Tribology

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Lecture No. # 21

Solution of Reynold's Equation

Welcome to twenty first lecture of video course on Tribology. Today's topic is solution of Reynold's equation. I can use a word as a solutions because we do not use only one solution depends on kind of a Reynold's equation comes out from the problem, we can use different solutions.

In previous lecture, we started with a finite difference method, indicating that whole bearing surface can be divided in number of nodes; and using Tellus series we can estimate temperature or we can estimate pressure not temperature pressure profile on the bearing surface.

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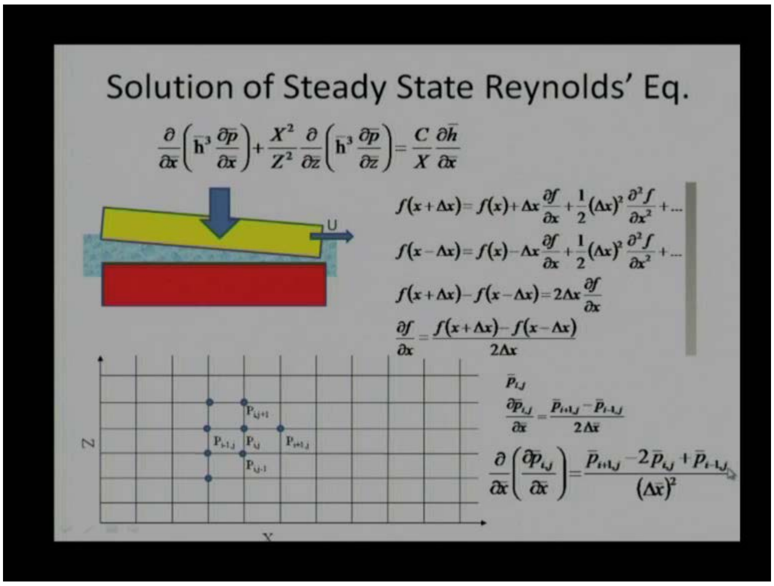

We consider only the steady state Reynolds equation, where the time dependent term was neglected. The dh by dt is negligible for a steady state Reynold's equation; and we considered rejection as in viewing the surface is rigid there will not be any pressure generation due to variation in surface velocity. Surfaces are rigid, u is constant it is not a function of any space variable.

This was a figure, which we showed that two surfaces, and this was a grid which we showed to be used to estimate pressure P i j pressure at any i n j node. This expression was used to indicate as related to tellus series and tellus series was expressed over here at f function of x plus delta x and function f as a function of x minus delta x tellus series gives expression when we subtract equation 2 from the equation 1 we get this expression and this gives an approximate value of first variation or first derivative of f with respective to x.

That was utilized to approximate pressure gradient, first order pressure gradient and second order pressure gradient and get our all expression something like that. For second order e I plus 1 j minus 2 P i j plus P i minus 1 j. For the gradient in z direction we have to write i j plus 1 P i j will remain same while in case of z variation or variation with respect to z you write I comma j minus 1.

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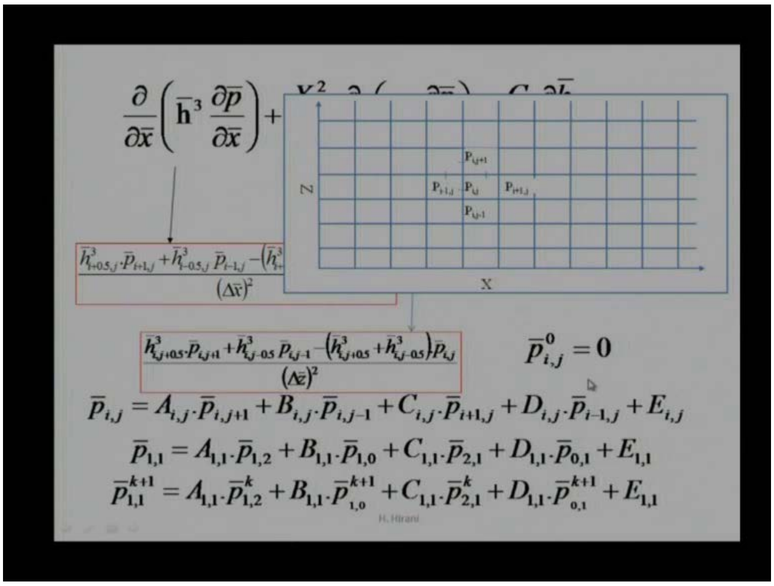

Substituting that this value and we can rearrange equations something like this, P i j the pressure at any node in x direction and y direction when the cross section comes we can write in terms of four pressures P i j plus 1 P i j minus 1 P i plus 1 j P i minus 1 j and a constant term at that node, that is e i j others multiplication factors with all these pressure terms will remain constant at any one node. It will not change, however the pressure will continuously change as per the iterations.

It may increase if we are using the initial pressure as a 0 pressure as a gauge pressure is 0 pressure or z pressure is atmospheric pressure, then we will slowly get more and more pressure at the node and finally, it will reach to a 1 saturation limit and that saturation limit what we use name as a conversions limit.

That was indicated in a previous lecture and repeated over here, we are saying that at any point or at any iteration may be say iteration number k plus 1 we take values from a previous iteration that P 1 2 from previous iteration. However, P 1 0 will be known to us, that means we can take from the present iteration while P 2 1 from previous iteration P 0 1 again will be known to us then it will be at the present iteration. While all others are constant for that nodes will remain same will not change with any iteration it will be just repeated.

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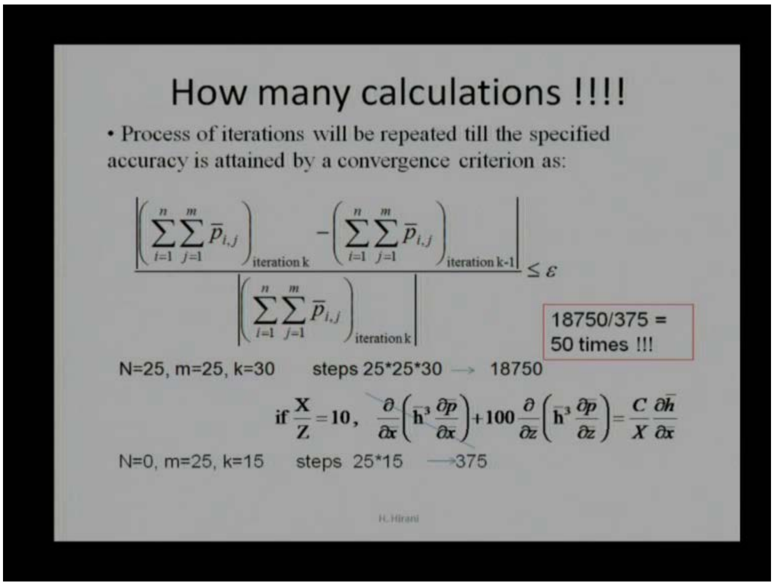

And for we can use a boundary condition as an initial condition not boundary condition initial condition P i j is 0 at all nodes, then this is a convergence criteria we are have been we are using. We say that at any iteration k whatever the summation pressures or non dimensional pressures for all the nodes, you say that when node vary from; in x direction 1 to n.

