Non-Linear Vibration

Prof. S. K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 2
Derivation of Nonlinear Equation of Motion
Lecture - 3
Some Equation of Motion for Some Other System

So, in the last two classes, we have studied about how to derive the equation motion of non-linear systems.

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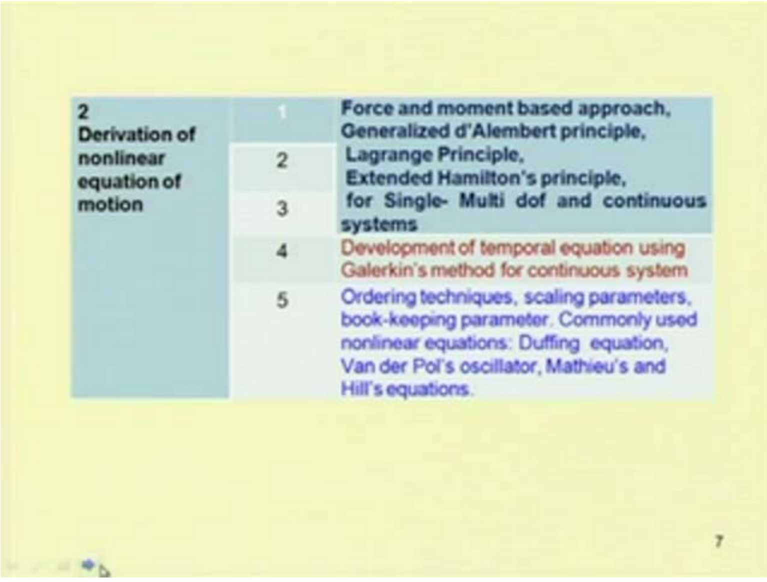

So, in this case we have studied mainly two different types of methods; one is inertia or force base method and second one is energy based methods. So, in force or momentum base method we have used Newton's second law or d'Alembert principle and in case of energy base principle, we are using Lagrange principle or extended Hamilton principle. So, last class we have derived some equation of motion for different discrete systems using this Lagrange and Hamilton principle. So, today class also will continue with the same and we will derive some equation of motion for some other systems. So, will study or will derive the equation motion for a simple pendulum, a double pendulum, also some spring mass system and one continuous system today.

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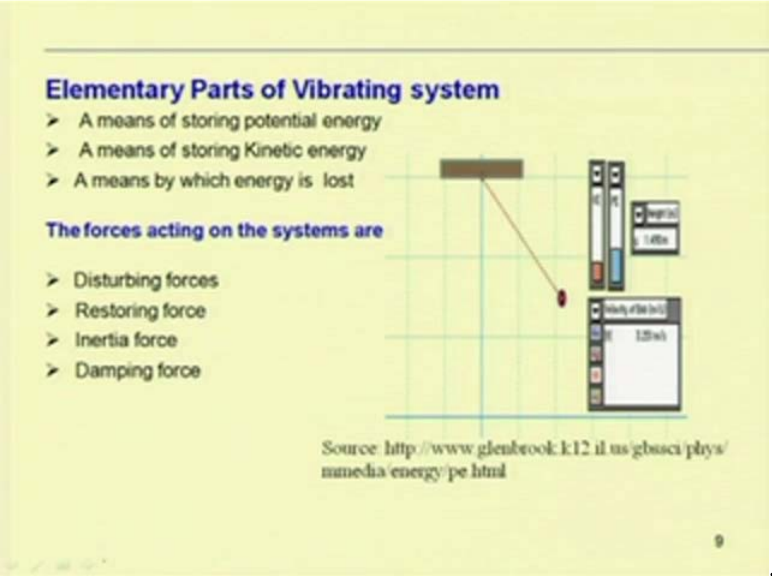

So, in the last class we have seen for these vibrating systems, this is the simple pendulum in which this kinetic energy is converted to potential energy and vice versa. So, the total energy of the system remains constant. So, by using this energy based principle also, we have to derive this equation motion.

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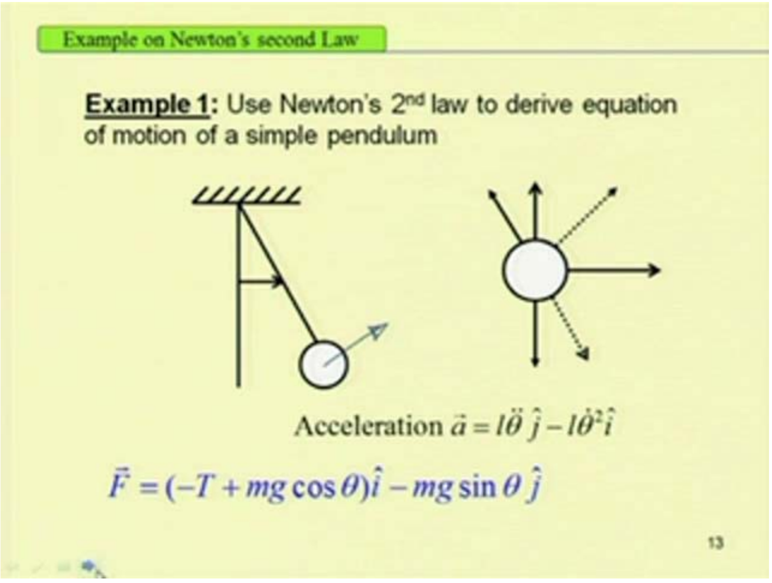

So, initially we have derived the equation motion of the simple pendulum by using this Newton second law where, we have found the force and we have obtained the acceleration of this bob. And taking this acceleration and equating this or finding this inertia force. External force equal to inertia force and we have found the equation motion. But, by using this so, today class will derive the equation motion of the simple pendulum by using this energy based principle. So, this is the simple pendulum.

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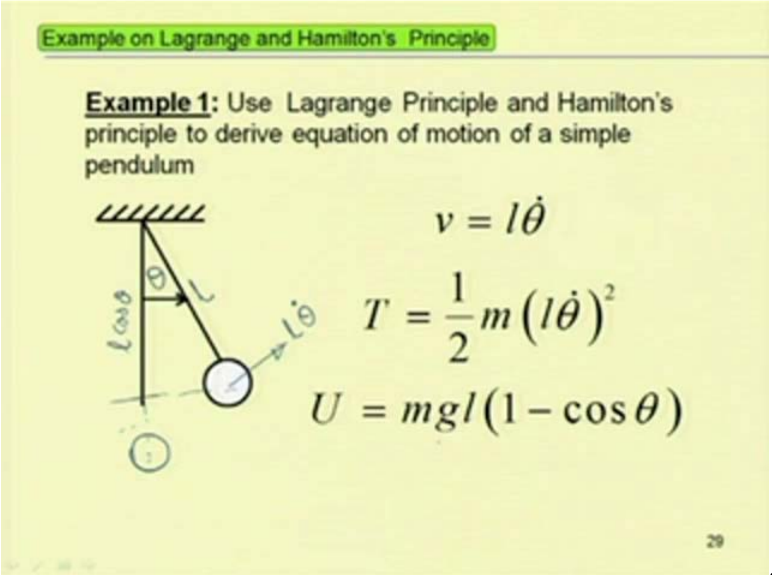

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So, the velocity of this pendulum so, this mass so, velocity so, if it is moving in this direction data time t equal t equal to 0 it is at this vertical position and at t equal to t it has rotated by an amount theta. So, it has come to this position and at this position if the length of the pendulum if l then, its velocity will be equal to theta dot. So, the velocity of the pendulum can be or velocity of this bob of this pendulum can be written as l theta dot. So, one can find the kinetic energy and potential energy. So, kinetic energy equal to half mass into velocity square so, velocity equal to l theta dot so, it will be l theta dot square and potential energy.

So, taking these as the reference so, potential energy can be found so, potential energy it is change in height into mass and into g. so, change in height becomes so, if this length is l so originally the bob was here so, now it has come to this position so, this is the change in height. So, the change in height can be this to this so, this distance become l minus l sin theta so, l minus l cos theta so, this is l cos theta this l sin theta so, this portion is l cos theta so, l minus l cos theta is change in position. So, due to this change in position this potential energy and this potential energy equal to m into g into l into 1 minus cos theta.

