**Non-Linear Vibration**
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**Module - 1**
**Introduction**
**Lecture - 3**
**Commonly Observed Nonlinear Phenomena**

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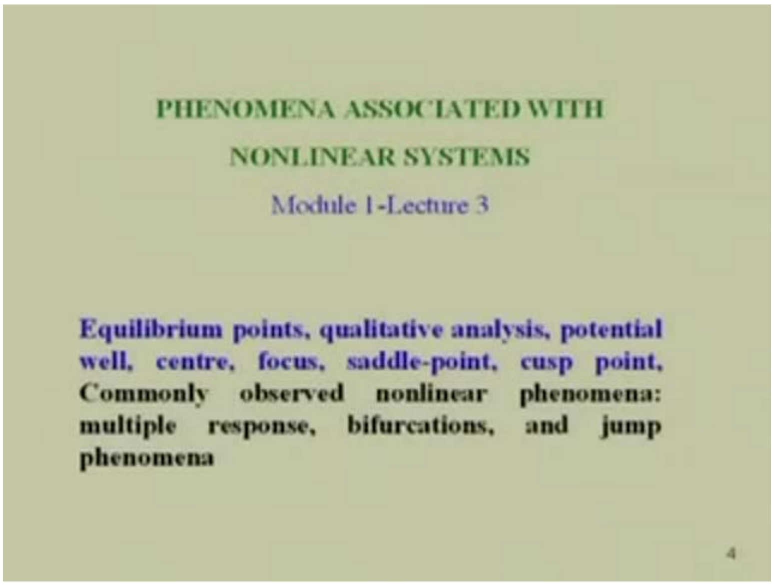


Welcome to the third class on this non-linear vibration. So, today also, we will continue with the introduction of the non-linear vibration and I will tell about the phenomena associated with this non-linear systems. So basically, I will tell about the equilibrium points. So, in these equilibrium points, I will tell about the center, focus, saddle-point, cusp point and commonly observed non-linear phenomena. So, here I will tell about this multiple resonance, bifurcation, jump phenomena and other phenomena associated with the non-linear systems.

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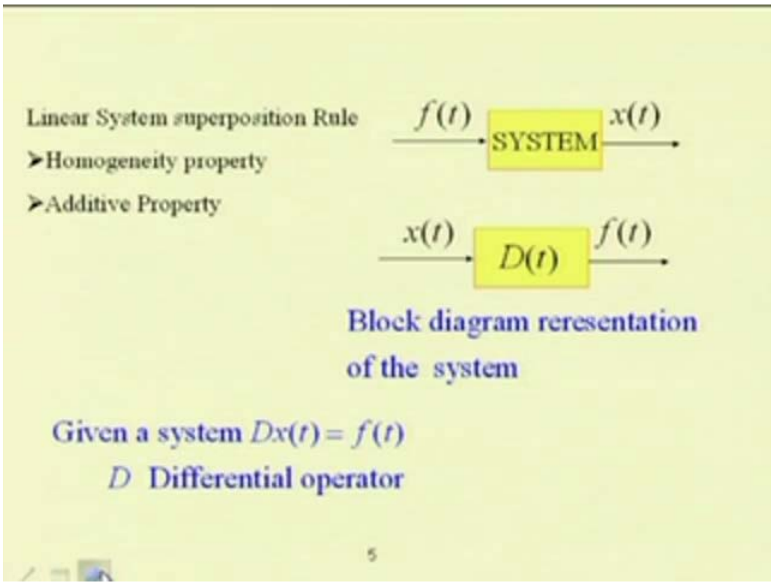


So, last class we have studied or we know about the linear difference between linear and non-linear systems. I told that a linear system can be identified by using the superposition rule. So, in superposition rule, it has to satisfy the homogeneity property and the additive property. So, if we have a system and given an input force f t, if its output is x t. So, in case of the additive rule if I will apply force of alpha f t, the response would be alpha x t.

Similarly, in case of additive rule so, if I am applying two inputs that is f 1 and f 2, the outputs would be x 1 and x 2. And incase of homogeneity property, if I am applying a force of alpha f t, the outputs would be alpha x t. So, a system can be represented by a differentials equation, and that thing can be represented by D x t equal to f t. So, D is the differential operator. So, using that thing, we can apply this homogeneity property and additive property to check whether the system is linear. So, if it is not satisfying these two properties or the superposition rule, then you can tell the system to be non-linear and we have seen one example also.

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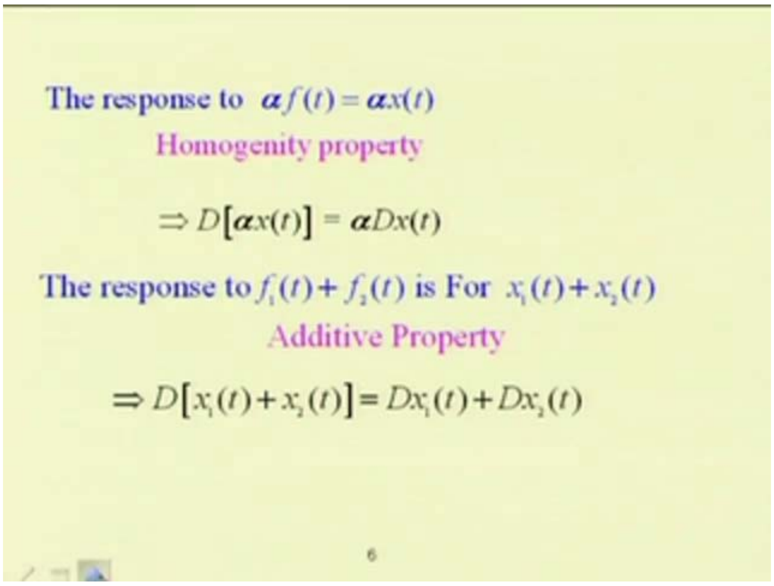


So, in case of homogeneity property, it has to satisfy this D of alpha x t should be equal to alpha D x t. And in case of additive property, when you are applying a force of f 1 t plus f 2 t, then its outputs would be x 1 t plus x 2 t; or in terms of the differential equation, I can write this d of x 1 t plus x 2 t should be equal to D of x 1 t plus D of x 2 t. So, separately the addition of these D of x 1 t plus D of x 2 t should be this operator, differential operator of x 1 plus x 2 t.

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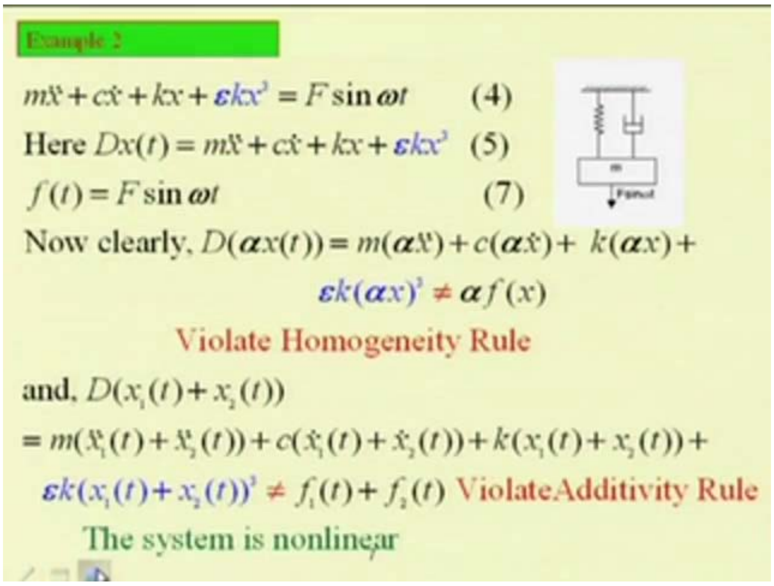

So, we have taken this example and we have studied these. For example, let us take the spring mass system. So, in this case of the spring mass system, if I will take a non-linear spring, then this equation motion can be written in this form that is m x double dot that is the inertia force, plus c x dot that is damping force and k is spring force.

And if these spring is non-linear, so I can add one extra term that is alpha that term is epsilon k x cube so, this will be equal to F sin omega t. So, here the D x t can be written as m x double dot plus e x dot plus k x plus epsilon k x cube and the forcing term f t equal to F sin omega t. So now clearly, if I will take this alpha x t, then it will be equal to m so, in place of x, I will substitute it by alpha x, so it will be m alpha x double dot plus c alpha x dot plus k into alpha x plus epsilon k alpha x cube.