While in g direction in z direction from j 1 to m and we are saying that this is not case sensitive, it can be capital n or small n. The value will remain same. It will be subtracted from previous iteration and overall will be absolute value we are not talking about the negative value, the overall it will be absolute value divided by absolute value of summation at the k iteration.

If you are getting this value smaller and smaller that may saturation is coming closer and closer that is indicative and that indicate with a number of epsilon. Epsilon can be 0.01, can be 0.005, can be 0.001 depend on a solution like accuracy required solution accuracy.

We took a example, you say that may be assume the number of nodes in x direction 25, number of nodes in z direction in 25 and it requires of a thirty iterations to reach to this solution accuracy. In that situation we will be requiring number of steps that is 25 into 25 into 30 that will be 18,750.

However, if you take a approximation, reason being that the dimension is very large in x direction compared to z direction. In that situation what will happen the second term will be dominating, it is more like an 99 percent higher in domination or we say 99 times higher not 99 percent, it is 99 times higher.

Then this term can be neglected for the, to reduce the number of steps and if we neglect this term we require only terms in, on number of nodes in z direction. We do not require any node in high direction as it is independent node and due to this what will happen number of iteration will reduce. We are saying iterations may turn out to be only 15 instead of 13 to form a final solution.

If we do that which is a 25 into 15 is turning out to be 375 that means, we are going to get benefit of 50 times when we are using some logic. We say this second term is very much dominated compared to first term. First term can be neglected and second term only can be considered that will give better results to us not better results the faster results to us.

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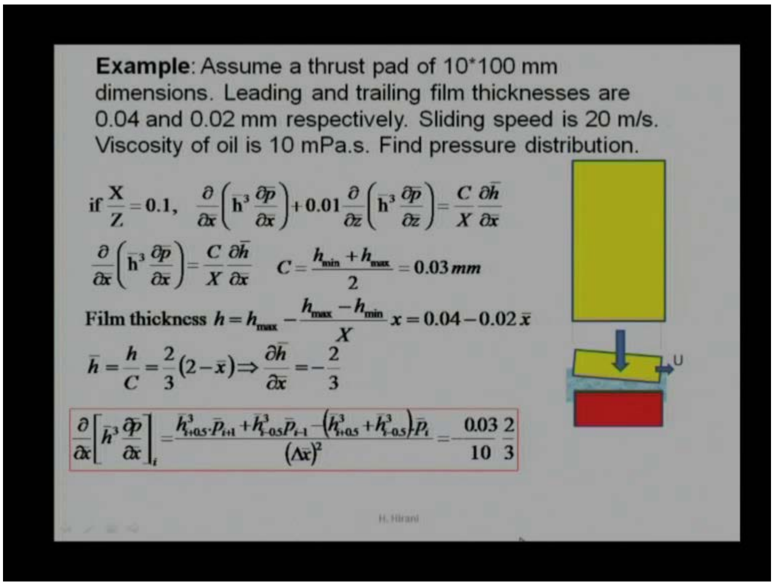

Then we say that we will be covering this example in present lecture. So, we are starting here the pad dimension invention may 10 into 100 mm .10 m m is in x direction and 100 mm in z direction. So, this is x direction, this is top view of, this front view and we are able to see this plate. That is z direction is having a 100 mm that is much larger compared to x or m.

Then in addition to this 2 year dimensions we have a leading dimension from leading film thickness dimension and telling from thickness dimension, what is the meaning of that? Film thickness at the entrance is known and from thickness at exit is known, indirectly we know the inclination.

Once we know these dimensions, we know we will be able to figure out what will be film thickness at the any cross section, at any value of x what will be the film thickness. Velocity sentential velocity is provided in question and viscosity of lubricant is also provided in this expression.

So, what is required to find out the pressure distribution, how pressure will distribute on this surface? Using this dimension we say x by z x is 10 mm z is 100 mm. So, it will turn out to be 0.1. We use this 0.1 over here that is x by x square by z square will turn out to 0.01 and this term can be neglected, compared to one unit over here. One verses 0.01 naturally this is negligible we can neglect it and we can use only this term.

We know the clearance it can be the average value of overall film thickness it is a 40 micron and 20 micron film thickness and then you can find c. The c is 30 micron over here. So, c is known to us capital x is known to us what do we need h bar. To find out the h bar we need a film thickness expression.

And that film thickness expression can be given easily we say that the film thickness is decreasing to us x direction, as the value of the x is increasing film thickness is decreasing. That can be represented with a simple friction we say that h is h maximum minus h max minus h minimum divided by capital x. Capital x is a dimension in x direction which in the present case is a 10 mm.

So, here the value is already known the 40 micron, minimum value of the film thickness is 20 micron. So, 40 micron minus 20 micron will be known to us as 20 micron. 20 micron divided by 10 mm. So, if we give in terms of meter it turns out to be value is 20 micron divided by 100 sorry 10,000 micron the ration will be known.

However, we know very well that we require x bar we are doing a dimensional and x bar is simply x divided by capital x. So, that is why it is written clearly over here, x divided by capital x is x bar and this 20 micron can be represented in terms of mm that will turn out to be 0.02 and this 40 micron will turn out to be 0.04.

Know we can non-dimensionalize this from thickness using capital c value that is in our case is thirty micron or 0.03. So, h bar in our case turn out to be 0.04 divided by sorry 0.04 minus 0.02 x bar and divided by 0.03.

