Computational Continuum Mechanics
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Introduction – Origins of Nonlinearities
Lecture – 01 and 02
Origin of nonlinearities - 1

So, welcome everyone. This is the 1st lecture on this course on Computational Continuum Mechanics. So, the first two lectures are on introduction and the sources of nonlinearities which are present in the system.

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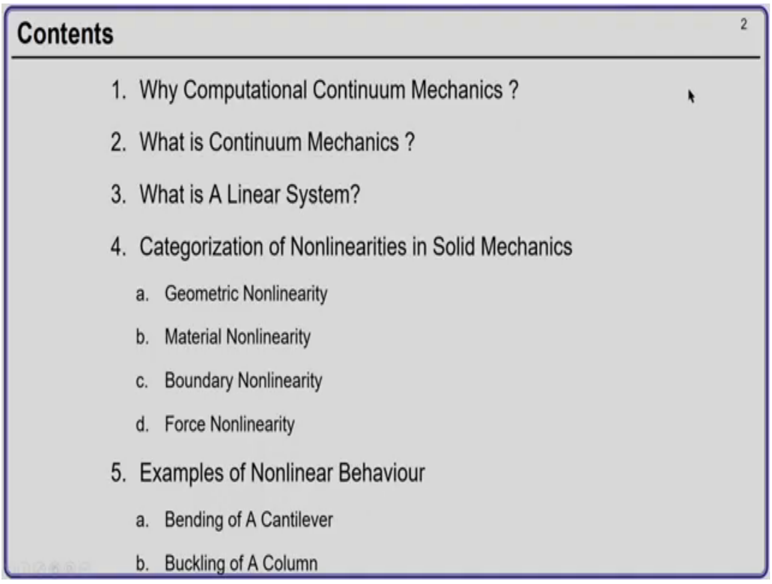

So, the contents of the first two lectures are as follows. First is, why we should learn computational continuum mechanics? So, this question will naturally arise because of the title of the course. And next we will look into what is meant by quantum mechanics?

So, this is not a course on continuum mechanics, so but to give you a glimpse of actually how important the field of continuum mechanics is with respect to computational mechanics, we will look into just a brief details about the flow charts, and what the in fields encompass computational mechanics in this particular topic.

Next before actually going into nonlinear system, we first should look into what is meant by a linear system ok. So, after we have done this, we will go into characterization of some of the nonlinearities which are present in solid mechanics problem.

So, we will just deal with some of the nonlinearities which are present in solid mechanics problem. So, we look into what is meant by geometric nonlinearity, what is meant by material nonlinearity, what is meant by boundary nonlinearity, and finally, what is meant by force nonlinearity.

And finally, we look into some of examples – very simple examples of how nonlinearity can come into very simple systems. For example, bending of a cantilever beam which will be the first example that we will look into, and also the buckling of a simple column. So, with this content in mind, we proceed to our first topic which is why one should learn about computational continuum mechanics.

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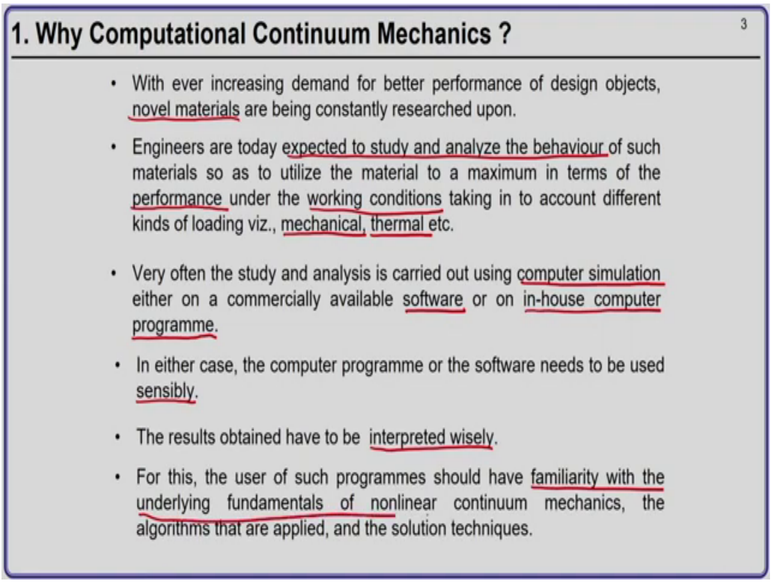

So, as you are aware that with increasing demand every day, there is always say a demand about better performance of the design objects like for example, cars for which very novel materials. So, which we have I highlighted here, novel materials are to be constantly researched upon ok. So, to improve the performance of a certain design object like cars, new materials have to be designed ok.

So, and, we, as a engineers are expected to study. So, we are expected to study and analyze the behaviour of such materials ok, so that we can utilize the material to a maximum in terms of its performance ok. So, in terms of the performance, I mean we want to extract the maximum performance of the material under various working conditions ok.

So, there are various working conditions under which for example a design object may work ok. It may be subjected to what is called the mechanical loads, thermal loads, etcetera ok.

Now, currently most of this study and analysis is carried out using what is called what is done on computers so which is called computer simulation ok. So, either one uses what is called the commercially available softwares ok. So, they are commercially available softwares, and many a times these softwares do not have those capabilities for new materials ok. So, at that case people tend to develop their own in-house computer codes ok.

So, either you will be using some software or you will be using an in-house computer code you which you yourself would have been coded or you would have got from your own seniors ok. And you need to build upon those codes. So, in either case these computer programs have to be used very sensibly. So, this word is very important using the software or the computer program very sensibly ok.

And the results that are obtained have to be interpreted very wisely ok. So, you have to interpret the results very wisely. Whatever you are getting out from the computer program or from the software, there will be, those will be some numbers and those numbers have to be interpreted very wisely ok.

And how you can do that? For this, the user of such programs should have familiarity with the underlying fundamentals of nonlinear continuum mechanics the algorithm which are applied ok, and various solution techniques that are available inside these softwares, or these in-house computer codes.

So, for example, in a computer code in-house, you will have these algorithms or the solution techniques already available or you may have to code it yourself ok. So, before you could apply these algorithms and solution techniques, you should be very familiar with the underlying fundamentals of the nonlinear continuum mechanics ok.

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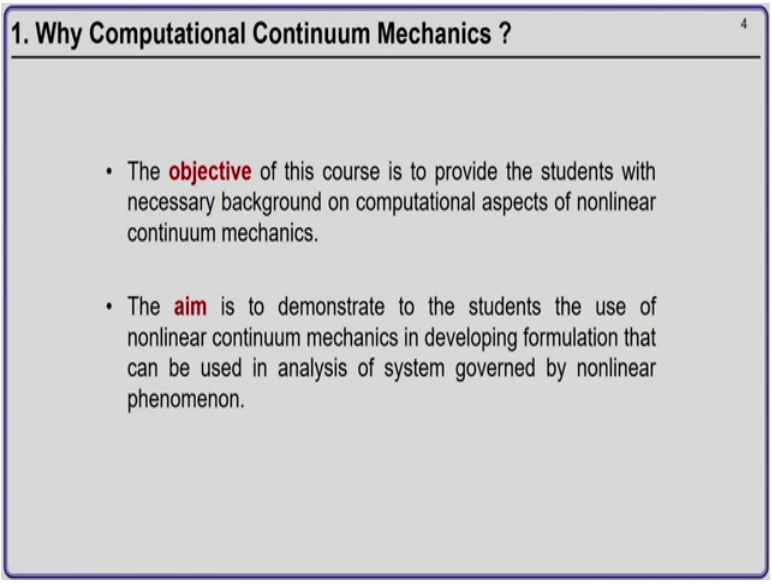

So, the objective of this course hence is to provide the students with necessary background on computational aspects of nonlinear continuum mechanics. And the aim is to demonstrate to the students the use of nonlinear continuum mechanics in developing formulation that can be used in the analysis of system governed by nonlinear phenomenon ok.

