Advanced Matrix Theory and Linear Algebra for Engineers

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Lecture No. # 20

Linear Transformations-Part 4

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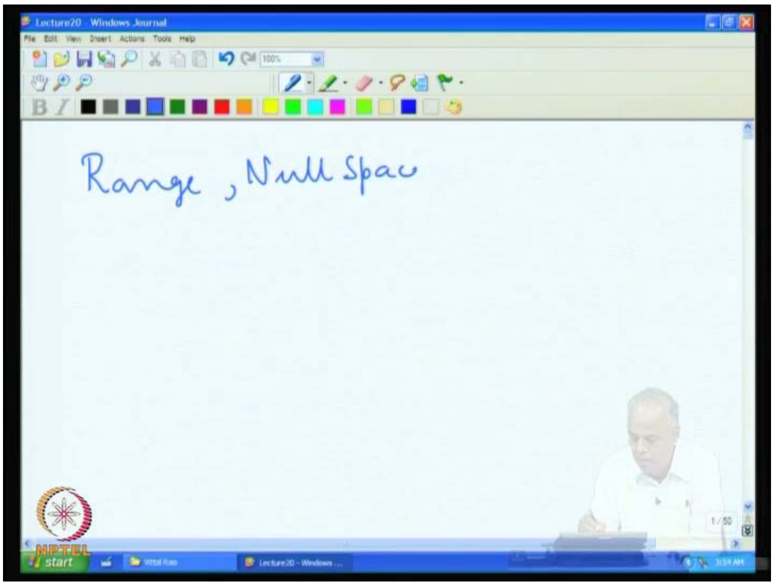

In the previous lecture, we introduce the important notions of the range and null space of a linear transformation. Let us recollect and now focus on a finite dimensional space.

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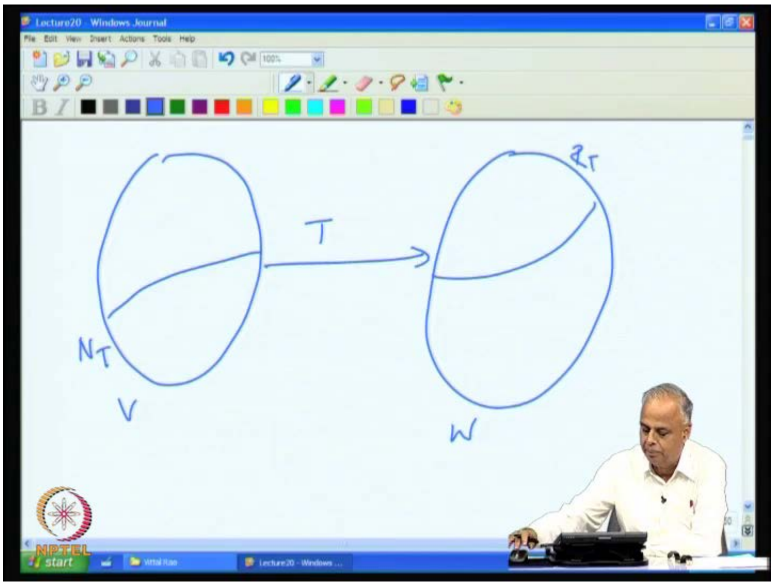

We have a finite dimensional space V and another finite dimensional space W and we have a linear transformation T from V to W, then a part of this is what was known as N T and part of this, what was known as range of T.

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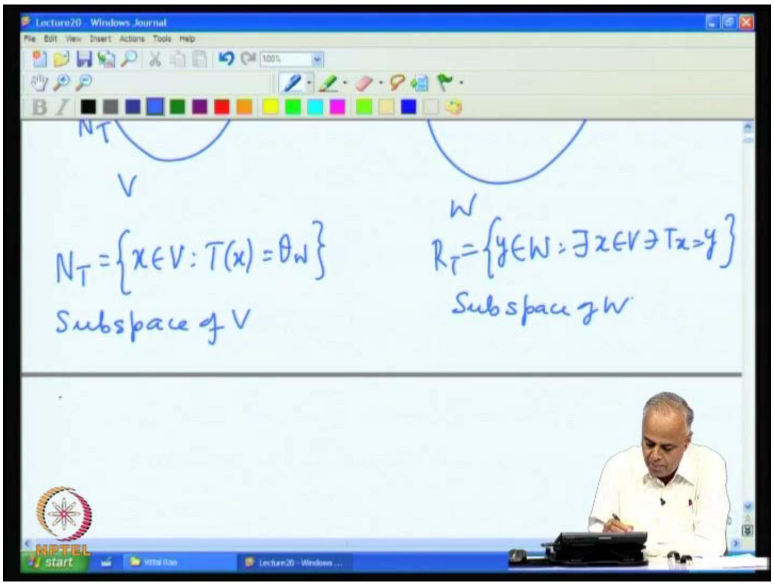

Now, let us recollect the definition, N of T is all those vectors in V which get mapped to the 0 vector and let us recollect R T on this side is the collection of all those vector y, y in W. Such that there exists a x in V with T x equal to y, then the N T is the subspace of V and R T is the subspace of W, so we have the situation where we have linear transformation from finite dimensional vector space to finite dimensional vector space in domain side. We have the null space of T and the co-domain space we have the range of T.

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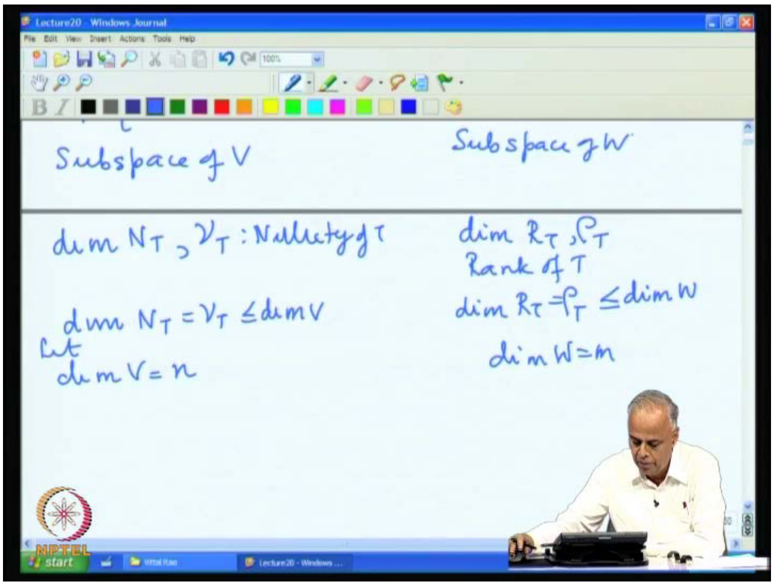

The dimension of n T is what we call as nu T, And it was the nullity it is called the nullity of T. The dimension of range of T is what we called as rho T denoted by rho T and this is called rank of T and clearly N T being subspace of V dimension of N T. Which is V T les than or equal to dimension of V and dimension of range of T, which we call as rho of T must be less than or equal to W. Let us, to take dimension of W dimension of V to be n and dimension of W to be m.

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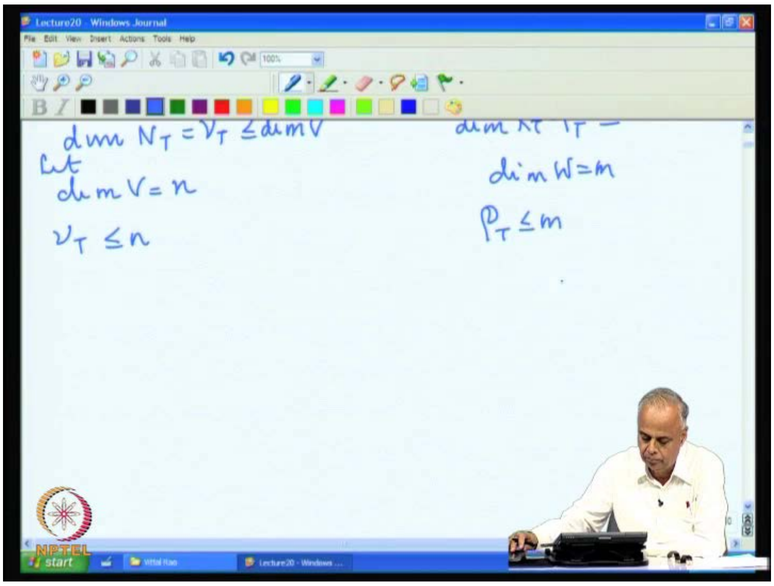

We have nu of T less than or equal to n and rho of T less than equal to m.

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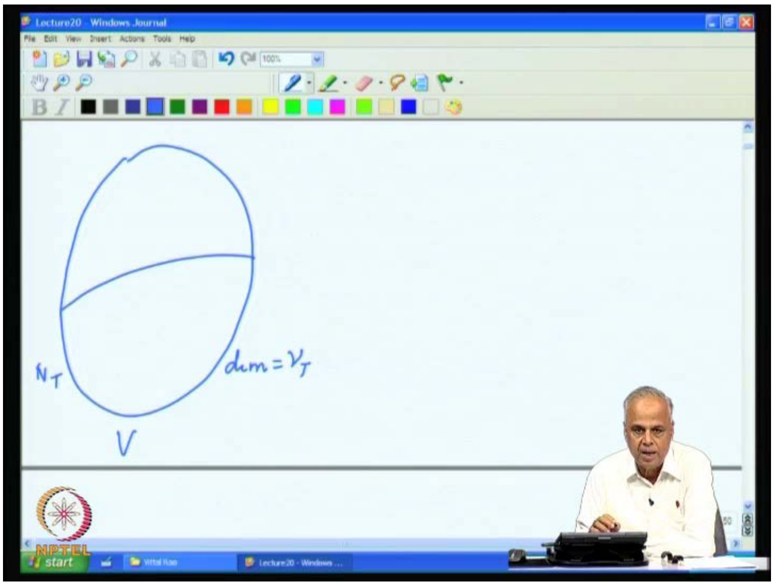

Now, let us look again space V in this space V is what we marked of a portion is the null space T and its dimension was mu T. This space in V we have a portion of V a subspace of V, which is called the null space of T and it has dimension mu T. If it has dimension mu T any dimension basis for N T must have exactly mu T vector.

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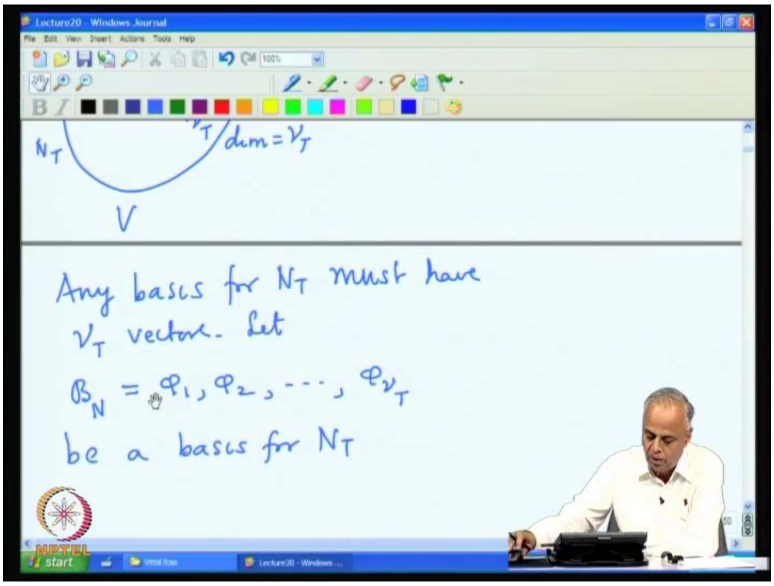

So, any basis for N T must have mu sub T vectors. To take one set basis called b n basis for the null space to be phi 1 phi 2 and there should be phi mu T be a basis for the null space of T. We have phi 1 phi 2 and so on, then phi mu T these are vectors in null space of T these are linearly independent they span in T at least they form a basis for B N. If we look at the space V, it is an n dimensional space and we have these vectors linearly independent space there are mu T of them in an n dimensional space any linearly independent set of vectors can be extended to be a basis by appending suitable number of vectors. In this case, the dimension of V is n we already have a mu T vectors we need n minus mu T vectors from outside N T. They should be linearly independent and together.