So, this is not equal to this alpha of f x this term for presence of, due to presence of this term that is epsilon k alpha x cube so, this is not equal to alpha x. So, this is not satisfying the homogeneity rule. Similarly, one can check that by taking these D of x 1 t plus x 2 t so, this will be not equal to, this is not equal to f 1 t plus f 2 t as the term containing this epsilon term so, this violates these additive rule. So, in this way one can check due to the presence of this non-linear term so, this is not satisfying the homogeneity rule and additive rule.

So, this equation cannot be considered as a linear equation and this equation should be a non-linear equation. And this non-linear equation can be solved in different ways those things I will tell you later. And what is the equilibrium position? So, in this case of the spring mass damper system or you can see a different spring mass damper system also here so, in the spring mass damper system you just see so, this is a 2 degree of freedom system.

So, in this 2 degree of freedom system, this is the base so, the base is on my palm and these are the springs and this is so, this can be considered as the model of a 2 storied building or it can be different can be considered as a different part of a machine. So, I can take out one spring so, you can see one of the spring is here so, this is 1 spring so, this spring can be a linear spring or it can be a non-linear spring.

So, if in the forth displacement diagram, the force is proportional to this displacement then, in that case it will be linear otherwise, if it is not satisfying that thing then, it will be non-linear spring. So, in case of using this non-linear spring you can write the equation motion in this form but, for this 2 degree of freedom system you can write 2 independent equations or you can have coupled equations also. So, already we have studied about the 2 degrees of freedom system this is the equation for single degree and you have studied the equation for 2 degree of freedom system and you know the principle of vibration isolations also.

So, in case of, in case of principle of vibration isolation, let this be the primary system so, in case of the primary system to attenuate the vibration of this primary system you can add another spring and mass system to the system. So, you have to take the spring and mass in such way that the excitation frequency should be equal to the additional route over or the excitation frequency should be route over k 2 by m 2. So, if you add this secondary system.

So, let this is the primary system the bottom one is the primary system and you are adding this secondary system that is this spring and mass then, the system can be the vibration of the primary system can be isolated completely or can be observed completely. So, vibration observed completely when you are choosing the spring and mass in such way that the excitation frequency of the primary system equal to the root over of the equivalent stiffness of the spring and the mass.

So, root over k 2 by m 2 if it equal to omega then, this system can be used as a vibration observer or tune vibration observer. So, those things we have considered before. Similarly, you can have a continuous system so, this is a continuous system, this is a aluminum beam or aluminum panel so, it can be considered with different boundary conditions. So, you can fix one end so, in case you are fixing this end, this is cantilever beam so, if you are simply you can make it simply supported beam by supporting at this 2 ends also, you can make it fix by fixing at these two ends. So, this system can be considered as a continuation of a number of infinite numbers of spring and mass that is why this is distributed mass system.

So, in this distributed mass system you can find infinite number of natural frequencies but, for our applications we may consider only few natural frequencies depending on the situation. Similarly, you can have to attenuate the vibration of this simple continuous system like these this aluminum sheet. So, sometimes it is required to add additional elements dumping elements to this. So, here we have added a rubber pads in the aluminum.

So, these are 2 skin between two skins you we have added the mass this is a rubber mass this is made up of disco-elastic material. So, this is a sandwich beam where, the core is made up of rubber and the skins are made up of aluminum. Similarly, you can modify this rubber or this disco-elastic material. So, when putting this rubber like disco-elastic material, weight is more and to reduce this weight you can go for foam type of material. So, you can see this is very light weight so, you can use a foam type of material in between these 2 skins, but in all these cases when you are making this light you can observe that this cantilever beam or this beam. So, if I will keep a tap here it goes and it continues to vibrate.

So, when you are making it lighter and lighter or we are making it flexible that time, the vibration of the system increases. So, to reduce those vibrations, one should do proper analysis. The analysis for these can be made in a number of ways for example; one can consider this as a simple spring mass system. Considering a point load here, one can find its stiffness, the stiffness can be or stiffness will be equal to can be calculated by finding its deflection. So, this deflection for a small load w, the deflection will be equal to w l cube y 3 e I so, knowing this deflection one can find its stiffness.

So, after finding the stiffness one can find its natural frequency. In that way, you can find its fundamental frequency. But when the vibration is more, then you can consider this as an Euler Bernoulli beam and in this Euler Bernoulli beam you can find a number of natural frequencies for the system. So, when you are considering this as a non-linear system when the deflection is more. So, let the deflection be more so, this curvature this curvature is more. So, in that case, you cannot consider this deflection to be less. So, in that case your Euler Bernoulli beam can be replaced by the Euler Bernoulli beam equation with some additional non-linear terms. So, those things we can consider in due course of time.

So, in all these cases, for example, in the spring mass system, when I will put this x dot or the time derivative of the displacement to be 0, I can find this k x for this linear system consider this linear system m x double dot plus c x dot plus k x equal to 0 for free vibration. So, if I will put x dot and x double dot equal to 0 then, the resulting value becomes k x equal to 0 so, x equal to 0. So, x equal to 0 represents so, x equal to 0 represent the equilibrium position.

So, in this equilibrium position or for this equilibrium position one can find whether this equilibrium position is stable or whether this equilibrium position is unstable or whether equilibrium position can be attain attainable or not. So, we can study about these equilibrium positions by finding the response of the system. The response of the system can be considered by studying this as a free vibration when this f sin omega theta equal to 0 or it can be considered as a force system when this we are considering this force.

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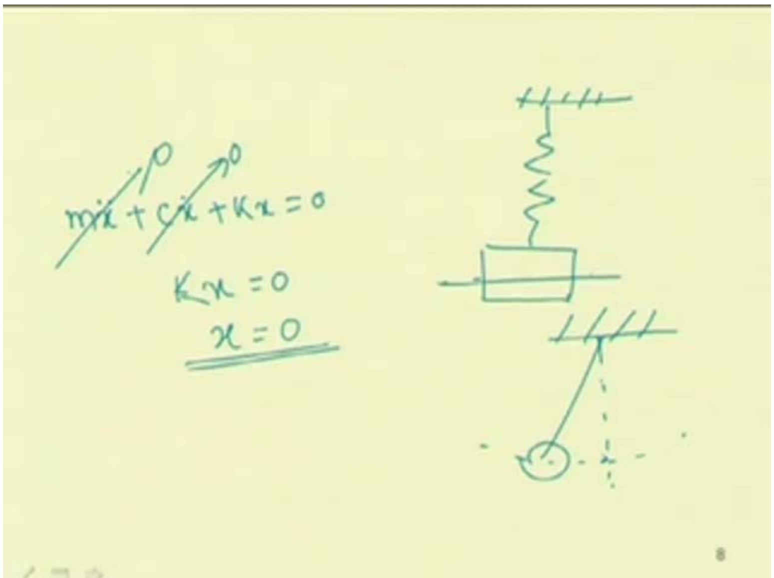


So, in this spring and mass system in the spring and mass system I can have these are the equilibrium position by keeping this m x then, the equation becomes m x double dot plus c x dot plus k x equal to 0. So, for steady state which is independent of time, I can put this x dot equal to 0, this x double dot equal to 0. So, the resulting thing that is equal to k x equal to 0 so, from this as k is not equal to 0, I can put x equal to 0. So, x equal to 0 is the equilibrium position. Similarly, for the pendulum case, it can oscillate about the mean position so, that will be the equilibrium position. So, we have to study the equilibrium position of different systems. So, in case of spring mass damper system with non-linear spring, we have written the equation in this form m x double dot plus k x plus c x dot plus alpha x cube equal to 0.