So, we are not looking into systems which are governed by what is called linear phenomenon. We will demonstrate to the students how the concepts of nonlinear continuum mechanics can be used in developing the formulation for the analysis of systems which are governed by this nonlinear phenomena ok, so that is our aim.

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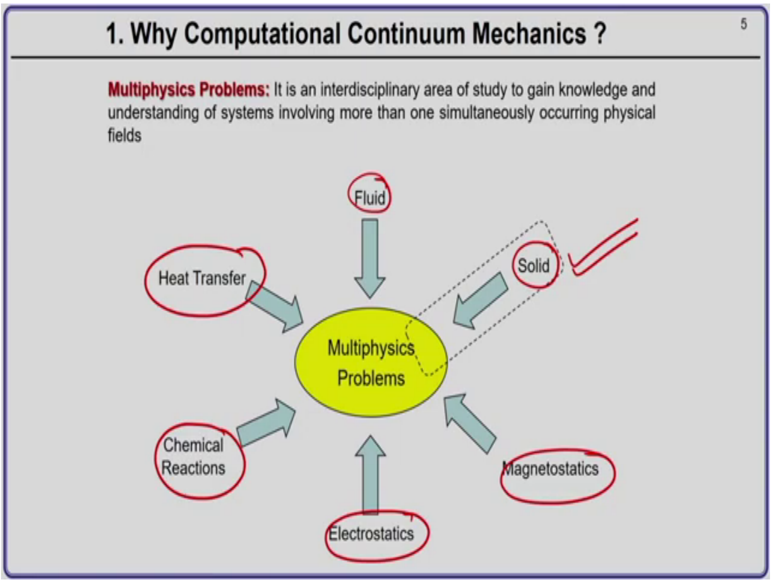

So, before we move into what is meant by computational continuum mechanics, we first look into definition of what is a very prevalent term these days which is called multiphysics problems. What are multiphysics problems? Multiphysics problems are inter disciplinary problems ok, where you need to study systems or problems where more than one simultaneously occurring physical fields are present.

For example, you could have a body which is undergoing large plastic deformation ok, so you would be having plastic deformation. You would be having body which is deforming, and also because of this deformation of the body because of the plastic work there is a lot of heat generated inside the body. So, you have thermal fields ok, thermal fields and you have mechanical field. So, you will have a thermo mechanical problem. So, there are many such problems where different fields come into picture ok.

So, this figure here. So, at the center you see that is what is called multiphysics problem domain ok. And this multiphysics problems can have input from for example fluids can have input from solid mechanics, they can have inputs from magnetostatics, electrostatics, chemical reactions, heat transfers like this ok.

This is not an exhaustive list, but more or less you can have different kind of physics that can come into picture. So, because of the limited time duration of this course, we will look into only the solid mechanics part of the multiphysics problem. So, we are not going to deal into the completely into multiphysics problem, we will just look into one aspect where the nonlinearity is because of the solid mechanics field ok.

So, next we come into what is continuum mechanics. So, you will see that the title of this course is computational continuum mechanics. So, computational continuum mechanics that the last two words stand for I mean our continuum mechanics ok. So, what is continuum mechanics?

There is a whole course which can go on continuum mechanics, and we are not going to dwell much into continuum mechanics. But before we dwell into some aspects of continuum mechanics, we need to see the unified picture of what continuum mechanics stands for.

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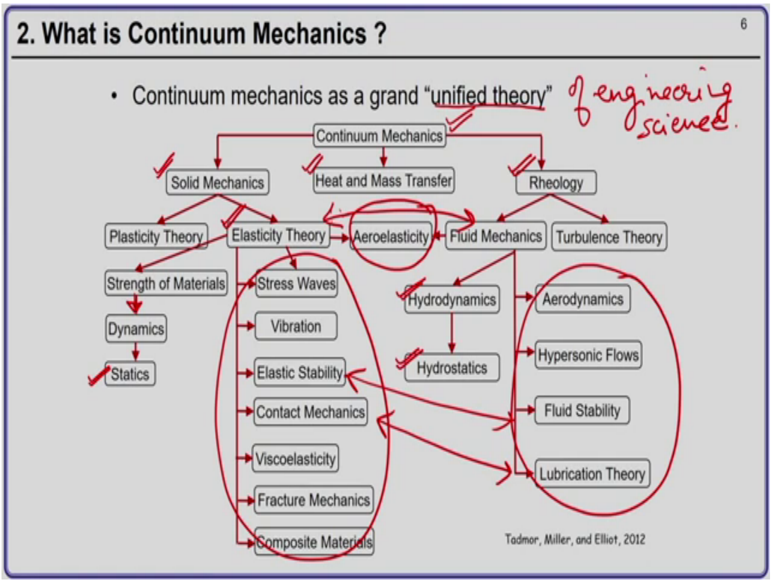

So, continuum mechanics can be seen as a grand unified theory of engineering science ok. So, the flow chart that you see here, we have put continuum mechanics at the very top ok. So, now continuum mechanics the governing equations ok. So, the governing equations are continuum mechanic of fully coupled ok.

It does not make distinction between what is solid or what is fluids ok. So, there are one set of equations; and the governing equation describe the behaviour of both solids and fluids. At the next level, you will see we have put solid mechanics, heat and mass transfer, and rheology which is like study of flow of complex fluids.

So, solid mechanics is a branch of continuum mechanics where we deal with deformation of solids ok. And heat and trans heat and mass transfer is a field where we study the heat and mass transport across rigid bodies. So, solid mechanics we will deal with deformation of the bodies; heat and mass transfer we study the heat and mass transfer to rigid bodies. And rheology is where we study the flow of complex fluids.

Further down, if you see we can have some more assumption. So, one of them assumption in solid mechanics is that the deformations are very small for example, and then you can use what is called derive what is called elasticity theory ok. So, in elasticity theory, you can have the body or the solid which is behaving elastically ok.

So, the relation, so the deformations are not too large; and the relation between stress and strain are basically elastic which means once you apply the force the body deforms; and once you leave the force remove the forces the body comes back to its original shape ok.

So, this elasticity theory can be further simplified into what is called strength of materials ok. And then if you further simplify what you get is called dynamics ok. So, dynamics is where the deformation of the body is completely taken of ok. So, you will do not consider that the body is deformable, and we consider that body is totally rigid ok.

And once you dynamics itself can be simplified further by removing what is called the time component. Once you remove the time component from the dynamics equations what you get is called statics ok.

So, the rheology coming to the other part, other side; the rheology can be broken into simplified into fluid mechanics ok. So, fluid mechanics is usually in terms of I mean we deal with Newtonian fluids ok. And this can be further simplified into hydrodynamics and hydrostatics which are the counterpart of statics and dynamics from the solid mechanics sites ok.

