Mechanical Vibrations

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Module No #07
Multi DOF
Lecture No. #02

Properties of Vibrating Systems: Flexibility and Stiffness Matrices, Reciprocity Theorem

An IITG person promises only what he can deliver; an IITG person delivers what he promises.

So, today we are going to study about this multi-degree of freedom systems. So, last class we have studied how to determine the equation motion of this multi-degree of freedom system and you know, different methods of finding the equation motion. The methods are either you can you use the inertia base principle, that is, Newton's methods or energy based principle, that is, Lagrange principle or Hamilton's principle or extended Hamilton principle.

Also, I have started another, I told you another method to find the equation motion; motion, that is by using the flexible influence coefficient method. And we have seen the definition of flexibility influence coefficient. So, two different types of flexibility influence coefficient, I told, one is the displacement flexibility coefficient and other, it is stiffness flexibility coefficient.

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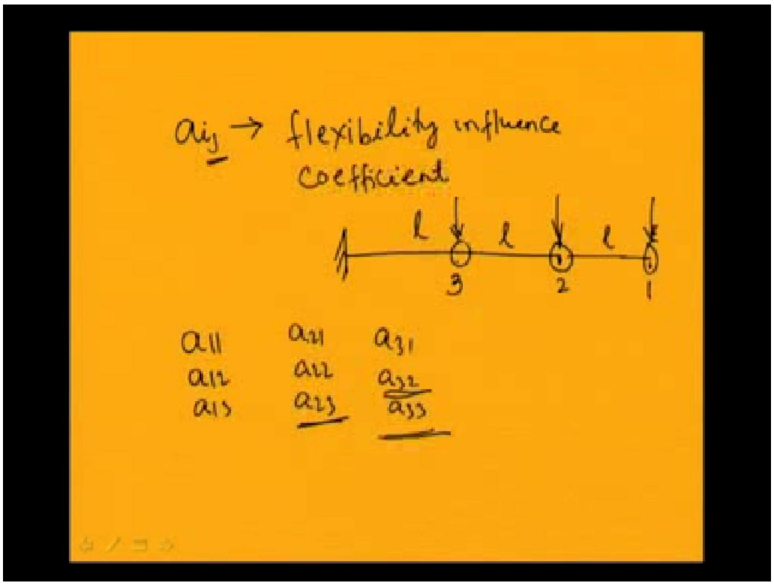

So, that is flexibility influence coefficient, ai j plus, plus, I have defined it as the displacement at i. So, it is the displacement at i due to a unit force applied at j when the other forces are 0. So, ai j is the flexibility influence coefficient. So, this, the displacement flexibility, flexibility, influence coefficient and we have defined these flexibility influence coefficient as the displacement at i due to unit force at j when the forces at other places are 0. Also, we have taken one example to derive or to find the influence coefficient.

So, in that example I have taken three mass, so they are placed at same distance, that is, l, so this is l, l and l. So, I can take the mass, same mass, m 1, m 2, m 3 or m. So, I have taken, so this station 1 and this is station 2 and this is station 3 and I have found the influence coefficient a 11, a 12, a 13, a 21, a 22, a 23 and a 31, a 32, a 33. So, a 11, if the displacement at 1, displacement at 1 due to a unit force at 1; similarly, a 12 if the displacement at 1 due to a unit force applied at this position, that is, 2 and a 13 is the displacement at 1 due to a unit force applied at 3. Similarly, a 21, a 21 is displacement at 2. So, this, the displacement at this position, that is, 2 when a unit force is applied at 1. Similarly, a 22 is the displacement at 2 when a unit force is applied at 2. Similarly, a 23 is the displacement at 2 when a unit force is applied at 3 and the forces at other places equal to 0. Similarly, a 31 if the displacement at 3, displacement at 3 when a unit force is applied at 1 and the force at the other two places are 0. Similarly, a 32 is the displacement at 3 when a unit force is applied at 2 and forces at other places are 0.

Similarly, a 33 is the displacement at 3 when a unit force is applied at 3 and at other forces are 0.

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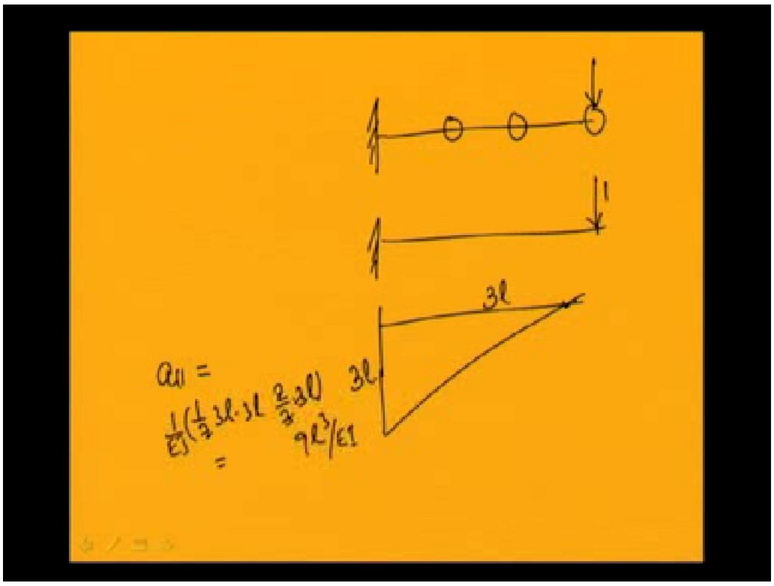

And I have derived, that those expressions by drawing the Vending Venn diagram. So, displacement of a, so I can consider these as a contileaver beam in 3 mass. So, 1, 2 and 3 mass and when a unit force is applied at here, so I can find the displacement. So, I can find the displacement at this position by applying a unit force at this. So, that thing can be obtained by drawing the Vending Venn diagram.

So, the Vending Venn diagram, so here, Vending Venn diagram, you have like this. So, this is the length is 3l, so this is 3l. So, to find a 11, one can find a 11 by finding the area of this diagram area. So, area of this diagram equals to half 3l into 3l and taking the moment of these area. So, moment will be, so it is acting at its length of two-third of 3l. So, this becomes, so 3, 3 cancel. So, this becomes 9l cube by, so this by 1 by EI. So, this becomes 9l cube by EI. So, in this way we have determined the influence coefficient a11, so which is equal to 9l cube by EI.

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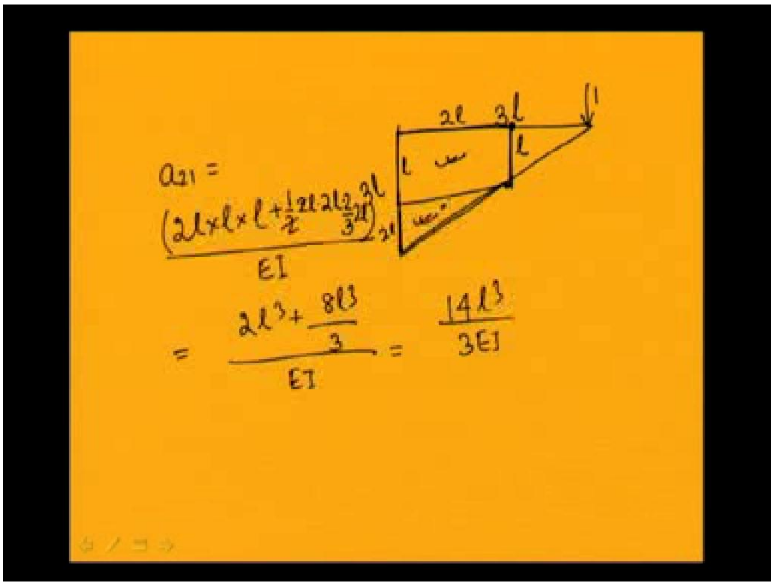

Similarly, you can determine the other coefficients also. For example, let me determine a 21. So, a 21 is the displacement at 2 due to a unit force at 1 and forces at other place equal to 0. So, when a unit force is applied here, so the displacement can be obtained from this Vending Venn diagram. So, this is the Vending Venn diagram, length is 3l as you are applying a unit force here, vending moment at this position will be 3l. So, the vending moment at, are the displacement at this position, can be obtained from this area, vending moment of this area. So, you can obtain it, vending moment of this area. So, a 21 will be…

So, this area you can divide into 2 parts, so this is the rectangular part and this is the triangular part and a 21 you can obtain from this area as, by taking the moment, if, of this point, for this rectangular part. So, this rectangular part, this length is l. So, this is l and this 2, this length is l. So, this area will be equal to 2l into l into moment. So, you have to take the moment, so this distance is 2l. So, it is at a distance l and for this triangular part it will be half, this is 2l into, so this is 2l and this is also 2l. So, 2l into 2l into, from this to this, centroid of this point is at this, so this is two-third of, two-third of 2l by EI. So, this becomes, so this is 2l cube plus, so 2, 2 cancel, so this becomes, 2 and 2, 4, 28, so 8l cube by 3 by EI. So, this becomes 6 plus 8, this is 14l cube by 3 EI. So, in this way in the last class we have determined the flexibility influence coefficient of all the points.