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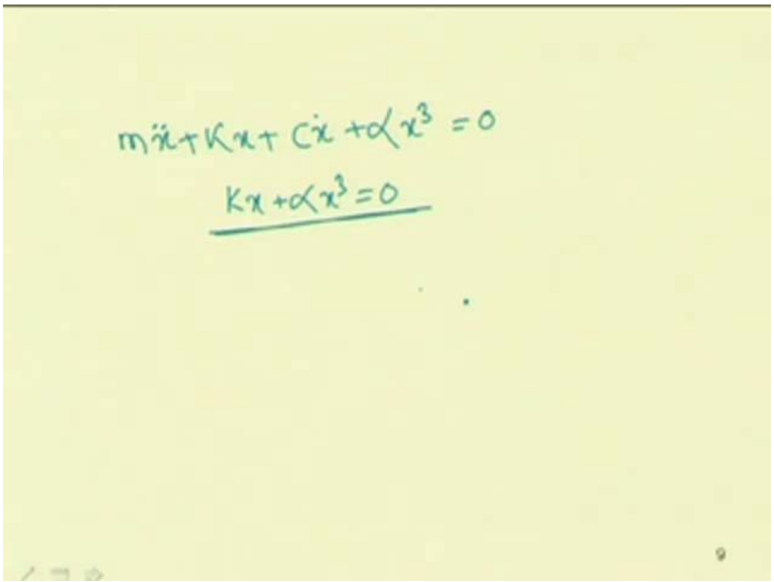


So, in this case for steady state I can have this k x plus alpha x cube equal to 0 and by solving this equation I can find the equilibrium position. So, one can make the qualitative analysis to study about the equilibrium position. How above the equilibrium position with time of the response behaves, that thing can be studied by using this qualitative analysis or quantitative analysis. So later, we will study about this qualitative and quantitative analysis.

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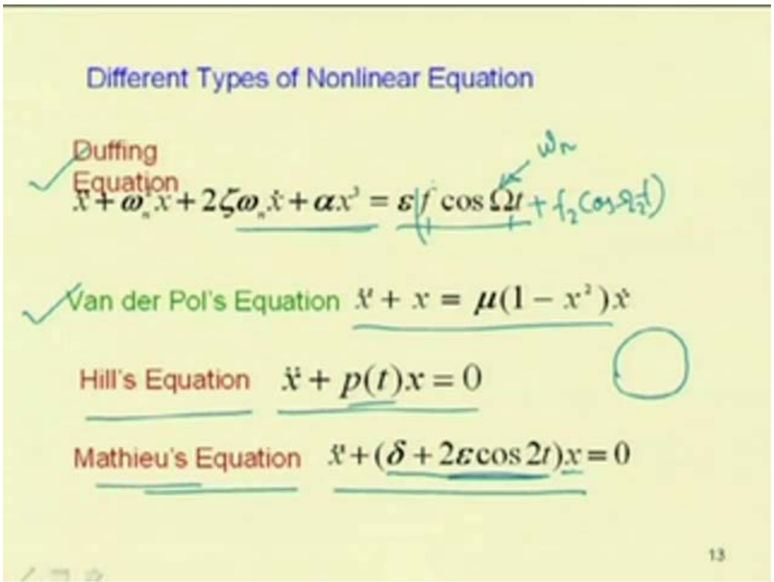


And now let us see some of differential equations used in these non-linear systems. So, first one what I have shown you just now, that is the spring mass damper system.

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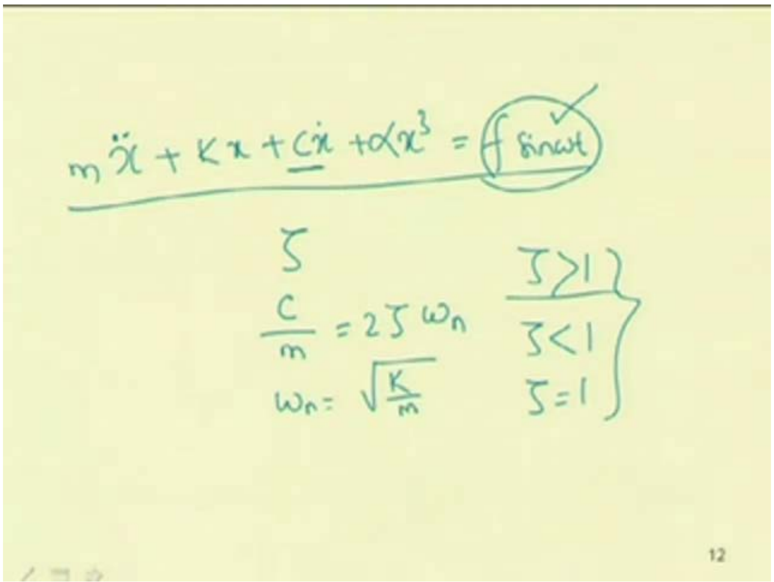


So, the equation can be written in this form x double dot plus k x plus c x dot plus alpha x cube equal to f sin omega t. So, this equation is known as duffing equation. So, in this duffing equation, I can take this force equal to 0. So, that will be the duffing equation for a damped free vibration system. So, if I will consider the forcing function, then this will be the duffing equation for a harmonically forced un-damped system. So, I am considering these as un-damped system depending on the value of c previously, we have studied. So, when this for what value of c this will be un-damped, over damped or critically damped. So, one can find the value of zeta. So, that thing one can find. So, one know this c by m equal to 2 zeta omega n from this expression and omega n equal to root over k by m. So, in this case one can take this m.

So, k by m equal to omega n and the c by m equal to 2 zeta omega n. And from that one can find what the value of zeta is. If zeta greater than 1 then, the system is over-damped, zeta less than 1 this is under damped and zeta equal to 1 the system is critically damped. So, depending on the value of zeta the system can be under damped, over damped or critically damped and accordingly one can study the behavior of this duffing oscillator. So, most of the spring mass system or most of the equipments subjected to a force vibration can be considered in the form of this duffing equation. So, another form of equation, one can find in this general system is the van der pol equation also. So, in this case one can have this x double dot plus x equal to mu x dot minus x square.

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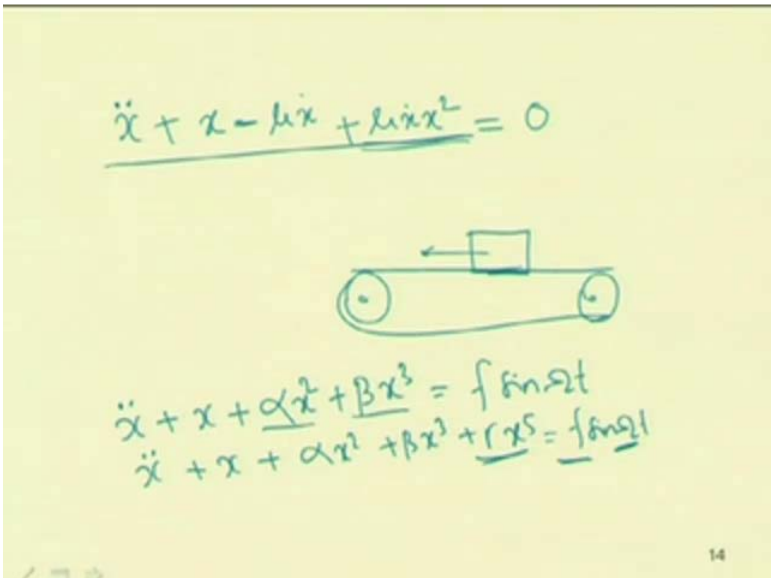


So, this equation if I will write this is x double dot plus x double dot plus x, so x double dot plus x let me take the other part this side. So it will be mu x dot and mu x dot you can see this is mu x dot minus mu x dot plus mu x dot plus mu x dot into x square. So, this is the non-linear term associated with the system. So, this mu, so here you can put a negative damping, so due to the presence of negative damping this is a self excited system. So, in the self excited system due to the generation of energy, due to this damping force, the system will behave in a particular way. So, one can get periodic response in the self excited type of motion.

So, for example, it can be this type of system that can be obtained by considering a belt so, by considering a conveyer belt system in which it is carrying the blocks or carrying some material so, when this is moving on this it can be the force, can be modeled as a duffing type of equation. Also one can add the duffing or this non-linear term with this van der pol's equation and one can get this van der pol duffing type of equation. Similarly, in this case of duffing equation here, you have seen a cubic term and one may take a quintic term or quadratic term also along with those terms. So, in that case in the addition of a quadratic term, the duffing equation will be reduced to this form, x double dot plus x plus alpha x square plus beta x cube equal to F sin omega t. So, here you have a quadratic term and a cubic term similarly, you can have a quintic term also. So, in that case you have x double dot plus x plus alpha x square plus beta x cube plus gamma x to the power 5 equal to f sin omega t. So, 'f' is the amplitude of the forcing and omega is the frequency of the forcing term and these are the quintic non-linear term.