And then directly under the elasticity theory, you will see there are various courses where subjects like stress waves, vibration, elastic stability, contact mechanics, viscoelasticity, fracture mechanics, composite material, which deal with various aspects of solid mechanics under certain assumptions ok. And the same goes with fluid mechanics we have aerodynamics, hypersonic flows, fluid stability, lubrication theory. So, this aerodynamics and hypersonic flows can be taken like they are the fluid mechanics counterpart of stress waves and vibration; the fluid stability is the counterpart of elastic stability – so these two. The lubrication theory for example as a counterpart of contact mechanics and solid mechanics.

You can also have in aerospace engineering we have something like aeroelasticity which is studying of aeroelastic behaviour which combines elasticity theory with the fluid mechanics theory ok. So, you have fluid structure interaction kind of studies which is part of aeroelasticity subject. So, you can see that continuum mechanics as a whole encompasses all of these fields ok. So, a very good understanding of continuum mechanics is usually required to study as a whole these subjects ok.

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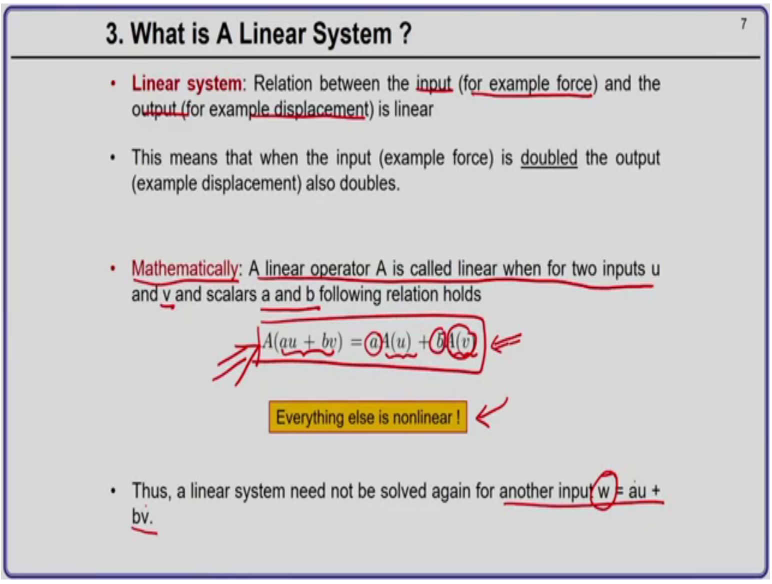

Next we come to what is called linear systems ok. We define linear system. We have been we have been mentioning a lot about nonlinearities, but the term nonlinearity it will contains the term linearity. So, first we have to look into what is meant by linearity, what is meant by linear systems ok. So, by definition a linear system is one where the relation between the input ok, for example, force and the output which is for example, the displacement is linear. So, there is a linear relation between the input and the output.

So, this means that what does linearity means it means that once you double the input, say for example, you double the input; you have applied a certain amount of force. Now, if you double the force, so from f say you go to 2 f, then the output the displacement u say the displacement is u at a point, then the displacement will go from u to 2, 2 u ok.

So, in a linear system mathematically what you can write a linear operator A is called linear when two inputs u and v, and these inputs u and v can be scalar can be vectors can be tensors ok. And so you have two inputs u and v and given two scalars a and b the following relation will hold.

The operator A when it operates on input linear combination of input u and v which is given by a u plus b v the output that you get is the linear combination of the outputs of when the operator A operates on the input u and operator A operates on the input v.

So, if this condition which is here shown in the equation is not valid, then what you get is a nonlinear system. If this condition does not hold you get what is called a nonlinear system ok. So, you would guess from here that say if you have already solved this system for two inputs u and v.

Now, if somebody gives you another input which is w which is au plus bv. So, you need not solve this system again for the getting the solution corresponding to this input u. All you need to do is apply this linearity principle which is this equation you know the solution for the inputs u and inputs v. So, all you need to do is multiply the output corresponding to u by scalar a, and the output corresponding to v given by a u with b to get the final output ok.

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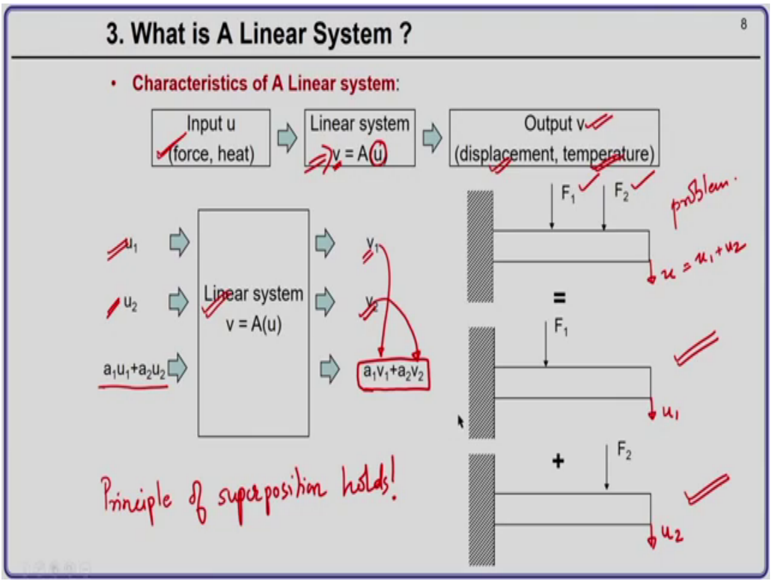

So, some of the characteristics of a linear system is shown here. So, you have an input u ok, you have an input u in terms of maybe force or heat. And you have a linear system which for an input u gives you an output v ok. So, the output that you get may be in terms of displacement or temperature ok.

So, this figure shows here. So, you have a linear system where you have an input u 1. Once this input u 1 is given to this linear system, you get an output which is v 1 ok. You have another input say you give another input u 2. You get an output which is v 2.

Now, if you give a linear combination of these two inputs u 1 and u 2 which is given by a 1 u 1 plus a 2 u 2. So, you know the output of input u 1 is v 1; the output of u 2 is v 2. So, we applying and we know that the system is linear ok.

So, if we get an input of the form a 1 u 1 plus a 2 u 2, we note that because the system is linear we need not solve the whole system again to get the solution. All we need to do is we need to just take the linear combination of the outputs v 1 and v 2, where the factors the coefficients of v 1 and v 2 are a 1 and a 2 ok.

So, how I mean where can this linear system we apply there is one example say consider you have a cantilever beam. And there are two point forces F 1 and F 2 which are acting on this cantilever beam at a certain location ok, two different locations. For simplicity, we have just taken point forces. It can be a combination of distributed load or point forces or different distributed loads ok.

Now, this system can be thought of as equivalent to a system cantilever beam where only the load F 1 acts, and another system where only the load F 2 acts. If you know this solution of the first case, and if you know this solution for the second case, the solution for our problem which is given here can be applied ok.

See you want to find out the displacement at the tip u. So, now, if you know the displacement at the tip u 1 in the first case, and you know the displacement of the tip and the second case u 2, so the final displacement u will be nothing but u 1 plus u 2 ok.

So, a very important characteristic of a linear system is that the principle of superposition holds ok. So, for linear system, this principle of superposition, so you can superimpose two different solutions to get another solution which is can be thought of as break up of two different solutions ok.

