and if you simplify here we have USV transverse, transverse USV transverse and we can say that it is nothing, but V S transverse, V transverse here we have utilized that U transverse U is identity matrix because U is an orthogonal matrix.

So, we can simplify and we have S trans S is basically what S is a diagonal matrix. So, we can write it V as S square v transverse where S square is basically it is S transverse S. And S transverse S is an n cross n matrix with diagonal entries sigma 1 square sigma, 2 square sigma, r square as its diagonal entries.

So, A transverse A is given as V S square V S transverse S V transverse and with the help of this a follows that v transverse A transverse A, V equal to this diagonal matrix similarly to find out B we calculate A transverse A.

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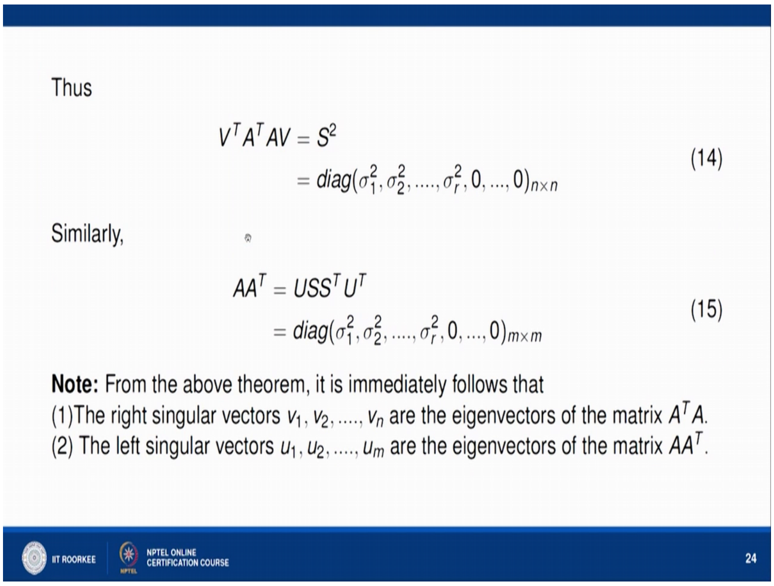

So, here we calculate A transverse as U S, S transverse U transverse and with the help of this we can say that that B U, U transverse A, A transverse U is given as diagonal matrix of size m cross m whose diagonal entries are given as sigma 1 square, sigma 2 square, sigma r square, 0 comma 0. Now if you look at equation number 14 and 15, we can say that right singular vectors V 1 to V n are the eigenvectors of the matrix A transverse A.

So, basically the name write singular vectors are coming from this fact that 14 follows, which implies that the column vectors of V basically eigenvectors of A transverse A. Similarly, column vectors of U which are U 1 to U m are eigenvectors of A A transverse.

So, here we can say, we can say that the left singular vectors U 1 to m are the eigenvectors of the matrix A A transverse, and this remark justifies the name of the right singular vectors and left singular vectors because of this 14 and 15 here.

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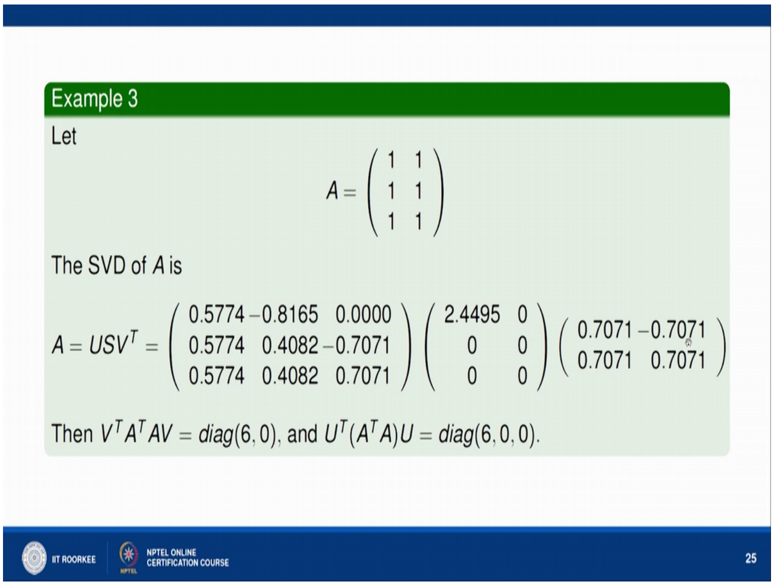