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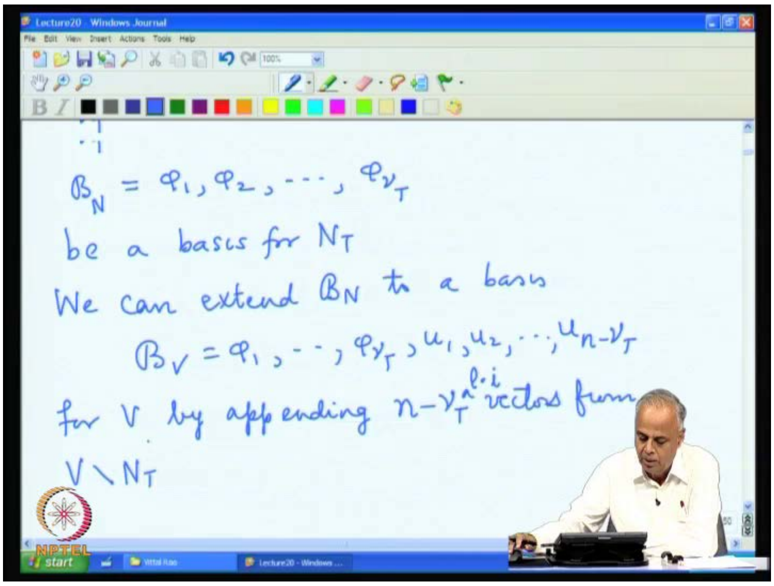

We can extend B N to a basis of B V, which consist of all these vectors and we append exactly n minus mu T vectors in order that we get totally n vector to form a basis for V. Now, we can extend B N to a basis B V for V by appending n minus mu T vectors and they should come from outside N of T. This should be linearly independent n minus mu T linearly independent coming from V, but outside N of T.

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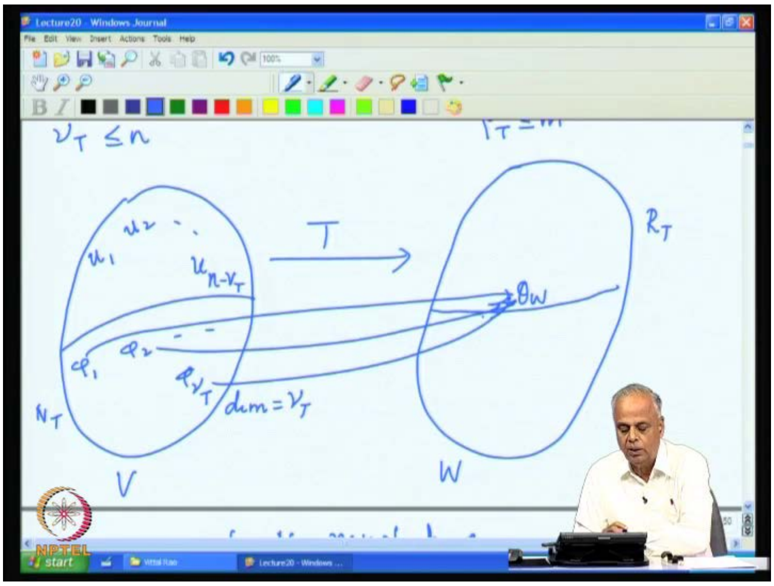

Now, we are going to choose u 1 u 2 and so on u n minus mu T. So n minus mu T vectors outside of N T, but in V which are linearly independent then this set these set of vector and these set of vector together, will form a linearly independent set. Since n of them and they will form a basis for B.

What we have done is? We have started from null space of T we have extended to the basis for whole space V. Let us look at on the T made side T is going from V to W and this side we have range of T and we know theta W is part of range of T. We have already seen that range of T is subspace of W and any subspace must contain 0 vector that we know. Therefore, theta W belongs to range of T and now what happens to this vector phi 1 under the mapping T. Since, phi 1 is in the null space of T it will get mapped to theta W phi to will get mapped to theta W phi mu T will get mapped to theta W. In fact everything in N T will get mapped to theta W. In particular, this these and all these fellows are going to focus on theta W all the phi is are going to map to theta W.

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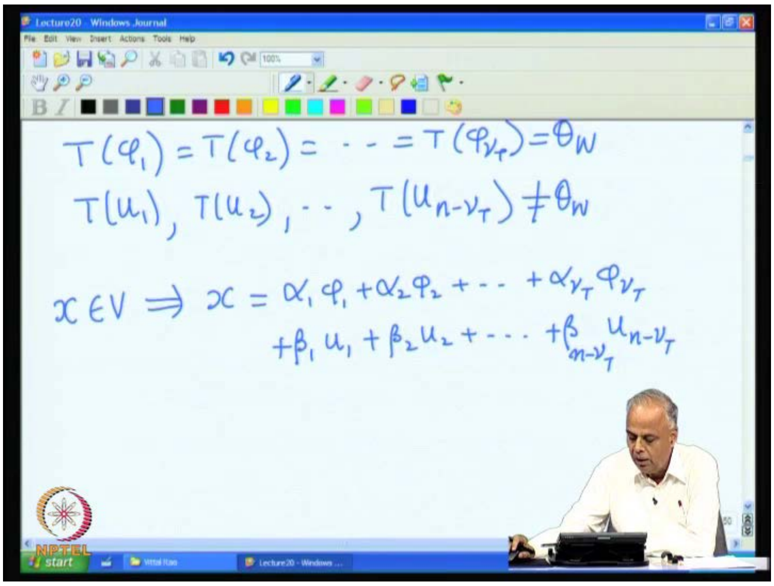

We have first observed, that T phi 1 T phi 2 T phi mu T are all going to be the 0 vectors so that is the first observation, because all these vectors are in the null space T. Let us look at u 1 T of u 1 will be the value of T at the point u 1 therefore, it will be in the range but it will not be 0 it will be in the range T of u 1 will be in the range. But it will not go to theta because, if it has go to theta u 1 must be inside the N T, but u 1 is outside n T u 1 is going to R T. In case of R T a vector different from the 0 vector similarly, u 2 is going to non 0 vector u n minus mu T is going to go to a non 0 vector. The u 1 will go somewhere, there u 2 will go somewhere there and so on.

Then there will avoid theta W, so we have T phi 1 T phi 2 T phi m T is theta W u T u 1 T u 2 and T of u n minus mu T are all different from theta W. Now consider any vector x in V, if will take x in V any vector in V can be written in terms of basis vector is a linear combination. Your basis is consisting of these phi's and u is and therefore, we must be able to write x as linear combination of the basis vector namely. Therefore, we can write it as alpha 1 phi 1 alpha 2 phi 2 plus alpha mu T phi mu T plus, those combination which will involve the u 1 vectors also b 2 u 2 plus et cetera b n minus mu T u n minus mu T. Any vector x in V can be written as a linear combination of vectors in the basis, we are now chosen the specific basis B V and B V consist of five vectors and u vectors therefore, we are able to write x as a linear combination of this 5 vector at the u vectors.

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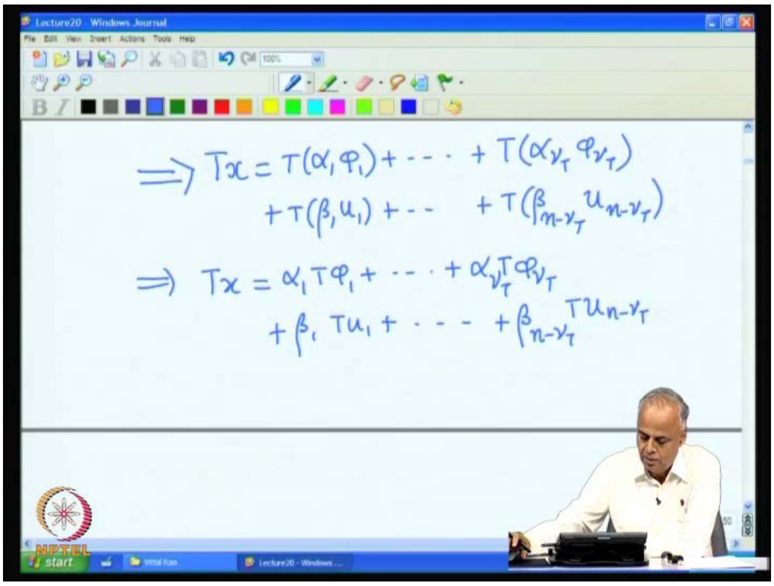

That says, T x must be the T of the right hand side. Now the right hand side is the sum and le linear transformations preserve addition and scalar multiplication in particular they preserve addition. Therefore, T of x plus y is T of x plus T of y, then T of the sum is the sum of T. So, we can take T individual terms this will be T of alpha 1 phi 2 plus T of alpha 2 phi 2 plus T of alpha mu T phi mu T plus T of beta 1 mu 1 plus T of beta n minus mu T u n minus mu T. Because T preserve addition T is a linear transformation and therefore, T of a sum is sum of a T that is equal, now T also preserves scalar multiplication.

The T of alpha 1 phi 1 will be alpha T phi 1 T of alpha 2 phi 2 to will be alpha 2 phi T phi 2. To pulling out all these scalar we get finally, this is equal to T alpha 1 T phi 1 and alpha 2 T phi 2. Then finally, alpha a we should make it as mu T alpha mu T phi mu T plus beta 1 T u 1 plus beta 2 T u 2 plus beta n minus mu T u n minus mu T. So T x for any vector x T x will have this form.

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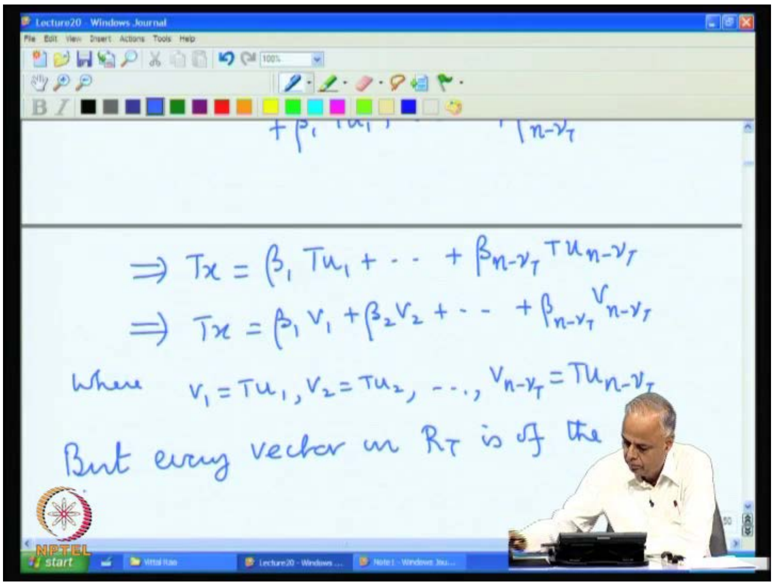

Which means the following remember phi one is in the null space of T. If you recall the picture we had phi 1 phi 2 phi mu T, then where all in null space of T. They were all going to 0 vectors under T therefore, we had absurd T 1 T phi 2 T phi mu are all 0 vectors. Now using that fact, we get these alpha 1 T phi 1 must be 0 ups alpha 2 T phi 2 must be 0 alpha n mu T phi mu T must be 0. All these are 0 vector they are not going to contribute anything, so what we have is just p 1 beta 1 T u 1 plus et cetera beta n minus mu T t u n minus mu T. We will write it as beta 1 V 1 plus beta 2 V 2 beta n minus mu T V n minus mu T, where V 1 is T u 1 V 2 equal to T u 2 and so on V n minus mu T is T T of u n minus mu T.

What are we achieved? What we are shown now? Is if you take any vector in V T of that will have to be of this form, so any vector in x x in V T of x must be this form, but all the vector in the range of T are of the form T of something T of some vector in V. Therefore, all the vector in the range of T must be of this form, but every vector in range of T is of the form T of x for some x in V.