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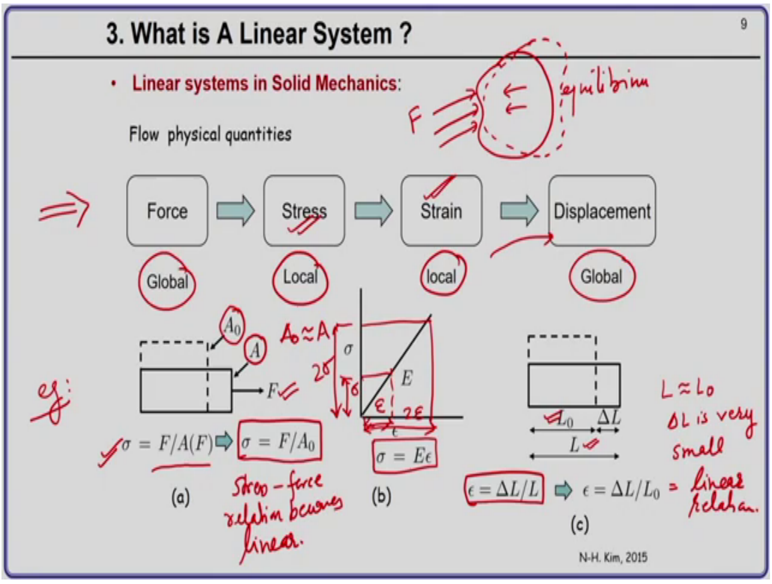

Now, next we come to how linear system manifest themselves in solid mechanics ok. So, the top figure shows here the flow of various physical quantities ok. Say you may have a body ok, and you may apply some force on the system. So, this force is applied in what is called global sense ok. If you are applying the force globally ok, this results ok, so the body will deform. So, you guess that the body will deform.

And at certain point the body will try to come in equilibrium with the external forces F. And how will this happen the body will generate stresses ok. So, these stresses are generated at the local level at each and every point locally inside the domain or inside the body, we will have stresses which are generated.

And these stresses will be generated through what is called generation of strain ok. So, you will have strains which will get generated. And these strains will manifest themselves in the form of displacements ok.

So, you have body here; finally, when the body comes into equilibrium if you will have something like this dotted configuration as the equilibrium configuration of the body. And these displacement will manifest themselves globally. So, you can actually see the body has changes its position ok.

So, for example, and for linear systems we take an example of this uniaxial tension of a bar ok. So, consider you have a bar shown by these dotted lines. The initial area is A 0 – cross sectional area is A 0.

And you apply a force F ok. So, what would happen? The length of the bar will change. And because of the Poisson's effect, the area of the bar will also change ok. So, let the after the equilibrium is achieved, let the area be A ok. So, the initial area is A 0; and the current area after the equilibrium is achieved is A.

So, what is stress? So, stress is given by force divided by area A ok. So, stress will be force divided by area A. Now, this area A itself depends on the force F ok. So, larger the force I mean correspondingly the area become smaller. So, there is a relation between force and area. But if the force is small enough, so that A 0 is nearly equal to A ok. So, the cross sectional area before the deformation is nearly equal to the cross sectional area after the deformation. So, one can write that the stress is equal to force divided by the initial area A 0.

And then the relation between the stress and the force ok, so this relation between the stress and the force become, stress force relation now becomes linear ok. So, this holds. So, this linear relation holds only when the initial area and the final area are nearly the same ok.

Coming next to the stress-strain relation ok. So, for many metallic materials, you would have observed that the stress-strain relation has a linear part ok, where the stresses are related to strain by this relation sigma equal to E epsilon ok. So, what this means is if the stresses are doubled ok, so if you have initial stress sigma and then if you have another stress which is 2 sigma.

So, what the strain here if you have epsilon corresponding to sigma, then if you change this stress to 2 sigma, the strains that you will get will be 2 epsilon ok, so then you will have a linear relation between stress and strain.

Coming to the strain displacement relation ok. So, as you pull the bar the length of the bar changes let the initial length of the bar be L 0, and let the change in the length be delta L, so that the final length of the bar is L ok. So, the strain can be defined as the change in length divided by current length ok.

Now, if the deformations are very small which means that L is nearly equal to L 0, the bar as delta L which means delta L is very small, in that case L can be approximated. So, in this equation the length L, final length L can be approximated by its initial length L 0. In that case, you will see that this relation L 0 epsilon is delta L by L 0 is a linear relation ok.

So, in a linear system in solid mechanics, you would have a linear relation between stress and forces, you will have a linear relation between strain and stresses, and you will have a linear relation between displacements and strains ok. So, if any one of these relation becomes nonlinear, it does not follow the principle of linearity which means that principle of superposition cannot be applied, in that case your system will no longer remain linear, it will become a nonlinear system ok.

So, for example, you could have a linear strain displacement relation, you could have a linear force stress relation, but the relation between stress and strain will become nonlinear ok. In that case, you cannot apply the principle of superposition to get the solution of your system. You cannot apply the principle of linearity. You have to go for what is called nonlinear principles, you have to apply nonlinear principle.

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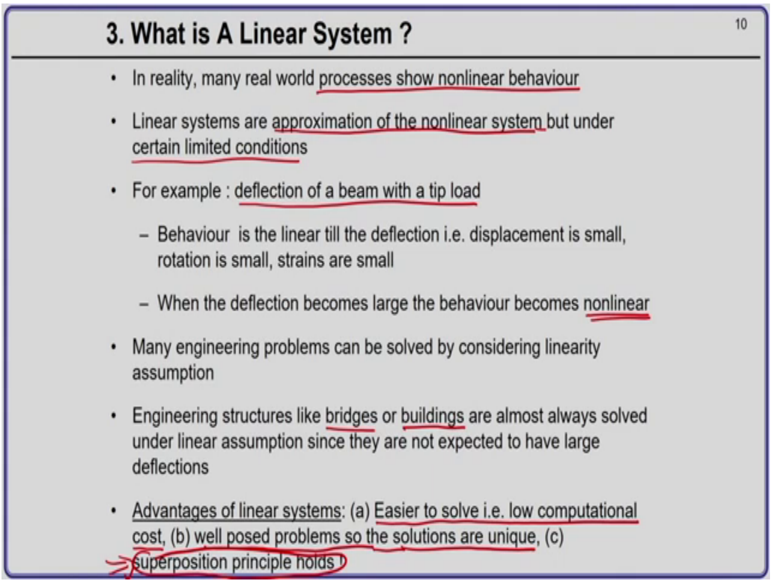

So, summarizing, we can say that many real world problem they show nonlinear behaviour ok. So, almost all the real world problem they show nonlinear behaviour. So, these linear systems are only approximations of the nonlinear system under only certain limited conditions ok.

So, if you know how to solve a nonlinear system ok, you can always solve any problem ok. But if you know nonlinear system, but you also know that there are certain conditions which hold ok, then instead of solving that nonlinear system you can as well solve the system using linearity principles ok.

For example, deflection of a beam with a tip load ok. So, behaviour of this cantilever beam under the action of the tip load will be linear till the deformation is small ok, the deflection is small which means the displacements are smalls, and the rotations are small, and the strains are small. If the deflection becomes too large ok, then you cannot apply the linear system you can apply cannot use the linear approximations to solve the get the solution of this system. So, what you need to do is apply the nonlinear principles ok.

Now, many engineering problems can be solved simply by applying the linearity principle. For example, engineering structures like bridges and buildings, they are almost always solved using linear assumptions, because we do not expect these engineering structures to have a very large deflection ok. So, you can initially study this system by applying linearity principle. So, there has to be a certain advantage of solving a linear system over nonlinear systems.

So, if you want to solve a system using linearity principle, what are the advantages that you get? So, the first advantage that you have is these systems are easier to solve, that means, the computational cost which means the time and the amount of resources – computational resources, the computed time that you need to solve this systems are very small ok. It is much easier to small.