Now, moving on just to so that this result whatever we have seen is true for that let us take this example a as 1 1 1 1 1 1. And we can find out SVD of A, as A as USV transverse that easily calculate from mat lab or the theorem we have given, and we can easily calculate that V transverse A transverse AV is equal to diagonal matrix.

And U transverse A transverse A is given as this again there is a mistake here it is U transverse A A transverse U equal to diagonal matrix 6 comma 0 comma 0 here. So, let us verify this so using mat lab, so let us look at here your what is your matrix A, so here we let us say that.

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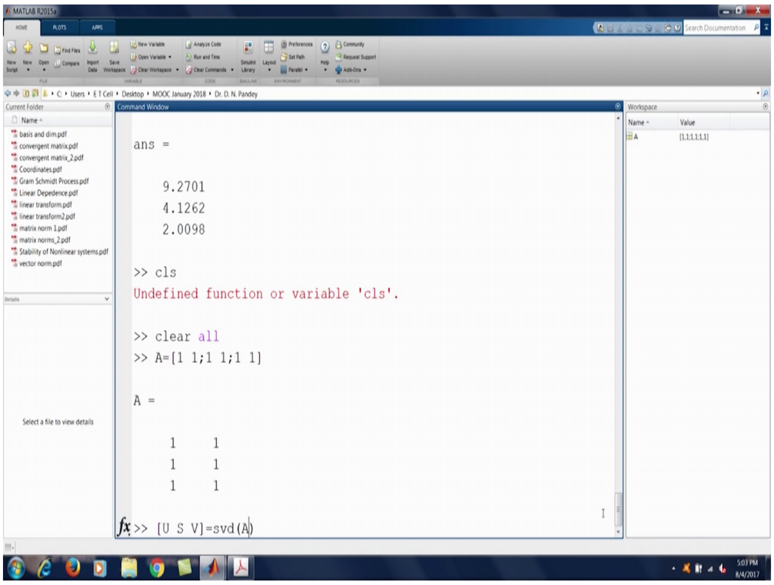

Let us denote as B or we can say that let us clear all clear all. So, now, let us say that A is going to be matrix 1 comma 1 1 comma 1 1 comma 1. So, A is our matrix a now let us calculate U S V, so here U S V is equal to svd of this matrix A.

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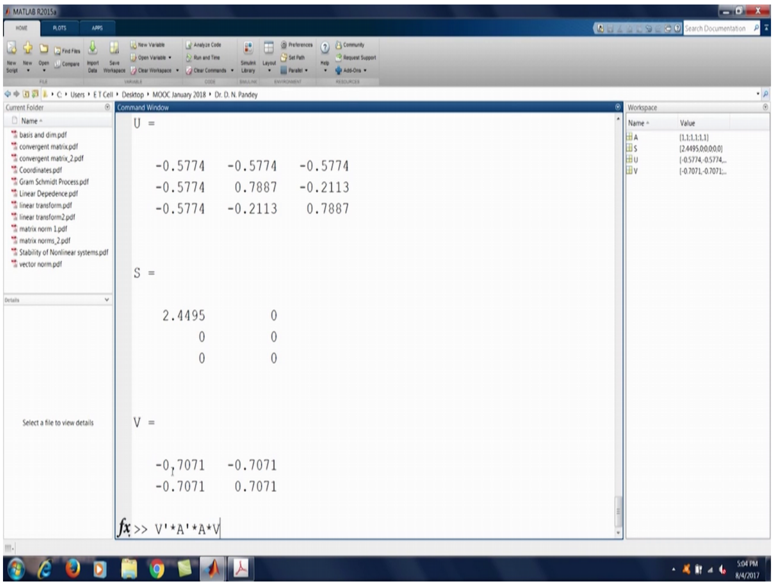

Which is coming out to be this, so here U is this S is this and V is this, and that you can verify here and you can say that your U A S and V is given here. So, U is 0.5774, and so on. And it is looking look that here we are getting this, so now; to see that this is true let us calculate V transverse. So, here V transverse and A transverse and A and V is.