We can say simply it will be 4 divided by 3 minus 2 divided by 3. 2 can be taken common and 3 can be taken common. So, it will turn out to be 2 by 3 in bracket it will turn out to be 2 minus x bar. If I differentiate it, if I differentiate h bar with respect to x this will turn out to be 0 minus 1 and we know the outside bracket is 2 by 3. So, this derivation of h with respect to x bar will turn out to be minus 2 by 3. Once I know this expression I know, what is the value of the source term I can directly substitute this equation, and carry forward higher integration on numerical analysis of finest difference method. First we will consider finest difference method.

So, we know this term and we know how to represent this pressure term this derivation of the pressure term in terms of finest difference equation that was learned in last lecture that will turn out to be h cube, h bar cube i plus half node P i plus 1 similarly h cube, h bar cube I minus point 5 and P i minus 1.

This will a summation of the film thickness i plus 0.5, i minus point 5 or we say film thickness at this node h cube of that and h cube of this that node. Now h is will be known to us we can divide overall value of x, capital x in 10 division or 11 division.

So, we will be knowing what will be the value at each node, that because we will be knowing what will be the x bar. x bar will vary from 0 to 1. If I divide in a 11 or 10 division, we say 11 number of nodes then I can say x bar is x 0 then 0.1 then 0.2, 0.3, 0.4, 0.5, 0.6 2 y and that when we substitute those values I can figure out what will be h i, h 1 in this case h bar 1 will be 0, h 1 bar 2 will be point based on the point 1 of value of x.

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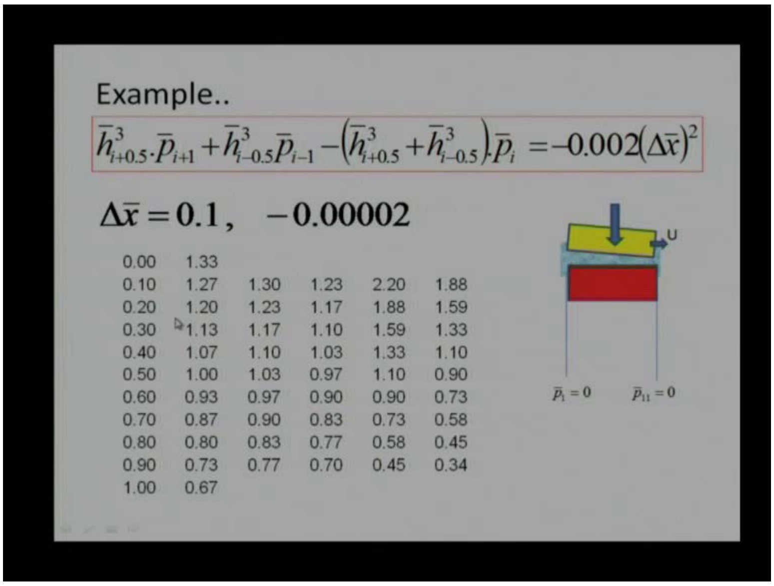

So, we can calculate when that and expression can be written something like this, we can rearrange or as I mentioned that we are taking in step size as a 0.1, this is non dimensional number vary from 0 to 1. So, we are taking a step of 0.1, if that is the case we can directly substitute this 0.1. It will be 0.01 after you get squaring it and that expression will be modified and formed like this, our right hand side will be 0.0002

And this is, these are the values for x bar. On x bar at the initial node, node number 1 submits is 0, node number 2.2.1, node number 3.3, node number 2 is a 0.1, node number

3 will be0.2, node number 4 will be 0.3 and node number 11 it will be 1.0 and these expressions are the film thickness expressions.

The film thickness at that node that means, when we substitute value of x bar what we get is something like a 1.33, again this is a non dimensional number. This film thickness will continuously decrease because of the conversions. There is a taper angle or that there is a taper surface or there is inclination that is why this film thickness will be maximum at the entrance and the minimum at the exit which is as shown in this case.

Now, we need the film thickness of the half the step and the half a step we can find out what will be the h i minus, initially it will be h i minus 0.5. So, that is turning out to be like something like this we are talking about when a h bar is equal to 1 what will be the film thickness at step 0.5 towards a negative x, in that case it turning out to be 103; however, the positive sign step one is step ahead it turnout to be 1.23.

And this is what we can arrange value of h value of h step ahead of that sorry in this case it will be negative sign. So, it will be h i minus 0.5, it will be h i plus 0.5 and this is a cube, this is a cube of h i minus 0.5 cube of that; that means 2.2 is a cube of 1.3.

Similarly, 1.88 is a cube of 1.23 and similarly we can move ahead and the 1 good point is that whatever the film thickness is moving ahead then a what will be the node value is 1.23 is a same, which we have calculated in previous step. 1.17 is same as a 1.17 earlier, say similarly 0.77 and 0.77 is same that is the obvious.

Whatever, the way we are increasing the value of i by 1; so, wherever at any point we are getting i minus 0.05 we had plus 1 over here that will turn out to be i plus 0.5. So, that is way we calculate and now this values will not change, that will remain same for iteration to iterations these values are not going to change that is why which is always preferable to evaluate this value keep away from iterative loop.

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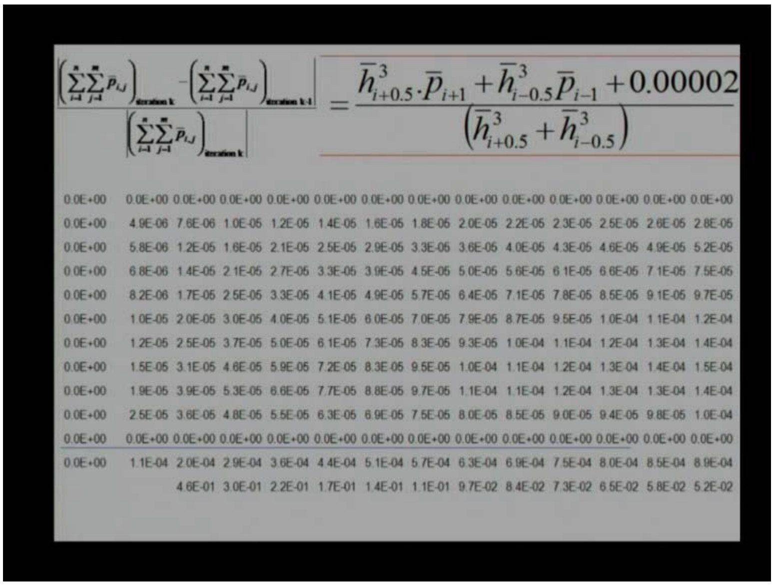

That's why we do not require too many calculations once these values are stored and this values should be repeated again and again in a iteration loop, that iteration loop will come from the pressure and pressure is something like this, the P i at any iteration can be given something like this i plus 1, i minus 1 and i minus 1 will be known to me or known to us.