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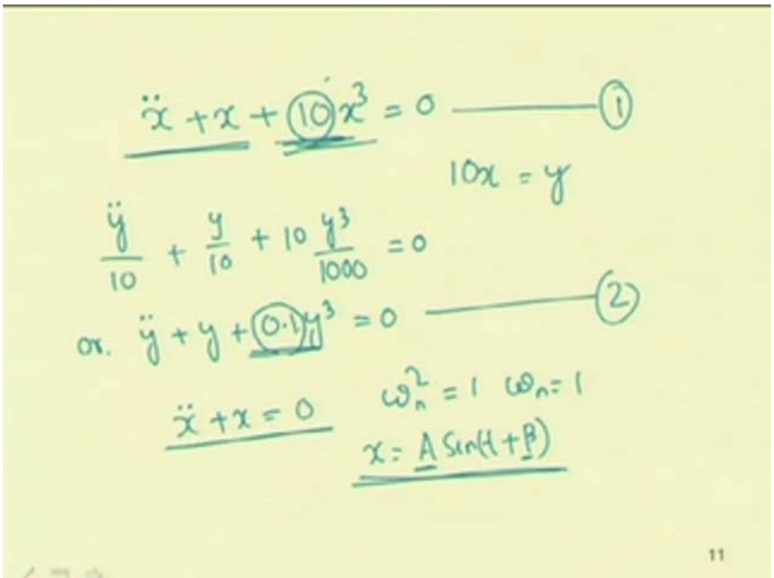


So, for example let me take a term in which or let me take it in this way, x double dot plus x plus 10 x cube equal to 0. So, now, I can modify this equation by taking another parameter. I am taking one parameter, this let 10 x equal to y so if I will taking 10 x equal to y in that case, this equation can be replaced by or I can replace this x by y by 10 or if I will put y by 10 here, then these equation will be y double dot by 10 plus y by 10 plus. So, here I will take in place of x, I will put so, this will be equal to y cube by 1000 equal to 0 or I can write this equation in this form, y double dot plus y plus. So, let me mod multiply 10 everywhere. So, if I will multiply 10 then, this will becomes 0.1 y cube equal to 0.

So, both the forms are identical. So, here the equation is x double dot plus x plus 10 x cube equal to 0. But in the second case, I have written a equation which is y double dot plus y plus 0.1 y cube equal to 0. So, this 10, the coefficient 10 I have replaced it by 0.1. Now, analysis of both the systems if one considers then, in this case the coefficient is 10 and in this case the coefficient is 0.1. So, the coefficient 10 is very-very large than the coefficient of x. So, if I will take the non-linear part only.

So, linear part is x double dot plus x and its solution equal to. So, one can have the solution for these x double dot plus x equal to 0. So, here the frequency equal to so, omega n square equal to 1 so, omega n equal to 1. So, the solution x equal to, I can write the solution x equal to a sin as omega n is 1. So, I can write this is a sin t plus beta where, beta and a can be obtained from the initial conditions. So, initial conditions are t equal to 0 if I know the displacement and the velocity then, I can find what is x. So, this is the linear part. So, these term, this 10 x cube, as this 10 is very-very higher larger than the coefficient of x this cannot be considered as a small increment of this x or this non-linearity cannot be considered as a small non-linear term added to the system. But, in the second case by taking this 10 x equal to y I have a term that is 0.1 y cube.

So, this y plus point as this 0.1 is very-very less than the 10 time less than the coefficient of y so, in this case it can be considered as a weak non-linear system but, in this case in the first case it can be considered as a strong non-linear system. So, I can convert a strong non-linear system to a weak non-linear system by suitable taking a scaling parameter, by taking a scaling parameter that is 10 x equal to y so, I have converted this strong non-linear system to weakly non-linear system. So, one can easily consider this weak non-linear system to study or to find its response by using different perturbation method.

So, today class also I will tell you about what are the different methods available to solve these equations and to find their response and what are the different equilibrium conditions available and their possible bifurcations. So, we have considered this duffing equation and the van der pol equation and also in case of duffing equation, you have seen or you can have cubic non-linearity, quadratic non-linearity or quintic non-linearity and you can have a harmonic force or the force can be changed to different other different type of forcing instead, of a single force you can have a multiple number of forces with different frequencies. For example, if you can add this f 1 epsilon f 1 omega 1 t plus f 2 cos omega 2 t so, this will be two frequency excitation.

Similarly, you can have multiple frequency excitations in this case of duffing equation or you can have a series of harmonics in terms of cos and sin applied to the system. In actual case, the response to be may or may not be periodic but, you can convert to that of a periodic response and you can convert a periodic response to that of a harmonic by using the Fourier series.

So, for a general system you can convert the actual forcing to that of a periodic and again from periodic forcing to that of the harmonic and you can have a number of harmonic forces acting on the system. For a liner system one can easily find its solution but, in case of the non-linear system as it is not satisfying the superposition rule so, for each case you have to find the solution and you have to find the response of the system.

So, in case of a linear system when the equation is x double dot plus omega n square x plus 2 zeta omega n x dot equal to f 1 cos omega 1 t. So, you can find its response to be equal to x sin or x cos omega t, the excitation frequency becomes equal to the excitation frequency omega 1, when it becomes equal to the natural frequency of the system or nearly equal to the natural frequency of the system the system has resonance. But in case of this non-linear equation non-linear equation, this may or may not happen.

So, the resonance may happen at a frequency lower than the natural frequency or it may take place at a frequency higher than the natural frequencies. And in case of this linear system, this frequency omega is independent of the amplitude depends on the frequency but, frequency does not depend on the amplitude frequency, amplitude frequency is an independent parameter. So, at resonance, this omega for a linear un-damped system is slightly less than that of the natural frequency of the system. But, in case of the non-linear system the resonance frequency is a function of both the applied frequency and the response amplitude. So, it depends on the excitation amplitude, excitation frequency and also it depends on the response amplitude of the system. So, in case of non-linear system also superposition rule is not applicable. So, for each of the forcing term one can find the response and then, one can study the response of the system in a different manner than it is considered in case of the linear system.

And in case of this van der pol equation which is observed in case of a damped non-linear system, one can find a limit cycle or one can find a response periodic response suddenly appearing in the system. So, suddenly a periodic orbit or periodic response will generate from the self excited system. Already I told you one can add the non-linear terms to this van der pol equation and one can convert this equation to a van der pol duffing type or many other equations can be generated by adding this forcing term, non-linear term to these equations. So, another set of equations one can obtain in this non-linear system is of this type that is, x double dot plus p t x equal to 0 where, p t is a periodic function. This equation is known as hill's equation, which he has developed while calculating or while studying the motion around the lunar motion system.

So, if one can replace this p t, this periodic function by this delta plus 2 epsilon cos 2 t then, this equation is known as Mathieu equation. So, this Mathieu equation is a special case of hill's equation. So, from this equation, one can tell this x double dot plus delta plus 2 epsilon cos 2 t x equal to 0 as Mathieu equation. In this case you can observe that the coefficient of x that is coefficient of the response is a periodic term that is 2 epsilon cos 2 t here also the coefficient of response is a periodic function. So, as the coefficient of response is a periodic function or this as a parameter of the response that is why these equations are known as parametrically excited system. So, in case of parametrically system the periodic function is represented as the coefficient of the response. So, you can study different type of parametrically excited system.