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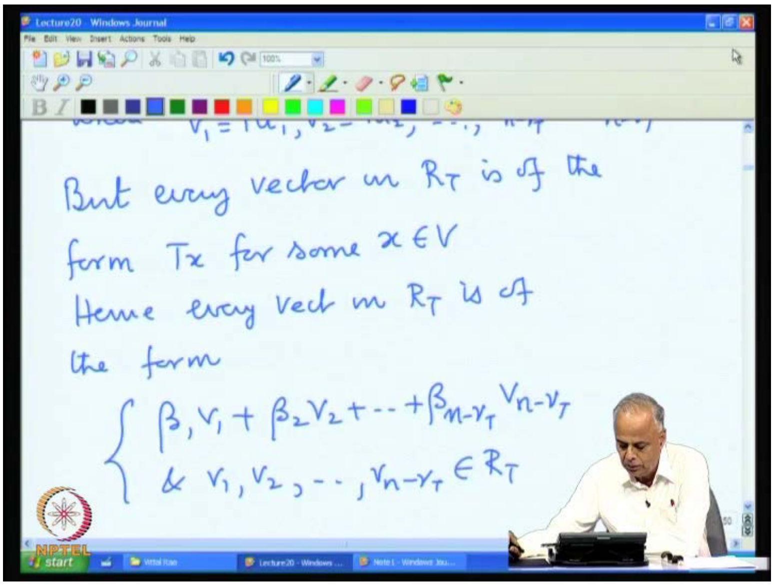

Hence, every vector in range of T it is of the form x and T of x must be of this form, so every vector in the range of T is of the form beta 1 V 1 plus beta 2 V 2 plus beta n minus mu T V n minus mu T and these vector V 1 V 2 are all T something under V 1 V 2. Therefore, come to the range of T and V 1 V 2 etcetera n minus mu T belongs to range of T. Then we have here, let us get back to the picture we have in the picture now u 1 went to V 1 that is T u 1 T u 2 is V 2 and so on. We get V n minus mu T in the range of T we bought hold of n minus mu T vectors and every vector in the range of T is the linear combination of the b 1 b 2 b n minus mu T.

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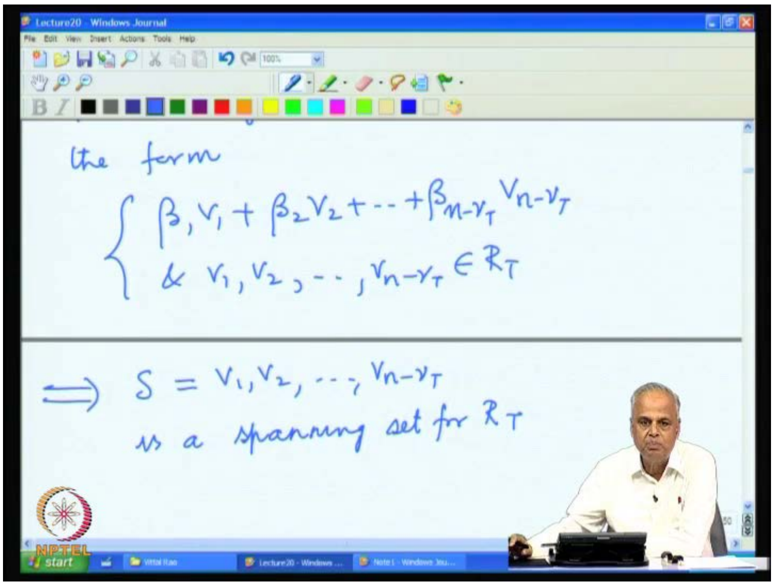

If we call the set S V 1 V 2 V n minus mu T, since every vector in the range of T linear combination of this is a spanning set for range of T. Once we have starting from a basic for null space of T which are phi vector, then we extended it to basis for whole space V and looking at the image of basis vectors we got hold of basis for spanning set T.

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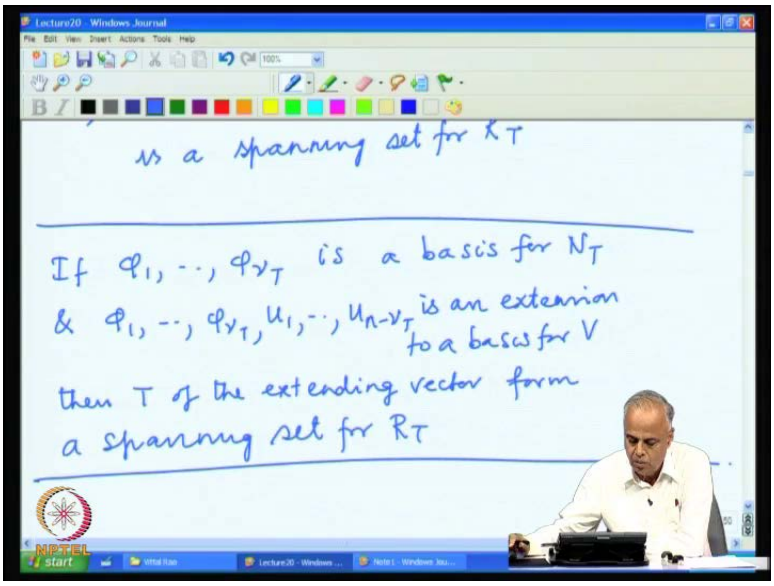

Therefore, if phi 1 phi 2 phi mu T is a basis for null space of T and phi 1 phi mu T u 1 u n minus mu T is an extension to basis for V. Then T of the extending vectors what are extending vectors u 1 u 2 u n minus T. These are extending vector if you take T of them u 1 V 1 V 2 V n minus mu T they form spanning set for range of T this what we have now V 1 V 2 V n minus mu T is a spanning set for the range of T. Once, we have a spanning set you wonder whether this is going to be a basis when will it be a basis is a spanning set will be a basis if it is also linearly independent, because linearly independent spanning set is called a basis which already a spanning set.

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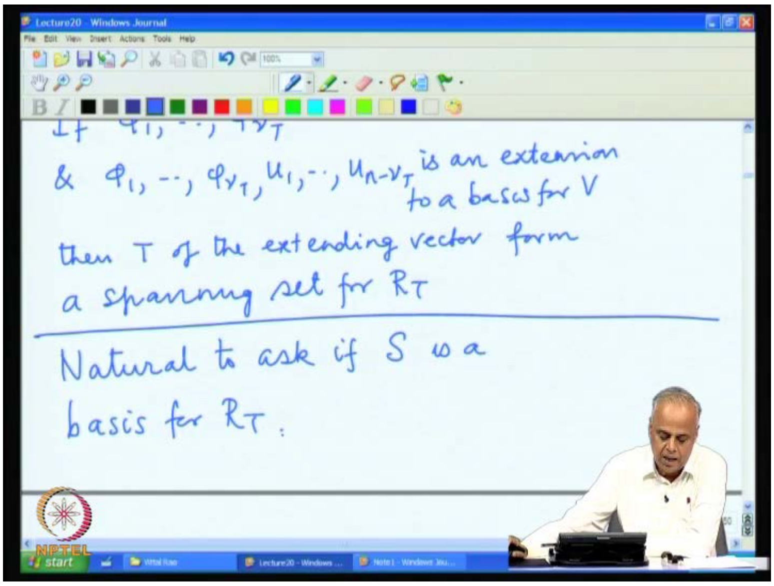

We would like to ask, if the set S is basis for range of T, then already we know it is a spanning set you want to know whether it is a basis.

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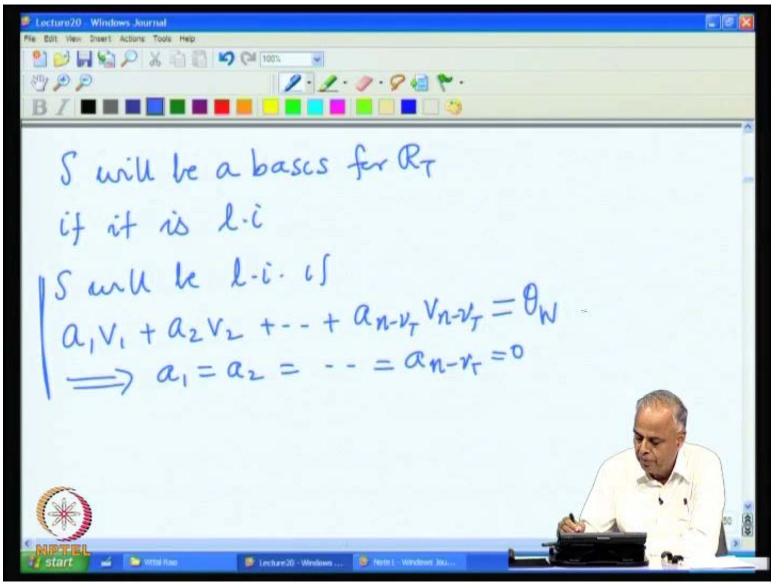

Now S will be a basis answer will be S therefore, S will be a basis for a range of T if it is linearly independent. We have to check whether it is linearly independent and then will be it linearly independent a set will be linearly independent. If the only linear combination that produced 0 vectors is the linear combination in which all the coefficient are 0.

The S will be linearly independent, if we look at any linear combination these vectors in s. If it produces the 0 vector 0 vector of what these are all vectors in W and therefore, it produced 0 vector W then it must imply all the coefficient must be 0. That is the only linear combination which can produce, then if this is true then S will be linearly independent let us check that whether this is true.

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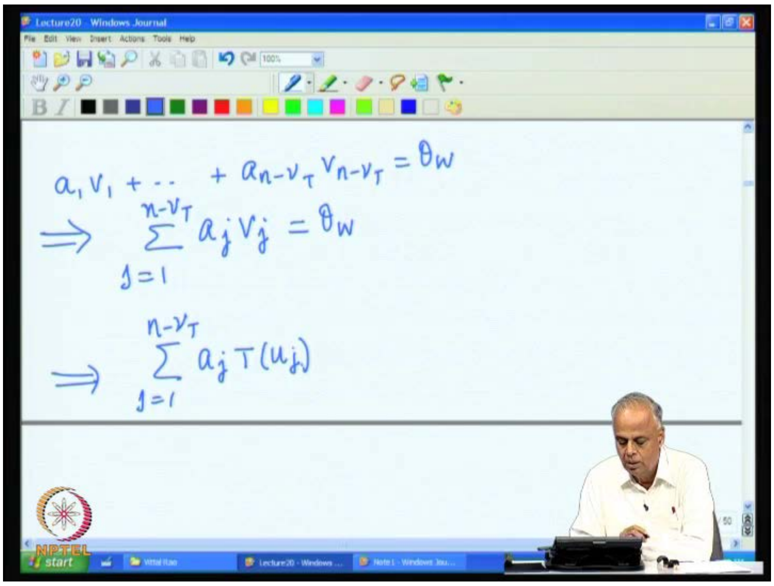

Let us start with a 1 V 1 plus etcetera a n minus mu T V n minus mu T equal to theta W. Then to say writing we do summation notation this is summation j equal to 1 to n minus mu T a j V j equal to theta W, now what does that mean we know that V vector are the images of the u vector and T so remember V j was T of mu j.

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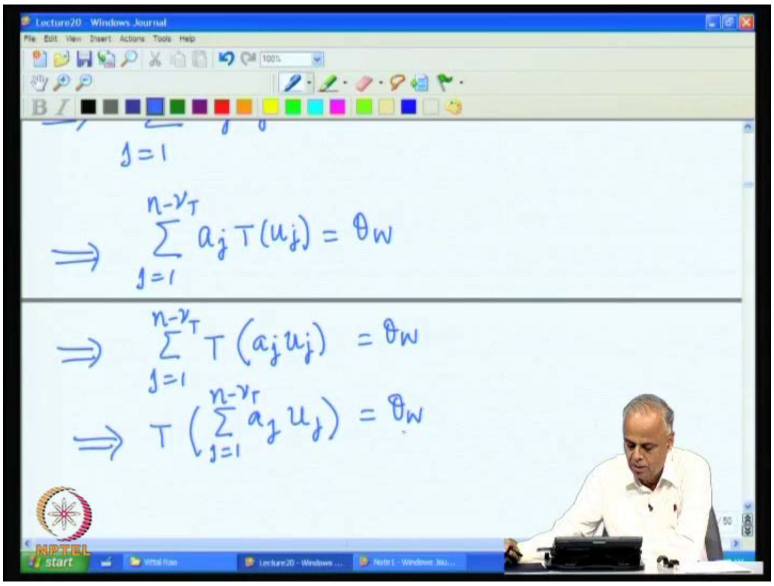

This must be equal to be 0 that says j equal to n minus mu T. Now scalars can be put in and out of the linear transformation, because linear transformations preserve scalar multiplication. We can write it as T a j u j equal to theta W again since T is a linear transformation T of the sum is sum of T. So we can pull the T out of the notation and we get a j u j equal to theta W now mu 1 mu 2 all these vectors are in V and therefore, any combination of them will also be in V.