So, we can use advance iterations, this will be the past iteration or we say that if I use a P i k plus 1 iteration number this will value will come from P i P k iteration or P i plus 1 will come from the k iteration while P i minus 1 will come from the present iteration because the 1 values already been calculated.

And this is what, we have already this values this h value, the summation value what is the only going to vary P i plus 1 P i minus 1 and from iteration to another iteration as I mentioned earlier when we do not know initially any value of P we write the value as a 0 and that is indicated over here 0 iteration. 0 iteration we are using P all the pressure value as a 0 0 0 .

Then we comes to the first situation and another thing is that we are using 0 value of pressure at the entrance and the exit those surfaces those interfaces are open to environment or pressure will be 0 at those interphases.

So, pressure at the initial value when x bar is equal to 0 pressure will b 0. Similarly pressure at a value of x bar equal to 1 will be 0. Now remaining pressures will be evaluated initially based on the term, constant term value in this case, that is a constant term over here 0.000002 that will be going to decide what will be the pressure given at any point ofcourse by after dividing these are the finite numbers we will be getting some variation in our value and that is coming over here.

Now, second value of pressure is going to get influenced with the first pressure value this is what we are say if I write a P i 2 P i 1 is already known to me and that this to evaluate P i 2 I already have this value. So, sequentially we go ahead and we get the value. We are able to see the pressure values continuously increasing; there is already one source term plus increase in a pressure term. Compared to 0 it will be higher value. So, that is why the pressure value is increasing is increasing to maximum up to the node 10 number node. Of course, 11 number node is a boundary value and we have already specified that the pressure value will be 0 in there, that will not changed with any iteration it will remain same.

Coming to the second iteration what we are getting some change in this is increase in a value instead of 4.9 we are getting 7.6 that is obvious, here the P i is getting influenced with the P i plus 1 of previous step; that means, it is getting influenced with the this term we are calculating in this P i ,even though P i minus is a 0, but here the P i plus 1 is already present to us that is getting me to influence let me say the pressure value will increase at the second iteration.

And this train will continue and the interesting thing what we can observe here earlier the pressure was maximum at the 1 node lesser than what we say the boundary node or the here in this case you can see the maximum value of pressure happening at a known number 9 itself. Here the pressure was maximum is 10 number node here the pressure maximum is a 9 number node.

And this is a what boundary conditions of acting boundary condition is 0 here, 0 here and that is affecting the results and that is required also because we know the pressure profile in this kind of a tribo surface will be a parabolic otherwise it will not turn out to be parabolic it will be turnout to be some linear profile at the exit.

And we can use this iteration on, say we keep adding finding the iterate ratio summation that is happening over here after iteration number first your evaluating a summation of all the pressures which is turning out to be 1 point 1 into 10 is to minus 4. After that we are again finding the pressure summation and the iteration number 2. Here it is a more than earlier one because all the pressures are increasing.

But interesting thing is that we need to find out what is the difference? What is a difference between the 2.0 10 is to minus 4 minus 1.1 10 is to minus 4 and it should be divided by this 2.0 10 is to minus 4. So this is the iteration loop or this conversion criteria need to be certified.

When we do this kind of conversions criteria we write 2 minus instead of this term we write 2.010 is to minus 4, instead of writing this term we write 1.1 10 is to minus 4 and substituting value of this that is the 2 .0 10 is to minus 4.

What we get overall is a ratio that is coming around 0.46; that means, there is a 46 percent increase after iteration number 1, we continue it. Iteration number 3 as I mentioned first node and last node will be at the 0 value these are the boundary condition we are not bearing it and remaining other terms are increasing like instead of 7.6 now pressure at the known number 2 has reached to the 1.0 10 is to minus 5 and these all are the non dimensional numbers.

It’s really slowly increasing and you are able to see the same change maximum pressure occurs at the node number 9 instead of node number 10. So, pressure maximum pressure is a 5.3 and summation also we can see earlier summation was 2.010 is to minus 4 while here the summation is increased because they have increased in value of the pressure 2 .9 and 10 is to minus 4.

However we are able to see even the pressure value of summation is increasing, but the difference is decreasing, earlier the this ratio was 0.46 now we are getting point 3 that means earlier the variation pressure variation was the 46 percent and now pressure variation is 30 percent.

That means we are going towards the convergence side that indicates the iteration this convergence this value is decreasing that indicates clearly that we are moving at right direction solution is the stable and we are going to get solution may be after a number of iteration that depends which kind of a scheme we are using.

That this is a after additional iteration we can say after this iteration this epsilon valve is turning out to be 0.22 which is a better that 3.0 that is why we can we say we prefer this convergence and after doing a number of iterations may be saying we can count may be say 13-14 iterations, what we are getting this convergence value as a 0.05; that means, this is a 5 percent. It started, a first iteration was 46 percent and after 10-12 iteration it has reached to the 5 percent; that means, improvement is significant. We continue it if we have a time.

And we understood how the pressure profile is changing, maximum pressure profile maximum value of pressure is come here somewhere here it is not I mean is a node 11, node 10, node 9 and is in node 8 it has to move to the node 8 earlier it was in the node 9, all the cases its 6.6 compared to 5.9, 7.7, 8.8, 9.7, 1.1, 1.1, 1.2, 1.3, 1.3.

And here in the final in this iteration it has reached to the 1.5. That means, its shifting towards entrance side. Of course, away from the midpoint away from the sorry slightly away from midpoint and away from the exit point, it is coming somewhere from in between.