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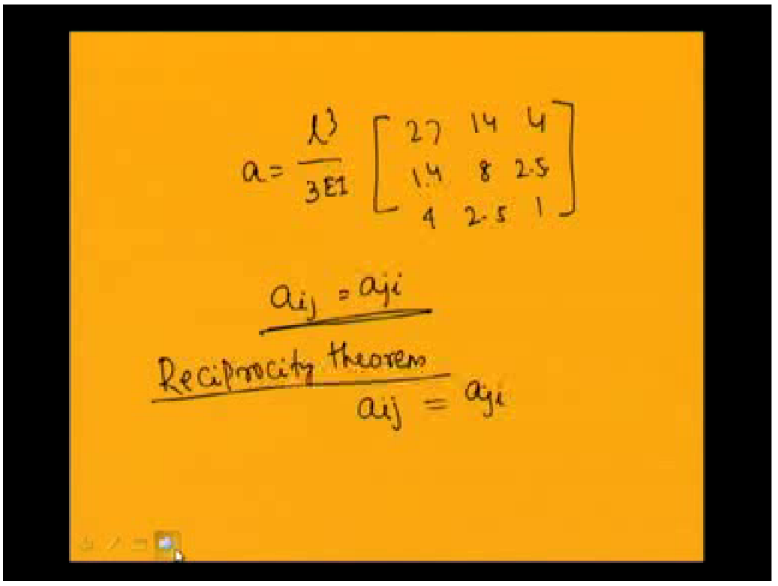

So, we have determined the matrix and that matrix is given by, so that matrix already you have seem and that matrix, it is given by l cube by 3 EI into 27, 14, 4, 14, 8, 2.5 and 4, 2.5, 1. So, in this case you can observe, that this aij, aij equal to aji. That means, a 21, so this a 21 equal to 14, your a 21, a 12 is also 14. Similarly, a 31, so this is a 13 equal to 4 and a 31 is also 4. Similarly, a 32, a 23 equal to 2.5, a 32 also equal to 2.5. So, you can see, that are observed, that this aij equal to aji. So, this is known as reciprocity theorem, reciprocity theorem. So, for a linear system, so this is a linear system we have considered, so for a linear system you can tell, that this aij, flexibility influence coefficient aij equal to aji. That means, displacement at i due to a unit force at j equal to displacement at j due to a unit force at i. So, to prove that thing let us take a system.

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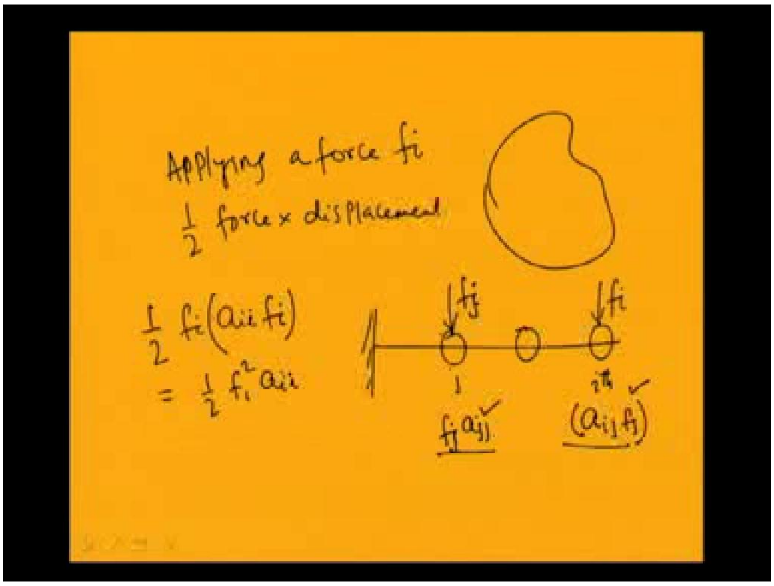

So, let this is a system or the system what you have, just now you have derived. So, let us take the system and you apply a force, apply a unit or apply a force at ith station. So, let this be ith point applying a force at I, applying a force fi at i. So, the work done will be equal to half force into displacement.

So, when we are applying a force fi here, so let this is the fi force applied here, so that at other place you are not applying any force. So, the displacement at this position will be fi into displacement will be fi into aii as due to a unit force at ith station the displacement will be aii. So, due to a force fi applied at ith position, the displacement will be fi into aii. And other places at force equal to 0, the work done will be equal to summation of force and displacement at all these places, so as the forces at other places are 0. So, work done will be equal to half into, so this is the force fi into displacement, fi into aii. So, this becomes of, fi square aii.

Now, after applying this force fi lift, again we will apply another force fj at jth point. So, let this fj force is acting at j. So, due to this force fj, this point j will undergo a displacement of fj into ajj and this ith point, which has already a displacement of ai, I will have a displacement of aij, aij be the displacement at i due to a unit force at j. So, the total displacement at this position due to this force fj will be aij into fj. So, the work done will be due to the displacement at this position and due to the displacement at this position. So, the displacement at this position is aij fj and this position, it is fj into ajj.

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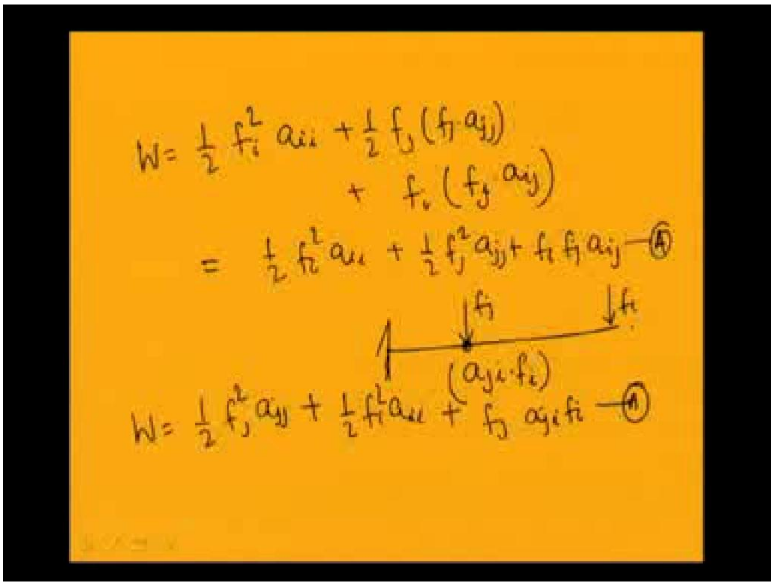

So, the total work done will be half fi square aii plus half, fj into, fj into ajj plus half fi. So, already a force fi is acting at this position and this point is ongoing a displacement of aij into fj. So, due to that, that is the force at, due to that force the work down will be fi into, f I f j I, fji into aij, force fj into aij. So, this becomes so the work done w equal to half fi square aii plus half fj square ajj plus, so this point will undergo force into displacement, so this becomes fi. So, this becomes fi fj aij.

Now, let us alternate the application of force, let us first apply the force of fj, let us first apply the force fj and then apply the force fi. So, if we apply the force fj first, then the work done will be equal to half fj square ajj and by applying a force fi after that will have a displacement of, displacement of aji. So, this is the displacement at j due to a unit force at i and as I am applying a force fi, so this point will undergo another displacement of aji into fi and here a force of fj is applied to the system. So, this work done will be equal to total work done half fj square ajj plus half fi square aii plus half plus force into displacement, so this in, so this force fj into aji into fi. So, we can, so work done, if we are applying the force at j first and then at i, then the work is half fj square ajj plus half fi square aii and plus half fj fi into aji.