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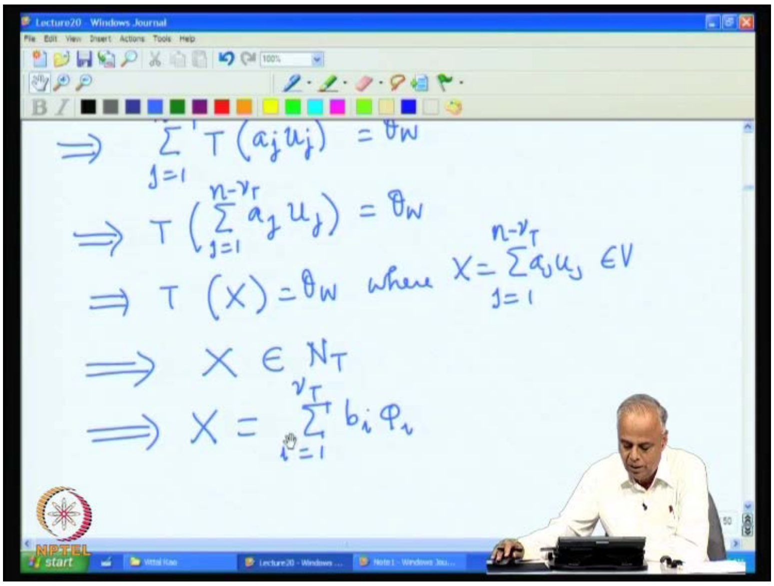

Let us call that whole thing inside as some x equal to theta W where x is equal to summation j equal to 1 n minus mu T a j u j is a vector in V, then x is a vector in V and it is get mapped to 0 vector. What we have got is if a 1 V 1 a 2 V 2 a n minus T V n minus mu T 0 vector, then this gives rise to a vector x which gets mapped to the 0 vector if that is getting mapped to the 0 vector and x must be in the null space of T. If x is in the null space of T why is x in the null space of T because T x in the theta W.

Now, phi 1 phi 2 phi mu were basis for a null space of a T x is a vector in the null space of T and any vector in the null space of T can be expressed as a linear combination of null space T basis vectors. The x can be written as a linear combination i equal to 1 to mu T b i phi i this is because, phi 1 phi 2 phi mu T form a basis for the null space T on the one hand x equal to this and on other x equal to this and therefore, these two must be equal.

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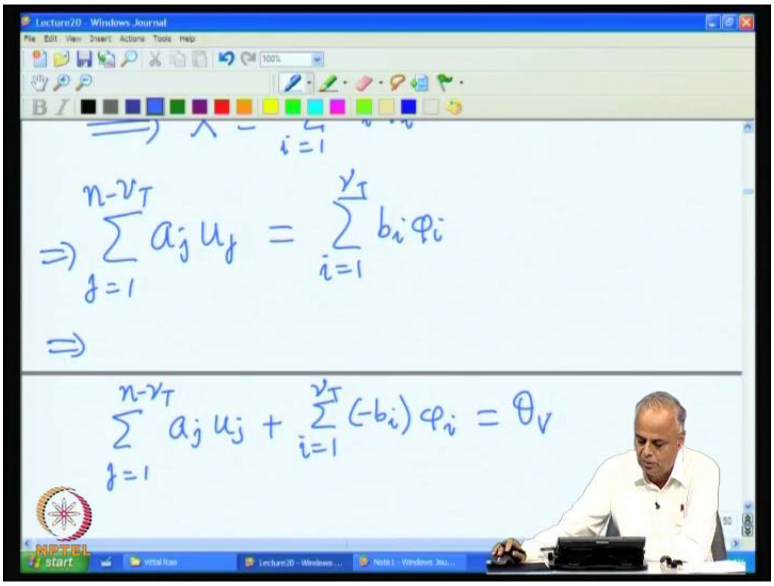

Hence, we have on the one hand we had x is equal to j equal to 1 to n minus mu T a j u j on the other hand, we have x is equal to b i phi i. So these two must be equal to each other now this can be written as j equal to 1 to n minus mu T a j i j plus summation i equal to 1 to mu T minus b i phi i equal to all these are in V. Therefore, V 0 vector we have a linear combination of the u vectors and phi vectors which gives the 0 vector recall that the u vectors and V vectors together form the basis for b space look at the way we constructed it we had this picture in which we had the u vectors and b vectors together forming a basis for V.

Since, u vectors and b vectors together form a basis they must be linearly independent vectors and therefore, any linear combination of them vanishes only all the coefficients are 0, then we have here linear combination of the u vectors and V vector and the phi vector to be 0 and since these are linearly independent.

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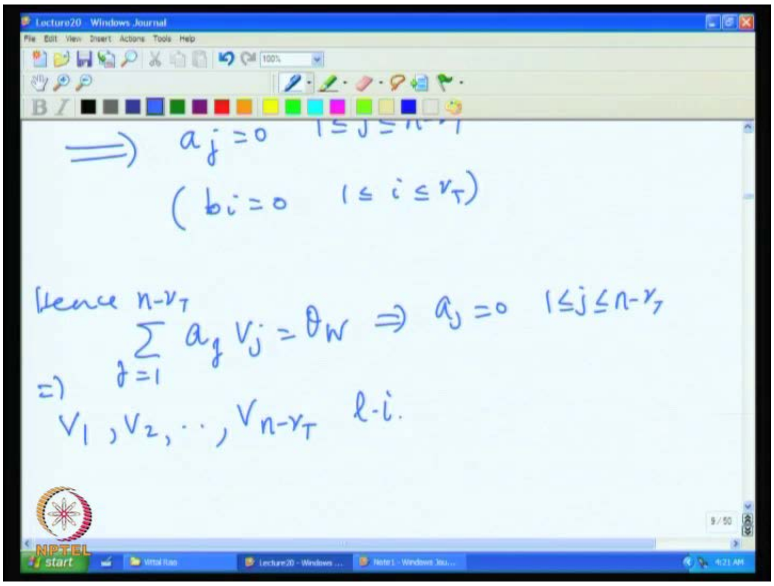

We get the j is to be equal to 0 in particular we also have b i equal to 0 for one less than are equal to 0. Thus, what is the conclusion we started with a linear combination of V is to be 0 and concluded all the coefficients must be 0. Hence summation j equal to 1 to n minus mu T a j V j equal to theta W implies a j is are all 0 hence V 1 V 2 et cetera V n minus mu T are linear, we have already seen that they form a spanning set for the range of T and now we are seeing that they are linearly independent.

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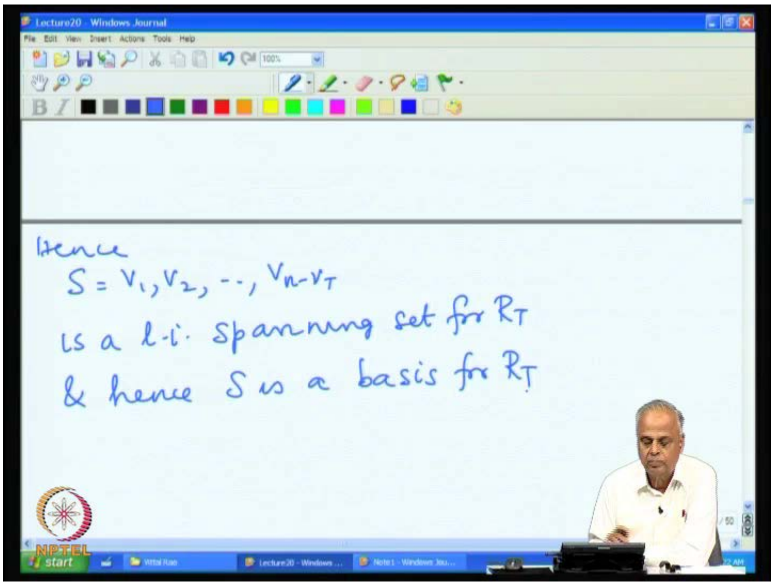

Therefore, S is equal to V 1 V 2 V n minus mu T is linearly independent spanning set hence linearly independent spanning set for R T and hence S is a basis for range of T.

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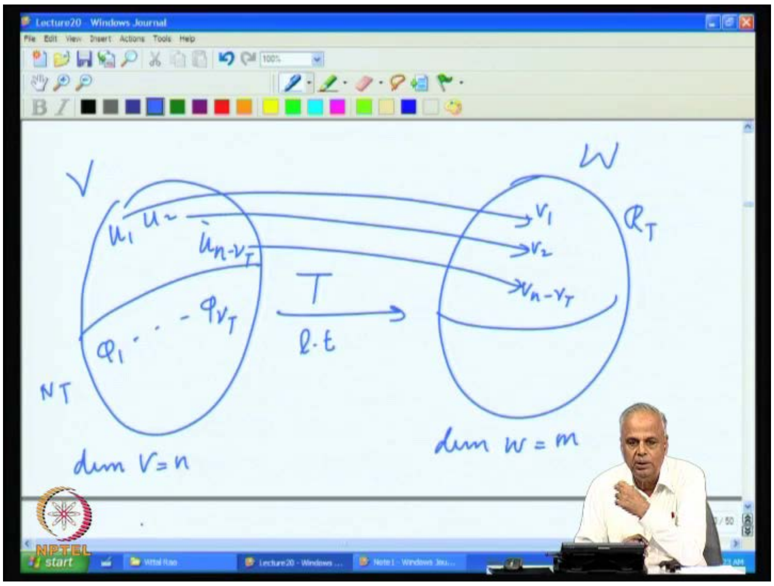

So the conclusion is we star with V in the space W this is n this is m and T is a linear transformation, then we pick a part of it which is a null space of T. We start with a basis phi mu T extend it to a basis u 1 u 2 u n minus mu T from there we get together basis for V. Look at only the images u 1 u 2 u n minus mu T they give V 1 V 2 V n minus mu T they are all in the range of T and they form a basis for range of T now what do we get from this V 1 V 2 V n minus mu T is a basis for range of T therefore, number of vector in the basis must be dimension there are exactly n minus mu T vector in the basis therefore, dimension of range of T must be n minus mu T.

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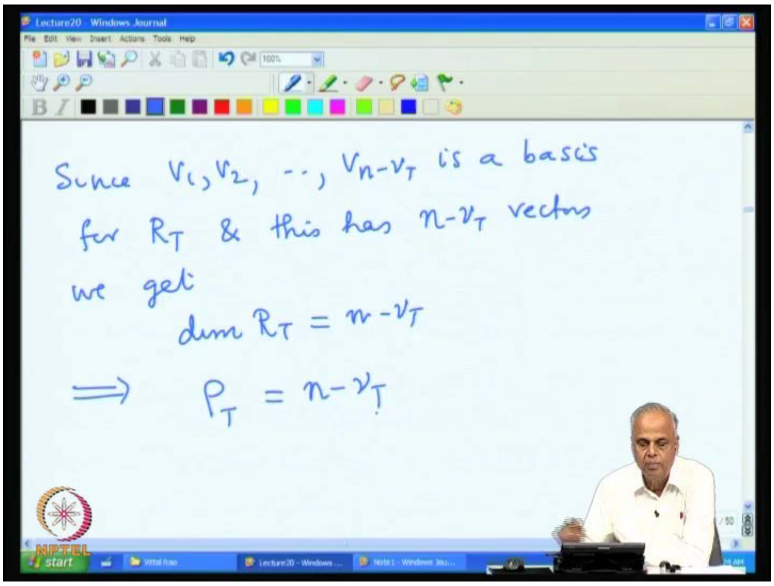

Since V 1 V 2 V n minus mu T is a basis for range of T and this has n minus mu T vectors, then we get dimension of range of T is to equal n minus mu T, but dimension of range of T is what we call as rank of T.

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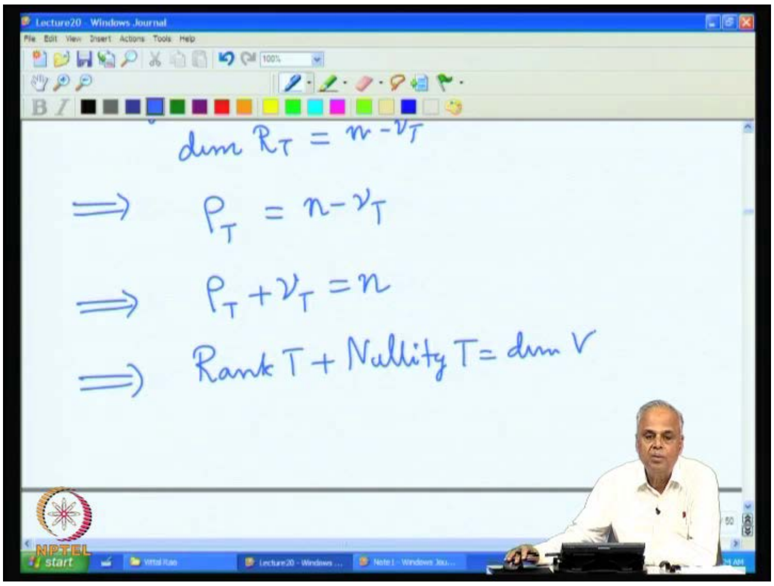

The rank of T is n minus mu T or bring mu T to this side rho is the rank of T mu sub T is the nullity of T and n is the dimension of V so this simply says rank T plus nullity T equal to dimension of V this is called the rank nullity theorem.