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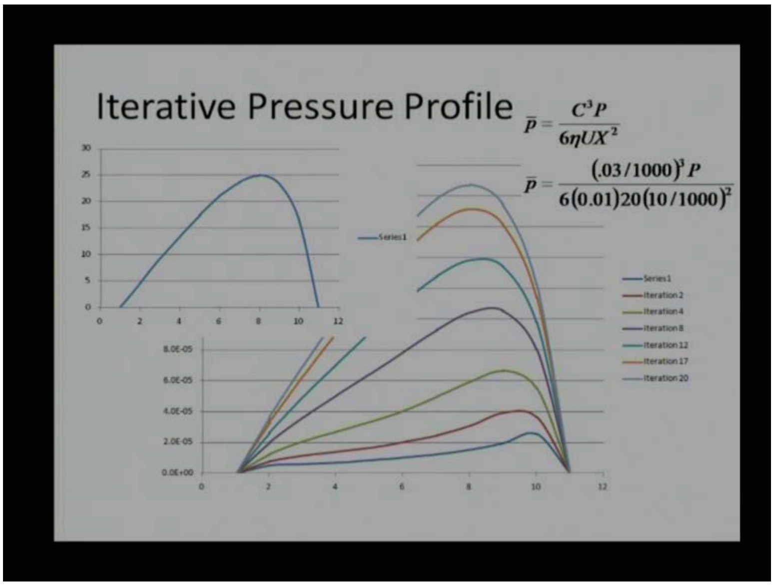

And this is a pressure profile on a how we can see how the improvement is happening, as we can see in the series 1 is the iteration number 1. Initial iteration is a pressure 0 this is the what we are showing we have selected the 11 number nodes, the node number 1 what is the pressure will be 0 always that is a boundary condition. Node number eleven again pressure will be 0 because that is a boundary condition.

In between pressure is continuously wearing initially it was maybe say maximum value was the 10 number node then slowly to move to the 9 number node and is finally, moved to the 8 number node. You can see there is a parabolic distribution and if I take this as a mid point, it is away from the midpoint and away from the may be say exit point and is coming somewhere in between midpoint and exit point pressure is continuously increasing with iteration that is a clear.

And when we use some additional technique like over induction technique and all then there is a possibility of change sudden change in pressure radiation while; however, if you are not using any iteration technique then it will take slightly more time in convergence time, but there will be a definite profile there will be continuous improvement, even the improvement may be slight, but there will be continuous improvement.

So, this gives an indication on how the Iterative loop, iterative cycle works for the tripo surface we are using the finest difference method and we have taken approximation in this case because of the dimensional dimensions were large in x direction compared to dimension in z direction and we say that with this kind of loop there will not be much problem.

With this assumption there will not be much problem, thus x by z square was in this case dimensional in x direction was a lesser. So, it was a negligible of fact and that is why we consider only pressure variation in x direction. Whatever we have done in this case is non dimensional number, but finally, at the designer stage we require dimensional numbers for solution purposes, for initial getting a feeling of which term is more dominating, we are non dimensionalize it.

But coming to the final when we substitute and we want to get a result in a dimensional numbers and that is possible using this solution we use this for non dimensionalization of a pressure. You see this is a clearance because there is viscosity, velocity and maximum value in x direction. When we substitute this we know there was a viscosity is a 10 mille Pascal and mille Pascal second velocity is a 20 meter per second and clearance is 30 micron and in meters it will turn out to be 30.03 divided by 1000 and cube of that, this is the 10 mm at capital x the 10 mm.

So, 10 divided by 1000 you make a value and we can find the pressure profile at the iteration number 20, what will be pressure profile; Of course, in natural case pressure will continuously increase and it will reach to the final 1 saturation limit and that is after substituting this values for we find the pressure somewhere 25 some maximum value over here after number of iterations. Iteration n may be say twentieth iteration we are able to get this value as a roughly 25 bar or which is 25. 25 bar in this case (( )) is a Newton per meter scrap.

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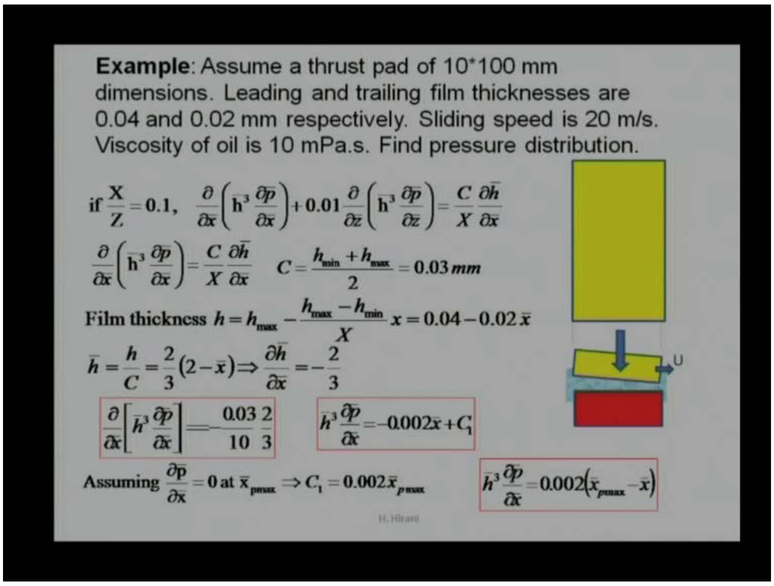

Now, we can solve this equation using some sort of integration technique that can reduce the number of iterations, that is why I am repeating this example this is again we are using the same dimensions 10 into 100 mm; that means, the dimension in x direction is a negligible compared to dimension in z direction. Relate to velocity is same and maximum film thickness and minimum film thickness have been kept same. What we need to find pressure distribution and if we are able to solve it analytically using a center of integration technique then we can really say iterative loop we can find conversions without going through conversions loop.

To do that again we can take a few steps, there will be some repetition of the steps. So, in this case if x by z is point 1 and if this term is negligible and we are getting only this term. Further we know the what will be the clearance that is turning out to be 30 micron and we can express h in terms of x, which was written in earlier case also. this h is in terms of x, but it is in mm. We need to non dimensional number and non dimensional number is turning out to be 2 by 3, 2 minus x bar.

While as a gradient is known to us now after differentiation because there is x term here and we can find that is a gradient is a negative 2 by 3. We can substitute this now, what we have we have terms in terms of sorry this expression in terms of x bar we can integrate twice and get the value.

The first integration gives 1 integration constant as nothing as a here it is a constant term it will turn out to be 0.002 into x bar plus c 1 that is integration constant and we need to determine it; however, we do not, if you dont know anything we should not well define while we can take approximation we know the pressure will be maximum somewhere. And pressure some location maximum, some location pressure will be maximum we can use this as a gradient based approach you say wherever the pressure will be maximum this gradient will be 0.

So, we need to find, estimate x will be having some constant value were the pressure will be maximum. So, I can say pressure will be maximum at a some location x bar p max using that condition assuming this gradient is equal to 0 at x bar p max which is a, we have given as a symbol or in the normal, we have given the nomenclature to the location of maximum pressure that gives value of c 1 in terms of this location.