Now, the Lagrange of the system L can be written as T minus U so, already you have this equal to half m l square theta dot square and U become the potential energy equal to

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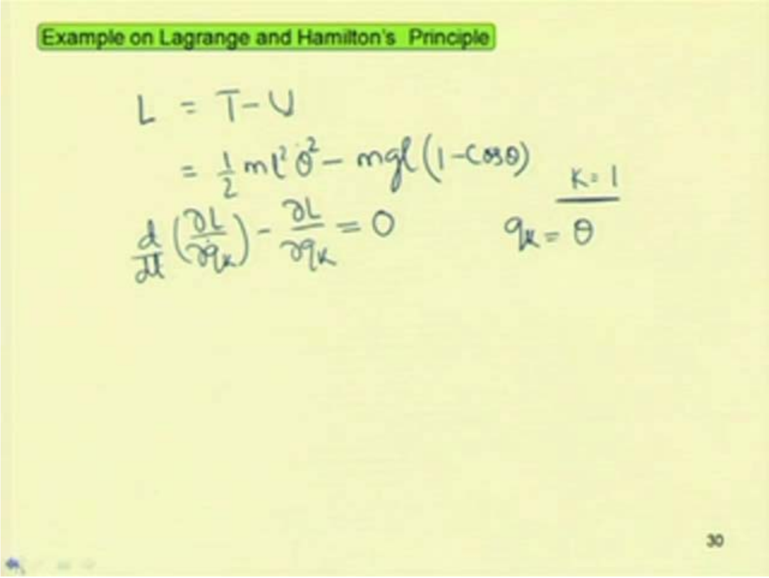

So, already you have found the potential energy this is m g l into 1 minus cos theta. Now, has to use this Lagrange principle and then Hamilton principle to derive this equation of motion. So, using Lagrange principle our equation motion can be obtained in this way so, d by d t of del L by del q k dot minus del L by del q k so, as no external force is acting on the system so, this will be equal to 0 and in this case q k that is the generalize coordinates so, k equal to 1.

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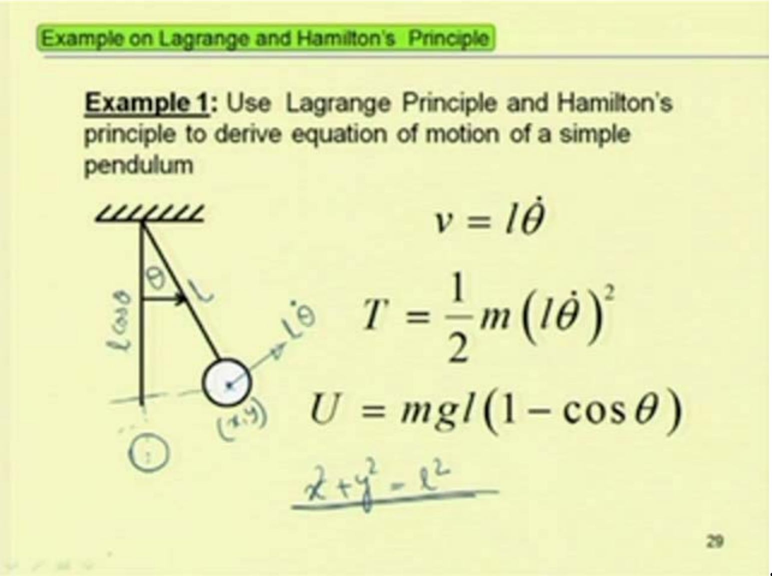

So, this is single degree of freedom system. So, k equal to 1 and q k is nothing but theta. So, one can take different coordinate systems also one can take x and y coordinate system but, if one takes x and y coordinate system to locate this position of this pendulum that time the equation can be retained in terms of x or y as x and y are related by this constant equation. That is, if this coordinate is x y then one can write x square plus y square equal to l square.

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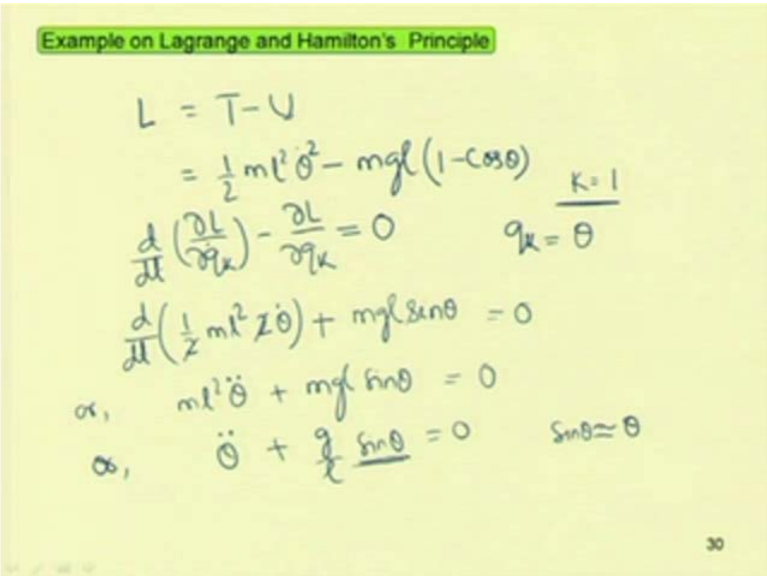

So, either one can use the generalized coordinates x or y or theta to determine this equation motion. So, in this case we are using theta as the generalize coordinate to determine the equation of motion. So, here q k is theta so, I can derive this equation in this way so, d by d t of del l by del q k dot that means d l by del theta dot so, it will be half m l square into 2 into theta dot so 2 theta dot. So, and if you differentiate this thing with respect to theta dot then it will be 0 so, this part becomes this then minus del l by del theta will give us so, differentiation of this part with respect to theta equal to 0 and differentiation of this part now, it becomes minus minus plus.

So, this becomes m g l and differentiators on of this minus cos theta equal to sin theta so, this is equal to 0 so or one can write this 2, 2 cancel. So, m l square theta double dot plus m g l sin theta equal to 0 or this theta double dot plus g by l sin theta equal to 0. So, we know the sin theta can be so for small angle theta it can be retained as sin theta will be equal to theta approximately equal to theta.

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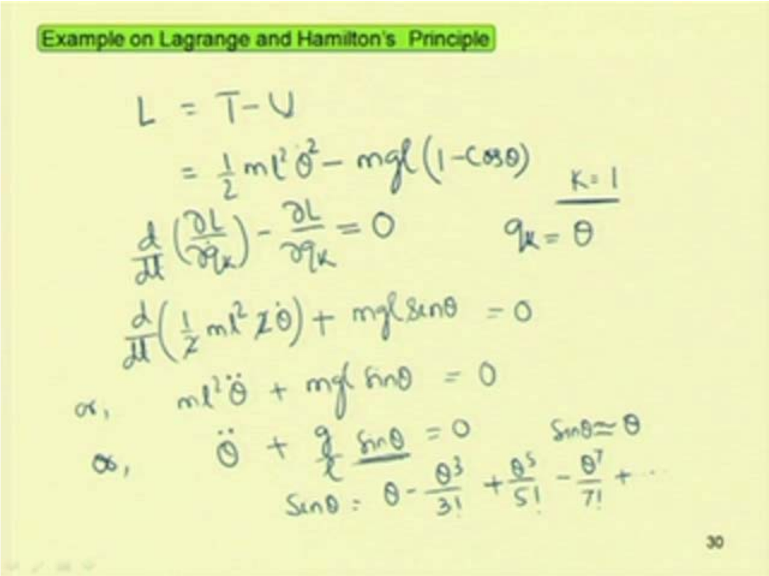

But, if you take large amplitude of oscillation then, we may write the sin theta equal to theta minus theta cube by factorial 3 plus theta 5 by factorial 5 minus theta to the power 7 by factorial 7 and one can go on adding the terms.