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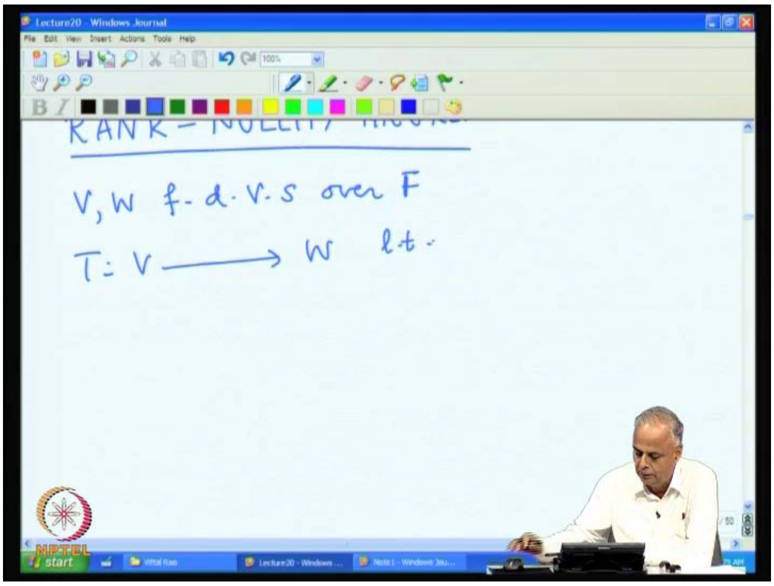

To summarize, then we have the rank nullity theorem rank nullity theorem is for V W finite dimensional vector spaces over a field F T mapping V to W linear transformation.

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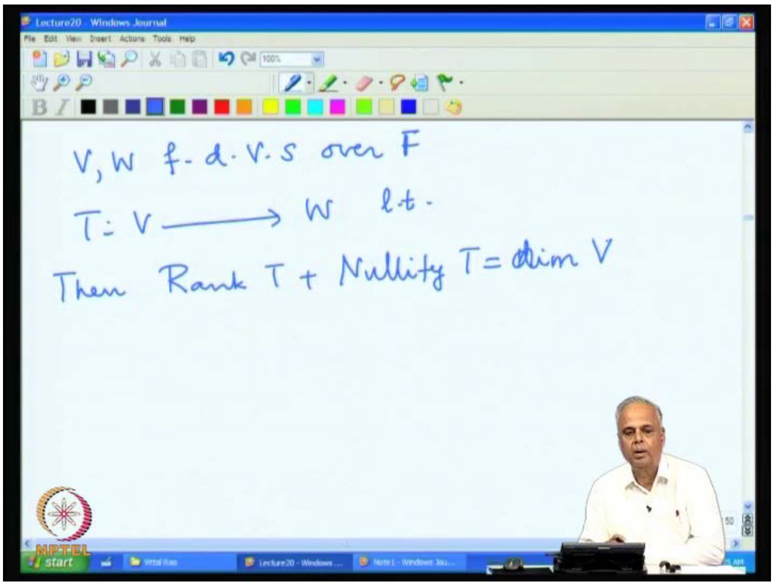

Then rank T plus nullity T is equal to n is equal dimension V this is known as the rank nullity theorem. This is the important connection between the dimension of range of T which is subspace of W and dimension of nullity which is subset of V subspace of V.

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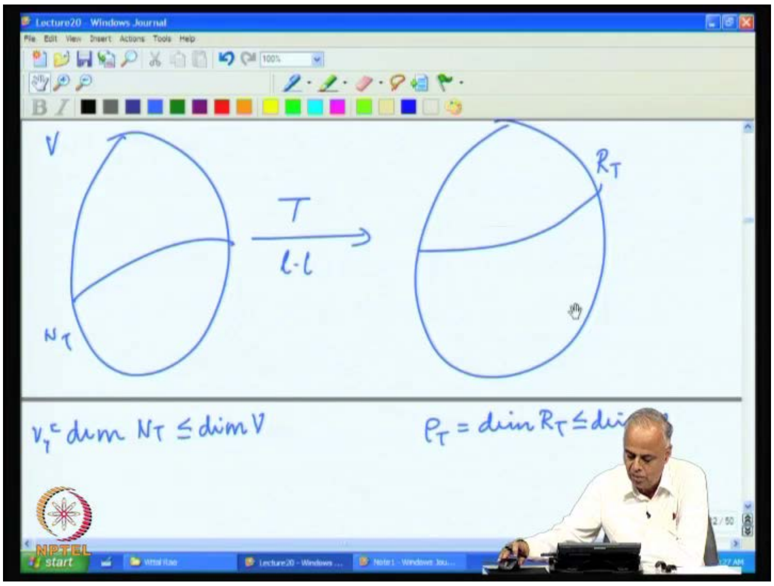

So again look at the picture, we have V we have W and then null space of T is a linear transformation and null space of T is carved out of the space V. The range of T is carved out of the vector space W and what it says is we already had, because n T is part of V we had dimension of n T which is mu T. Since it is a part of V it must be less than or equal to dimension of V and the rho T, then which is the dimension of range of T must be less than or equal to dimension W.

Because it is a part of W now what we have shown is that this mu T plus n T mu T plus rho T is equal to n we also had rank nullity theorem which says the rank plus nullity is equal to the dimension of V so this is rho T this is mu T is equal to dimension of V mu T is a non negative it is a dimension it is a number so it is a non negative quantity and therefore, when we add non negative quantity to rank of T with that dimension of V so rank of T smaller than or equal to dimension of V.

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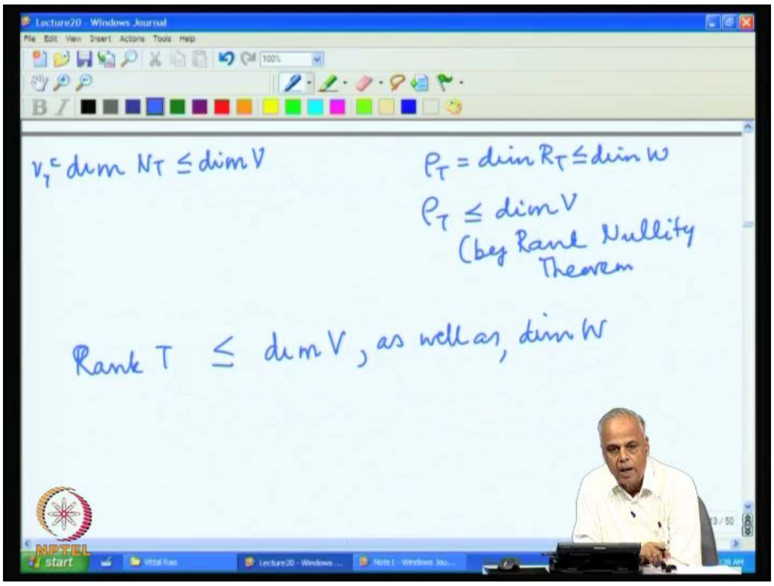

We also get rank of T must be less than or equal to dimension of V by rank nullity theorem. So we have rank cannot exceed the dimension of space W it cannot exceed the space V and hence rank of T is less than or equal to both dimension of V as well as dimension of W. It cannot exceed the dimension of V and it cannot exceed the dimension of W the rank of T is something which controls both the sides dimension of V as well as the dimension of W.

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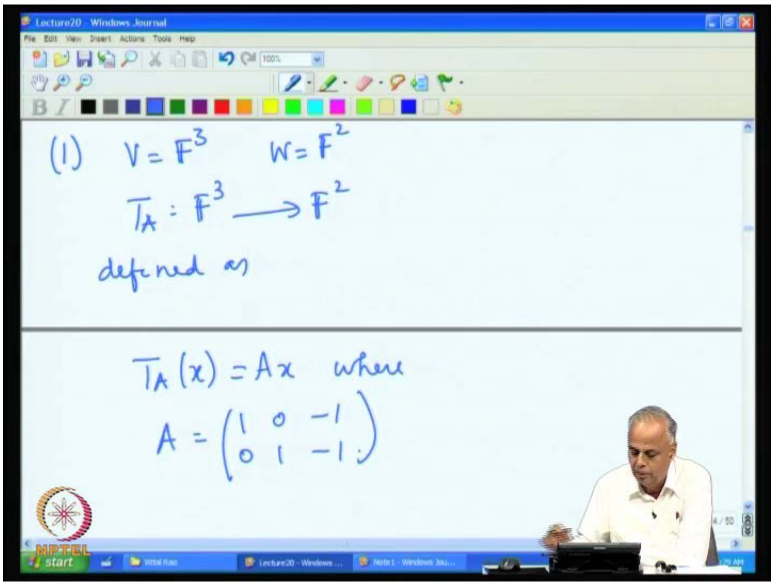

Let us look at some examples of this rank nullity theorem if you recall we had the example, V equal to F 3 W equal to F 2 and it linear transformation from F 3 to F 2 and defined as T A of x equal to A x where was the matrix 1 0 minus 1 0 1 minus 1 we had seen this example in the last lecture.

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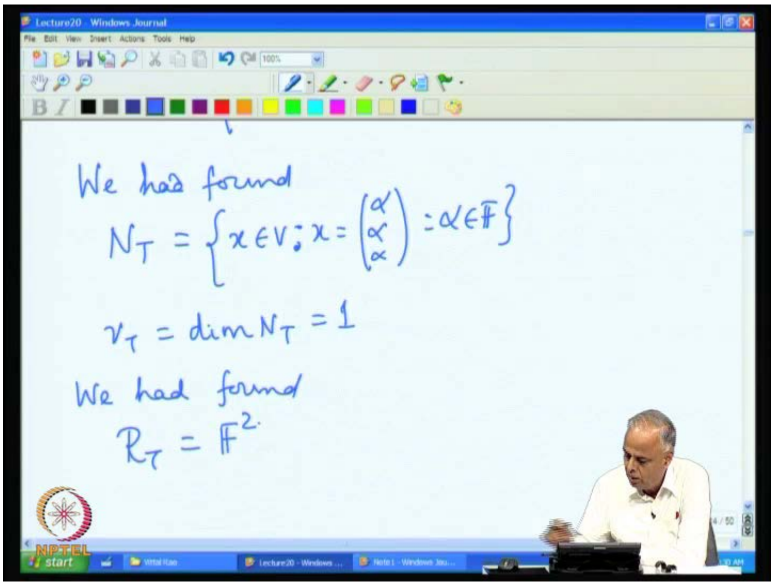

And we had found the null space of T to be the set of all vectors in V, such that x is of the form alpha alpha alpha alpha belongs to F. We found that mu T which is the dimension of N T is 1 because, this dimension is one because the space is stand by one single vector namely one we had also found the range of T is all of F 2.

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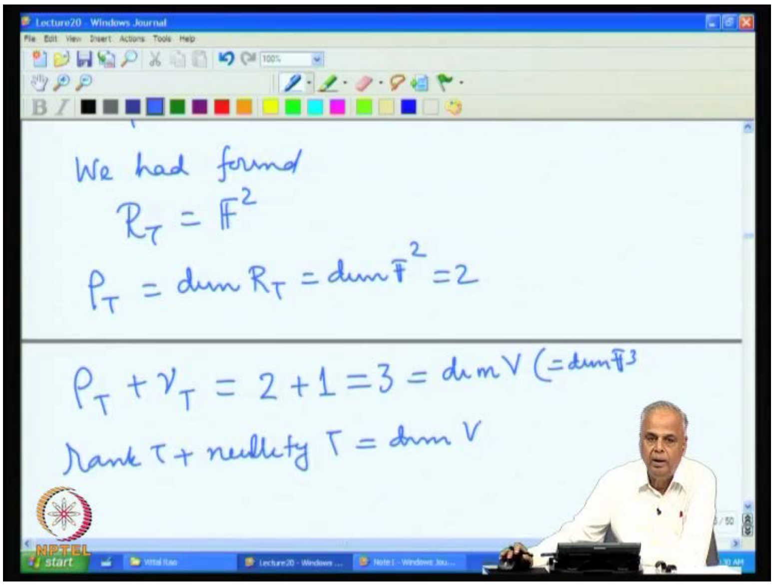

Hence rank of T dimension of range of T which is dimension of F 2 was two. Now we add the rank and the nullity the rank is 2 nullity is 1 which is equal to 3 which is precisely dimension of V. Because V is F 3 thus we have seen that rank T plus nullity T is equal to dimension V thus verifying rank nullity theorem for us.