But it helps us, we know this is the constant value it can be no further integrated without much problem right and when you substitute this value we get a this expression this need to be integrated once more to find out the pressure value because in this expression is giving us only the pressure gradient, it is not giving us a pressure value we require pressure distribution we do not require a pressure gradient.

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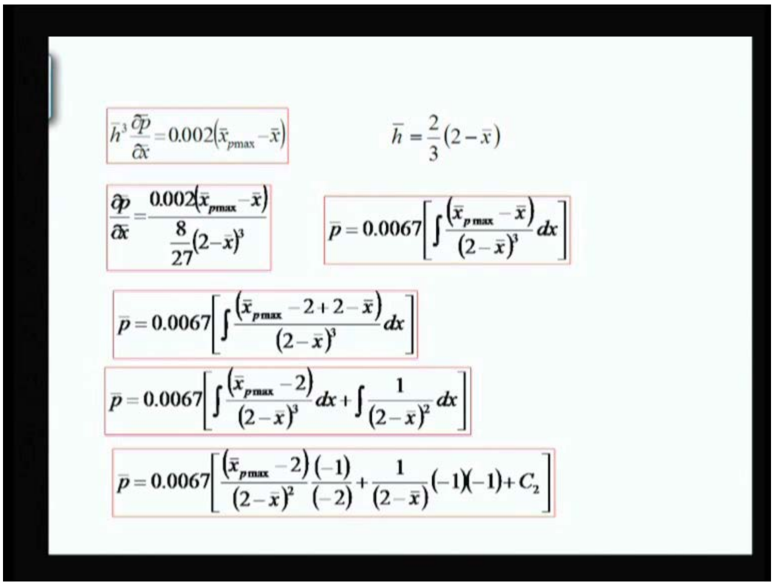

To do that and we require expression of h that is already we determined that is a 2 by 3, 2 minus x bar substitute this rearrange equation we know is h bar cube. So, cube of 2 two will turn out to be a 8, cube of 3 will turn out to be 27 and this will be 2 minus x bar cube. We can rearrange this equation something like this and then integrate it you say, x bar x bar max minus x bar integrate with the respective x and this is also depending term, this is a function of x bar.

So, this term has a function of x bar, this term has a function of x bar and remaining will be constant will be out. So, out of the integration bracket it is a turning out to be this integration in this constant in the 0.0067 and whatever we get from this expression that was going to decide what will be the pressure.

We can do some manipulation, we can say we, let us put subtraction minus 2 and addition 2; that means, minus 2 plus 2 will make it 0 we are not loosing anything, but; however, we are able to make 1 bracket it will turn out to be 2 minus x separate expression and this will turn out to be constant. So, we can get a 2 integration terms something like this is x bar p max minus 2 divided by 2 minus x bar in a whole bracket is a powered 3 while in this case 2 minus x is already there. So, that why 2 minus x will be cancelled out and remaining term will be 2 minus x bar to power 2.

Now, you can integrate easily this, I can integrate easily this and once we integrate we can use some sort of integration constant that can be resulted and that can be evaluated based on the boundary condition which we are in now.

So, if we do integration for the first term this is a same, this is a constant term x bar p max minus 2 while the integration of this will be a 2 minus x bar power 2, this is a minus 2 will be here and this is a negative x bar. So, it will be negative; so minus 2 and minus 1.

Coming to the integration of the second term this is 2 minus x, this will give power minus 1 and x is also negative. So, it will be minus 1 plus integration constant which we need to determine later. Now, I can rearrange this equation and we can say this is a overall equation terms of x bar or this equation to express pressure in terms of x bar.

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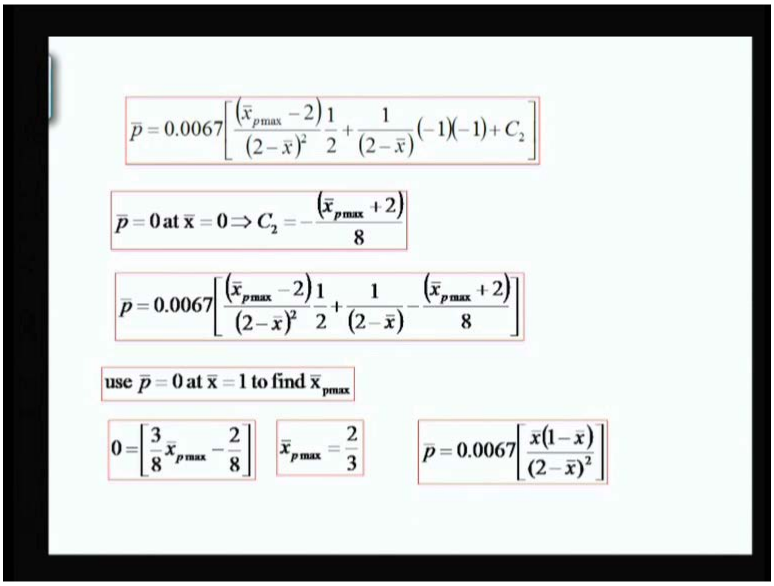

So, rearrangement can be done; however, we have 1 value x bar p max which is not known to us. So, what we can do we can say we can find out by some boundary condition similarly we have another constant c 2 which is also not known to us. So, that also can be determined by pressure boundary condition; that means, we require 2 pressure boundary conditions and we know at the entrance and the exit pressure will be 0. So, we have 2 boundary conditions, we require 2 boundary conditions. So, there is definite solution for that.

So, we write first condition is a P bar is 0 at x bar 0, that is going to give with a this x bar is 0, x bar is 0, substitute ratio is 0 rearrange the c 2 constant or constant c 2 will be in negative x p max that is a location of maximum pressure minus 1 divided by 2.

Now, we have, we can substitute this value in this expression to simplify the expression and, but we get our result something like this. Now we can use the second boundary condition that is a pressure 0 at x bar equal to 1 and that is going to give us location of maximum pressure when we substitute it, it turn out to be x bar P max is a 5 by 3 that is a non dimensional term and this is a value given over here.

Now, we can substitute, we get result something like this and rearrange or directly we can use to find out the results. If we try to use 11 values say the let us take a x bar as 0, x bar as point 1, x bar as a point 2, point 3, point 4, point 5 it up to 11 node the way we have done for the numerical analysis, finite difference analysis then we can compare the results.