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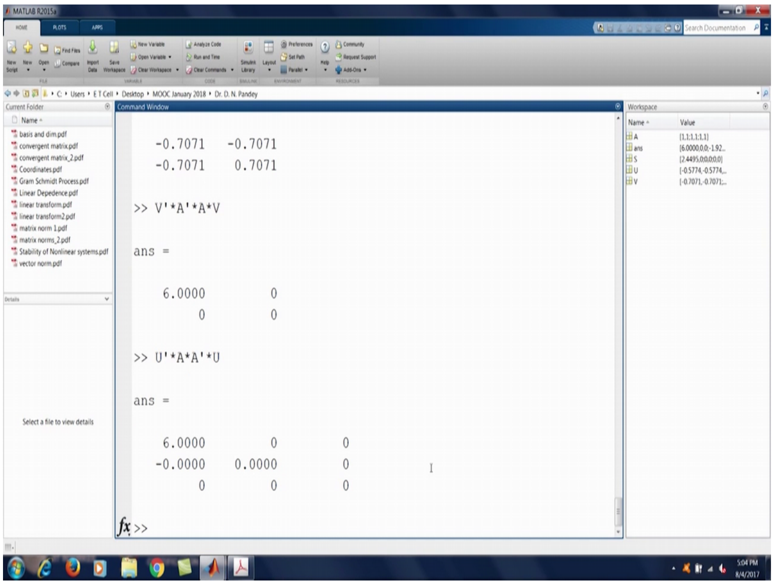

Let us say that what is this and it is coming out be diagonal matrix of size 2 cross 2 here n is basically 2. So, it is coming out to be 6.000 and so on, and similarly we can calculate the second component that is U transverse A A transverse U.

So, for that U transverse A, so here multiply A multiplied by A transverse and then U and it is coming out to be a diagonal matrix 600. So, here we are getting this U, U transverse A transverse U is equal to diagonal matrix 6 comma 0 comma 0. So, now let us consider some very small result based on the given theorem and which says that.

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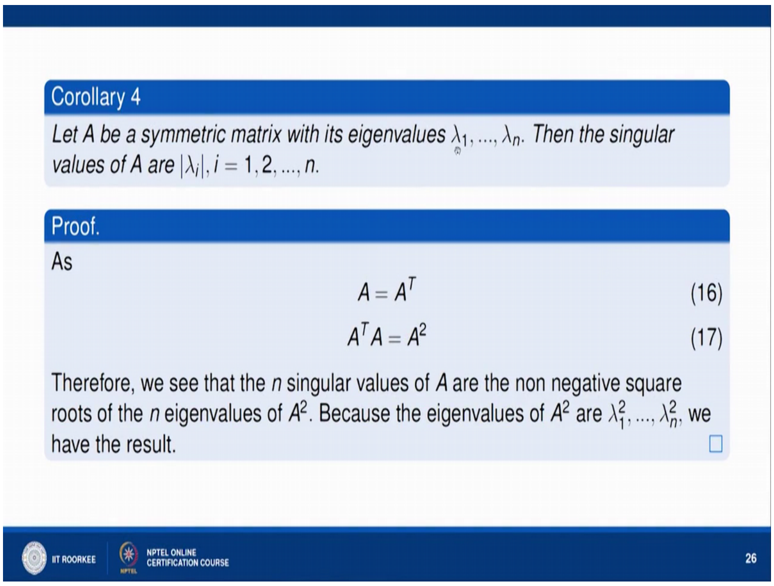

Let A be a symmetric matrix with its eigenvalues lambda 1 to lambda n, so here the size of A is given as say n. So, it is a square matrix, so here we, we say that that the singular values of a are nothing, but modulus values of lambda is where lambda 1 lambda is are the eigenvalue of this matrix a and proof is quite simple, because if you look at A A equal to a equal to A transverse because we have a symmetric matrix.