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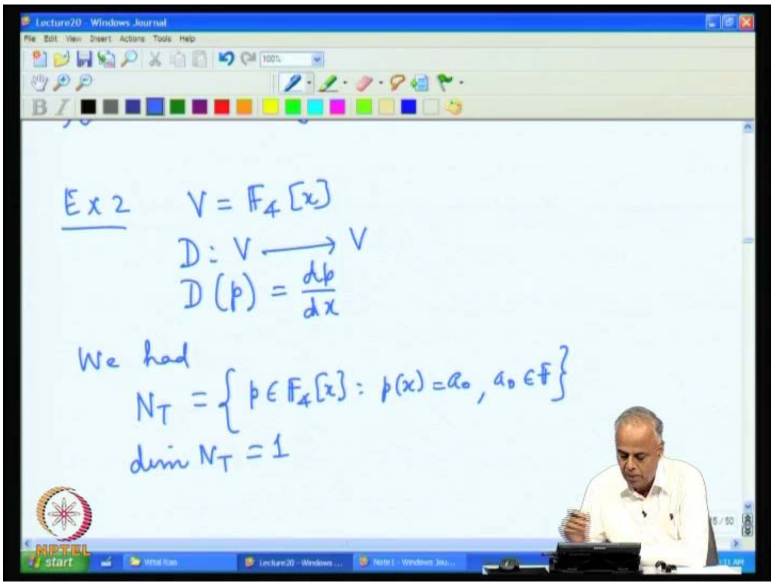

Another example, recall we had the V as F 4 x and then we looked at the linear operator from V to V defined as the different ion operator dp equal to dp by dx. We had found null space of T consist of all constant polynomial where px equal to a naught belongs to F and dimension of N T is one again, because it is spanned by constant polynomial one.

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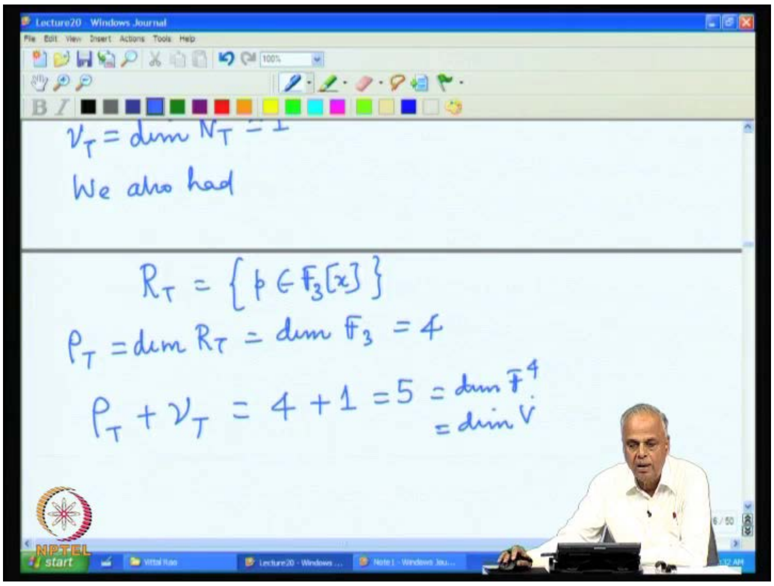

Therefore, mu T is 1 and we also had the range of T to be set of all polynomial which are in F 3 x. Because when we differentiate we lose one degree and therefore, the dimension R T is exactly the dimension of F 3, which is four and this is what we call as the rho sub T the rank of T. So we have rho sub T plus the mu sub T which is 4 plus 1 which is 5 which is dimension of F 4 which is what the dimension of V in this case V was because V was f one again we see rank T plus nullity of T is a dimension of V.

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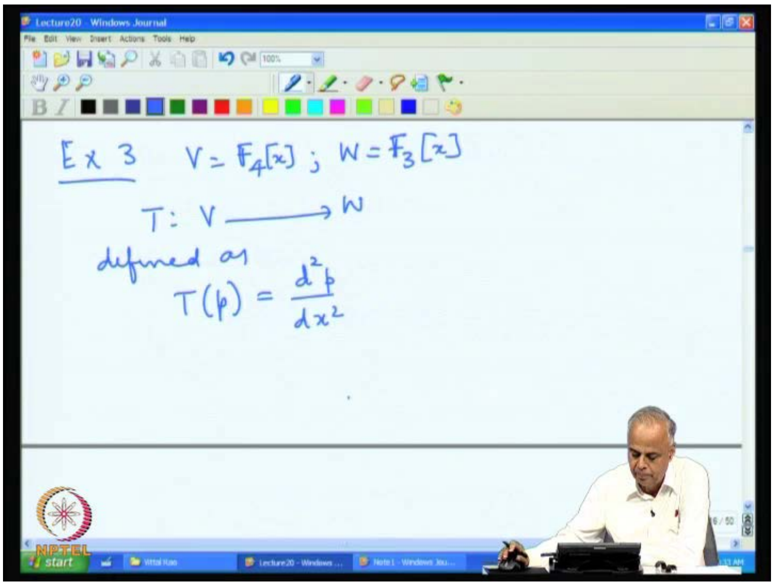

The last week example, we had we understand we took V equal to F 4 and we took W equal to F 3 x polynomials of degree less than or equal to 4. In the vector space V polynomials of degree less than or equal to 3 in the vector space W, then we had this linear transformation defined as T of p is equal d square p dx square the second primitive of T.

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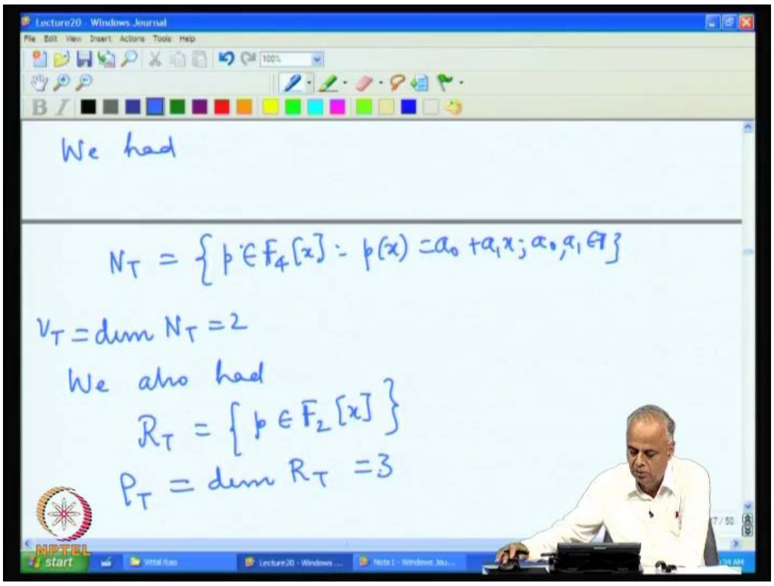

And we found the null space of T consisted of all linear polynomial p belonging to F 4 x, such that p x is a naught plus a 1 x a naught a 1 scalars and the dimension of this space is two. Because the polynomial one and polynomial x expand the space and they are linearly independent and these two form a basis dimension of n T we found as two and that is what the nullity of T. We also had the range of T, because we differentiate twice we lose degree the two degrees and this lead us to the fact that this is a space of all polynomial of degree less than or equal to 2.

Therefore, the dimension of range of T which is the rank was equal to 3 because F 2 the polynomials of degree less than or equal to 2 for this subspace, then we had the polynomial one the polynomial x and the polynomial x squared form a basis and there are 3 of them.

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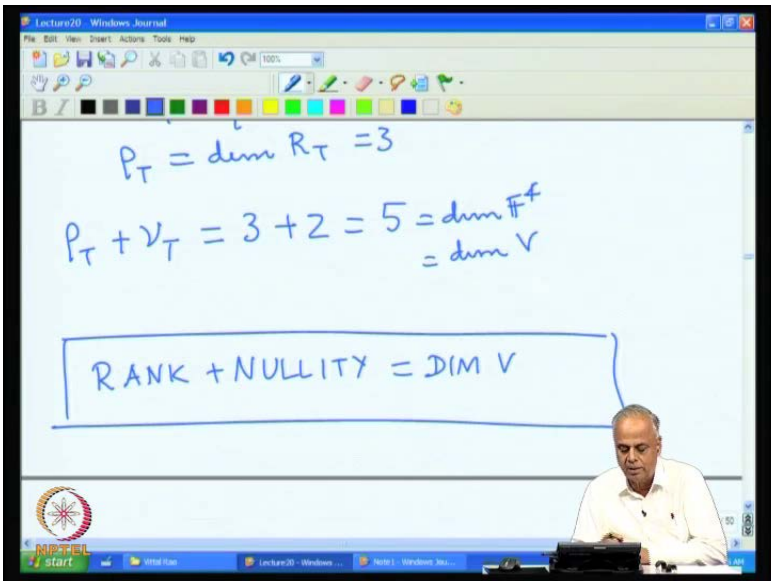

The dimension is three consequently we get rho T the rank of T plus mu T the nullity of T is we had rho. We have rho of T is 3 the nullity of T is 2. So 3 plus 2 which is 5 which is the dimension of F 4 and which is what dimension of V is because, V in this case was F 4 again we have rank plus nullity is equal to dimension of V this is a very important theorem which is very useful in many of your our proofs later.

This rank plus we conclude again by restating thus rank plus nullity equal to dimension of V the dimension of domain space for any linear transformation rank plus, nullity equal to dimension of V which will now use this fact and look at the linear transformation in various angles.

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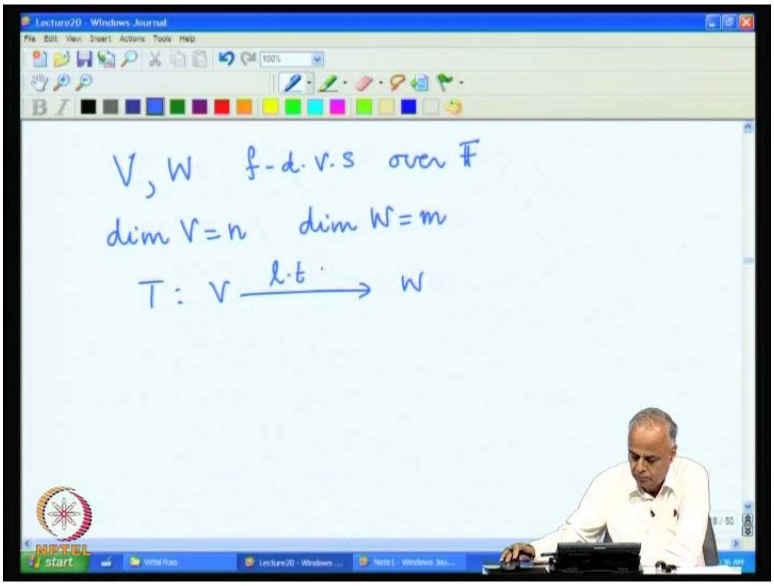

Let us look at now, a vector space V and a vector space W deal with finite dimensional vector spaces for the time being over a field f. Let say dimension of V is equal to n at the dimension of W is equal to m. So we have two vector spaces both are finite dimensional one of them with dimension n the domain space and the co domain dimension spaces as W and we have a linear transformation.

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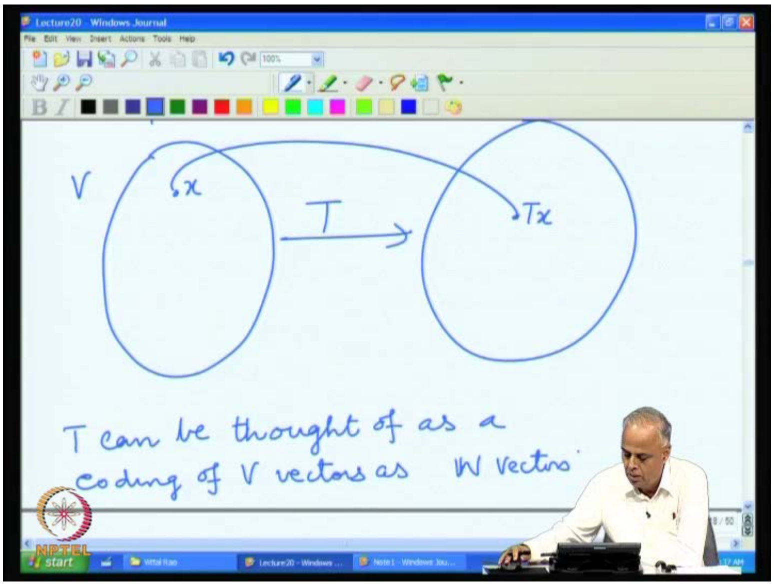

We have vector space V and the vector space W, then T is a linear transformation from V to W. What does T do? This T takes a vector x and maps it to a vector T x here, so we can think of T as coding it codes V vector as a W vector the vectors in V are all coded as vectors in W. The T can be thought of as a coding of V vectors as W vectors V vectors as W vectors.