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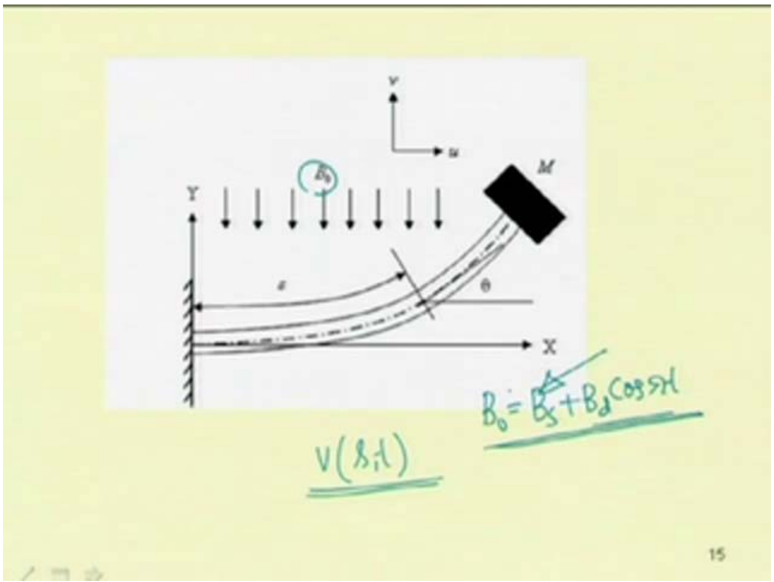


So, for example, in this case, let us see let us take different systems. So, let us take a cantilever beam subjected to magnetic field. So, if it is subjected to magnetic field so, due to this magnetic field if you have a conductive skin then, a force will be generated. Force and movement will be generated due to the edicurrent set up in the skin. Already, you know that in a magnetic field if we have a moving in a magnetic field, if we have a moving conductor then, a current will be set up in the conductor. And also we know in a magnetic field if we have a current carrying conductor, the conductor will be subjected to some force. So, due to these effects the beam is subjected to a force and axial force and movement. So, in this case, in this case of this beam, if this displacement is large then, one can model this as a system like this. So, one can model this as a system like this.

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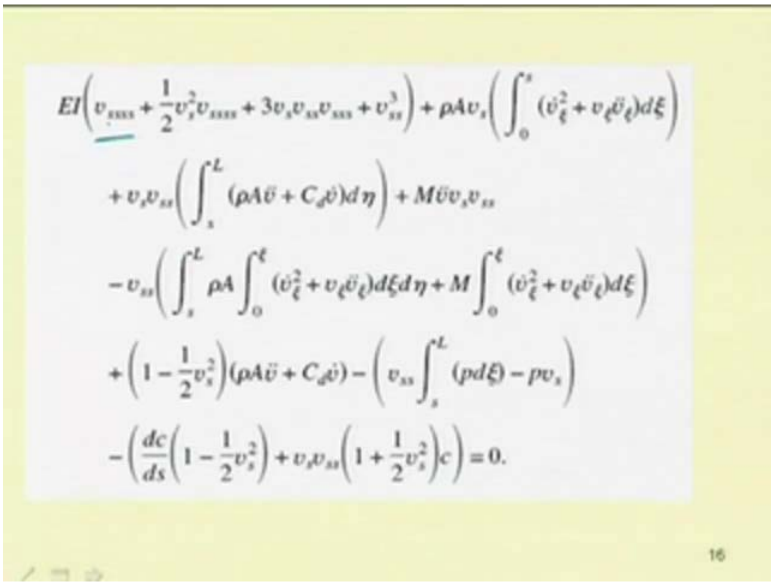


So, here if one takes only this as Euler Bernoulli beam, one can have only this term that is EI del forth v by del s forth plus. So, you have one more term that is rho del square v by del t square. So, we can have only two terms in case of a linear system. But, in case of this non-linear system, when you are considering large displacement along with this force due to this magnetic field the equation can be written in this form. So, detail of this thing will study later but one can write this equation. So as this is a continuous system one can write this or this displacement transpose. Displacement v can be represented as a function of both space and time. So, this is a function of both space and time. So, one can write this v as a function of space and time and later one can reduce it to that of a time or temporal form.

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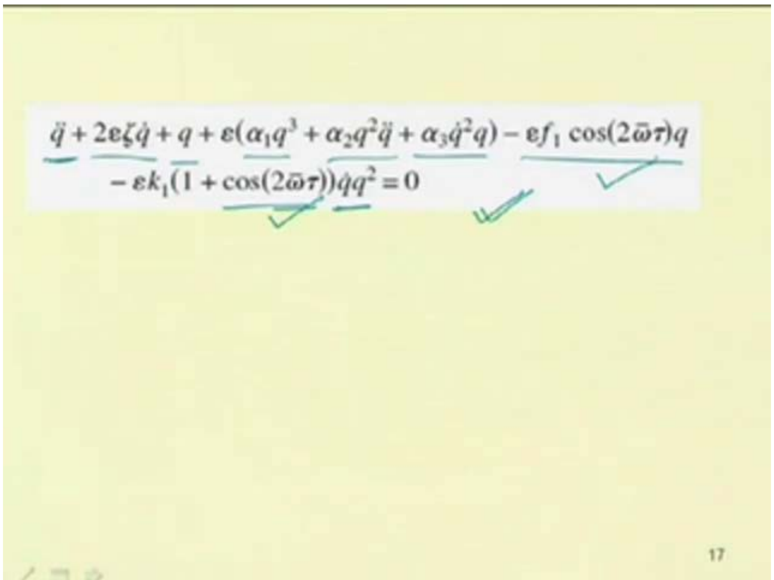


So, this is the equation in temporal form. So, this is the term representing the inertia force. So, this is the damping force here. So, this is the force due to stiffness and we have different non-linear terms also. So, this non-linear term this is due to this geometric non-linearity that is, when the beam is bending due to its geometry it will resist this motion. So, in that case it will be. So, in this case in this case, this alpha x cube term will come also when it is moving due to its inertia the non-linear terms also coming to picture. So, these 2 terms are inertia non-linear term and this geometric non-linear term and due to the application of the magnetic field we can have these additional terms.

And one may note that this magnetic field one can apply a magnetic field this, one can apply a constant magnetic field or one can apply a magnetic field in this for be 0 can be of a constant magnetic field B s plus B d cos omega t. So, one can have a variable magnetic term or magnetic field also. So, one can have a constant magnetic field or one can have a time varying magnetic field.

So, depending on that one can have, one can have different type of different type of equation. So, in this case for this cantilever beams subjected to magnetic field one can have this type of governing equation. So, in this case you can observe that the coefficient of q is a periodic term. So, this is a parametrically excited system along with cubic, geometric and inertia non-linear terms. Similarly, you can have another non-linear term also so, this q dot q square this is a non-linear term, this is a, this q as a coefficient cos 2 omega bar t that is a periodic term. So, you have a linear parametric term and a non-linear parametric term also in this equation.

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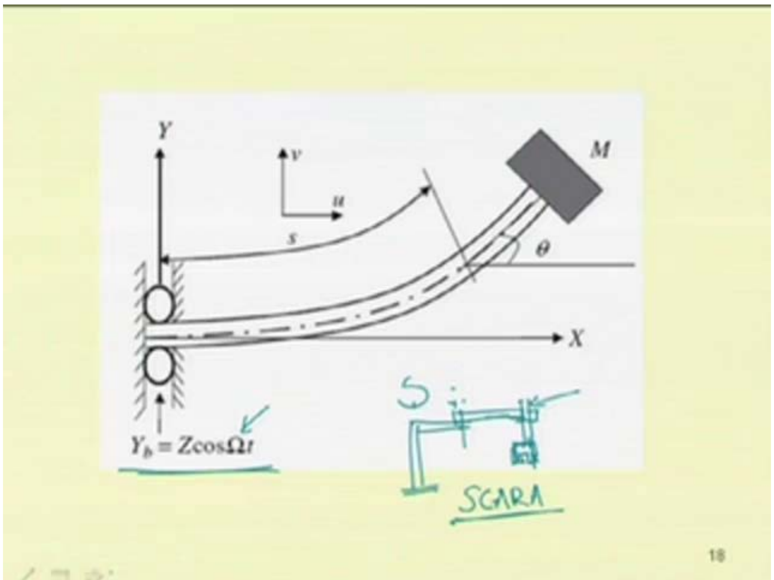


Similarly, you can take a base excited, base excited cantilever beam, which can be modeled as a Cartesian manipulator. So, in case of a transverse motion of a robotic arm for example, in case of the SCARA robot in case of the SCARA robot it can be represented like this, this is a, in brief I can draw this SCARA robot. So, in this case this is a joint prismatic link it can move up and down so, the base can be so, this is rotating. So, you have very valued joint here, another revalued joint here and you have a prismatic joint and finally, the end effecter has a rolling motion.

So, this is a SCARA robot. So, in this case you have a joint with prismatic motion. So, you can have a if, you consider a prismatic motion in this cantilever beam then, you have one additional base excited term that is y b equal to z cos omega t, in which the z the frequency of this oscillation and omega is the frequency of this oscillation and z is the amplitude of this response. So, instead of taking a small response, if one considers a moderately large response in that case the equation can be written in this form.