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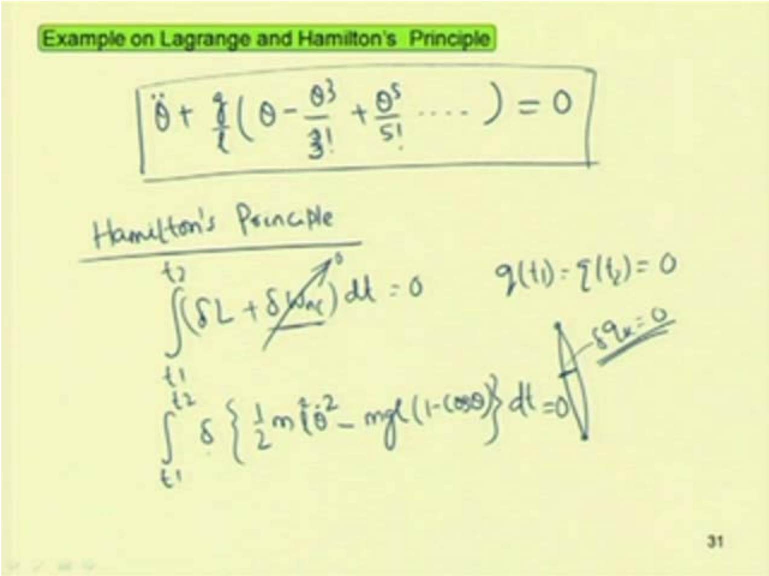

So, the resulting equation becomes theta double dot plus g by l into theta minus theta cube by factorial three plus theta 5 by factorial 5 and one can go on adding these terms so, this is equal to 0. So, this is the non-linear equation of motion for the simple pendulum we have derived using Lagrange principle. We can also use this Hamilton principle to derive this same equation motion so, here using Hamilton principle.

So, one can write this Hamilton principle integration t 1 to t 2 del L plus del W n c d t equal to 0 and where this so, q t 1 will be equal to q t 2 equal to 0 so, here this t 1 and t 2 are the time where the true path becomes equal to the barite path so, at this position this delta q k equal to 0 delta q k that is the variation in the displacement or variation in the generalized coordinates becomes 0. So, q t 1 equal to q t 2 equal to 0 in this case q our q equal to theta so, we can derive this equation by using this Lagrangian L so, here delta W n c equal to 0 as there is no force acting on the system. So, in this case this part equal to 0 delta W n c equal to 0 no external force is acting on the system so, one can write t 1 to t 2 for this l, I can write so, this is equal to half m v square v square is l theta l square theta dot square minus so, I can write this U so, this l equal to t minus u so u equal to m g l into 1 minus cos theta.

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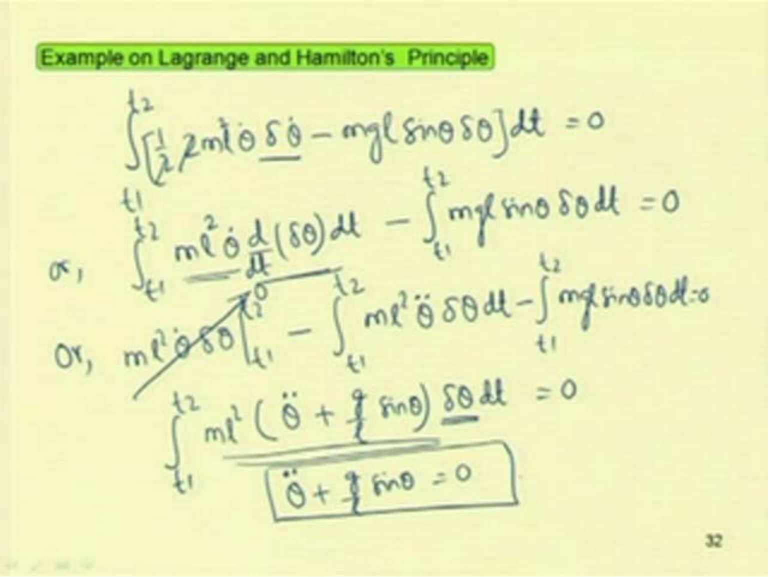

So, using this del operator into d t so, this should be equal to 0. Now, one can use this del operator in the system and derive the equation motion so, anything this del operator one can write so t 1 to t 2 half so this is half then m l square into 2 theta dot. So, i can write this into 2 m l theta dot into del theta dot minus m g l. So, this term using del operator this term becomes m g l minus sign is there before so, del operator with one this becomes 0 and this becomes plus sin theta so, m g l sin theta into delta theta d t so, this will become 0 or one can write this as t 1 to t 2 m l square so, this is l square so, this is m l square theta dot so, I can write this delta theta dot into d by d t of del theta so, this is d t minus integration t 1 to t 2 m g l sin theta del theta d t so, this will be equal to 0 or one can write so, one can integrate this by parts.

So, taking this as the first function and this as the second function so, one can write first function as it is so, integration of the second one, so integration of the second one gives delta theta t 1 to t 2 then minus integration t 1 to t 2. Differentiation of the first function so, this will give m l square theta double dot then delta theta d t minus integration t 1 to t 2 m g l sin theta delta theta d t equal to 0. But, we know this delta theta at t 1 equal to 0 and delta theta at t 2 equal to 0 so, these terms becomes 0 so, the remaining terms becomes t 1 t 2 so, if I will take this m l square common so, then this becomes theta double dot plus g by l sin theta delta theta d t equal to 0. As this delta theta is arbitrary that is this virtual rotation so, this cannot be 0. So, the coefficient of this that means these terms should be equal to 0 that means one can write this theta double dot plus g by l sin theta equal to 0 so, this gives the equation motion of the system.

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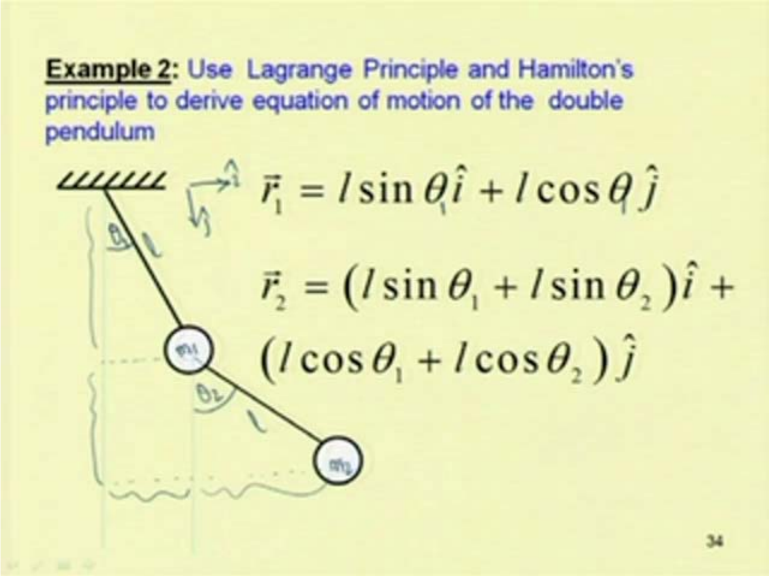

So, in this way one can use either Lagrange principle or Hamilton principle to derive the equation motion of a system. So, let us see that of a double pendulum now. So, previously we have seen for a single degree of freedom system but, now let us take a 2 degree of freedom system so, in this case let us take length both length same but, the mass different so, I can take this mass equal to m 1 and this mass equal to m 2 so, let at time t it has rotated the first link has rotated by an among theta 1 and the second one has rotated by an among theta 2 from the vertical position. So, the position vector of this point that is mass m 1 can be retained as r 1 so, it will be equal to l sin theta 1 so, it will be equal to m sin theta 1 i plus l cos theta 1 j. So, let me take these are the direction i and these are the direction j.

So, positive i direction and j direction I can take in this way. So, the coordinate or the position vector r 1 of this mass can be written as l sin theta 1 i plus l cos theta 1 j. Similarly, the position vector of this point can be written as the x component will be from this to this, that means this plus this and this part becomes already we have found this part so, this is l sin theta 1 and this part becomes l sin theta 2. So, one can write this position vector x component of the position vector equal to l sin theta 1 plus l sin theta 2 i and this vertical part can be written so, addition of this part plus this part so, these becomes l cos theta 1 plus l cos theta 2 j. So, in this way one can find the position vector of this point and the second point so, from these physical coordinates or from this position vector one can find the velocity of the system.