So, A transverse is nothing, but a then we can say that A transverse A is nothing, but A square. So, we can say that the n singular values of A are the nonnegative square roots of n eigenvalues of A square.

So, here we can with the help of this and the result that since A is a symmetric matrix real symmetric matrix. So, we can say that a can be diagonalizable and we can say that this A transverse A equal to A square means that the eigenvalue singular values of A are nothing, but nonnegative square roots of n eigenvalues of A square and because of the eigenvalues of A square is nothing, but lambda 1 square, lambda 2 square. lambda n square.

We can say that singular values of A are nothing, but modulus of lambda 1 modulus of lambda n here. So, this corollary 4 says that if A is a not rectangular matrix, but it is a square matrix symmetric matrix then singular values of A is nothing, but modulus values of the eigenvalues of this A. So, this is the relation between eigenvalues and singular values of A square matrix now corollary 5 says that an n cross n matrix.

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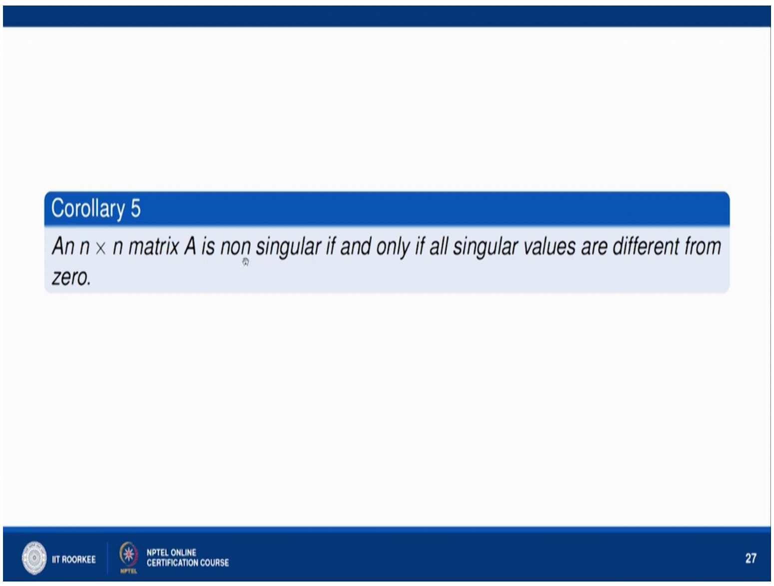

A is non-singular if and only if all the singular values are different from 0. So, this again we can say that this can be achieved by the relation between A singular values of A and singular values of A transverse A. So, to prove this corollary 5 please recall the fact that determinant of A transverse A is nothing, but a determinant of A square. So, it means that if A transverse A is non-singular then determinant of A is also nonzero.

So, it means that if A transverse A is non-singular then is non-singular and when A transverse A is non-singular. If all the eigenvalues of A transverse A is nonzero or we can say that this is same as thing that A is non singular if and only if all the singular values of A are different from 0.

So, this corollary 5 will follow for the fact that determinant of A A square is nothing, but determinant of A transverse A and the relation of non singularity with the eigenvalues of A transverse A. So, here the proof of corollary follow 5 follows from the given fact now let us come to theorem 6.