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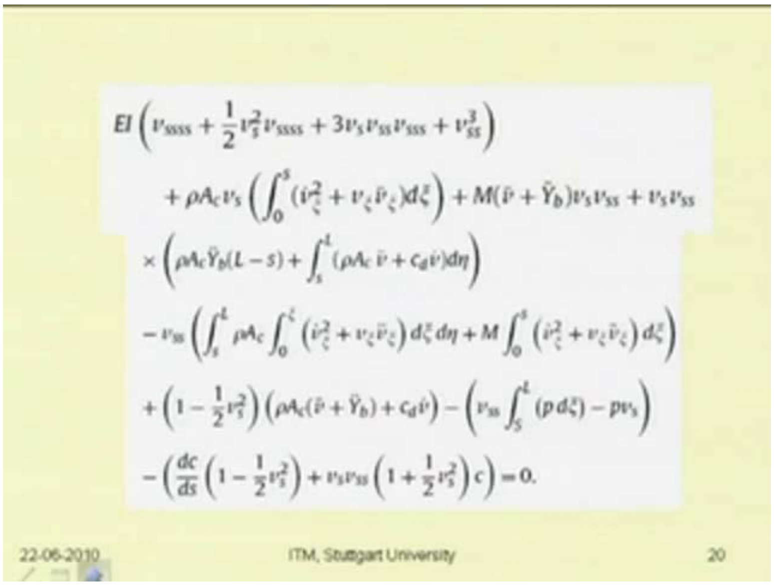


So, one can write the equation in this form. So, what we have seen in the previous case, in that equation we have to add that of the base excitation and in temporal form you can write this equation here.

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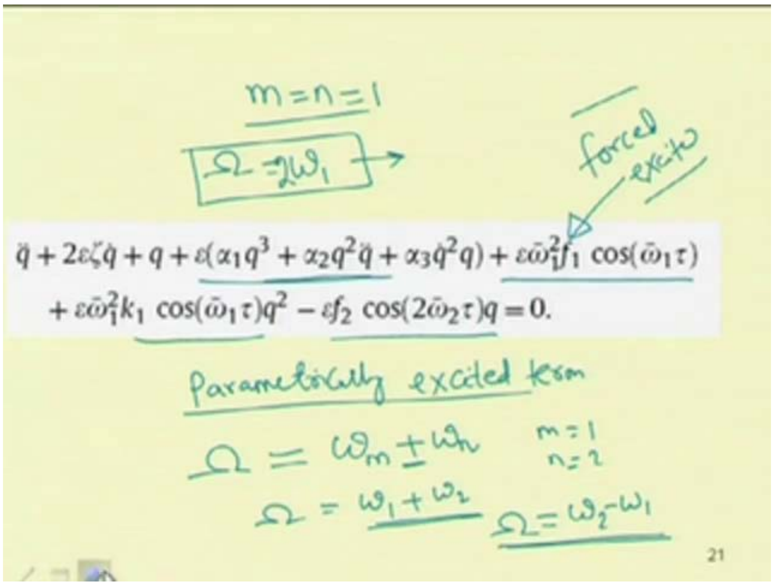


So, similarly one can apply a magnetic field and go on adding different terms in the equation. So, in this case you can observe that it contain so, this is a force term that is epsilon omega 1 square f 1 cos omega t tau and these 2 terms, this term and these 2 terms.

These two terms are parametric parametrically excited these two are parametrically excited term and this one is force excited term. So, already I told that incase of the force vibration when this omega equal to the external frequency equal to the natural frequency of the system resonance occur. But in case of parametrically excited system, one can find the resonance, when this excitation freq external frequency equal to plus or minus of this omega m plus or minus omega n, where omega m and omega n are the natural frequency of the systems. So, already you know for a continuous system you have a large number or infinite number of natural frequency.

So, in this case if you two take any two natural frequencies. For example, let m equal to 1 and n equal to 2. So, if you take omega equal to omega 1, this omega equal to omega 1 plus omega 2. So, this is known as combination resonance of first and second type. Similarly you can take the difference, you can write this omega equal to omega 2 minus omega 1.

So, this is combination resonance of some type or you may have this when this m equal to n. So, when m equal to n, you can have omega equal to omega. Let for m equal to n equal to 1 so, this becomes 2. So, omega equal to 2 omega 1. So, this is known as principle parametric resonance. So, the resonance condition will appear here, when m equal to n, that will be known as principle parametric when m not equal to n. So, then it will be combination type. So, in case of combination type we may have combination resonance of some type or combination of combination resonance of different type depending on whether you are adding or subtracting different frequencies. So, in case of this parametrically excited system now you know that resonance may occur at a frequency when the natural frequency m, when the excitation frequency may or may not be equal to that of the natural frequency.

But in case of force vibration, in case of force vibration when excitation frequency equal to the natural frequency the system oscillate in the resonant mode. But in case of parametrically excited system, you can have two different types; one is principle parametric and other one is combination parametric. But when you are adding the non-linear terms again, the resonance may occur at a frequency which may be away from the natural frequencies also. So, you may have sub resonant or sub harmonic super harmonic resonance condition.

So, now, you know 4 different types of resonance conditions. So, that is sub harmonic resonance condition, super harmonic resonance condition, principle parametric resonance condition and combination parametric resonance. Conditions in addition to the resonance, which occurred at a frequency equal to the natural frequency that is known as simple resonance condition. So, 5 different type of resonance condition you may observe incase of non-linear system. But, in case of linear system you can observer only the resonance when the excitation frequency equal to the natural frequency of the system. So, already I have shown you different type of systems.

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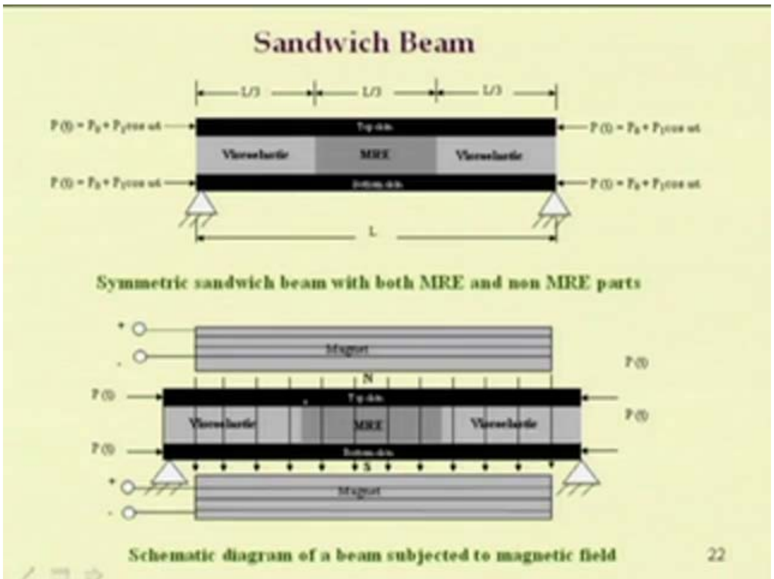


So, this is a sandwich beam. So, in case of the sandwich beam, you have a core and you have the skin and this core you can modify by taking different patches. So, you can add pejualact layer also, you may add magnetoriologicall Elastromer core also. So, in case of this magnetoriologicall elastromer core or can sort MRE for adding then, you can actively or passively you can control the vibration of the system.

So, in case of a MRE embedded beam so, this is one MRE embedded beam. So, here magnetic field is applied to the system and if you are applying axial load in addition to this then, the equation motion of the system can be that of a parametrically excited system. So, you can study the non-linear vibration of a sandwich beam under free vibration then, its behavior may be that of a duffing equation. So, you can consider this as a parametrically excited system when you can apply force at the end or you can have a base excitation then, you can consider this as a non-linear parametrically excited system. Also in case of this non-linear system you may consider single frequency or multi frequency excitation. This is one experimental set up in which by putting accelero-meter, we are finding the natural frequency of the system. So, this is a cantilever beam the, cantilever sandwich beam so, here accelerator is placed and it is fixed at this end so, when it is subjected to excitation then, one can find its response.