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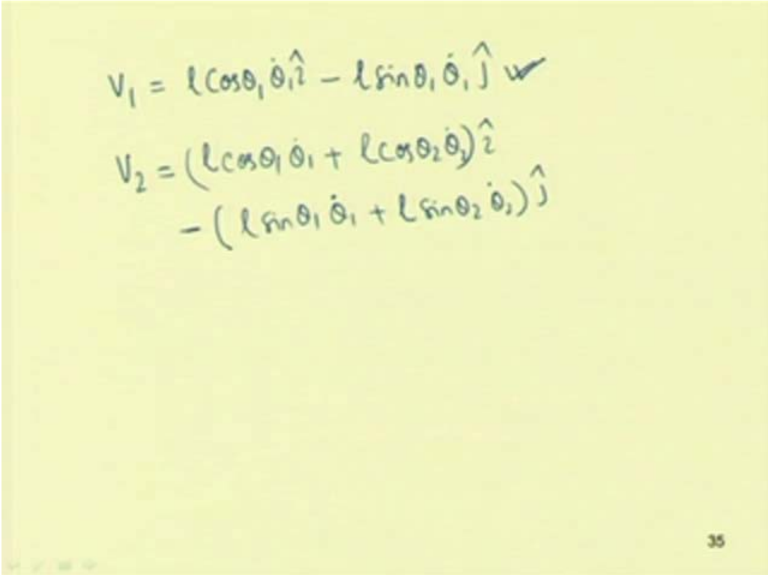

So, velocity V 1 can be obtained by differentiating this one so, by differentiating this one can write V 1 will be equal to l cos theta 1 so, it will be l cos theta 1 into theta 1 dot this is in i direction plus so, this l cos theta 1 differentiating this thing one can write so, it will become minus l sin theta 1 so, this becomes minus l sin theta 1 theta 1 dot j and one can write the velocity V 2 equal to by differentiating this thing one can write the velocity of the second mass. So, the velocity of the second mass can be so, this becomes l sin so, differentiation of l sin theta 1 this becomes l cos theta 1 theta 1 dot plus l cos theta 2 theta 2 dot. So, this is in i direction plus in j direction differentiating that thing one can find so, this becomes minus l sin theta 1 theta 1 dot. So, the sign will be different so, this is l sin theta 1 into theta 1 dot plus l sin theta 2 plus into theta 2 dot so, this is in j direction so, we got the velocity in i and j direction for the first mass.

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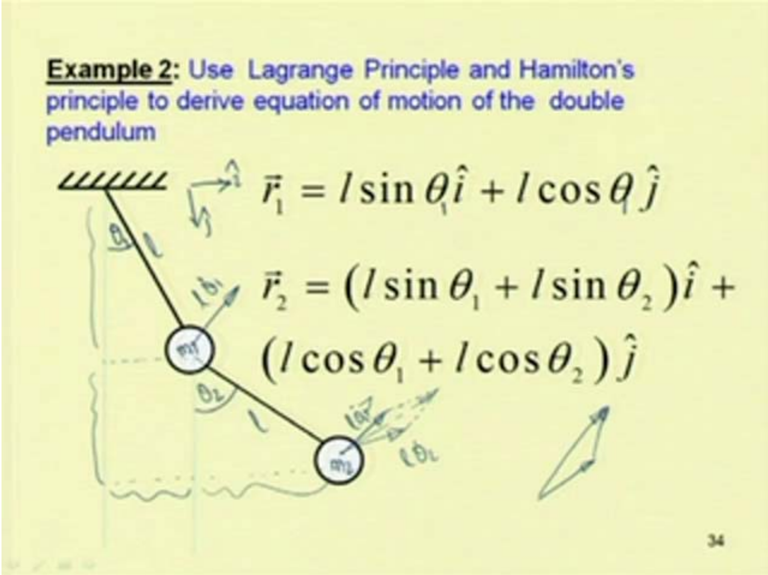

And then for the second mass or one can directly also write the velocity of these so, velocity of this is nothing but, this is equal to l theta 1 dot so, the velocity of this point equal to l theta 1 dot and velocity of this point will be velocity of this, of the first one parallel to this one can draw a line parallel to the first one plus the velocity of this point with respect velocity of this point with respect to this so, which perpendicular to this link.

So, one can find so, this is l theta one dot and this is l theta 2 dot. So, the summation of these two so, one can find so, this is l theta one dot and this is l theta 2 dot. So, one can find the resultant of these 2 so, the resultant by using this parallelogram theorem one can use this parallelogram theorem to find the resultant of these 2. So, this is the resultant of these 2 velocity so this is the velocity of this mass. So, in this way one can find the velocity of mass 1 and velocity of mass 2.

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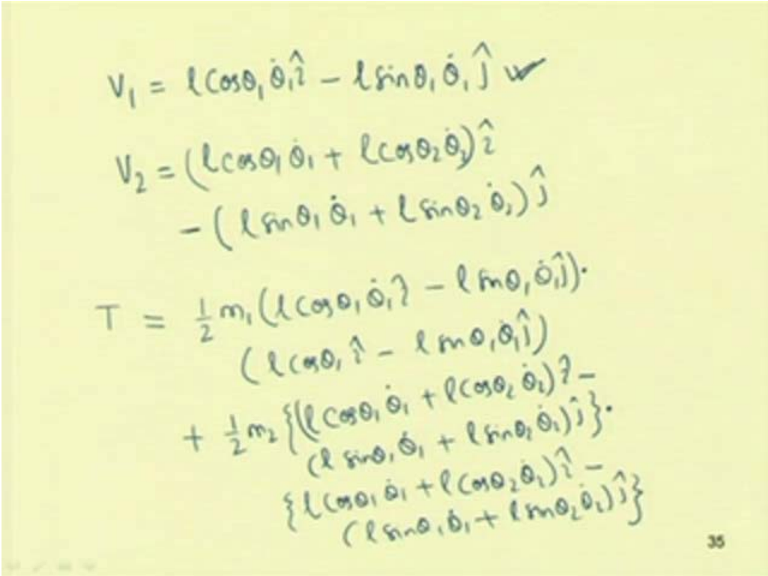

So, finding the velocity of the mass 1 and mass 2 so, one can determine the potential energy and kinetic energy of the system. So, the kinetic energy of the system t can be written as half mass for the mass 1 into so, one can find the velocity m V 1 dot V 1 so, it will be l cos theta 1 theta 1 dot i minus l sin theta 1 theta 1 dot j and a dot product i can give so, this will become l cos theta 1 minus l sin theta 1 theta 1 j so, this is for the first mass. Similarly, for the second mass the kinetic energy can be written as half m 2 into velocity of the second mass.

So, velocity of the second mass can be written in this form so, using this dot product again one can write this becomes l cos theta 1 theta 1 dot plus l psi l cos theta 2 into theta 2 dot so, this is i component and j component becomes minus l sin theta 1 theta 1 dot plus l sin theta 2 theta 2 dot so, this is j. Now, i can have this dot product so, with the dot product same thing I can write so, this is l cos theta 1 theta 1 dot plus l cos theta 2 theta 2 dot i minus l sin theta 1 theta 1 dot plus l sin theta 2 theta 2 dot so, this is j component. So, in this way one can find the kinetic energy of the system so, simplifying this thing now, as we are doing this dot product so, i dot i will be equal to 1, i dot j equal to 0. So, this will give l cos square theta 1 dot square plus l square sin theta 1 square theta 1 dot square.

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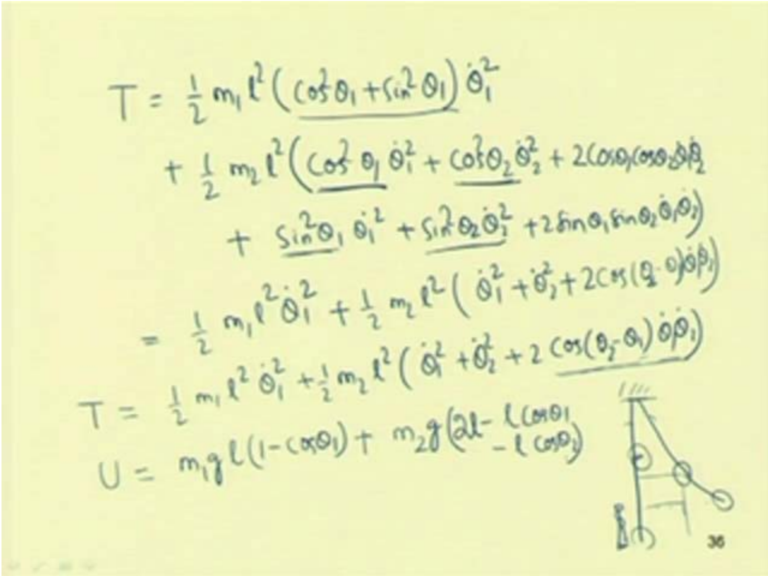

So, in that way for the first mass 1 can write t will be equal to half m 1 l square so, this is cos into cos so, this will give you cos square theta 1 so, this becomes cos square theta 1 plus sin square theta 1 then, for the second mass it will be equal to half m 2 l square. So, one can have this cos square theta 1.