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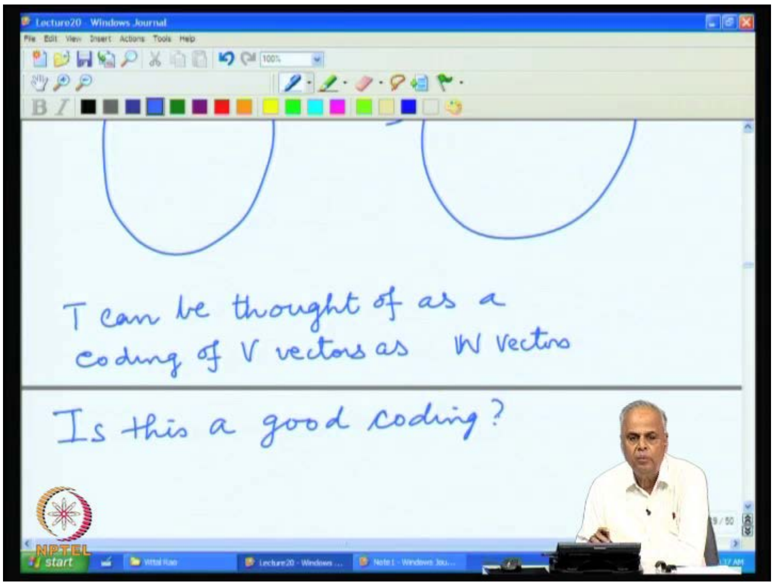

Now this is good coding. What you mean by saying is this a good coding? Let us ask let us ask simple questions about the code.

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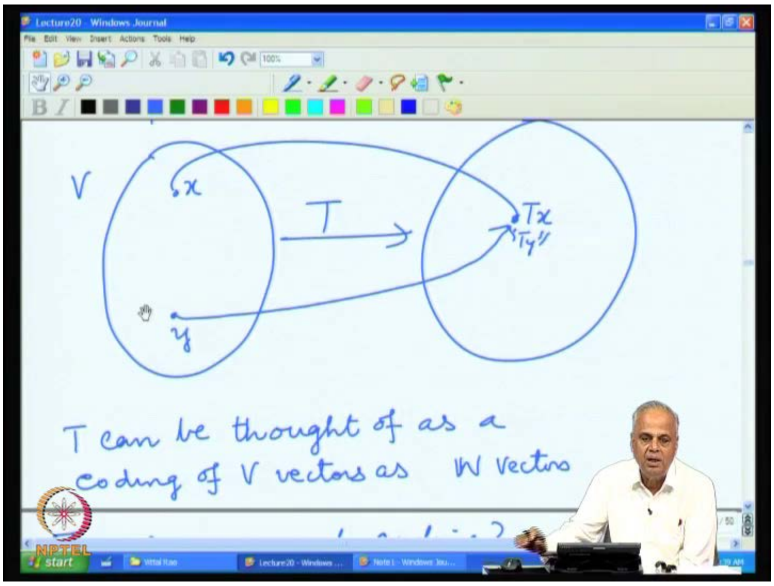

Suppose, I had a vector x which went to something here and vector y also went to some same thing T y is also equal to T x. Then what we would have done is we would have coded vector also x as T x you would have also coded y as T y.

Therefore, if at all any time we have to decode we will be in confusion whether to decode at this point whether to decode this point as x or to decode this point as y. In order to avoid this confusion you would like to have this linear transformation T to have the additional property that if x goes and sits somewhere no other fellow should go and sit there in other words if x goes to T x and y goes to T y and if x and y are different T x and T y should be different therefore, the T x will be equal to T y only if the y and x are the same if they are different they must be different.

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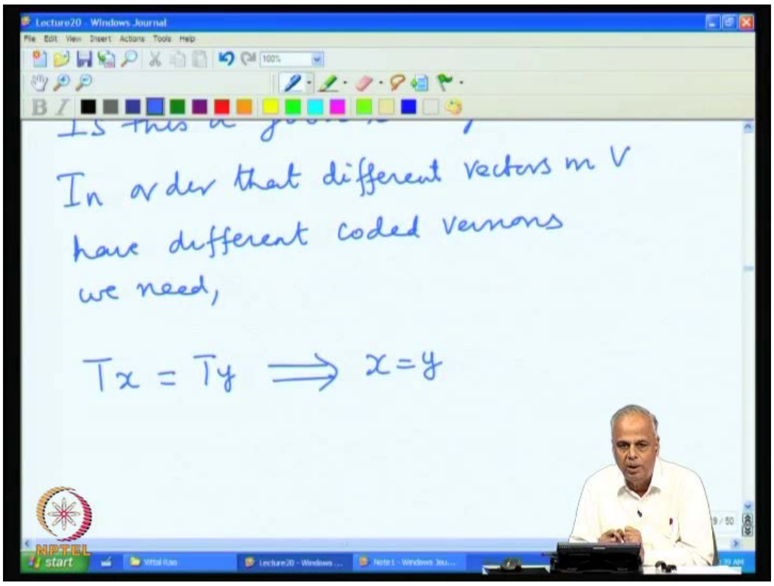

In order that different vectors in V have different coded version, then we need T x the code of x will be equal to the code of y. Only when x equal to y we would like to have T x equal to T y when x equal to y. Now not that all linear transformation likely there are bad codes and there are good codes. See if you are looking at point of coding then we would like to have T to be a good code and hence this property any linear transformation which had this property is said to be one - one linear transformation.

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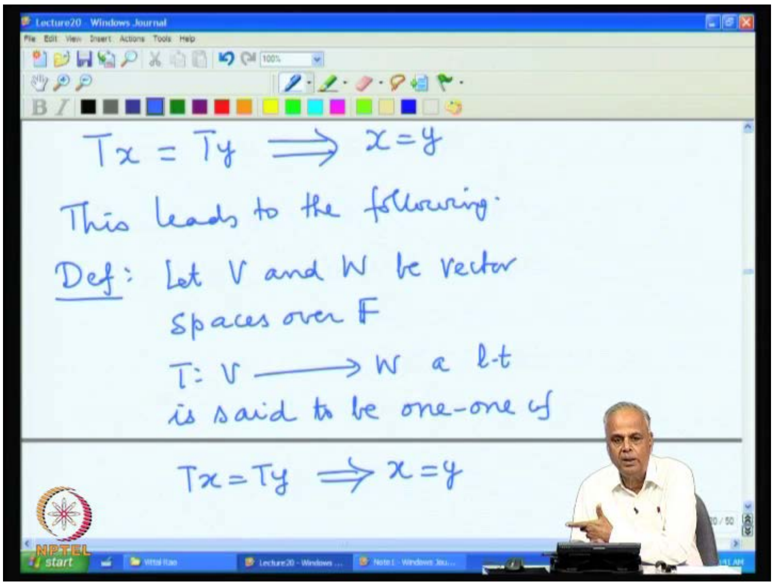

So this leads to the following definition, the definition let V and W in fact this definition did not ever used that V and W are finite dimensional space we only specialize later to finite dimensional spaces. Let V and W be vector spaces over F T mapping to W a linear transformation is said to be one - one if T x equal to T y implies x equal to y this same thing as saying different vectors in V will have different images in W if x not equal to y means T x not equal to T y this thing is same as saying that x is not equal to y since T x is not equal to T y.

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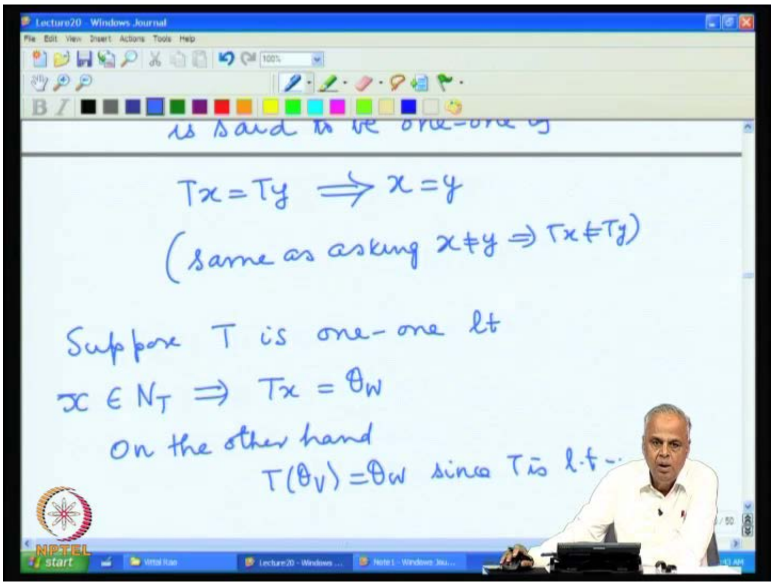

This is same as asking x not equal to y whereas, T x not equal to T y different fellow must have different images. Such a linear transformation is called one - one linear transformation, let as look at a simple property of such a one – one linear transformation suppose T is one – one V is a vector space W is a vector space.

Just like here V is a vector space W is a vector space T is a linear transformation at suppose, T is one-one what is that mean this let us now look at how the null space of T looks like so we have x belongs to null space of T implies T x equal to theta W because something that qualified to be in null space of T only when it gets mapped to the 0 vector on the other hand T of theta V equal to theta W since T is linear because linear transformation we saw always takes the 0 vector to 0 vector comparing this T x equal to theta W this theta V equal to theta W.

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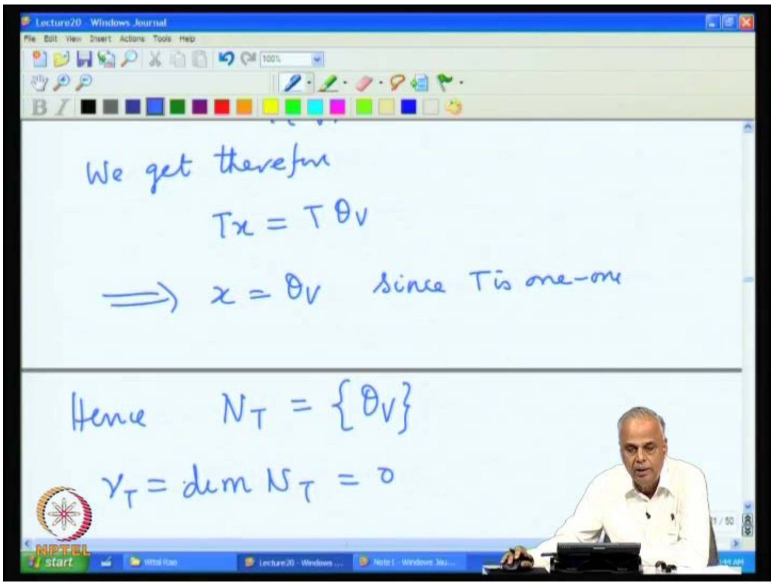

We get therefore, T x equal to T of theta V because both are theta W, but T is one-one T is one-one means different fellows should have different images here x and theta V has the same image and therefore, x must be equal to theta V. Because x were not theta V then x will have different image from theta V, but if x and V has the same image and therefore, they must be same because T is one-one since T is one-one that says the only vector which in the null space of T is the 0 vector.

Because if x belongs to n T it must be 0 vector therefore, we get N T must consist of only the 0 vector therefore, the dimension of N T must be 0 or the null space of the T or the nullity of T must be 0 the nullity of T must be 0 T if T is one-one.