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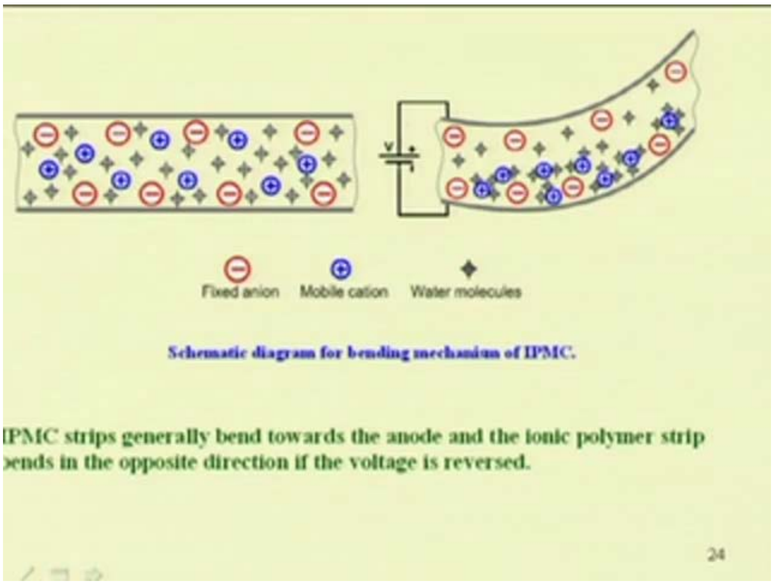


So, one can take also other different type system. So, in this case, we have taken one IPMC that is ionic polymer metal composite in which you have a fixed base Nepean base, you are having the ionic polymer inside. So by applying a voltage, you can actively change its response or you can have if you require the bending of this by applying this voltage, you can have this bending. So, in all these cases, you can have a non-linear system and these equation motions of the systems can be written in or as a non-linear differential equation motion.

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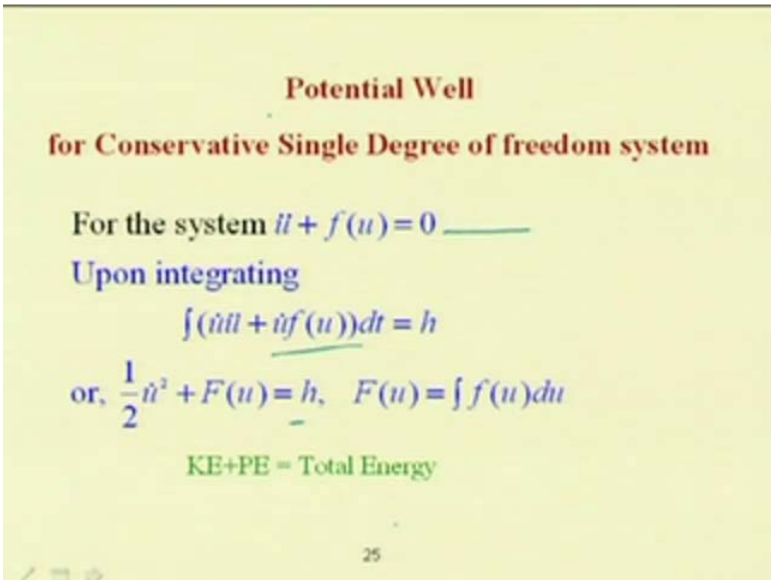


So, we can study the non-linear system by using is put by using qualitative method or quantitative method. So, let us study this thing by using a qualitative method. So, in this qualitative method let us consider a conservative system. So, in this conservative system I can write the equation motion of the system to be u double dot plus f u equal to 0. So, if integrate this equation so, this will become. So, let multiply u dot to this equation, u dot and u double dot plus u dot f u d t equal to so, integration of this 0 so, this will becomes a constant.

So, by integrating this u dot u double dot you can find this is equal to half u dot square plus. So, I can write integration of this term equal to capital F u and this is equal to h. So, you can write this term as the kinetic energy, F u as the potential energy and h is the total energy of the system. So, kinetic energy plus potential energy equal to total energy of the system. So, this differential equation can be so, this differential equation can be written in terms of the energy like kinetic energy plus potential energy equal to total energy of the system.

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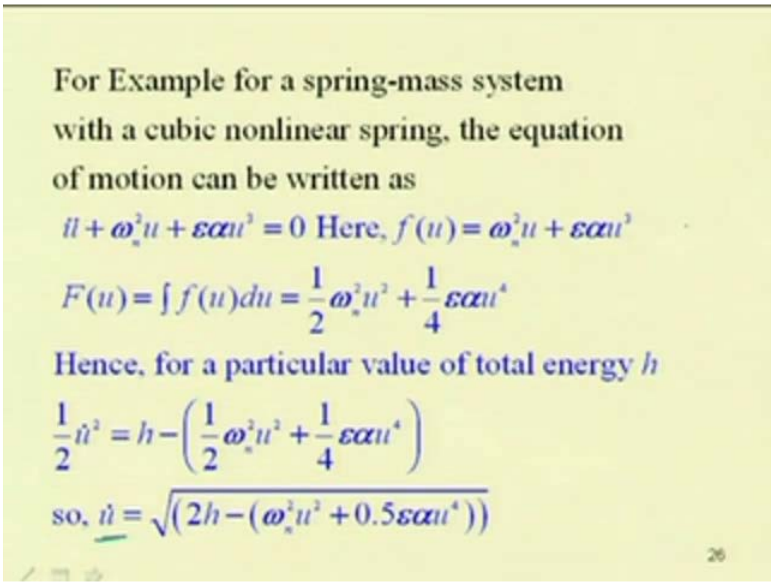


So, in this case for example, let us take the spring mass system. So, in this case I can write this u double dot plus omega n square u plus alpha epsilon alpha u cube equal to 0. So, here f u will be equal to omega n square u plus epsilon alpha u cube then, by integrating this thing I can find this f u capital F u that is the potential function. So, the potential function becomes half omega n square u square plus 1 by 4 epsilon alpha u forth. So, for a particular value of total energy h, I can write this is equal to half u dot square.

So, half u dot square will be equal to h minus half omega n square u square plus 1 by 4 epsilon alpha u forth. So, I can have this u dot u dot that is the velocity so, this velocity will be equal to root over 2 h, this 2 can multi multiplied here so, this is equal to 2 h minus omega n square u square plus 0.5 epsilon alpha u cube. So, for a given potential for a given potential function or for a given total energy, I can plot this u dot versus u that is the phase portrait that is known as the phase portrait.

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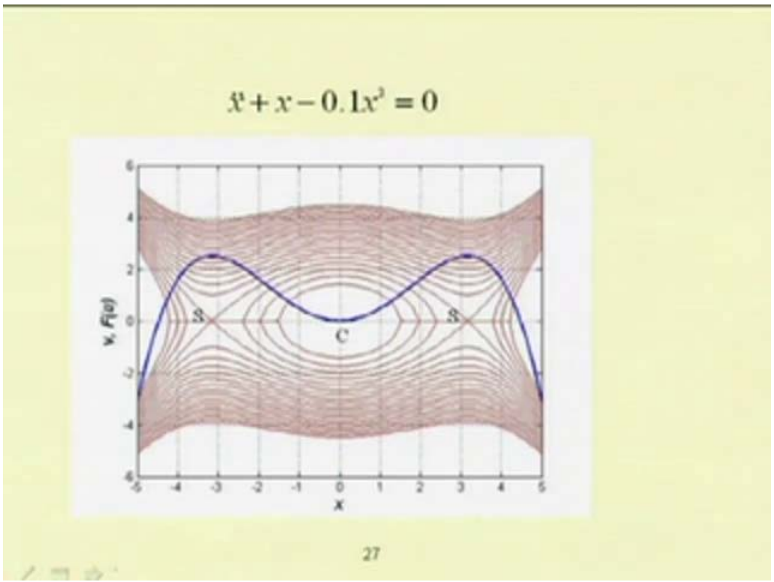


So, these are the phase portrait rotate for different value of h taking different value of h so, these are the phase portrait plotted. So, in this case, this is the example taken x double dot plus x minus 0.1 x cube equal to 0 where, alpha equal to minus 0.1. So, by taking that thing you can find this b. So, this is the potential function. So, you can observe that by taking a point higher than this potential function, let come back to this equation so, in this case if this part or so, this is a h minus f capital F. So, if the potential function is greater than the total energy of the system then, the system will not move the system will not move if the potential energy or potential function is higher than the total function. So, the system will oscillate only when this total energy h is greater than this. So, if h is greater than So, if h is greater than this function there will be oscillation or motion in the system or it will have a velocity otherwise, it will have no velocity.