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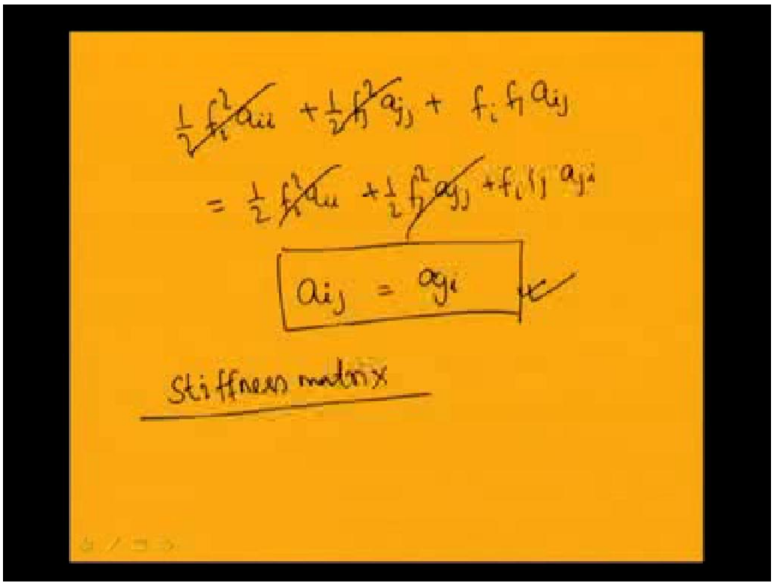

So, from these two, as the work down in both the cases will be same because it is, it does not depend on the application of the force or the, so we can write, so we can equate these a and b and we can write this half f i square aii plus half f j square ajj plus half fi fj fi fj aij equal to half fi square aii plus half fj square ajj fj square ajj plus half, there is no half here, so fi into fj into aji, so this is aji. So, by equating this you can got this and you can see, that aij equal to aji. So, for a, so this is the reciprocity theorem.

And for a linear system you can half aij, that is, flexibility influence coefficient aij equal to flexibility influence coefficient aji, that is, the displacement at i due to a unit force at j equal to a displacement at j due to a unit force at i. So, this is the reciprocity theorem. And now, we will study about the stiffness matrix, stiffness matrix or property of the stiffness matrix of a system.

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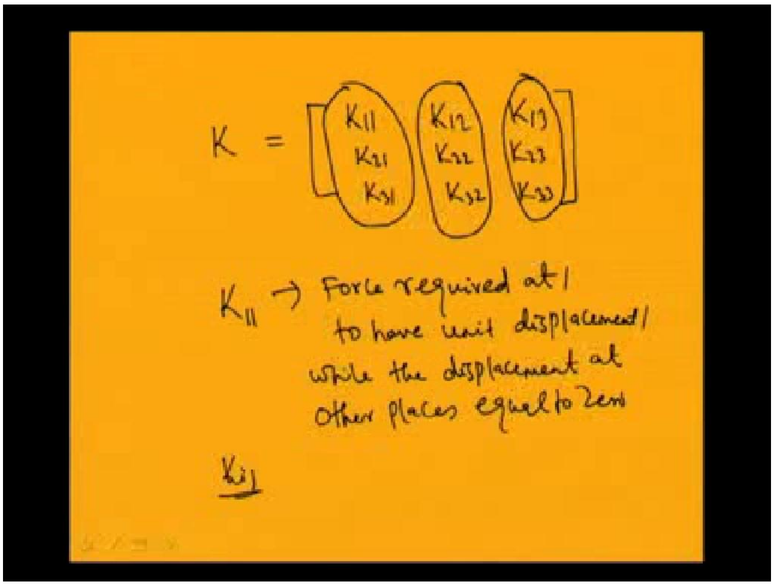

So, in previous system we have written this stiffness matrix K. Let us take a three degree of freedom system. So, the stiffness matrix K can be written as K 11, K 12, K 11, K 12, K 13, K 21, K 22, K 23 and K 31, K 32, K 33. So, this stiffness matrix, this K 11 or K 11 can be defined as, so this is the force required at 1, this is the force required at 1 to half, unit displacement at 1. So, this is the force required at 1, so this is the force required at 1 to have unit displacement at 1 while the displacement at other places equal to 0, at other places equal to 0.

So, similarly K ij, you can define as force required at i to have unit displacement at j when the displacement at other places equal to 0, this K 12. So, this 1st column, so this 1st column will represent, so this is force required at 1, 2 or 3 to have unit displacement at 1 while displacement at other places equal to 0. Similarly, this K, 2nd column will represent the displacement force required at 1, 2 and 3 to have unit displacement at 2. And similarly, this 3rd column will represent the force required to have unit, this force required at 1, 2 and 3 to have unit displacement at 3 while the displacement at other places equal to 0.

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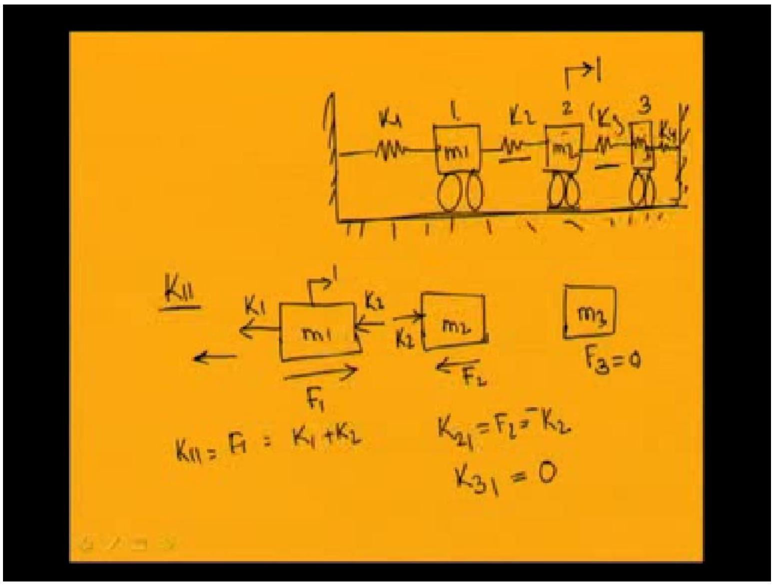

So, let us take an example. So, let us take a system with three mass and springs and let us determine the stiffness matrix of the system. So, this is K 1, this is K 2, this is K 3 and the system is supported and some wheels. So, the system has… So, this is mass m 1, this is mass m 2 and this has mass m 3.

So, to find K 11, that is, force required at 1 to have unit displacement at 1, we can draw the three body diagram of all these three masses and you can find that thing. So, in case of K 11, so it is the force required at 1. So, this is 1, this is 2, this is mass 3. So, force required at 1 to have, so this is the force required at 1 to have unit displacement at 1 and displacement at this position and this position, that is at 2 and 3, so could be 0.

So, to draw the three body diagram, let us draw the three body diagram to determine these. So, this is mass 1, mass 2 and this is mass 3. So, we have to find what is the force required at 1 to have unit displacement at 1; so, if it will have the unit displacement at 1. So, this spring will be stretched by (( )) 1 and so it will exert a force in opposite direction, that is, magnitude will be equal to K 1. Similarly, the 2nd spring K 2, so this spring will be compressed by 1 as this point is not moving at, the 2nd point is not moving, so this will be compressed by 1 and so it will exact a force K 2 in this direction.

So, for unit displacement of this mass 1, these 2 springs will be giving a force K 1 plus K 2 in this direction. So, they will exact a force in this direction. So, to have that displacement, one has to apply a force F 1, so that force one has to apply a force F 1 equal to K 1 plus K 2 to have unit displacement at 1 and displacement 0 here and 0 here. So, this K 11 becomes K 1 plus K 2. So, K 11 equal to your F 1. So, this F 1 equal to K 1 plus K 2. And now, let us see the 2nd mass, so as the displacement of the 2nd mass equal to 0. So, this spring K 2, due to this motion, unit motion of mass 1, so it will compress this mass m 2. So, it will exert a force K 2 and this mass m 2, and it is K 3 for K 3 spring as there is no motion of mass 2 and 3. So, this K 3 spring will have no motion. So, so it will not exert any force on this. So, the total force acting on m 2 equal to K 2. So, two half are due to this force.