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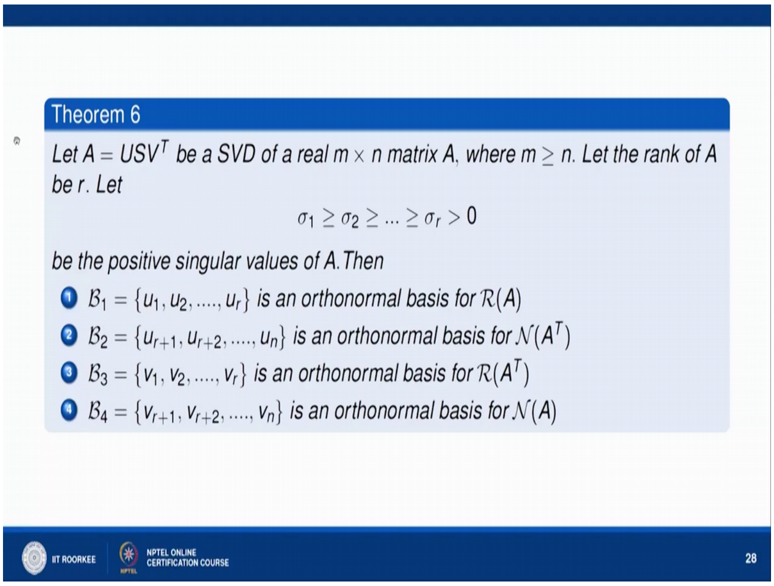

So, theorem 6 says that let a is U S V transverse be A, singular value decomposition of a real m cross n matrix A where m is greater than equal to n and let the rank of A be r and let sigma 1 to sigma r the r positive singular values of A, then this B 1 which consists of first r, columns of vectors matrix U then it is an orthonormal basis for range space of a B 2, which contains the remaining n minus r column vectors of U is an orthonormal basis for null space of A transverse and B 3 which consist of first r Eigen column vectors of V, and it is an orthonormal basis for range space of A transverse and B 4 which consist of the remaining n minus r vector is an orthonormal basis for null space of A so to prove let us observe this fact.

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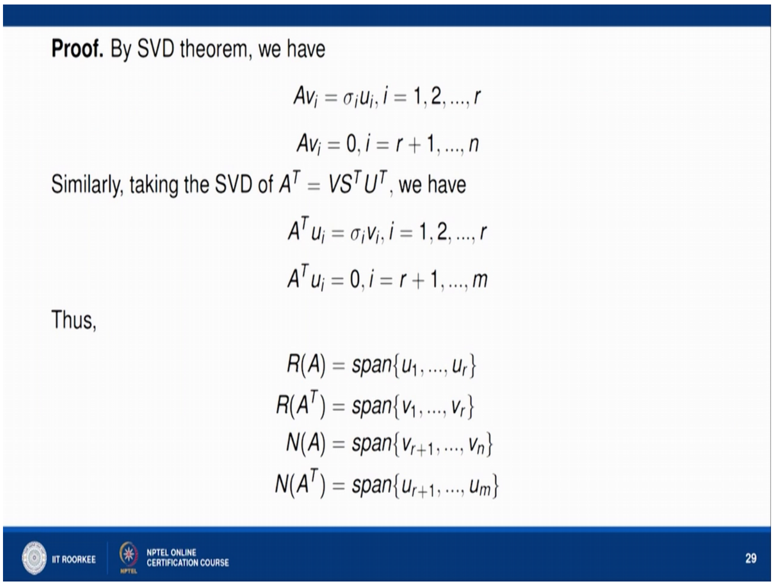

That by singular value decomposition theorem we have A v i, as sigma i U i for i equal to 1 to r and A v i equal to 0, if i is from r plus 1 to n this we have proved and so, if the same thing we can do for A transverse. So, it means that if we have singular value decomposition for a then we can use that for singular value decomposition for A transverse also.

So, A transverse is given by V S transverse U transverse, and we can say that A transverse U i is given as sigma i V i and it is given as for i equal to 1 to r. And A transverse U i is equal to 0 for i equal to r plus 1 to m and this is follows from the relation given here. So, here if you look at the A as USV and A transverse as VS transverse U transverse, and for A as U S V, U S V transverse this result follows, so for A transverse also these result will follow. So, here we can say that from the first relation we can say that span of U 1 to U r will span the range space of A.

So, range space A is nothing, but span of U 1 to U r and we have also seen that range space of A has dimension n dimension r. So, it means that these spanning vectors which are also orthonormal vectors will denote the basis orthonormal basis of range space of A. Similarly, if you look at range space of A transverse, so look at the range space of A transverse through this we can say that V 1 to V r will generate the range space of A transverse and again range of range space of A transverse has the dimension r, and we want to V r are also r orthonormal vectors of V.