And linear problems are always well posed which means their solutions are unique, you will always get a unique solution. You will not encounter cases where at a certain point you will have two different solutions possible ok. So, in nonlinear system such cases arise ok.

So, in linear systems, you have well posed problems and the solutions are always unique ok. And then very importantly the principle of superposition always holds ok. So, if you know the solution for given inputs, and then you are given a new input which can be expressed as a linear combination of the known inputs for which the output is known. You can without solving the system again you can guess or you can find out the solution of the this new input using the principle of superposition ok.

Now, there are many cases where linear system assumption will lead to erroneous results ok. The system may seem like you can solve this system using linear assumption ok. I mean this system looks like everything is linear I will just apply linear principles and I will get this solution, but the solution that you will get will not be correct, I mean you will have certain error, and this error may be very large.

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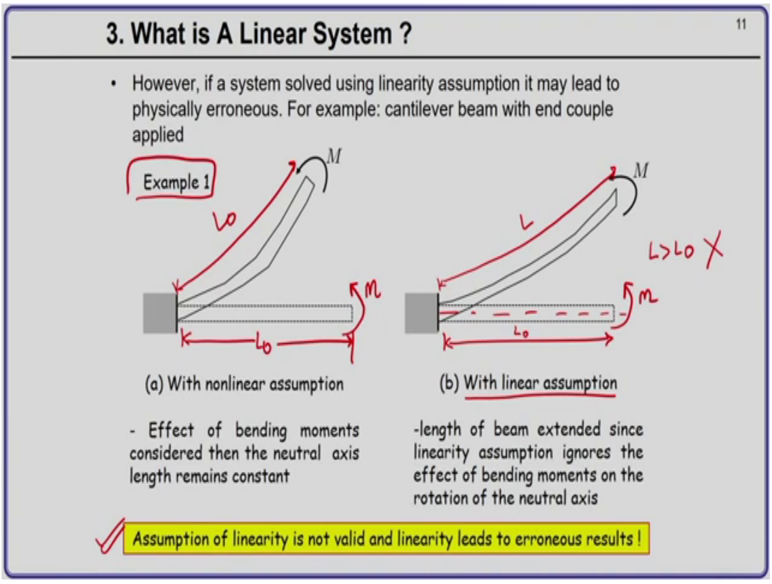

For example, we have shown here ok, one example where you have a cantilever beam and the initial position of the cantilever beam is shown by dotted line. Now, you apply a moment M at one end. What will happen? If you take the linear assumption ok, because in linear assumption the effect of bending moment on the rotation of the neutral axis. So, you will have a neutral axis of the beam ok.

So, the bending moment effect is not taken into account on the rotation of the neutral axis. So, what you will get is you will get the stretching of the beam ok. So, in this figure if you see the length of the beam L ok, so if the initial length of the beam was L 0, so you will get stretching of the beam. Now, this is not correct.

Now, if you take nonlinear assumption, because you can see I mean the deflections are not small here. Now, if I take nonlinear assumption which means I take into account the effect of the bending moments on the length of the neutral axis, the solution that I will get is something like what is shown in figure a here, where the length of the beam will remain constant. And this is what you will get when you use nonlinear assumption in solving this kind of problem.

So, I have shown here in this yellow box this statement that assumption of linearity is not always valid, and linearity may lead to very erroneous results ok. So, you have to be very careful when to apply linear assumption and when to consider nonlinear effects into account ok.

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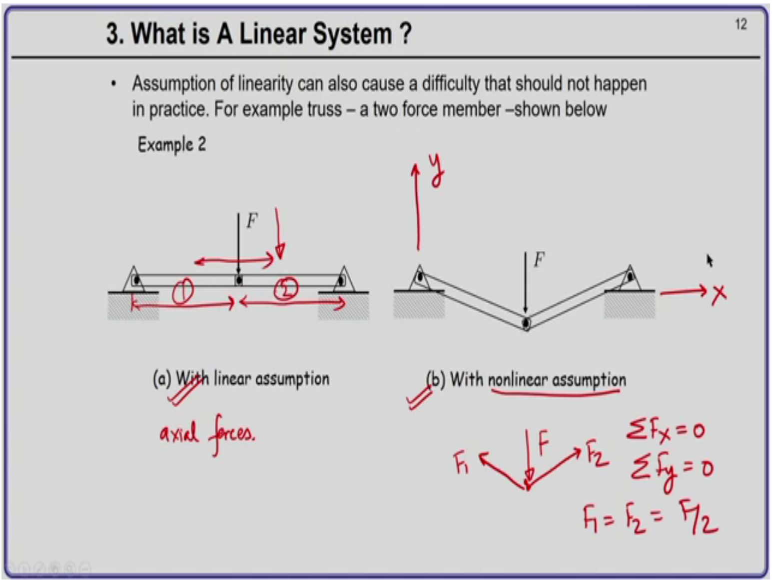

Now, one more example that I show here. So, this is two four I mean a truss ok. So, there are two members in this truss ok. You can see here this, this is member one and you have another member which is two they are both horizontal their pin jointed at the ends. And at the middle joint you have a force F, vertical force F which is applying ok.

Now, if you solve try to solve this problem using linear assumption, what will happen you will not be able to solve this problem. Why, this happens because the truss members 1 and 2 ok, they are all they can only take what is called axial forces ok, they can only take axial forces. So, when you try to take linear assumption at you in the truss members, you will only have axial forces ok.

Using this axial force, these axial forces, it is impossible to set equilibrium with a vertical force F. So, your axial forces are horizontal and you have a vertical force F. Now, you cannot equilibrate the horizontal and the vertical forces. So, under linear assumptions, you will not be able to solve this problem ok.

Now, if you take a nonlinear assumption and you take you allow the deformations to become large, in that case you will get what is shown here in figure b ok. So, at the joint pin joint ok, where the vertical force F is applied and we will have two forces F 1 and F 2 corresponding to truss members 1 into acting ok.

So, you can balance say the horizontal is x direction and the vertical is the y direction. So, you can balance the forces in the x and y direction, and you can show that F 1 will be equal to F 2 will be equal to F by 2 ok. So, with this you will be able to solve this truss problem.

So, here using this very simple you can see that problems which may look like very amenable to linear systems cannot be solve using linear assumption ok. Even for this simple problem, you have to go for what is called nonlinear assumptions taken into account. So, here you have to enable what is called nonlinear effects large deflection effects have to be considered, then only you can solve this problem.

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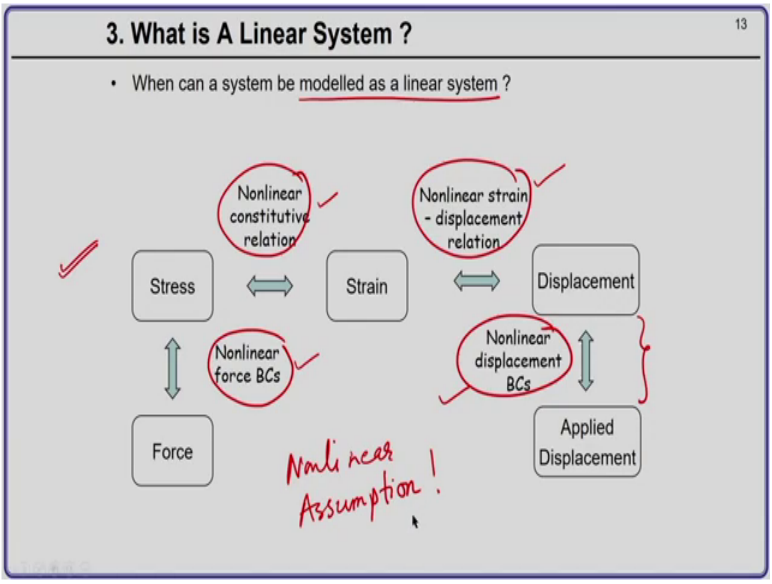

So, moving to the final slide on what is a linear system. So, when can you model a system as a linear system ok? So, this figure here shows the relation between different physical quantities in a solid mechanic system ok. So, you have relation between force-stress, you have relation between stress and strain, you have relation between strain-displacement, you have relation between displacements and the applied displacement ok.