So, i component so, i multiplied with i so, that will give cos theta cos square theta 1 then cos square theta 2 plus 2 cos theta 1 cos theta 2 so, product of this 2 terms also will be there. So, in that way one can write so, this becomes m 2 l square cos square theta 1 theta 1 dot square plus cos square theta 2 theta 2 dot square plus 2 cos theta 1 into cos theta 2 into theta 1 dot theta 2 dot plus so, this is from the i component. Similarly, from j component you can have this sin square theta 1 theta 1 dot square plus sin square theta 2 theta 2 dot square plus 2 sin theta 1 into sin theta 2 theta 1 dot theta 2 dot. So, this is cos square theta plus sin square theta equal to 1 so, this gives rise to half m 1 l square so, here you have also this theta 1 dot square so, this is theta 1 dot square plus this half m 2 l square so, this is cos square theta 1 plus sin square theta 1 this becomes 1 so, this becomes theta 1 dot square. Similarly, cos square theta 2 theta 2 dot square sin square theta 2 theta 2 dot square.

So, by adding this 2 so, this is reduces to theta 2 dot square and these 2 so, 2 sin cos theta 1 into cos theta 2 plus sin theta 1 into sin theta 2 whole multiplied by theta 1 dot theta 2 so, this becomes cos theta 2 minus theta 1 cos theta 2 minus theta 1 into theta 1 dot theta

2 dot. So, in this way one can write the total kinetic energy of the system equal to half m l square theta 1 dot square plus half m 2 l square theta 1 dot square plus theta 2 dot square plus 2 cos theta 2 minus theta 1 theta 1 dot theta 2 dot. So, this theta 2 minus theta 1 is nothing but, the difference between so, from this diagram one can see that is theta 2 minus theta 1 so, one can extend this line one can extend this line this theta 1 line so, this angle is theta 1 so, this angle this angle is theta 1 theta 2 minus theta 1. So, the kinetic energy of the system becomes half m 1 l square theta 1 dot square plus half m 2 l square theta 1 dot square plus theta 2 dot square plus 2 cos theta 2 minus theta 1 into theta 1 dot into theta 2 dot.

So, one can see that one can have a couple terms in the expression for the kinetic energy or one can have so, similarly, we can derive the potential energy of the system. So, potential energy of the system can be can be obtained by adding the potential energy of the first mass and for the second mass. So, potential energy is due to its position so, for the first mass this change in position so, you can write the potential energy becomes m 1 g into its change in position becomes l into 1 minus cos theta 1. Similarly, for the second mass the change in position becomes or the potential energy we can write m 2 into g into change in its position so, change in its position becomes so, initially it was in vertical position that means so, initially it was at this position now it has come to other position. So, one can find the equation or one can find this change in position like this so, this is the change in position which will be equal to so, this is l this is plus l so, l plus l this is 2 l, 2 l minus l cos theta 1 minus so, this becomes l cos theta 2 so, this becomes m 2 into g into 2 l minus l cos theta 1 minus l cost theta 2.

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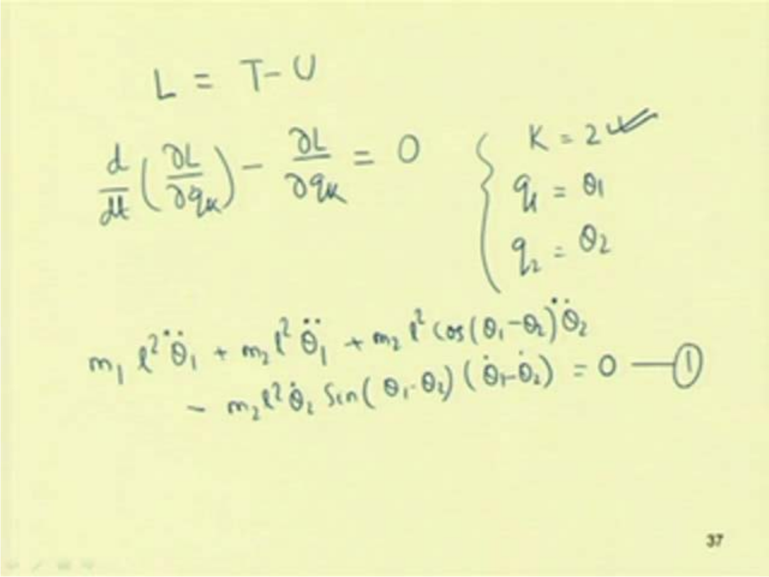

So, after getting this potential energy and kinetic energy one can find the Lagrangian of the system. Lagrangian of the system becomes T minus U. And now, one can use either Lagrange principle or this Hamilton principle to derive the equation motion. So, let us use this Lagrange principle to derive this equation of motion. So, in case of Lagrange principle it becomes d by d t of del L by del q k dot minus del L by del q k so, this becomes 0 as we do not have some external force acting on the system. So, k here equal 2 so, you have 2 degree of freedom system so, k equal 2 so, q 1 equal to theta 1 and q 2 equal to theta 2. So, in this case it may be noted that we have a 2 degree of freedom system unlike incase of this simple pendulum.

So, in case of the double pendulum k becomes 2 q 1 is theta 1 and q 2 equal to theta 2. So, we can obtain 2 equation motions in this case. So, first we will differentiate this or will take this q 1 and find the equation motion and second will find the equation motion using q k equal to or q 2 equal to theta 2. So, by doing that thing so, we can write the equation motion which will come in this form so, it will become or it will become m 1 l square theta 1 double dot plus m 2 l square theta 1 double dot plus m 2 l square cos theta 1 minus theta 2 into theta 2 double dot minus m 2 l square theta 2 dot sin theta 1 minus theta 2 into theta 1 dot minus theta 2 dot so this will be equal to 0.

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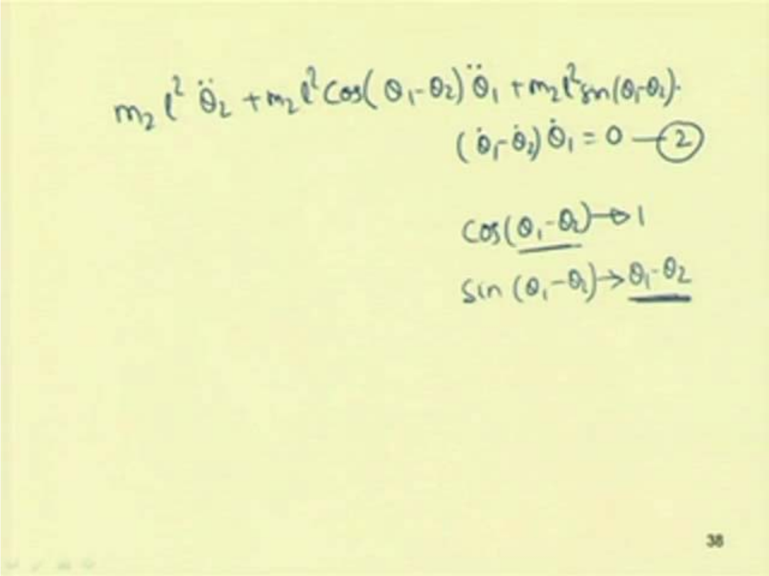

So, if you are taking q 1 equal to theta 1 so, we will get this equation. Now taking q 2 so, if you use this expression so, we can find the second equation so, which can be written in this form m 2 l square theta 2 double dot plus m 2 l square cos theta 1 minus theta 2 into theta 1 double dot plus m 2 l square sin theta 1 minus theta 2 into theta 1 dot minus theta 2 dot into theta 1 dot equal to 0. So, in this case you can see that when this theta 1 and theta 2 are very small then, this cos theta 1 minus theta 2 so, this theta 1 minus theta 2 will be very small and this will tends to 1 cos 0 equal to 1 and the sin theta 1 minus theta 2 will tends to theta 1 minus theta 2. And the resulting equation, this equation 1 and this equation 2 one can reduce to its linear form and one can write the equation in matrix form also.