So, we can say that range of A transverse A is the basis of range, range space of A transverse is given by V 1 to V r and, if you look at this relation that Avi equal to 0 for i equal r plus 1 to n, we can say that this V r plus 1 to V n will generate the null space of A. So, we can say that null space of A is given by span of V r plus 1 to V of n, now we know that that null the dimension of null space is n minus r.

So, and also that this V r plus 1 to V n are n minus r; orthonormal vectors and hence this will generate a basis for null space of A. Similarly, we can say that from this relation we can say that U r plus 1 to U m forms a null space of A transverse. So, we can say that null space of A transverse is given by span of U r plus 1 to U m and again by (Refer Time: 36:27) theorem we can say that the dimension of null space of A transverse is m minus r. And we have m minus r orthonormal vector which is span this space we can say that these will nothing, but this will generate the basis of null space of A transverse. So, basis of null space of A transverse is given by U r plus 1 to U m.

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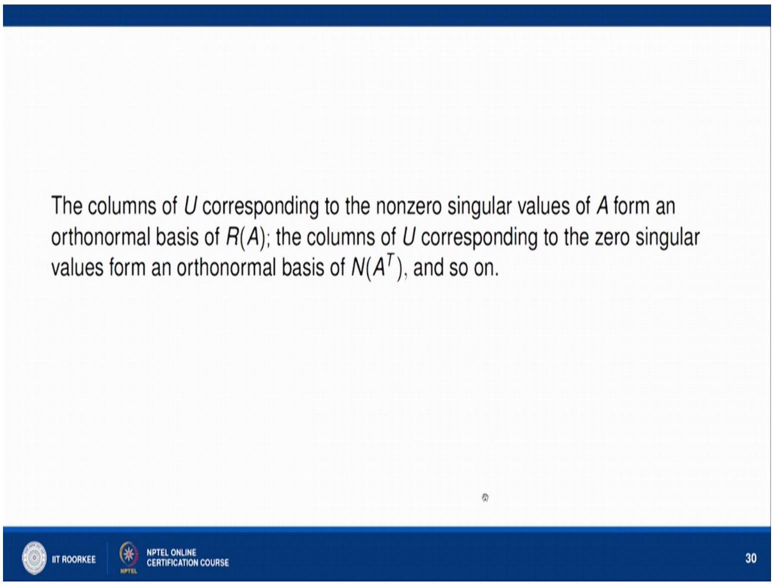

So, too we can say that the columns of U corresponding to the nonzero singular values of A form an orthonormal basis of r A which is given here. So, this U 1 to U r is basically what it is corresponding to nonzero singular values of A. So, that will generate the range space of A, so and the columns of U corresponding to the 0 singular values forms an orthonormal basis of null space of A transverse, that is again clear here that null space of A transverse is a has a basis U r plus 1 to U m and what are these are the vectors of U corresponding to 0 singular values.

So so this result follow similarly, we can look at the columns of v and we can have the similar kind of representation that the range space of A transverse is has a basis V 1 to V r, which is corresponding to nonzero Eigen values nonzero singular values of A and null space of A is the has a basis whose basis consist of Eigen vectors eigenvectors corresponding to the singular values of 0 singular values of A, so using this information let us consider 1 example.

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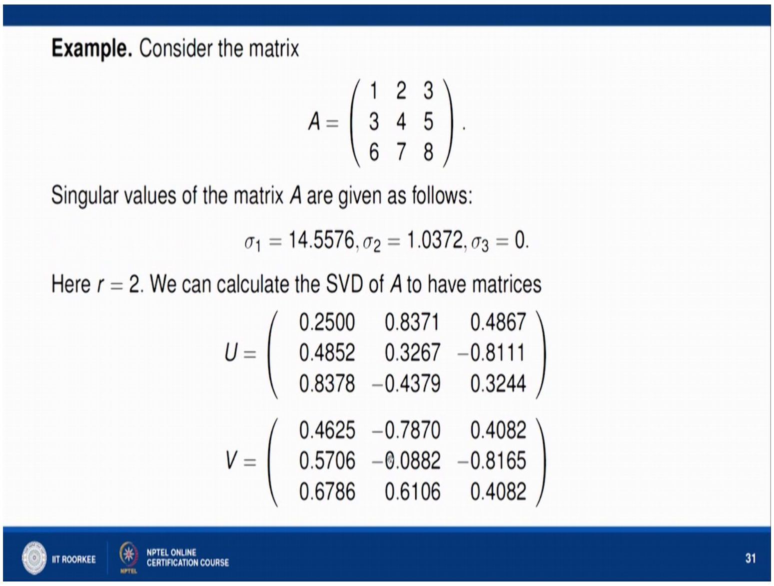