Now, you can have if any of the relation here between any of these physical quantities ok. So, say for example, you can have nonlinear displacement boundary conditions ok. So, you may have a linear relation between stress and strain, you can have linear relation between forces and stresses, you can have linear relation between strains and displacements ok. But if you have a nonlinear relation between displacements and the applied displacements ok, you have nonlinear displacement boundary condition. In that case you cannot model your system as a linear system ok.

So, if any one of these relation becomes nonlinear ok, if you have nonlinear force boundary conditions, or you have nonlinear constitutive relations, or you have nonlinear strain-displacement relation, or you have nonlinear displacement boundary condition, if any one of these relation is nonlinear then you cannot use or you cannot model your system as a linear system.

You have to go for nonlinear assumption ok. We have to go for what is called nonlinear assumption ok. So, it is just perfectly fine if you have only one nonlinearity present into system to make the entire linearity assumption go out of the window ok. So, even one is present, you no longer have linearity assumption valid yeah.

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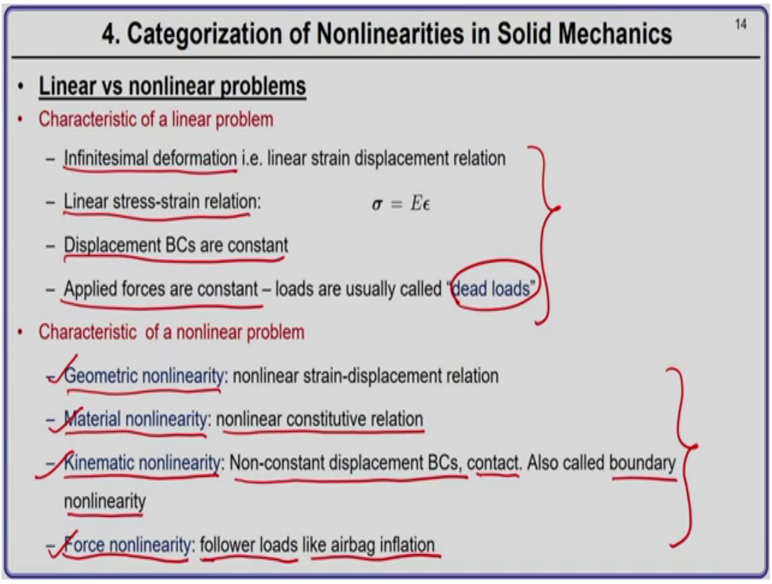

So, to characterize a linear and a nonlinear system ok, so what are the characteristics of a linear problem? In a linear problem, you will have infinitesimal deformation which means that the relation between strain and displacement will be linear, you will have a linear stress-strain relation.

You will have boundary conditions displacement boundary condition which will be constant; they will not change over time. And the applied forces will be constants. So, these kind of forces will be called dead loads ok. So, for a linear problem you will have these four conditions which will hold all the force have to hold simultaneously, so that you can apply the principle of superposition, you can have the linearity principle ok.

Now, what are the characteristics of a nonlinear problem? In a characteristic of a nonlinear problem is you will have nonlinearity present into the system. So, you may have a nonlinear strain-displacement relation ok. So, this kind of nonlinearity is called geometric nonlinearity ok.

You may also have nonlinear constitutive relation ok. So, the relation between stress and strain usually is called constitutive relation. So, this constitutive relation between relation between stresses and strain will be nonlinear. In that case you will have what is called material nonlinearity ok.

Additionally, you can have non constant displacement boundary conditions like contact ok. In contact problem, you can have non constant displacement boundary conditions ok. So, you can, so this kind of nonlinearity is called kinematic nonlinearity or also called boundary nonlinearity ok. Finally, the loads that you apply on the system will no longer remain constant.

Remember that force is a vector. So, a vector has a magnitude as well as direction. So, the magnitude of the force may remain constant, but the direction of the force may depend on the deformation itself. In that case you will have what is called follower loads; loads which follow the deformation such as in airbag inflation ok.

So, the way when you have a car which bumps against some other obstacle if you have air bags which are deployed. So, during that study of this air bag problem deployment of this air bag, you will have to consider what are called follower loads, loads which depend on the deformation. So, this kind of nonlinearity is called force nonlinearity ok.

So, you have for a nonlinear problem you can have either of these four nonlinearities present or you may have combination of these nonlinearities usually present in the system. So, these four nonlinearities are geometric nonlinearity, material nonlinearity, kinematic nonlinearity, and force nonlinearity ok.

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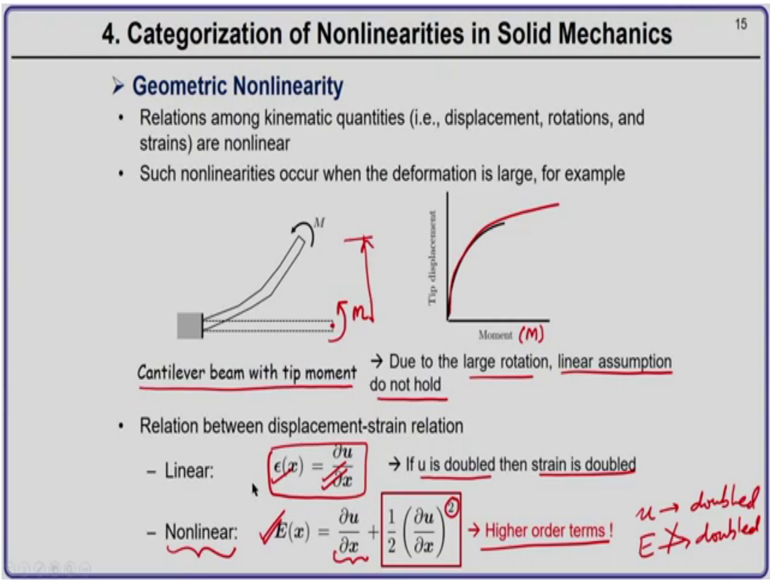

So, with this we come to the first kind of nonlinearity that we have ok. And this first nonlinearity now we look into each of these nonlinearity in detail, and the first one is what is called geometric nonlinearity ok. Obviously, I mean, we will not dwell too much into these nonlinearity which we will see later in the course; but here we will just give you an idea about what this nonlinearity stand for.

So, in geometric nonlinearity, the relation between the kinematic quantities. What are kinematic quantities? Quantity like displacement, rotations, and strains, these are called kinematic quantities. So, the relation between these kinematic quantities become nonlinear ok. And such kind of nonlinearities occur usually when your deformation is very large ok.

So, again you should take the problem of a cantilever beam with a tip moment. So, you have a cantilever beam, you apply a tip moment, and then the cantilever deforms, and the deformations are very large. The deflections you see here, the deflections are very large. The initial position which is shown by the dotted line here and the final position which is by the solid line are very different.