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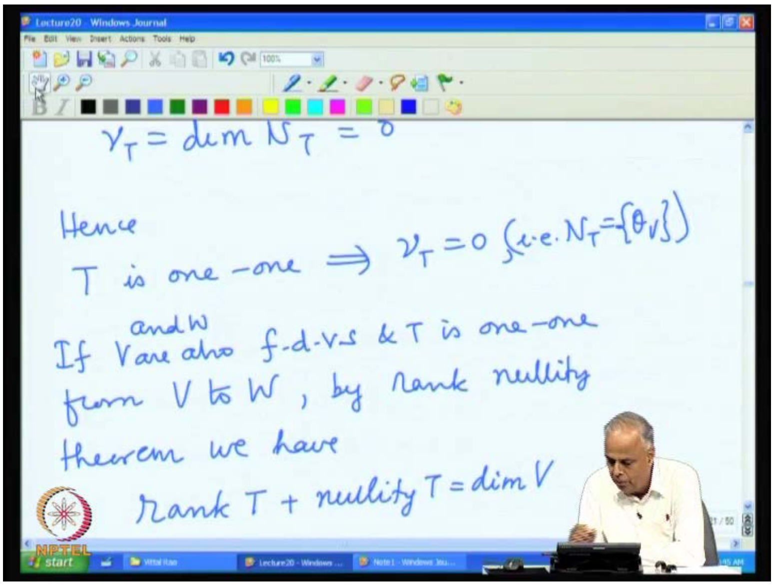

Hence we get T is one-one implies the nullity of T is 0 that is null space of T consist of only the 0 vector. Now, if V is also finite dimensional vector spaces and T is one-one from V to W we are not assuming W to be finite dimensional, then we are only assuming V to W is finite dimensional for assume both to be finite dimensional to be precise. So V and W are also finite dimensional vector spaces V and W are finite dimensional vector spaces by rank nullity theorem we have rank T plus nullity T is equal to dimension of V by rank nullity theorem now T one-one we have seen then the nullity is 0 if T is one-one we had nullity to be 0.

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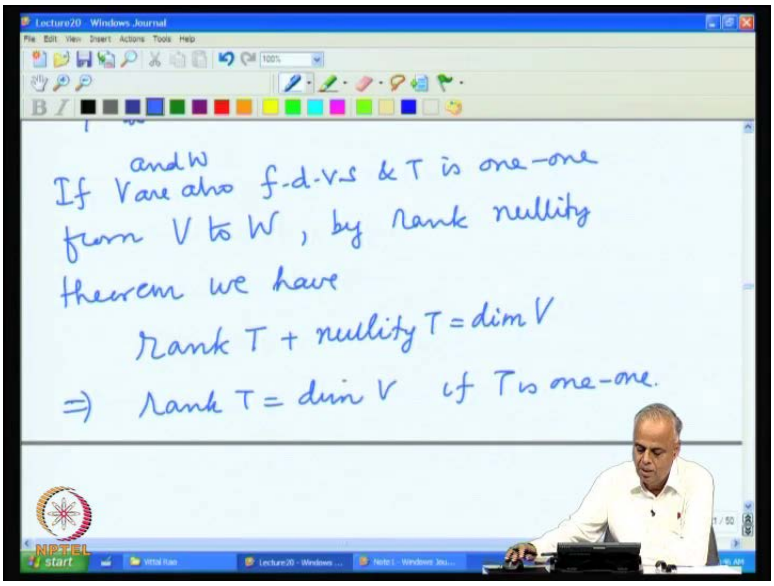

The rank of T is equal to dimension of V if T is one-one therefore, the conclusion is T mapping V to W one-one V and W finite dimensional vector spaces. We deal only finite dimensional spaces.

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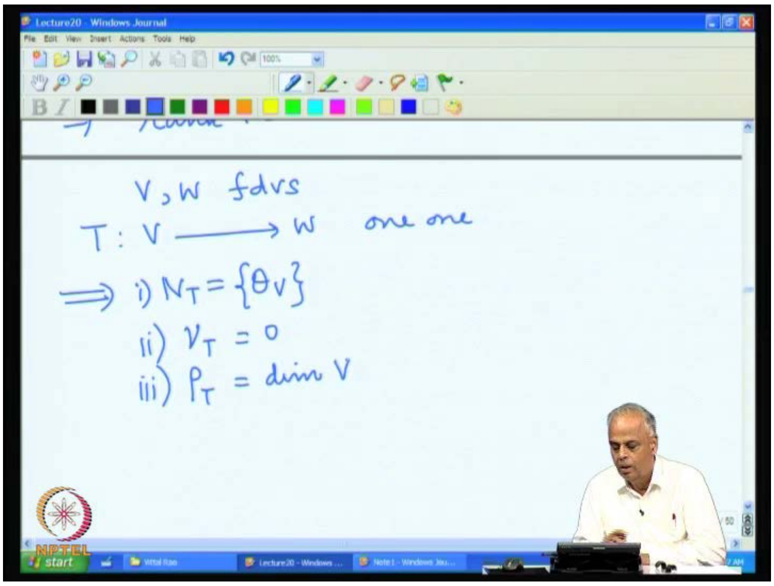

Now implies many things namely one the null space of T consist of one only one vector consequently. The nullity of T is 0 consequently the range of T the rank of T by the rank nullity theorem must be equal to dimension of V, then see the consequence of three.

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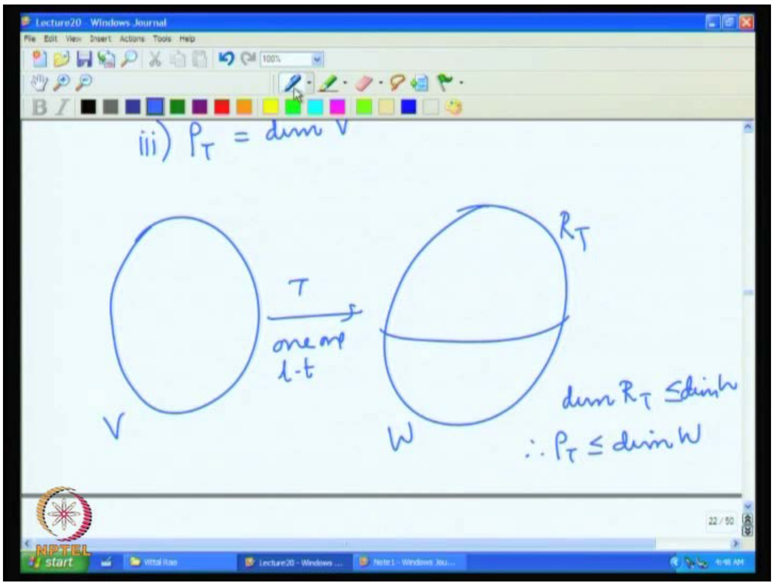

We have V here, we have W here T is one-one linear transformation the range of T is the subspace of this, but this must have dimension V. If T has to be one-one the dimension of the range of T which is the rank of T must be equal to the dimension of V, but we know that the rank of T must be smaller than or equal to dimension of W we have dimension of range of T less than or equal to dimension of W and therefore, rank of T must be less than or equal to dimension of W, but if T is one-one rank of T must be equal to dimension of V.

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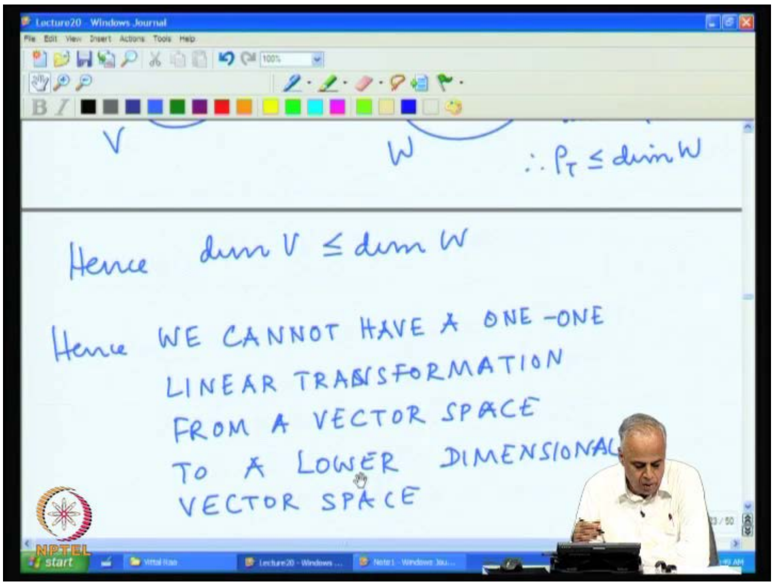

You must have dimension of V must be less than or equal to W, hence dimension V must be less than or equal to W therefore, if for any reason dimension of V is greater than dimension of W there is no chance of having a one-one linear transformation from V to W. Hence we cannot have a one-one linear transformation from a vector space to higher dimensional space to a higher dimensional vector space the dimension of W I am sorry to a lower dimension vector space a lower dimension vector space because we wanted dimension of V to be smaller than dimension of W. The dimension of W must be bigger than the dimension V if for some reason W has the dimension smaller than V a lower dimensional space, then there is no chance of having a one-one linear transformation from V to W we now look at important property of one-one linear transformation.

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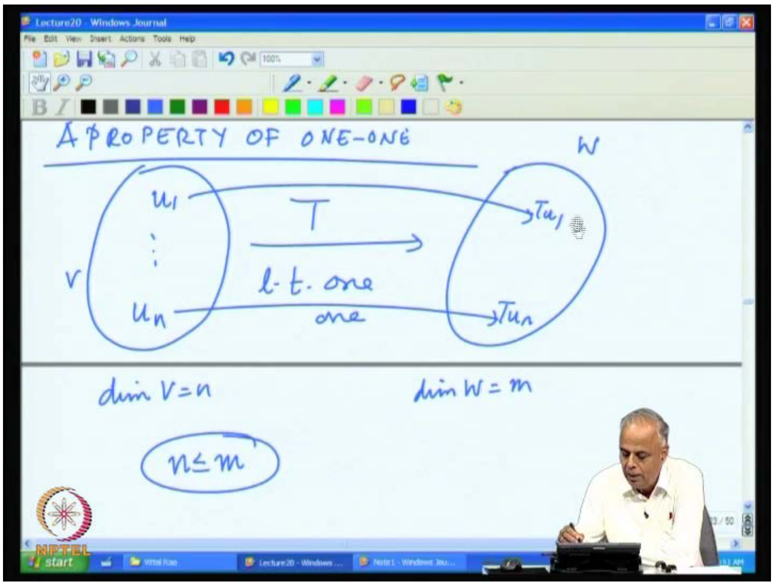

We just mention it first we look at, what that implies the property of one-one so we have a vector space V dimension of V is n and vector space W dimension of W is m T linear transformation which is one-one. Since we have one-one it is a one-one transformation a priory we must have that W must have bigger dimension than V, then we are assuming n is less than or equal to m.

Suppose, I look for a basis in V how many vectors it should have since the dimension of V is n it should have n vector, then let us say I have a basis if I have a basis for this and then I look at T u 1 and then I look at T u 2, then look at T u n this will all be different vectors in W. Because since T is one-one different vector go to different images if you look at T u 1 T u 2 to T u n u 1 u 2 u r s are linearly independent in V. So they form the basis they must be linearly independent they are linearly independent here and their image will they be linearly independent.

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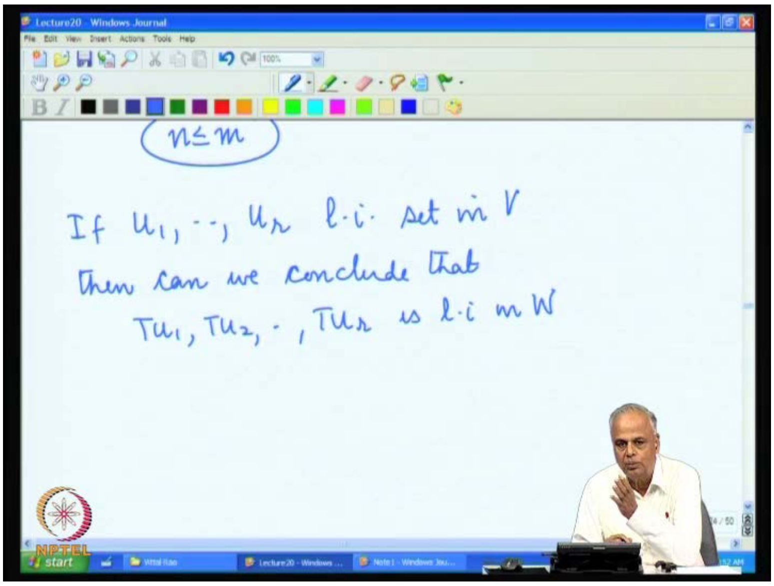

The general question, if u 1 u 2 u n not necessarily a basis look at linearly independent set in V, then we conclude T u 1 T u 2 etcetera T u r are linearly independent. So the answer is yes and it is one-one as that is gets us to the result. Now this would help us to connect again the rank and dimension of V similar to the notion of one-one is the onto linear transformation just as we saw one-one means different thing goes to different image an onto transformation means all the vectors in W will be used to coding and therefore, every vector in W is the coded version of some vector in V and what are the consequence of onto we studied leave it.