So, for that let us consider this matrix a it is 1, 2, 3, 3, 4, 5, 6, 7, 8, have we have just taken 1 example. And we can calculate singular value of the matrix A and it is coming out to be sigma 1, sigma 2, and sigma 3, if you look at we have 1 0 singular value, so we can say this is the third singular value.

So, we can say that here rank is given as two, so we can calculate the SVD of A, and it means that U V and S is given here and S is we can calculate. So, we can say that the first since r is equal to 2, so it means that corresponding to first 2 singular values U 1 and U 2 will generate the range space of A.

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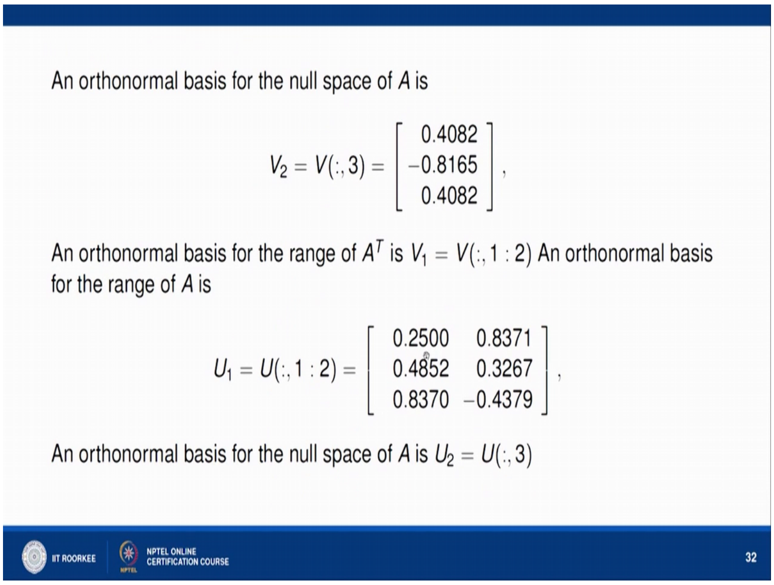

So, it means that this U 1 which is given by first 2 column of U will generate a range space of A. Similarly, the last column it means that columns corresponding to the 0 singular value of sigma 3 this will represent the orthonormal basis for null space of A transverse. So, null space of A transverse is given by this U 2 which is the third column of U here.

So, here we can say that first 2 column which is corresponding to nonzero singular values this will represent the range space of A. And the third vector third column of U it , it represent the orthonormal basis for null space of A transverse. So, that is that we follow from this here that that U r plus 1 to U m represent the basis for null space of A transverse. So, similarly this V 1 and V 2 represent the range space of A transverse.

So, we can say that orthonormal basis for the range space of A transverse is V 1, which consist of first and second column of V. So, it means that first 2 column represent the range space of A transverse and the last column which is corresponding to 0 singular value represent the orthonormal basis of null space of A.

So, that is we say that null space of an orthonormal basis for the null space of A is given by third column of V, which is given by 0.4082 and so on. So, here we will stop here, and in this lecture what we have seen here we have seen some examples of finding the singular value decomposition of a matrix A, and with the help of singular value decomposition of a matrix A, we have calculated the orthonormal basis for the range space of A, range space of A transpose null space of A, null space of A transverse with the help of the columns of the components of singular value decomposition U and V.

So, in next lecture we will continue our study of singular value decomposition and we will see certain more example.

Thank you for listening us. Thank you.