So, the mass, so moving towards right or to prevent this motion you should apply a force F 2 in this direction, this force F 2 equal to K, so that is equal to K. So, force required at 2 to have unit displacement at 1, displacement at other places equal to 0, so that is K 21 and so this will be equal to F 2 and it is equal to K 2. So, this will be equal to minus K 2 because we are taking this, the direction towards right projectry, so as we have to apply a force in the opposite direction. So, one has to write, this is equal to minus K 2.

Similarly, for mass 3, so this is K 3 and this is spring K 4, so as there is no motion of this mass 2 and 3. So, this spring will, so those spring force acting on the 3 equal to 0 and the spring force acting on 4 equal to 0. So, no force is required as there is no force acting on this mass m 3, so no force is required to, to have its 0 displacement. So, F 3 will be equal to 0. So, this K 31, that is, the force required at 3 to have unit displacement at 1 and displacement at other places equal to 0 will be equal to 0.

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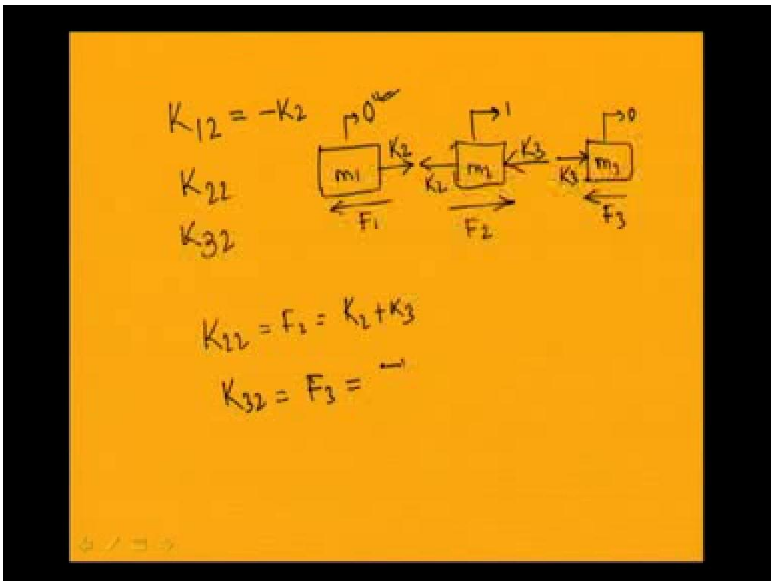

Similarly, you can find K, applying K. So, we can find K 12, K 21, K 12, K 22 and K 23, K 32. So, for this, the second mass will have to undergo a displacement of unity and other two mass will be at their equilibrium position. So, they will have no motion. So, for that I can draw the free body diagram.

So, in this case, free body diagram, so mass, so in this mass m 1 will have zero motion and mass m 2 will have unit motion. So, it will move unity and mass m 3 will have also zero motion. So, it will also not move. So, we have to find what is the force required to have this configuration. So, in this case I can draw the free body diagram. So, this, as this mass is not moving or it is stationary, so the spring will not exert the force on this side. But as the second mass is moving, as the spring K 2, the spring K 2 will be pulled by an amount K, so the spring K 2 will pull this mass m 1 with the force K 2. So, a force K 2 will act on this mass m 1. So, to prevent this motion of this mass 1 or to have zero displacement of this mass 1 when force K 2 which acting on this, so we have to apply a force or one has to apply a force F 1. You have this motion, have 0 motion here, so this F 1 equal to K 12, so this will be equal to, so as this is acting in opposite direction, so I can write K 12 equal to minus K 2.

Similarly, when this mass m 2 has unit displacement and mass m 1 and m 3 are zero displacement, this spring K 2 will be pulled by this mass m 2. So, this spring K 2 will exert a force in opposite direction and so, the free body diagram will be, so it will be K 2 and in the right side. So, the spring K 3, for this spring K 3, the K 3 spring will be compressed by this motion of mass 2 and as there is no motion of mass 3, the relative motion of spring K 3 will be unity. So, the spring K 3 will exert a force in opposite direction and this force will be equal to K 3, K 3 into 1, so that is equal to K 3.

So, a total force of K 2 plus K 3 is acting on mass 2 to, to have unit displacement. A force K 3 and K 2 is acting, so to have that, one has to apply force in this direction. So, that is your F 2 and this force should be equal to K 2 plus K 3 and that is K 22. So, K 22 equal to F 2 equal to K 2 plus K 3. Similarly, this mass 3, so at this time the mass 3 is subjected to a force due to this spring 3, K 3. So, this spring K 3 will push this mass 3 with a force of K 3, as the relative motion of this spring equal to 1 minus 0, that is equal to 0. And as there is no motion of this mass m 3, so K 4 force or force due to this spring K 4 equal to 0. So, the mass 3 will be subjected to a force of K 3 only and that force, so this is K 3.

So, to have this body in equilibrium position, the force required is K 3 in opposite direction, so that is F 3. So, this K 32 equal to F 3 equal to minus K 3.

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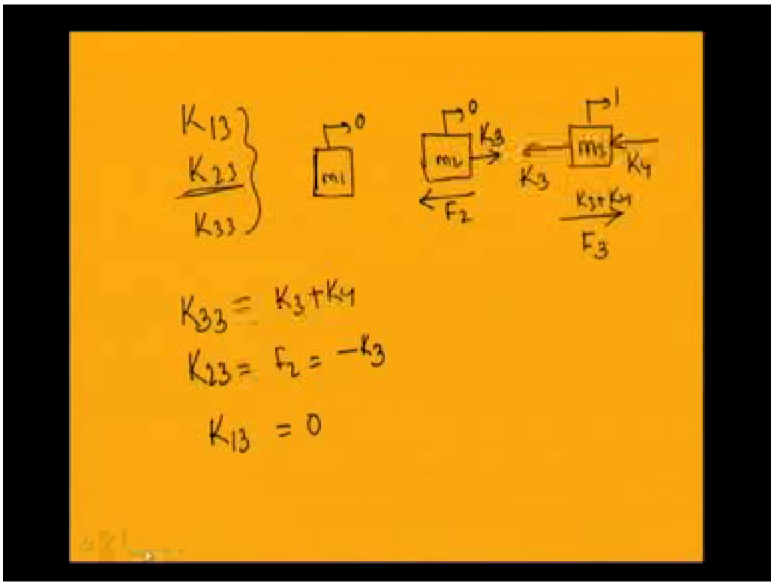

Similarly, one can find K 13, K 23 and K 33 by drawing the free body diagram when the mass 3 is having unit displacement and mass 2 and mass 1 having 0 displacement. So, this mass, this is mass m 1 having 0 displacement; mass m 2 having 0 displacement and mass m 3 having unit displacement. So, when mass m 3 has unit displacement, spring K

3 will be pulled by unity. So, it will exert a force in opposite direction, that is, K 3 in this direction. Similarly, the spring 4, K 4 will be compressed by unity and it will exert a force, also in opposite direction, so that is K 4. So, one has to apply or the force required to have unit displacement at 3 will be equal to K 3 plus K 4 and so that is equal to K, so this is the force F 3. So, that is equal to K 3 plus K 4.

So, this force is required to maintain or to have unit displacement at this 1. So, force required at 3, so this is K 33, force required at 3 to have unit displacement at 3. So, this is equal to K 3 plus K 4. Now, to find K 23, let us see this mass, free body diagram of this mass m 2. So, in this case, to have unit displacement at 3, the spring K 3, so in this case to have unit displacement at 3 plus spring K 3 will pull mass m 2 by m force K 3 in, towards right. So, so this spring K 3 will pull this mass towards right by a force K 3 as there is no motion of mass m 2 and m 1. So, this force or the spring K 2 will not exert any force. So, the total force acting on mass m 2 equal to K 3 towards right. So, the force equal to have this motion will be in opposite direction to this and this is equal to F 2 and this F 2 equal to K 23. So, K 23 equal to F 2 equal to minus K 3. Similarly, for mass 1, as there is no motion of this spring K 2 and K 1, so there is no force acting on mass m 1, so no force is required to have this motion. So, K 13 will be equal to 0.