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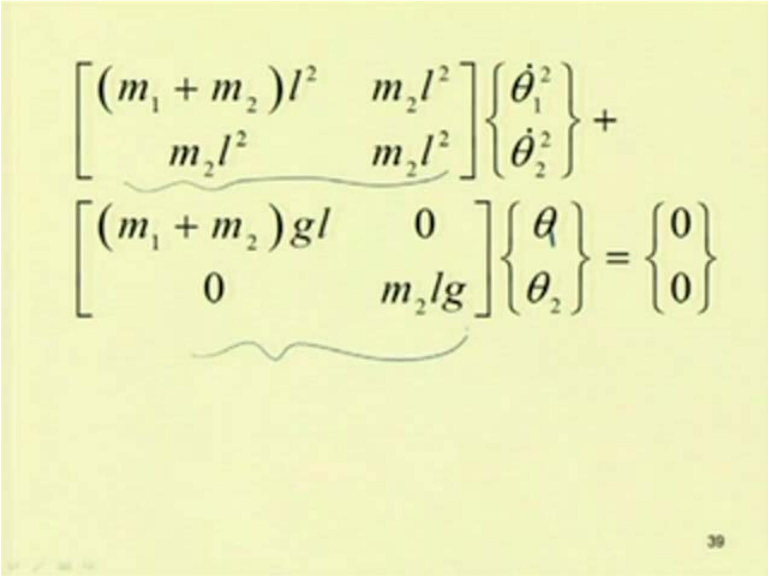

So, if one write the equation in matrix form it will be m 1 plus m 2 l square m 2 l square then, this is m 2 l square m 2 l square theta 1 double dot theta 2 double dot plus m 1 plus m 2 g l 0 0 m 2 l g theta 1 theta 2 equal to 0 0. So, in this case so, this part is known as the mass matrix of the system and this is the stiffness matrix of the system. So, one can observe that for this linear double pendulum equation motion the mass matrix is coupled that is, the diagonal terms are present but, the stiffness matrix is uncoupled. So, when the mass matrix is coupled then it is known as dynamically coupled system and if the stiffness matrix is coupled then it is known as statically coupled system.

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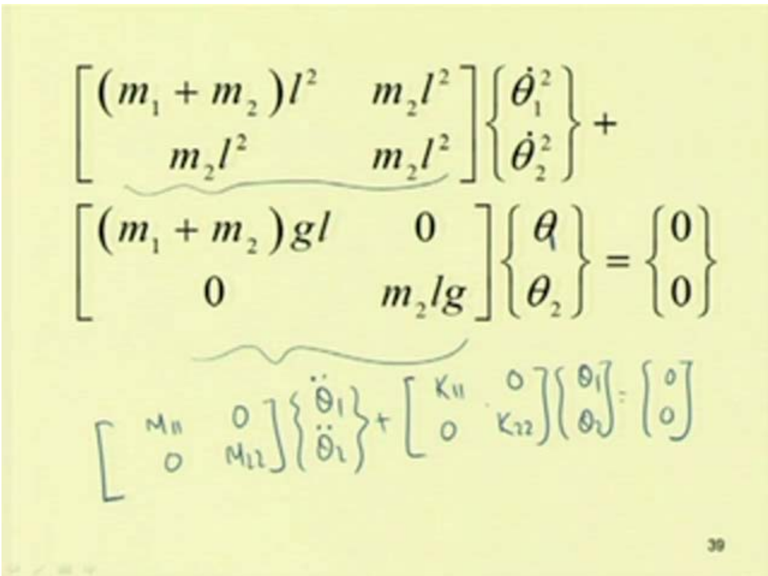

So, in this case we have a dynamically coupled but, statically uncoupled equation. The advantage of taking uncoupled equation will be so, if this equation can be written in this uncoupled form then, one write this one this way m 1 1 0, 0 m 2 2, theta 1 double dot, theta 2 double dot plus k 1 1, 0 0 k 2 2, theta 1 theta 2 equal to 0 0. So, in this case one can have 2 uncoupled equation for example, in this case it will be m 1 1 theta 1 double dot plus k 1 1 theta 1 equal to 0 this is the first equation and the second equation m 2 2 theta 2 double dot plus k 2 2 theta 2 equal to 0. So, this reduces to that of a single to a single degree of freedom systems. So, one can find the equation of solution of the single degree of freedom system equation very easily.

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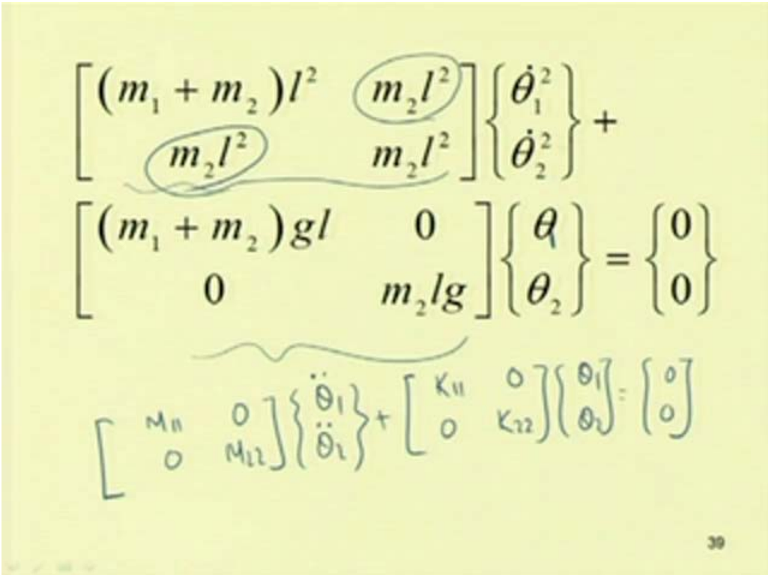

So, it is required to write the expression or write the equation motion or it is required to take the coordinate system in such a way that the half diagonal terms become 0 or the equation becomes uncoupled. So, in this case this term and this term give the coupling the mass matrix. So, one can decouple those equations by using model analysis method so, this is for a linear system but, actually if this theta is not small so, in that case we can have a different equation motion.

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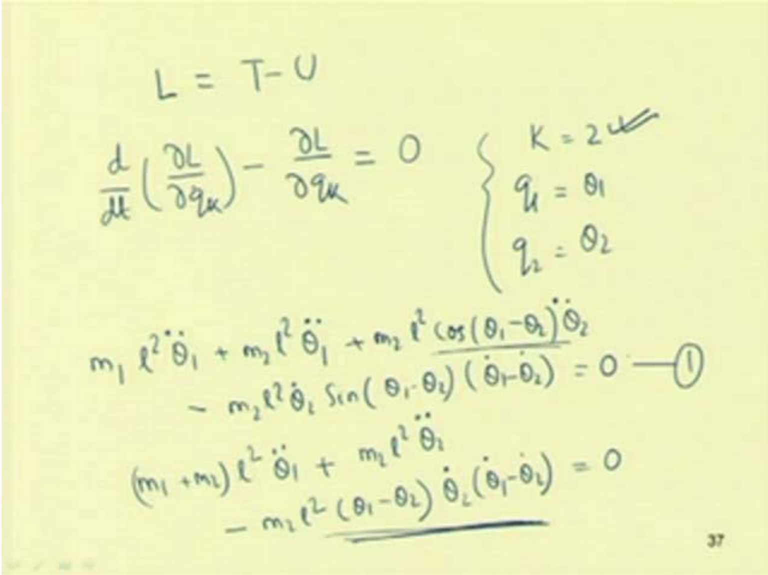

So, in this case one can write this equation in this form so, it will becomes m 1 plus m 2 l square theta 1 double dot plus so, one can have one can expand this cos theta 1 minus theta 2 term or m 2 l square cos theta 1 minus theta 2 into theta 2 double dot then this sin theta also one can expand so, this is m 2 l square so, one can see this term theta 1 minus theta 2 so, if I will take the small l also, this will becomes theta 2 double dot minus m 2 l square theta 1 minus theta 2 then, multiplied with this theta 2 dot multiplied with theta 1 dot minus theta 2 dot. So, one can have this cubic nonlinearity in this case or one can if one can expand this cos and sin for higher harmonic.