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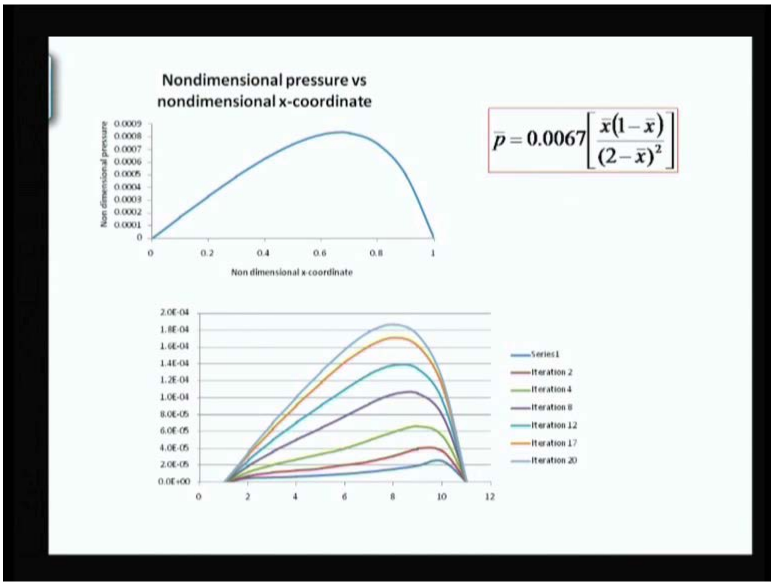

When we do that, we get a pressure profile something like this. This is a simple, you just substitute value get a pressure value, no iteration, in one iteration we are getting the solutions. So, this is maximum value is coming where here 0.7; that means, the node number 8, node number 11, node number 10, node number 9, node number 8 and this is a what we same value same similar expression or we say that same location we got a maximum pressure earlier and that is a non dimensional number and slightly more that point 0.00025 which we have already evaluated.

This is a series, is a iterations given we are comparing with the previous results that is a something like a maximum pressure again is a happening at somewhere here node number 8, this is also node number 8 only the difference in a value is happening because of the lesser number of iterations have been done in an finest difference method we require more iteration to reach to this (( )) that is possible. We can do more number of iteration to reach in this value; however, this does not require any iteration in 1 iteration or we say in the 1 calculation is set forward it gives the results.

So, efforts required to find out this curve is much lesser then efforts required to use this finite difference method and we have more reliable results here. It does not require any time to save as such we can reduce, we can increase a number of steps to the find out more smother curve compared to this curve.

So, in short this is a better method. If it is possible to integrate, we should the integrated we should get the results without going through the numerical calculations; however, there are some drawbacks also in this we say we know pressure profile only in x direction, we do not know pressure profile in a z direction.

We need to find out either we substitute some expression to find out the pressure profile in z direction; however, before coming to the that let us take another example just other dimension we say let us in this case we assume that dimension in x direction are too large compared to z direction.

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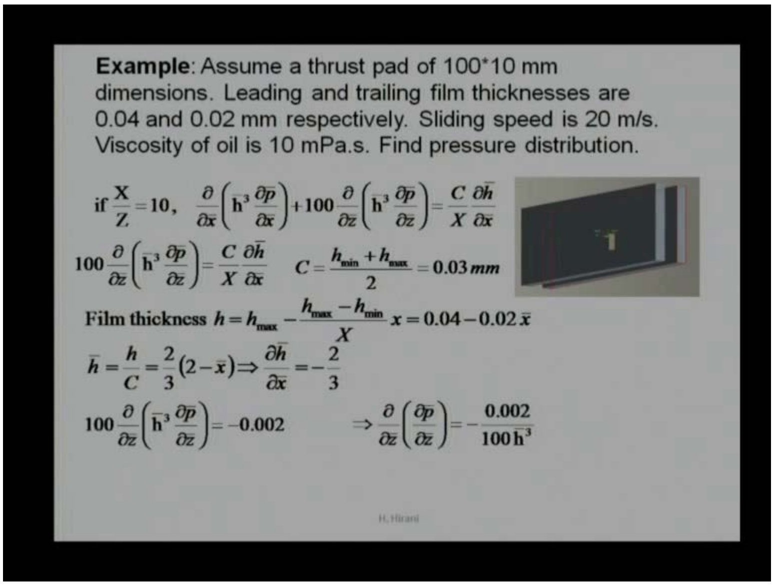

So, in this case we are just taking a separate thing we say let us take a thrust bearing pad or thrust pad is a 100 mm by 10 mm; that means, dimension in x direction in x direction is a 100 mm while dimension in a z direction is 10 mm. So, we have x by z is a 10 in this situation.

When x by z is 10 m m sorry 10 times then this second term is going to get dominated in earlier case this value was 0.1, while in the present case this value is 10. In earlier case we considered this term as a dominating term, we neglected second term while in present case we are going to consider the second term , we are going to neglect first term that is advisable to some extent because that is going to give good result to us.

Now, this term is neglected remaining two terms will be there left hand side pressure term, right hand side wedge term we know right hand side term can be easily calculated based on the theorems and finding the expression h that is given over here expression h in terms of h max and h min and variable x. When we do this and we can find out also gradient that is we have done this two times earlier. So, I am just once (( )) you saying that this is a minus 2 by 3 substitute this.

What is the advantage in this? h is depending on the x it does not depend on the z. So, integration in this case will be much simpler compared to the previous example, but the integration was slightly difficult, we really required integration it was not coming without constant; so, it coming with constant which we got some additional calculation.

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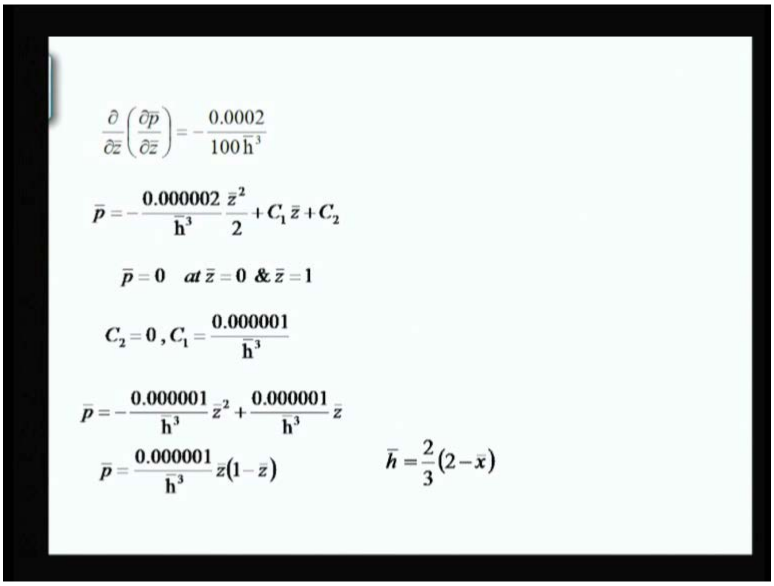

While in the present case integration is much faster you can simply take out h cube rearrange this equation in this term and this is a constant integrate 2 times after 2 times integration what we get here z bar square divided by 2 plus integration constant at the integration constant first we can multiply with the z bar and then integration constant 2. So, we have a 2 integration constants and we know we have a 2 pressure condition p 0 at z is equal to 0 p 0 at a z is equal to one.