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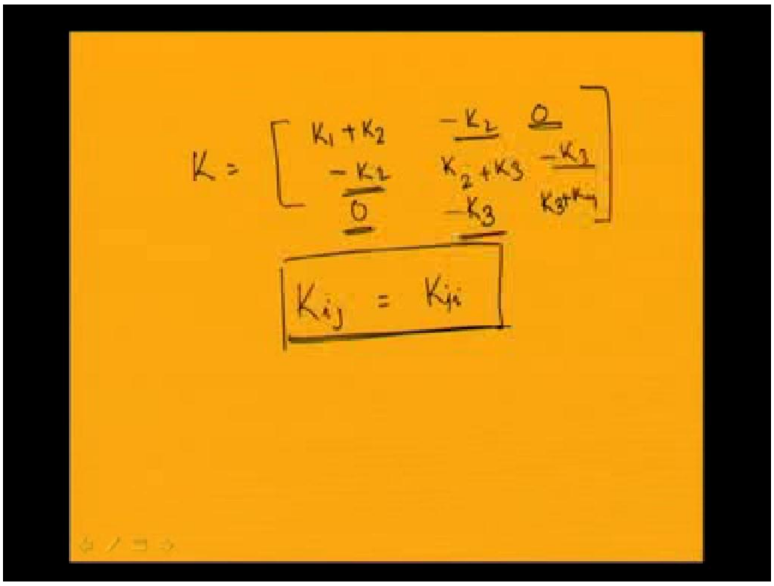

So, in this way we have determined all, the stiffness coefficient. So, by arranging those stiffness coefficients one can write, so this matrix equal to, so this matrix becomes K matrix equal to K 1 plus K 2 minus K 20 and this is minus K 2 K 1 K 2 plus K 3. So, this is K 2 plus K 3 and minus K 3 and this is 0 minus K 3 and K 3 plus K 4. So, here also you can observe, that K ij equal to, so it also follows the reciprocity theorem, that is, K ij equal to K ji, so for a linear system, so you can see, that this stiffness K ij, so K 12 equal to K 21. Similarly, K 13 equal to K 31 and K 23 equal to K 32. So, K ij equal to K ji. So, from the definition of the stiffness influence coefficient, you can determine the influence coefficient or all the elements of this stiffness matrix in this way.

So, in this case, in this example we have, as drawn the free body diagram of these three mass and we have the found the force required to have unit displacement to find K 11, we have applied a, so to find K 11. So, we have what is the force required to have unit displacement at 1 and displacement at other two places equal to 0, and we have found, that is equal to K 1 plus K 2 and similarly, we have found K 21.

So, in this case we have seen that this mass is subjected to a force K 2 to have a unit displacement at 1. So, as this mass is subjected to a force K 22, Hamilton equilibrium, the force required at station 2 equal to minus K 2. Similarly, we have seen, that at this position there is no force acting on this mass 3. So, K 31 equal to 0. So, in this way, one can determine the elements of this stiffness matrix by finding the force required at station, at ith station, to have force required at ith station to have unit displacement at jth station when the displacement at other places equal to 0.

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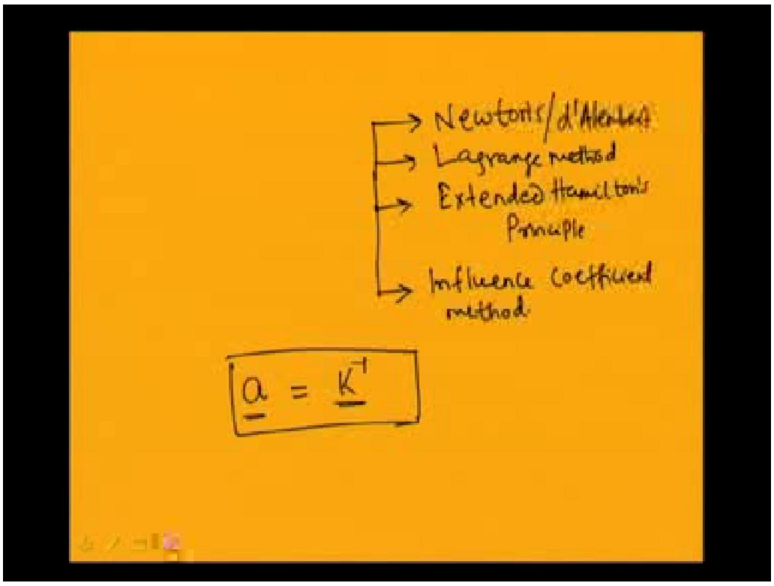

Now, till now we have studied how to determine the equation motion of the multi degree of freedom system. So, first, we know, by using the Newton's method we have determined the equation motion Newton's method or d'Alembert principle, and then we have used the Lagrange method and next we have applied extended Hamilton principle to determine the equation motion.

Also, we have studied this influence coefficient method. Now, we have studied this influence coefficient method, coefficient method to determine the mass, to determine the stiffness matrix and the flexibility influence coefficient matrix of the system. So, you can observe, that this displacement flexibility coefficient matrix equal to 1 by K, that is, 1 by this stiffness matrix or reciprocal of the stiffness matrix. So, this is equal to K inverse. So, influence coefficient matrix or displacement influence coefficient matrix or the stiffness influence coefficient matrix, they are related by this formula.

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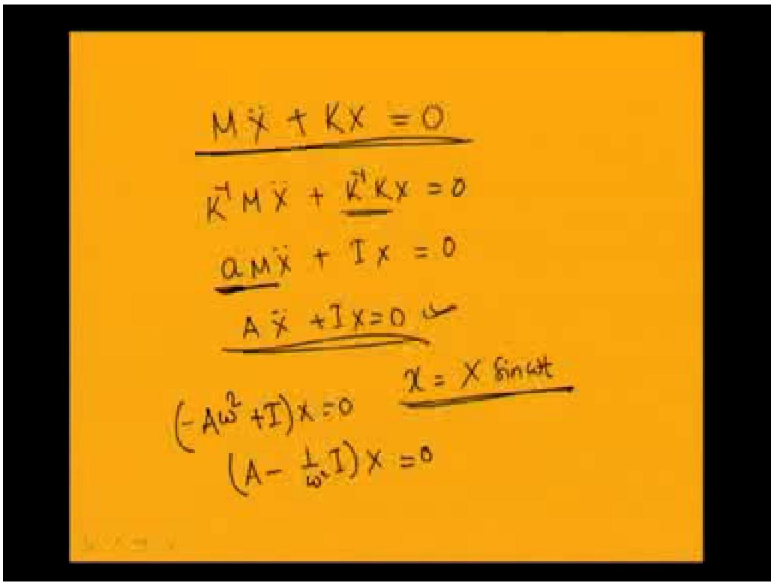

And by finding this displacement coefficient matrix or stiffness matrix one can write the equation motion, by writing this M X double dot plus K X equal to 0. So, already we have seen, this is the free vibration or this is the equation motion for free vibration of a multi-degree of freedom system, where m is the mass matrix, K is the stiffness matrix and X is the displacement vector. So, either one can write the equation in this form or one may write this equation by premultiplying K inverse. I can write this equation in this form, so it will be K inverse M K inverse M X double dot plus K inverse K X equal to 0.

So, K inverse K equal to I and this K inverse equal to A matrix. So, A M X double dot plus, so this is I X equal to 0.

So, if you know the flexibility influence displacement flexibility influence coefficient, so by multiplying this with mass matrix you can find another matrix, that is, A X double dot plus I X equal to 0 proceeding in the previous case to find the normal mode of the system. So, in this case also, we can substitute this X equal to, so we can substitute this X equal to X I X equal to X. So, sine omega t or so, by substituting this thing in this equation, so X double dot will becomes minus omega square minus omega square X.

So, this equation becomes, so I can write this equation, A minus A omega square plus I X equal to 0. So, in this way one can write this equation and you can, so minus A omega square plus I X equal to 0 or I can write this equation in this for A minus 1 by omega square A minus 1 by omega square I X equal to 0.