So, if you plot the tip displacement, so this is a tip here. If you plot the tip displacement against the moment M ok, the moment that you have to apply to get a certain tip displacement, you will see that this curve follows is of a nonlinear nature ok. This is because you have large rotations, and hence your linear assumptions are not valid anymore ok.

So, in linear strain-displacement relations ok, you will have strains which are given by the derivative of displacement with respect to position x ok. So, the strains are del u by del x. Now, this is a linear relation. So, if the displacement is double, then your strains are double ok.

But in geometric nonlinearity, you have to consider strains which have not only have this linear term, but they will have one additional higher order term which here is 1 by 2 del u by del x the whole square ok. Because of this quadratic term ok, we will have what is called the nonlinear strain. So, in this nonlinear strain measure strain, if you double the displacement ok, if the displacement is doubled ok, your strain is not doubled. So, this relation does not hold ok.

So, doubling the displacement need not necessarily lead to doubling of the strains ok. So, notice that we have used two different symbols for strains, epsilon for linear strain in capital E we will see later when we are discussing the kinematic aspects that this capital E stand for what is called the Green Lagrange strain tensor ok. So, there is a different symbol that you use ok.

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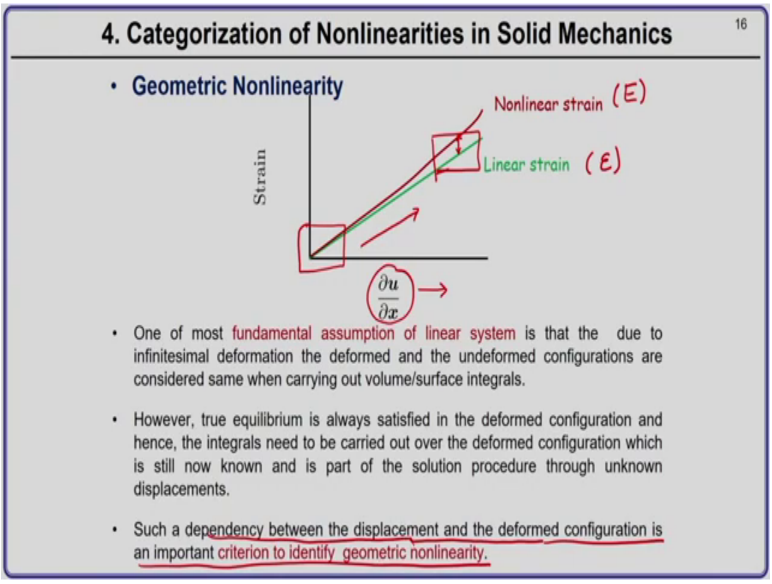

Now, if you plot both of these strain measures ok. So, the red one is your capital E which is a nonlinear strain, and the green one is your linear strain. See if you plot these strain with del u by del x. You will see that initially for very small values of del u by del x that is in this zone, you will see that there is not much difference between if you use nonlinear strains or if you use linear strains ok.

So, for very small values of del u by del x, you will have both these strains measures E and epsilon will give you the same value of strain. But as del u by del x increases ok, so as a del u by del x increases you will see that the later part of this graph, you will have some significant difference between the values given by these linear and nonlinear strain measures ok. So, there is an appreciable difference between these two strain measures ok.

So, one of the fundamental assumptions of the linear system is that the deformation is only infinitesimal that is this very small ok. So, when you apply forces on the system and when the system deforms, the final configuration after the equilibrium has been achieved and the initial configuration, they are very close to each other ok.

Hence I mean we will see later that we have to carry out integral over the deformed configuration. So, in linear system, the integrals which have to be carried out over the deformed configuration can very well be carried out over the undeformed configuration which is known ok.

And because they are very close to each other, this does not lead to too much of a error ok. So, that is what we do in linear finite element where the integrals are always carried out over the undeformed configuration.

However, you will know that equilibrium is attained in the deformed configuration. So, in a very strict sense, your integrals ok, all the integrals involved in the finite element formulations have to be carried out over the deformed configuration. What is the problem there? Problem is the deformed configuration itself is not known ok.

So, you cannot carry out this integration because the configuration itself is not known. How can you know the configuration? You first have to solve for the displacement. Once you have solved for the displacements and you add these displacement to the initial configuration, you will get the final configuration ok.

So, this dependency so that is what I have written at the end this dependency between the displacements and the deformed configuration is an important criteria to identify the geometric nonlinearity ok. So, if there is a relation dependency between the displacement and the deformed configuration, then you have what is called geometric nonlinearity present in the system ok.

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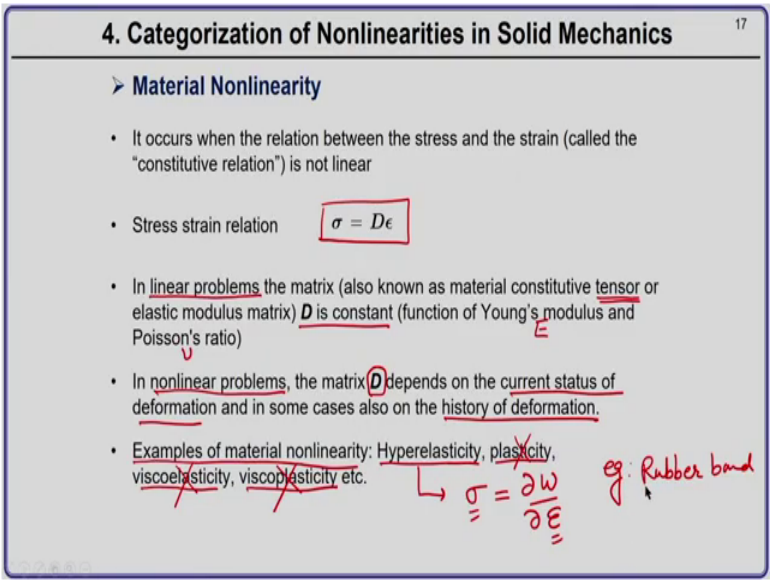

So, the next nonlinearity that we consider is the material nonlinearity. So, what is material nonlinearity? It is when the relation between the stresses and the strain. So, the relation between stresses and strain is also called the constitutive relations ok. So, these constitutive relations are not linear anymore ok.

So, you cannot simply write sigma equal to some constant time epsilon ok. And I am using very generic term here constant, but this constant can be a matrix or something. So, usually the stress-strain relation is given by sigma equal to D epsilon ok. So, D here is called the material constitutive tensor.

So, I am not described the word tensor till now we will come to it in the third lecture, but for the time being you just note that these called the material constitutive tensor or elastic modulus matrix ok. So, for linear problems, this matrix D is constant is just a function of Young's modulus E and the Poisson's ratio u ok. And these values are fixed ok. So, it makes this tensor D as a constant tensor ok.

So, once you know the strain, you can get the stress. You have new strain which can which is it says double of the original strain. So, the new stress you need not do D epsilon. You can just multiply the previous stress by two to get the new value of the stress. However, in the nonlinear problems this matrix D or the tensor D ok, material constitutive tensor D depends on the current status of deformation and also in some cases may depend on the history of deformation ok.

So, how do you reach that deformation? What path did you take to reach this deformation? It, so this matrix D depends not only on the current state, but also depends on the history of how you achieve that state ok. So, some of the examples of nonlinear material behaviour are hyperelasticity, plasticity, viscoelasticity, viscoplasticity, etcetera ok. So, these are different kind of nonlinear material behaviours.