So, in this case the system will be in equilibrium or it will have an equilibrium position when the potential energy is minimum. So, corresponding to this minimum potential energy that is x equal to 0, this is the equilibrium position when the system has a periodic response but, corresponding to the maximum corresponding. So, you just take this point and this point corresponding to this maximum potential energy you can see the system will have a saddle node point. So, this point is known as center point and these 2 points corresponding to the maximum potential energy when the system will not move. So, will have or the system will have an unstable response. So, that is known as saddle node points. So, these 2 points are saddle node points and this point is center point. So, in this case we have found three equilibrium positions. So, from these 3 equilibrium position, one position is stable and other two positions are unstable.

So, those equilibrium positions you can find by substituting this x dot equal to 0 and finding the value of x. So, you can find the value of x and corresponding x dot and you can plot the flow or the potential well so, this is known as the potential well. And corresponding to the minimum potential you will have a periodic response and corresponding to maximum you will have unstable response which corresponds to saddle point. So, either you can find this equation, this potential well by using these equations or you can find that thing by solving this differential equation by different methods numerical methods. For example, you can view this range-kutta method range-kutta method to find the response of the system. So, the second order differential equation can be written in the form of two first order differential equation and can be solved to find x dot and x. So, and one can plot that response.

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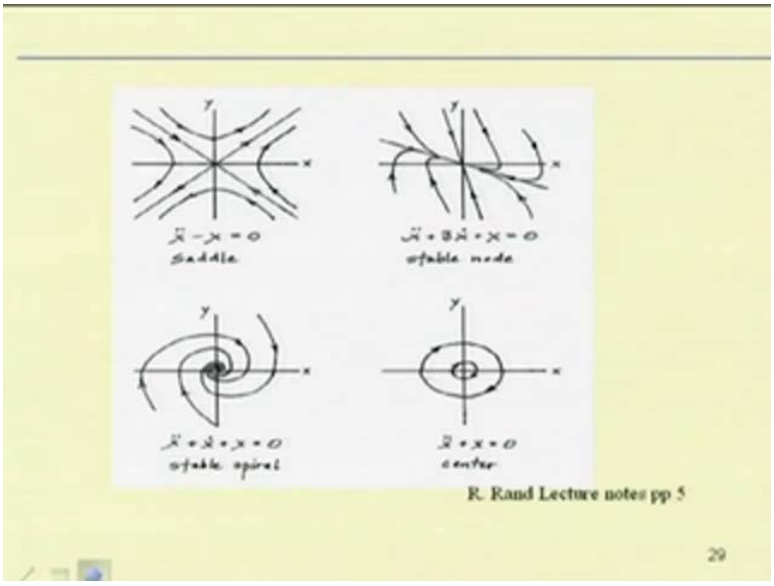


Similarly, you can find a saddle point here by considering this x double dot minus x equal to 0. So, near the saddle point you just observe that the flow will separate out that is why these are known as the separate tricks. So, these lines asymptote lines are known as the separate tricks, separate the flow around this equilibrium position. So, this is one saddle point. Similarly, you can have a stable node for this equation; you just consider this equation x double dot plus the three x dot plus x equal to 0. So, this equation is similar to that of the spring mass damper system with a damping. So, this damping factor equal to 3 in this case.

So, the response is all the response will come to this equilibrium position. So, this is a stable node position. Similarly, you can have a stable spiral so if the equation is in this form x double dot plus x dot plus x equal to 0. So, this is a stable spiral and you can have a center. So, x double dot plus x equal to 0 so, in this case the response will be. So, the response will be x sin omega t plus pi so, you can have a stable periodic orbit or you can have a center. So, one can study the bifurcation of all these equilibrium positions you can study the bifurcation of this equilibrium position in the next class.

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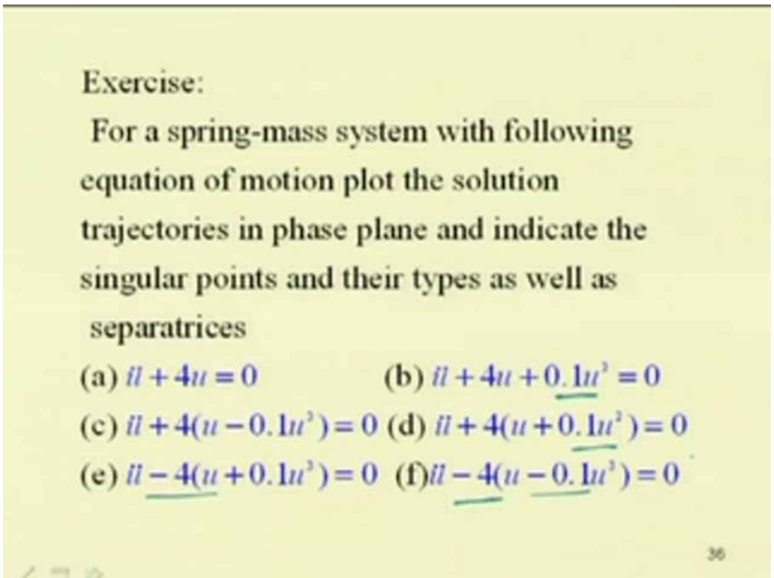


You can take these exercise problems what we had studied in this class, you can plot this phase plane and you can plot the trajectories in phase plane and indicate the singular points and their types as well as separatrices. For example, you just take this equation u double dot plus 4 u equal to 0. So, in this case this is the equation of a single degree of freedom un-damped system you can get a periodic response with a period equal to 2. You can find the trajectory.

So, this trajectory is a center type. Similarly, you can find the trajectory for this case, u double dot plus 4 u plus 0.1 u cube. So, in this case, you can observe the separatrices, you can observe saddle node separatrices and center and in this case u double dot plus 4

u. So, you just take this minus 0.1 u cube, this minus term will come when you have a sub-spring. So, in this case also you just find the potential well and also obtain the response and in this case this is a hard type of spring in which the alpha is different and we have taken a quadratic spring. So, in the previous case you have a cubic spring here you have a quadratic spring also you find the response. So, in this case, you just see I have taken this is equal to minus 4 u.

So, in case of minus 4 u, this is a system, this is unstable system because the stiffness is negative or the stiffness is taken to this negative. So, this we can find an unstable system. So, you just study its response by plotting the trajectory using these potential well methods. Similarly, here also you have a soft spring this minus, minus plus so, you have a hard spring with unstable. So, the linear part is unstable. So, this spring stiffness is minus 4 here. So, you will have a negative stiffness. So, in all these cases you just obtain the trajectory by using the methods two methods I told; one method you can find its equation u dot in terms of u and in second case you can solve this equation, this equations using Range-Kutta method to find their responses.

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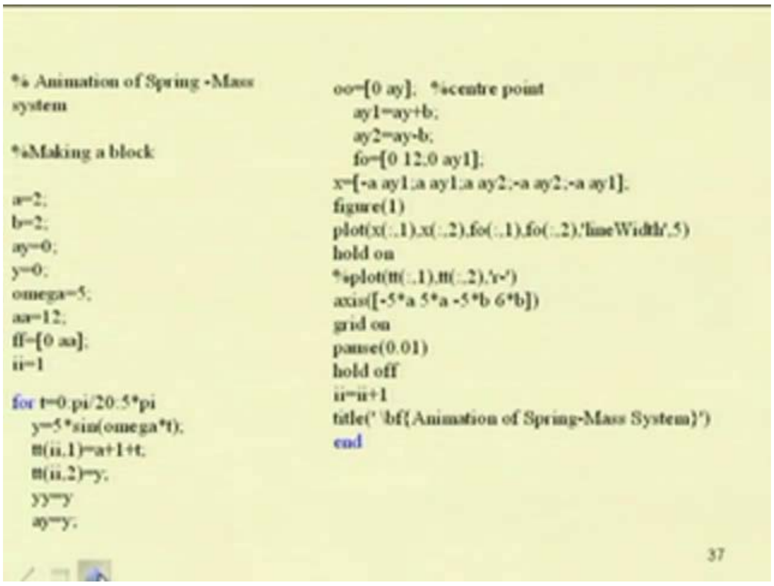


So, here one Matlab, small Matlab program is given to find the response of the system. So, this is for animation of a spring mass system, and also you can find or you can write a equation to find the potential well. We will study, in the next class, will study about different bifurcation, and how to determine the governing equation motion for different type of systems.