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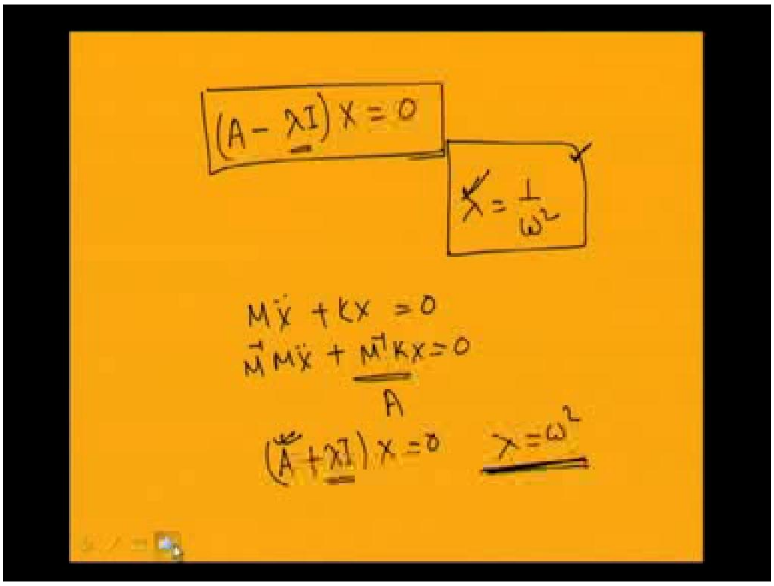

Or this equation can be written as A minus lambda I X equal to 0. So, in this case, in this case, this lambda equal to 1 by omega square. So, if you are finding the Eigen value of matrix A, so this will give you 1 by omega square, but in the previous case when you have taken the equation in this form M X double dot plus K X equal to 0 and you have multiplied it with minus M inverse, then you got this equation, M inverse K X equal to 0. So, that time also we have written this as A. So, it was reduced to this form, A plus lambda I X equal to 0. So, in this case this lambda equal to omega square, in the previous case we have seen.

So, the Eigen value of this matrix A equal to omega square and here we were proceeding from the flexibility coefficient, displacement flexibility coefficient, you will get a Eigen value, which is reciprocal of the actual Eigen value of the system. So, this, so in this case the highest frequency will correspond to the lowest Eigen value. And the lowest Eigen frequency will or Eigen value will correspond to the highest Eigen highest frequency of the system. So, you can solve the (( )), either from this by using this stiffness matrix method or by using this displacement influence coefficient method. In case of displacement influence coefficient method if you proceed, so you will get the Eigen value, which is equal to the reciprocal of this omega square and in this case you can find it, lambda equal to omega square.

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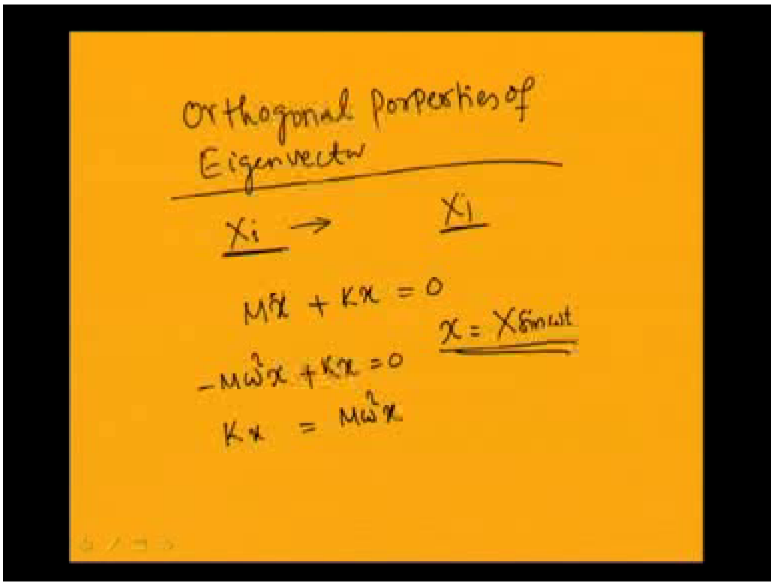

So, let us see the orthogonal properties of this Eigen vectors; orthogonal Properties of Eigen Vector. So, we have already seen, that this Eigen values corresponding to the, correspond to the square of the natural frequency and Eigen vectors correspond to the normal modes oscillation of the multi-degree of freedom system. So, for the ith mode, so for the ith mode I can write this Eigen vector X I. If this Eigen vector X I for the ith mode correspond to the normal mode of the ith mode and X j will correspond to the normal mode of the jth mode, so one can, so that these, these modes, ith mode and jth mode are at, these modes are orthogonal with respect to the mass matrix and stiffness matrix of the system.

So, in this case we are assuming that this mass matrix and stiffness matrix are symmetric matrix. So, already we know, that the equation motion can be written in this path and M X double dot plus M X double dot plus K X equal to 0 by substituting. So, M X double dot, I can write this small M X double dot plus 1 K X, so plus K X equal to 0. I can substitute this X equal to X sine omega t and I can write this equation minus M omega square minus M omega square X minus M omega square small x I can write, or capital X also the X can write, minus m omega square X will be, or plus K X equal to 0, or I can write this K X equal to m omega square X.

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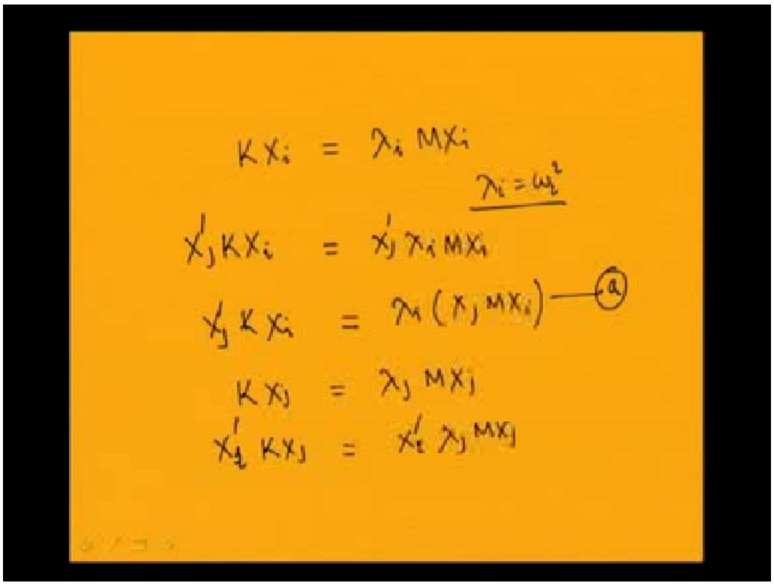

So, for ith mode I can write, for ith mode I can write this KX i will be equal to lambda i MX i, where this lambda i equal to omega i square. So, the previous case, this is omega square, for the ith mode I can write this equation equal to KX i. So, it will be KX i sine omega i t will be equal to M omega i square X i sine omega i t. So, this sine omega i t, omega i t will cancel from both side. So, you can write this KX i. So, this thing can be written KX i equal to lambda i MX i. So, here, lambda i equal to omega i square.

So, now, premultiplying this transpose of the jth mode. So, I can premultiply this equation by X j dash. So, this, the transpose of jth mode, so X j dash KX i will be equal to X j dash lambda i MX i, or this thing can be written X j dash KX i equal to lambda i into X j MX i. So, let this is equation a. So, let us start from the jth mode. So, I can write for the jth mode, K X j will be equal to lambda j MX j. So, let me premultiply this by X i transpose. So, this X i transpose, so X i transpose KX j will be equal to X i transpose lambda j MX j.

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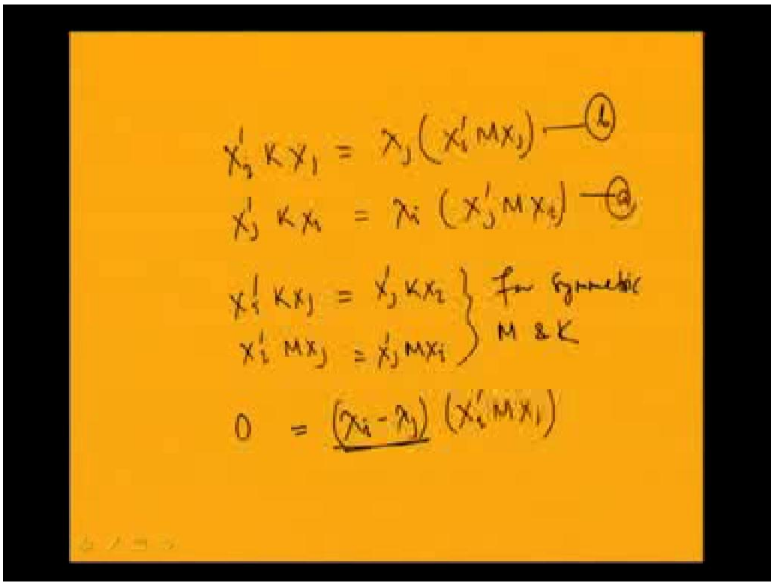

So, I can write this X i, X i transpose. So, this is X i transpose KX i transpose X j. So, this X i transpose KX j equal to lambda j into lambda j into X i transpose MX j; X i transpose MX j.