So, in this course, we will not deal with plasticity, or viscoelasticity, or viscoplasticity. We will deal with what is called hyperelasticity where these stresses can be obtained from a stored strain energy density potential ok. See have a stored energy potential, I mean we will come to it. If you take the derivative of that potential with respect to strain ok, we will get what is called stresses ok.

So, an example of this, yes, example of this hyperelasticity is say when you are say extending a rubber band ok. So, you have rubber band, I mean, it is made of rubber which are which is a hyperelastic material.

You extend, I mean you can stretch a rubber band twice or thrice its length, and once you have leave this forces, it will come back to its original shape. So, there is some elasticity because coming back to the original shape, but the relation between stress and strain is not linear ok. Then hence we call this kind of behaviour as hyperelastic behaviour ok.

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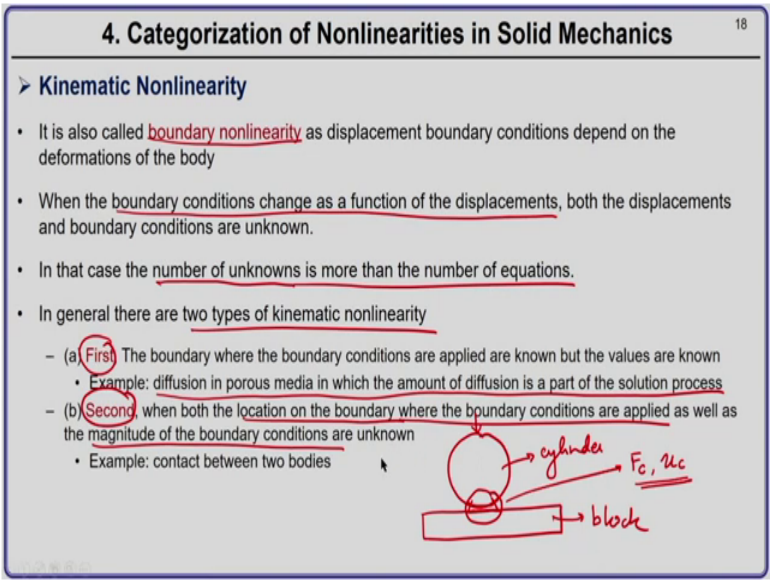

So, the next kind of nonlinearity is your kinematic nonlinearity. It is also called the boundary nonlinearity ok. So, this is where your boundary displacement boundary condition depend on the deformation of the body ok.

So, when the boundary condition change ok, so when the boundary condition change as a function of displacement, both the displacements and the boundary condition become unknown ok. So, there may be cases where your displacement boundary condition depends on the displacement itself which itself is a unknown ok. So, in that case you will have what is called kinematic for a boundary in nonlinearity ok.

In that case of such happens, you may have number of unknowns which will become more than the number of equations that you have ok. For example, in contact problems you will have more number of unknowns than the number of equations that you have to solve ok.

So, if your number of equations are say n, you will have more than n number of unknown. So, you cannot definitely solve, so you need to do something extra ok. So, that is not part of this course, but you may keep in your mind that the number of unknowns may become more than the number of equations in kinematic, non if kinematic nonlinearity is present into your system ok.

So, there are in general two types of kinematic nonlinearity. The first one is where your boundary conditions ok, where the boundary where the boundary conditions are applied that is known, but the values are unknown ok.

For example, diffusion in porous media in which the amount of diffusion is a part of solution process; you know the boundary where the diffusion occurs, but you do not know the amount of diffusion which has to be part of the solution process. So, here you know the boundary, but the boundary conditions are unknown.

The second kind of kinematic nonlinearity is one where both the location on the boundary where the boundary conditions are applied as well as the magnitude of the boundary conditions are unknown ok. So, where does this occur? For example, if you are studying contact between two bodies ok. Even if you are studying contact between two elastic bodies ok, then you will have these kind of second type of kinematic nonlinearity.

So, here what happens you have say, you are studying the contact between say a rigid cylinder a deformable cylinder will a rigid block ok. So, you have a block, and you have a cylinder. And the block is much more rigid than the cylinder ok. So, if you apply a force on the cylinder. So, what would happen? So, this part is what is the contact zone ok. So, there two bodies come in contact.

So, what you, so this zone is where you have to apply the displacement boundary condition, but you do not know at how much area of this cylinder you have to apply this displacement boundary condition and also you do not know what value of the displacement you have to apply ok.

So, here at the contact zone, what happens? Neither you know the forces contact forces F c or neither you know the contact displacements ok. So, you do not know both of them ok. So, you have more number of unknowns than the number of equations ok. So, these second type of nonlinearity kinematic nonlinearity is very difficult to solve ok, and is much more complex ok.

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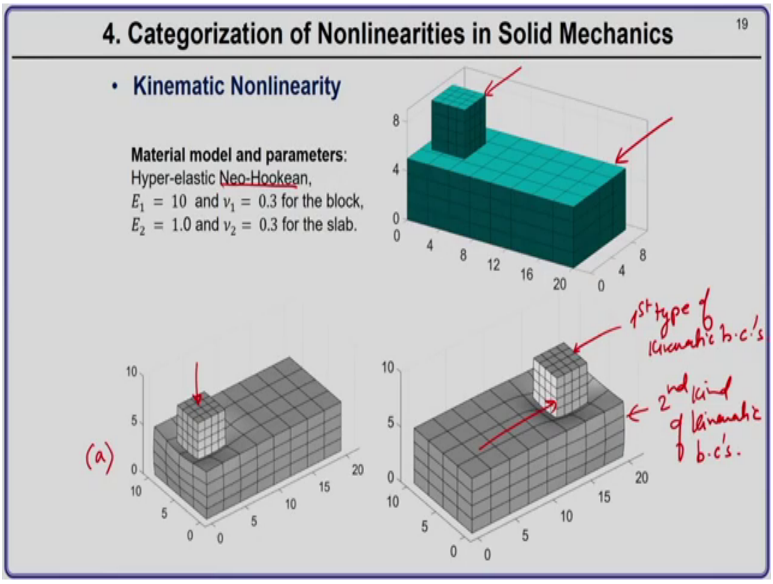

So, here I will show one example of kinematic nonlinearity. So, you have a deformable block ok. So, you see here you have a deformable block; you have another deformable block ok. And they both are made of hyperelastic material called a neo Hookean model and then this figure – figure a here shows you first compress the smaller block on the lower block ok. You see we apply.

So, here what happens the bottom block deforms ok. And then in the second part what we do, we drag this block ok. We drag the block top block against the bottom block. And you see the contact region changes. So, here there are very typical kind of kinematic nonlinearity.

On the top block, you know the contact zone you know the boundary where the displacement boundary conditions have to be applied, but you do not know the displacement. So, there on the top block you have the first kind of kinematic nonlinearity.

On the bottom block from the second figure you can know that as the lower block slides, the contact zone in the lower block changes over time ok. So, you do not know where exactly to apply the kinematic boundary condition ok. You do not know the where the displacement boundary conditions are applied, and what is the magnitude of displacement boundary condition.

So, for the bottom block you have the second kind of kinematic boundary condition ok. And for the top block you have the first type of; first type of kinematic boundary conditions ok. And these problems are very difficult to solve ok. So, usually if you have the second kind of kinematic boundary condition, these problems becomes very difficult to solve and very challenging, these are very challenging problems ok.