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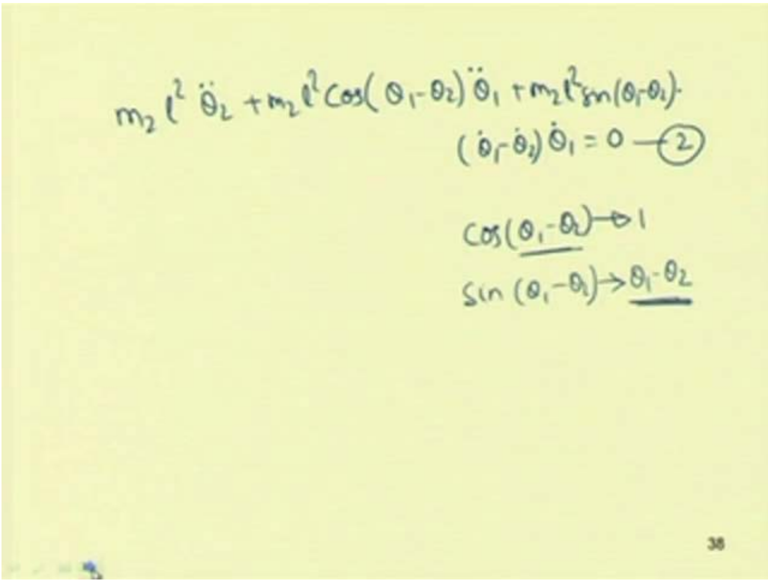

So, one can get quintic or higher order equation motion also, this is for the first mass. Similarly, for the second mass one can expand the sin theta 1 theta minus theta 2 or Cos theta one minus theta 2 and write the non-linear equation motion. So, in this way one can derive the equation motion by using either Lagrange principle or Hamilton principle.

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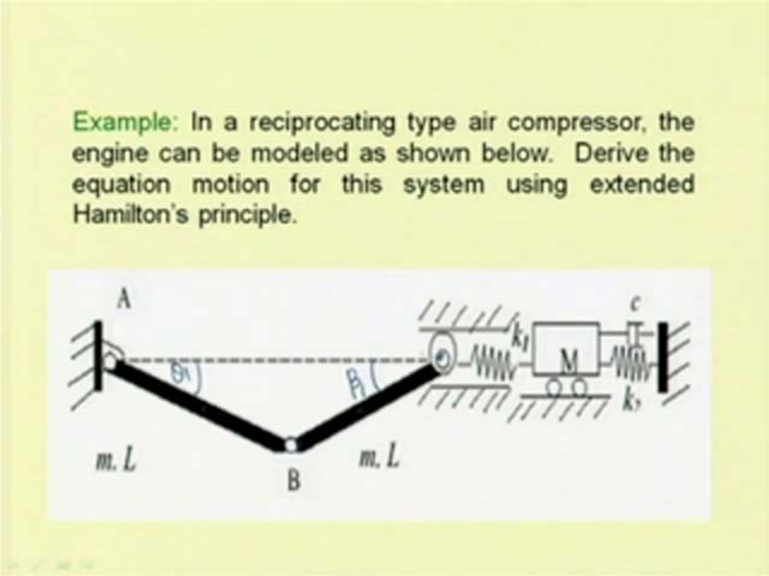

So, we can we have already seen this example so, in this case also one can find the equation motion using this Langrange or Hamilton principle. The first step is to write the equation, write the position vector of all the mass center so, here this is the mass center one can take this is the mass center and this point. So, one can write the position vector of all these points and then find the velocity after finding the velocity one can write kinetic energy of the system and potential energy of the system. And then using the generalized coordinate one can derive the equation motion of the system.

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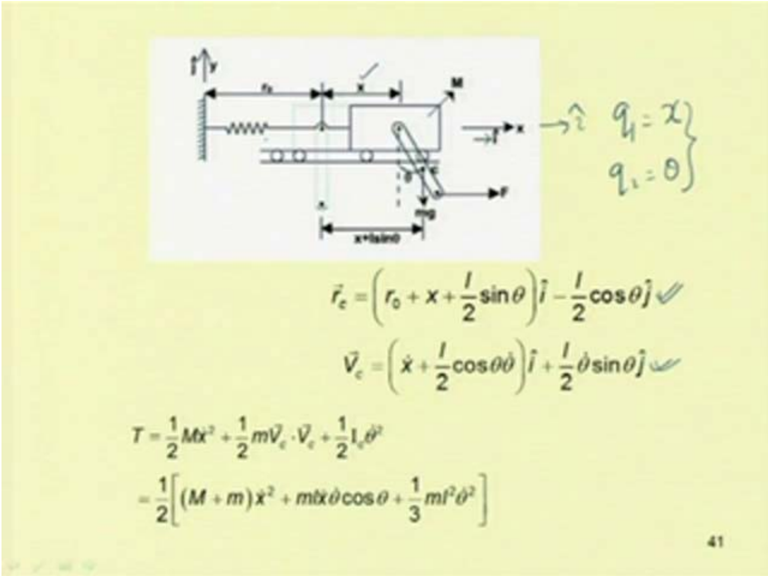

Similarly, this is another example so, in which one can derive the equation motion by writing the first step by writing the position vector. So, let us first write the position vector of this point c. So, in this case this is a cart containing, cart containing a rode which is oscillating and this one side of a cart is connected to a fixed rigid support by a spring. So, here one can use 2 coordinate system or 2 generalized coordinate: one is the x that is the change of the position of the mass center of this cart and the rotation of this bar with respect to this vertical. So, x and theta so q 1 one can take as x and q 2 equal to theta. So, initially one can write the position vector using Cartesian coordinate system by taking this as the x axis or i direction y axis or j direction.

So, one can write the position vector of this point as x plus r 0 plus x i. So, the position vector of this point c can be written as r 0 plus x plus so, if this total link l and the mass center is at l by 2 so, it can be written as r 0 plus x plus l by 2 sin theta i and as I have taken this j direction in the upward direction so, the y component will be minus l by 2 cos theta j. Similarly, one can find the velocity of this by differentiating this position vector. So, this gives the velocity of the system. So, the kinetic energy of the system can be written as half m so, m is the mass of this cart so, m x dot square plus half into m v c dot v c. So, v c is the velocity of point c. So, this gives the as this point has both translation and rotation so, the kinetic energy of this point will be, kinetic energy due to translation and kinetic energy due to rotation. So, by adding this 2 kinetic energy so, one can find the kinetic energy of point c. So, the total kinetic energy of the system can be written as half capital m plus small m x dot square plus m l x dot theta dot cos theta plus 1 third m l square theta dot square.

Now, the potential energy of the system can be written due to the potential energy of the spring and due to the change in position of these mass centers of this bar. So, the potential energy due to spring equal to half k x dot x square half k x square plus the position due to this change in position of the bar so, the potential energy will be equal to m g into so, change in position will be equal to 1 l by 2 into 1 minus cos theta.

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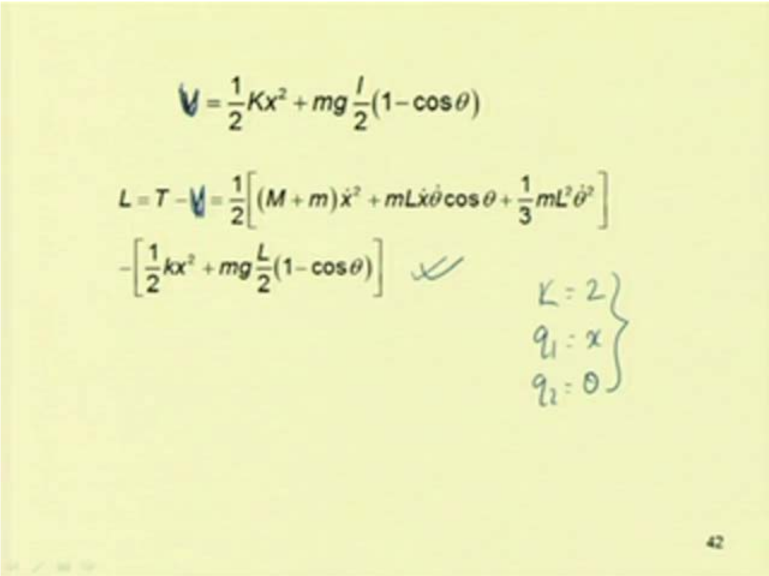

So, one can write L equal to T minus this v, that is or T minus one can write this as U so, T minus U so, T minus U will be equal to this. So, after getting this Lagrangian of the system now, one can use Hamilton principle or the Lagrange principle to find the equation motion. So, in this case you may note that k equal to 2 so, that is a 2 degree of freedom system here I can take this q 1 equal to x and q 2 equal to theta. So, by taking this so, I can derive the equation motion so, I can get 2 equation motion. So, due to the presence of this cos theta and sin theta the equation will be non-linear equation. So, next we can take another example also for a continuous system. So, in case of a continuous system so, let us derive the equation motion of a continuous system that is a longitudinal vibration of a rod.