So, now, from this equation a and b, from equation a and b one can write equation, this equation b and equation a is written in this part, X j dash KX i equal to lambda i X, we can see this things. So, this is lambda i X j, so this is X j dash. So, this is X j dash MX i. So, this is equation A. Now, the stiffness matrix and mass matrix are symmetric, then you know, that X i dash KX j will be equal to X j dash KX i and X i dash MX j equal to X j dash MX i for symmetric. So, this is for symmetric, so this is for symmetric mass and stiffness matrix.

So, from these two by subtracting a from b, so I can write, so if I subtract these things, so becomes, left side becomes 0. So, 0 equal to lambda i minus lambda j, lambda i minus lambda j into X i dash MX j. So, but this lambda i minus lambda j, that is, this Eigen values are the distinct numbers.

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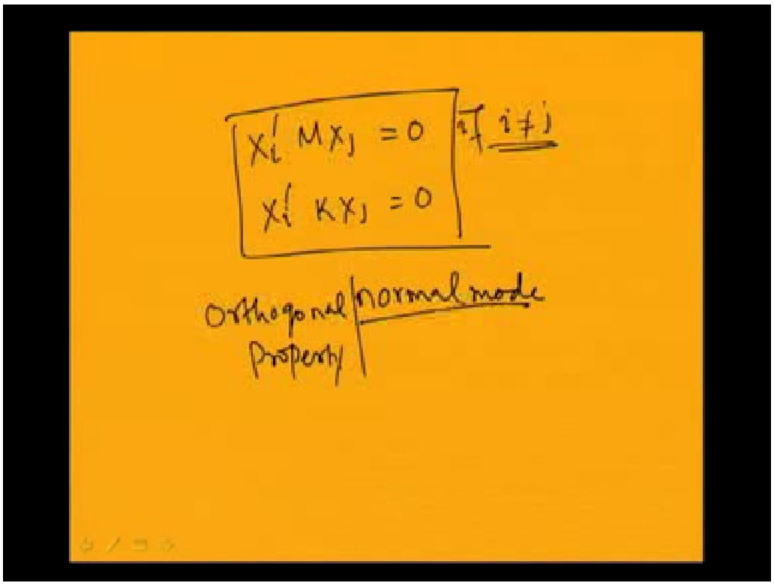

So, lambda i not equal to lambda j and as lambda i not equal to lambda j, then this X i show, it shows, that X i dash, X i dash MX j equal to 0, if i not equal to 0.

So, when i not equal to j, when X i dash MX j equal to 0. So, this is the source, the orthogonal property of this Eigen vectors. Similarly, you can show, that X i dash KX j also equal to 0. So, the source the orthogonal properties of this mass, orthogonal property Eigen vectors or the normal modes orthogonal properties of this Eigen vector, X i and X j or normal mode X i and X j, so this orthogonal property of the normal mode, orthogonal property of this normal mode.

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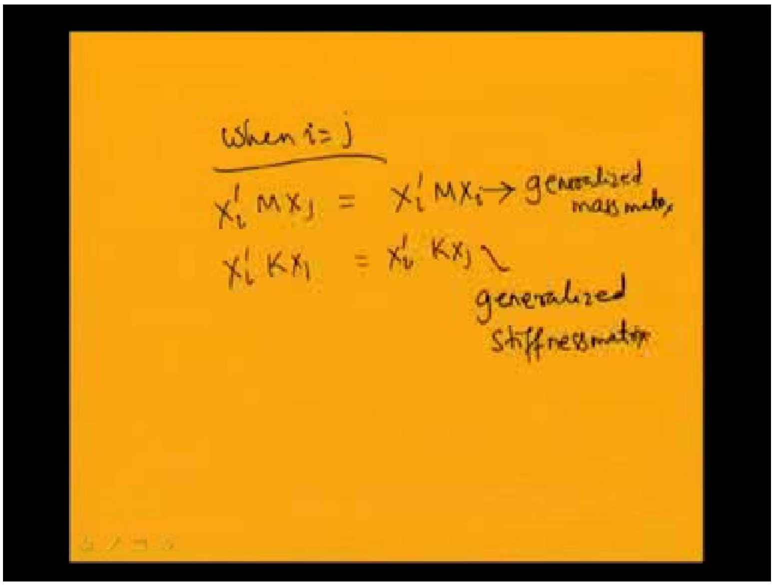

So, when i equal to j. So, this will reduce to, so this will reduce to, so when i equal to j, so when i equal to j, you can see that, so this part equal to 0. So, this will satisfy this equation. So, when i equal to j, this X i dash, X i dash MX j will be equal to X i dash MX i and X i dash KX j equal to, KX j equal to X i dash KX j. So, these are known as, so this is known as generalized mass. So, this will give a diagonal matrix and this diagonal matrix is known as the generalized mass and this is known as the generalized stiffness matrix. This will, if the generalized mass matrix and this will give rise to generalized stiffness matrix.

So, in this way you can obtain the generalized mass matrix and stiffness matrix by using the orthogonal property of this Eigen vector or normal modes.

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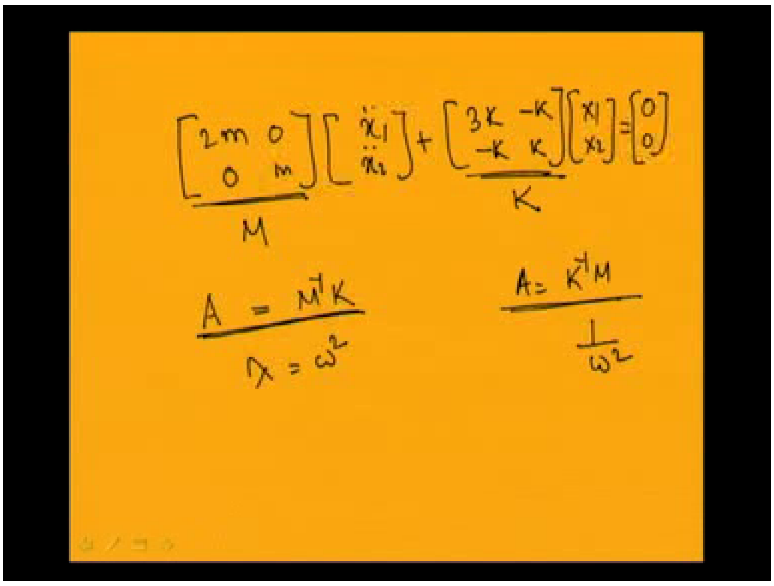

So, let us take 1 example. So, let us take a system. So, in the system, let 2, m, 0, mass matrix is 2m, 0, 0, m. So, the equation motion is retained in this for 2, m, 0, 0, m, x 1 double dot, x 2 double dot plus 3K, minus K, minus K, K, X 1, X 2, X 1 X 2, equal to 0, 0. Let us take this example and let us find, let us find this normal mode and check the orthogonality property of the system.

So, in this case, this is the mass matrix M and this is the stiffness matrix K. So, I can write this A matrix equal to M inverse K. So, either I can find this mass matrix, A matrix of this M inverse K or I can write form K inverse M. So, when I will write A equal to K inverse M, so Eigen value of this will give me 1 by omega square and when I will take A equal to M inverse K, Eigen value lambda will be equal to omega square. So, let me proceed in this way by taking M inverse K.