We already have this pressure condition we can find out, we can substitute this values and get the results. So, results are turning out to be something like this when we write z bar 0 z bar 0 pressures also 0 and that is why c 2 will be 0 that is given over here.

Second condition when z bar is equal to 1 and 1 over here. So, this is simple expression and c 1 turnout to be point naught naught naught naught 1 divided by h bar cube. You substitute this value we get this expression and off course we know this is a parabolic plus some addition from z is also accounted over here, this is the pressure variation in terms of z variable. We assume that pressure variation in x direction that not be that much important to us even the maximum value of pressure in the both the cases will be same.

However in this case we are getting advantage we have to use only 1 variable that is the z bar and again the way in previous example we consider the values in terms of x bar take the values x bar equal to 0.0, 0.1, 0.2, 0.3, 0.4 and based on that we can do this calculation of the pressure. We can find out the pressure distribution same thing can be done in this case also.

Now, we have z bar values which is known h bar is already known and we can find out pressure distribution. important thing in the present example is that we have x bar value also h is a term is expressed in terms of x. So, we have added advantage compared to previous example, added advantage in terms of that we have x bar value as well as we have a z bar value.

So, it gives a true, it gives a true dimensional pressure distribution; may not be 100 percent correct, but it is going to give us dimensional variation in a 2 direction in x direction as well as z direction. While in earlier case we found only the 1 profile that was varying in only in x direction, we assume the pressure variation in z direction is not important and we not use any pressure boundary condition related to z in that situation.

So, if you use this and may be say we better arrangement we rearrange this equation we take out this constant and we take out the h bar value may be expressed in terms of z bar and 1 minus z bar, this also gives a conformation to us whether we have calculated pressure profile right way or wrong way. In this case when we taking using this expression this clearly says whenever the z bar is equal to 0 pressure will be 0.

Similarly, whenever z bar is equal to 1 pressure will be 0. So, this is conforming what we have done that is a proper, it is not a we are not made any mistake in a evaluating this expression.

Now, already film thickness is known that is in terms of x bar and I mentioned clearly that this is better pressure profile compared to previous 1 because this is also expressing pressure in terms of x bar, it is not only the z bar it is the also expressing in terms of z bar in terms of x bar. So, it is a giving a 2 dimensional profile if you remember previous slides and we expressed a pressure only in x bar term, there is no z bar at all in this. So, this is given in the only the one dimensional pressure variation is not giving the two dimensional pressure variation compared to that this approximation is a better approximation.

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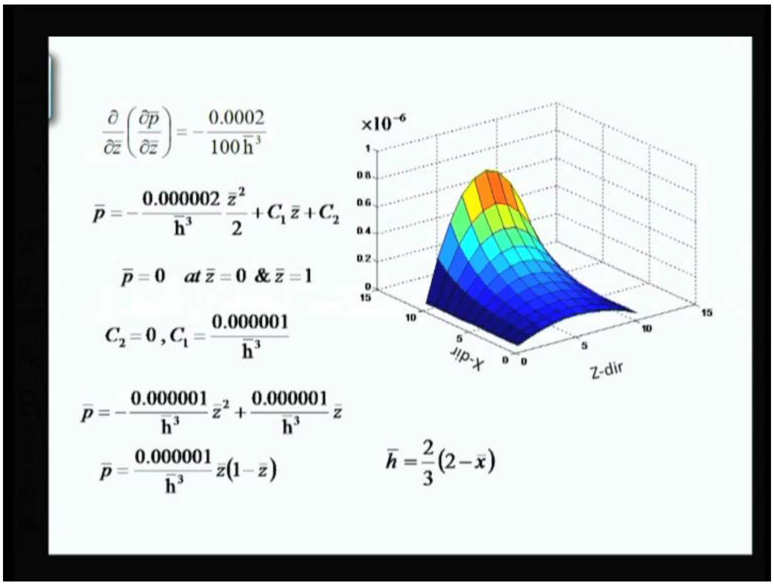

Only thing is that dimensional was a dimensions was to vary different in first example or previous example, we considered x dimension may be much lesser compared to the z dimension while in present example we are considering x dimension are much larger compared to z direction.

So, we get some pressure profile something like this, now this is a nice picture as such we can see z bar z direction and sorry x direction as well as a z direction we are consider the same number of steps, calculating at the when a value of x bar is 0.1, 0.2, 0.3 or we say number of nodes are 0.2211.

Similarly, in z direction also number of nodes are 11 we are point that and based on that we are evaluating the pressure profile. So, what is the difference in this even though it is a parabolic pressure profile at the every cross section, but there is a increase in a pressure in z direction also, it is a giving a good results 1 way bad results in other way.

We know pressure will be 0 whenever there is a value of x is equal to 0, the 0 value it will be 0 irrespective of value of z. When x bar is equal to 0 pressure should be equal to 0, it should not vary with z bar that is not indicated we can see there is a pressure profile over here which is a not right.

Similarly, pressure profile is not right in this case. So, that is a drawback, but it is a better than first approximation that is why we required some sought of hybridization we have in, we have getting some good results from one side and we are getting some good results from other approximation it will be always advisable to hybridize these 2 approaches and get a overall good solution from this solutions.

So, we will continue that kind of hybridization in our next lecture which will be basically hybridization of when we are neglecting one term and by getting pressure profile from that and neglecting second term and getting pressure profile from that and when we are getting two pressure profiles both have merits and when we combine in such a manner whatever the demerit these individual pressure profile have those should be eliminated one we another frame. So, we will discuss other kind of hybrid approach in our next lecture thanks for your attention. Thank you.