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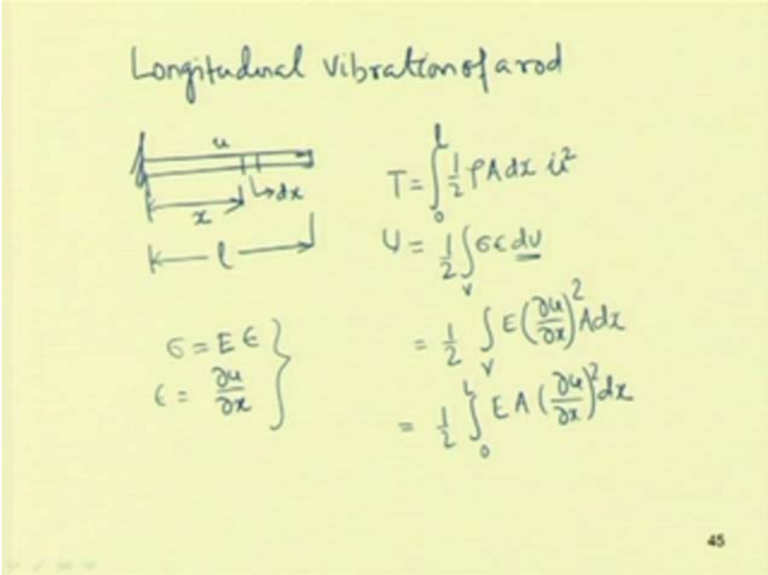

So, in this case so, this is the rod let us take a cantilever type. So, we are supposed to find the vibration of this rod. So, let us take a small element in this rod at a distance x so, at a distance x let its displacement becomes u. So, we are taking a very small element which is d x. So, we can write the kinetic energy of the system so, for the small element the kinetic energy will be equal to half so, mass of this small element so, mass of this small elements becomes if rho is the density of the small element, if the rho is the density of this rod then, it will be rho into A onto d x. So, this is the mass of this small element then, its velocity becomes u dot so, it becomes half mass into velocity square so, this is u dot square so, this is the kinetic energy of the system.

Similarly, potential energy of the system so, this is the kinetic energy of the small element. So, kinetic energy of the total system I can obtain by integrating this from 0 to l. So, l is the length of the rod so, you have taken l is the length of the rod. So, the total kinetic energy of the system will be equal to half rho into a into d x, rho is the density, A is the area of cross section and d x you have taken the length of a small element and u is the displacement at a distance x from the fix end. So, in this way one can find the kinetic energy of the system.

Now, the potential energy of the system U can be written as or can be obtain from the strain energy of the system. So, the strain energy of the system can be written as half so, this is volume integral so, volume integral stress into strength, stress into strain into d v so, the stress can be written so, already know that stress so, if it is subjected to a force f the stress can be written force by area or in terms of the strain you can write stress by strain equal to stress by strain equal to E. So, stress will be equal to E that is young’s modules into the strength one can write the strength equal to del u by del x so, strain is del u by del x so, stress equal to E into epsilon. So, one can write this potential energy equal to this is volume integral.

So, this becomes E into for epsilon E into epsilon into epsilon this becomes E into epsilon square so, this becomes E into del u by del x whole square into d v. So, this d v can be written if you are considering uniform cross section so, d v will be equal to A into d x. So, U can be written as half so, this is become 0 to L E A del u by del x whole square d x.

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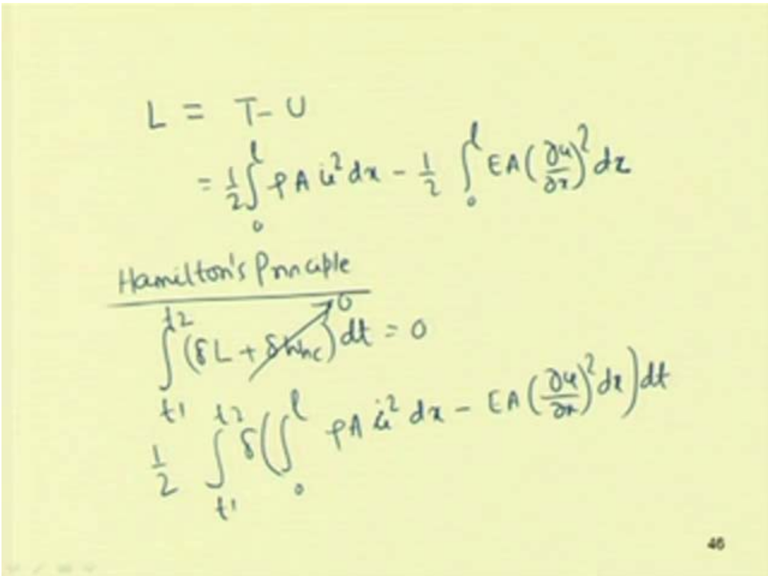

So, I can write the Lagrangian of the system equal to T minus U so, this T minus U can be written as integration 0 to so, this t already we have written so this is equal to 0 to l half rho A d x u dot square. So, I can write this is equal to rho A u dot square d x minus half 0 to l E A del u by del x whole square d x. Now, I can use let me use Hamilton’s principle to derive this thing so, if I will use this Hamilton principle. So, you can see that not only the equation of motion but, also the boundary conditions also I can get from this. So, to use this Hamilton principle already you know in this continuous system.

So, I can write this equal to del plus del W n c d t equal to 0. But, in this case this del W n c that is non conservative equal to 0. So, I can write this t 1 to t 2 so, this is from 0 to l t 1 to t 2 0 to l rho A u dot square d x u dot square d x minus so, this is half is there. So, I can take this half out so minus E A del u by del x whole square d x into d t. So, I have to use this del operator here.

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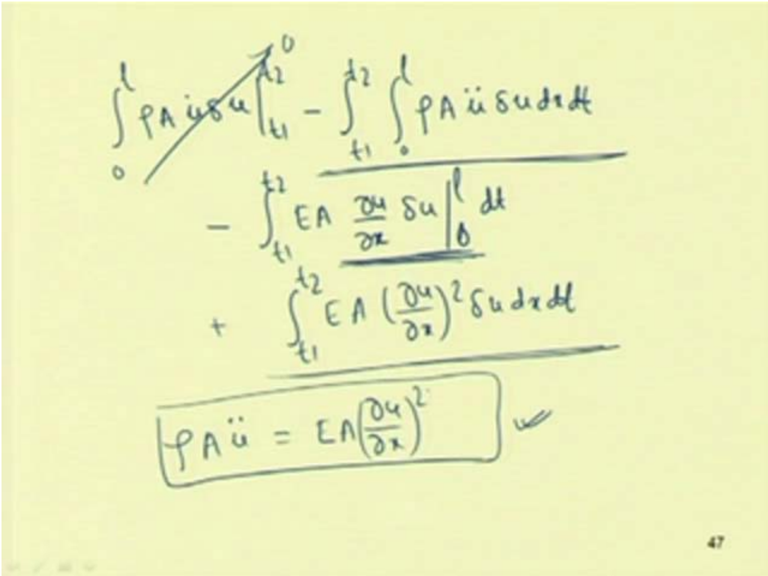

So, del of this equal to 0. Now, by using this del operator and using integration by parts you can write or it will reduce to 0 to l rho A u dot del u t 1 to t 2 minus t 1 to t 2, 0 to l rho A u double dot del u d x d t minus integration t 1 to t 2 E A del u by del x del u so, this is 0 to l d t plus t 1 to t 2 E A del u by del x whole square del u d x d t. So, already we know this del u at t 1 at t 2 this becomes 0 so, this term becomes 0 and for the equation motion we have these and this term as del u is arbitrary so, we have the equation motion this way so, rho A u double dot equal to E A del u by del x square. So, this is the linear equation what we obtain for this case and this term will give the boundary conditions. So, del u by del x at left end or that is the fix end either the del u by del x equal to 0 or u equal to 0 so, for the fix end slope will be 0 or displacement will be 0. Similarly, at right end one can find whether the slope is 0 or the displacement is 0.

So, in that way one can find the boundary conditions also in addition to equation motion by using Hamilton principle. So, today class we have studied or we have taken several examples using Lagrange and Hamilton principle to derive the equation motion. So, we have derived the equation motion for a single degree of freedom system that is simple pendulum, for a 2 degree of freedom system that is a double pendulum and a continuous system that is the longitudinal vibration of a rod. So, next class we will find the non-linear equation motion for the transverse vibration of a beam and there we will apply some non conventional type of forces like magnetic force or other type of forces to derive the equation motion. So, after deriving the equation motion will Linearize that thing to study the governing equation motion. Also, we will use this Galerkin principle to reduce the spatio temporal equation to its temporal form.

Thank you.