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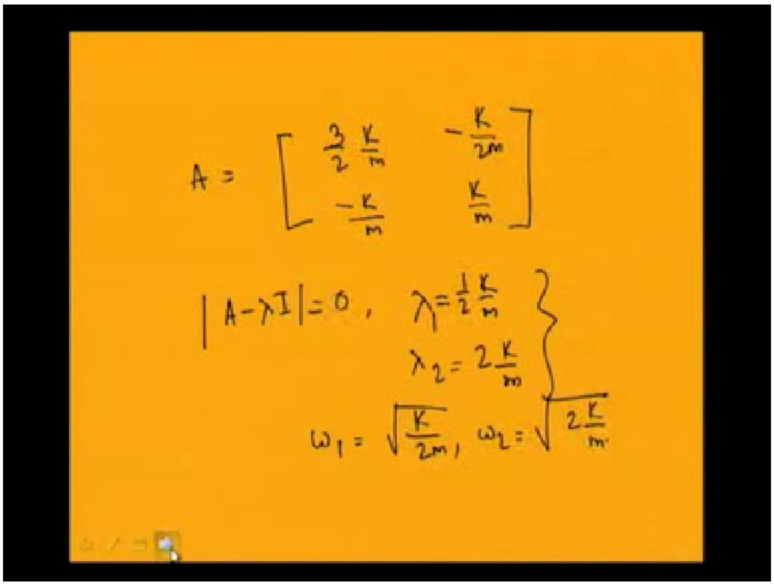

So, if A equals to M inverse K, so this matrix, again find, so this matrix will become, so A equal to M inverse K and this is equal to 3 by 2 K by M minus K by 2m, this is minus K by M and this is K by M.

So, to, I can find the Eigen value of this matrix. So, Eigen value, so I can find this Eigen value by finding the determinant of A minus lambda I equal to 0. So, this will give me lambda 1 equal to half K by m and lambda 2, lambda, so lambda 2 equal to 2K by m. So, lambda 1 equal to omega 1 square and lambda 2 equal to omega 2 square. So, the natural frequency of the system or the normal mode frequency of the system omega 1 will be equal to root over K by 2m and omega 2 will be equal to root over 2K by m. So, to find the normal mode of this thing, the system, I can find this normal mode by finding the adjoint of A minus lambda I matrix.

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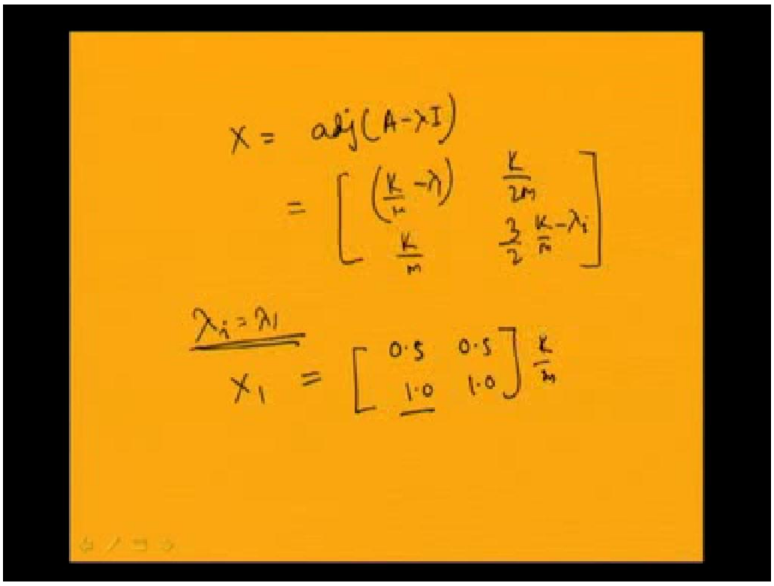

So, already you know this, X equal to this normal mode, X is equal to adjoint of A minus lambda i. So, I can find this adjoint matrix. So, adjoint of A minus lambda i becomes K by m minus lambda 1, this is K by 2m and this is K by m and 3 by 2 K by m minus lambda i. So, for lambda i equal to lambda 1, so I can find this X i or X 1, I can find the first normal mode, so X 1, so by substituting that thing and finding the adjoint of this thing.

So, one you can find, this X 1 becomes, so you will get two columns of these and you can check, that by substituting this lambda i equal to lambda 1, so this adjoint matrix, you will get two columns and you can see, that all the, you can verify, that normalize form of this both the columns are same. And this normalized form of both these columns will give this A minus lambda i. You can substitute this lambda i equal to lambda 1. So, you will get this. So, this is equal to 0.1, this is also 0.1, 1.0, 1.0, K by M. So, you can take, that both the columns are same.

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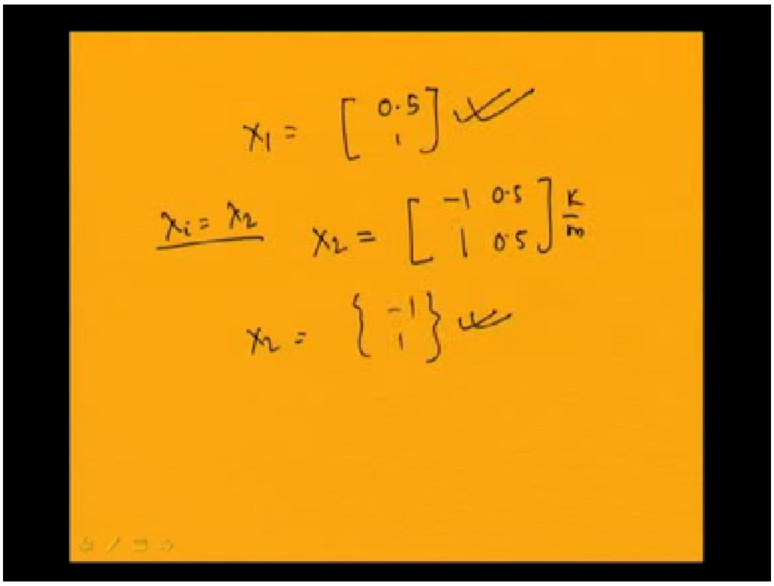

So, your X 1 equal to, so X 1 equal to, by normalizing you can write X 1 equal to 0.5, 1. Similarly, by substituting lambda i equal to lambda 2 you can find this X 2 and you can find the adjoint of A minus lambda i and that will give you minus 1.5, 1.5, K by M or the normalized value of X 2 equal to minus 1, 1.

So, you got the normal modes. So, the first normal mode is X 1, this is 0.5, 1 and the second normal mode is minus 1, 1. So, after getting these two normal modes you can now verify these normal modes are orthogonal.

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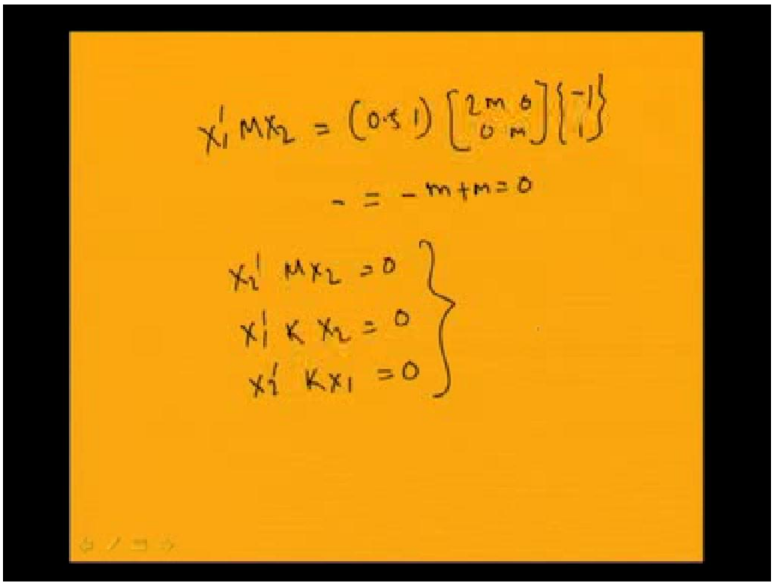

Now, you just find this X 1, X 1 dash MX 2. So, this will be equal to 0.5, 1 into 2m, 0, 0, m and minus 1, 1. So, this becomes minus M plus M equal to 0. Similarly, you can verify, similarly you can verify, that X 2 dash MX 2 equal to 0, X 2 dash MX 2 equal to X 2 dash MX 2 equal to 0 and X 1 dash K X 2 equal to 0 and X 2 dash K X 1 is also equal to 0. So, this was the orthogonal property of these Eigen vectors or normal modes.

So, today class we have studied about the, about finding the stiffness matrix of a multi degree of freedom system and the from, the definition of the stiffness matrix, each element of the stiffness matrix, or though K i j element of the stiffness matrix can be defined as the force required at i to have unit displacement at j, and displacement at other places equal to 0. And we have seen the reciprocity theorem of the influence coefficient and also, we have seen the orthogonal properties of these Eigen vectors or normal modes.

So, next class we study the free vibration of, free vibration of multi-degree of freedom system by using this modal analysis method.