Fundamentals of Convective Heat Transfer
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Module - 04
Convective Heat Transfer in External Flows - II
Lecture - 11
Laminar BL flow over flat plate: Uniform wall temperature

Hello everyone. So, today we will consider Laminar Boundary Layer Flow Over a Flat Plate with Uniform Wall Temperature Condition. In last class, we derived the momentum integral equation, and we derived the hardening boundary layer thickness, and the coefficient of friction. In today's lecture, we will consider the heat transfer where the boundary is maintained at uniform wall temperature, and we wish to determine the heat transfer coefficient, and the Nusselt number.

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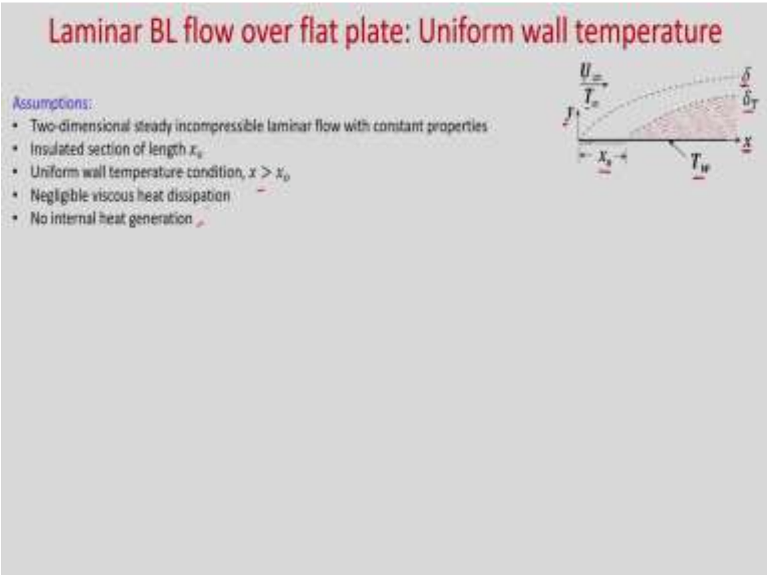

So, let us consider this flat plate x is in the in this direction, and y is perpendicular to the plate. You can see that up to $x = x_0$ it is insulated that means; there is no heat transfer from this surface. So, the free stream velocity is ∞ and free stream temperature T_∞.

So, in this region your wall temperature will be at T_∞. But from $x = x_0$ this wall is maintained at uniform wall temperature T_w. So, obviously, your thermal boundary layer thickness will start developing from x = x_0. And hardening boundary layer thickness obviously, will start developing from x = 0.

So, these are the assumptions we will consider, two-dimensional steady incompressible laminar flow with constant properties, insulated section of length x_0, uniform wall temperature condition T_w for x > x_0. We will neglect viscous heat dissipation and there will be no internal heat generation.

In today's lecture, first we will derive energy integral equation then we will put the temperature distribution and the velocity distribution in the energy integral equation and we will derive the expression for thermal boundary layer thickness, and then we will derive the local heat transfer coefficient and local Nusselt number.

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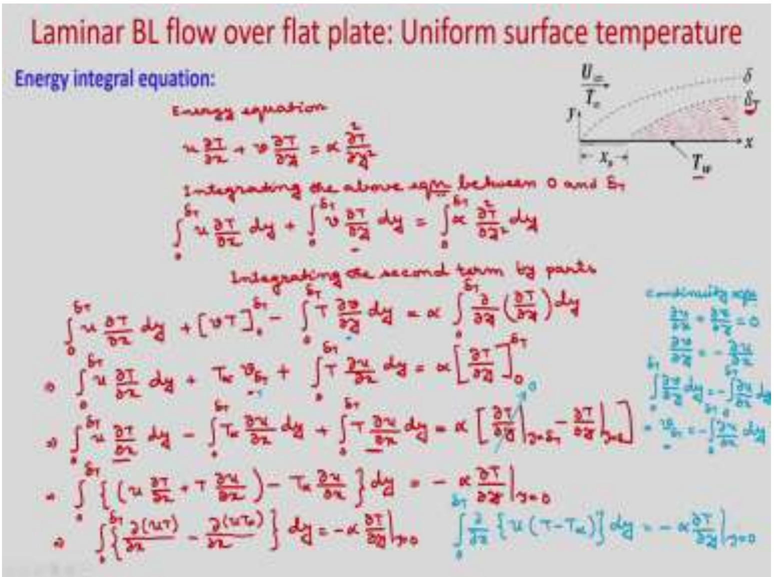

So, first let us derive the energy integral equation. So, we have the energy equation, you know $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$. So, let us consider today this flow over flat plate where from x =0 to x = x_0 it is unheated region, after that from x > x_0 you can see the flat plate is maintained at uniform wall temperature T_w.

So, velocity boundary layer thickness will start growing from x = 0. However, as it is insulated up to x = x_0 then your thermal boundary layer thickness will start developing from x = x_0.

So, today we will integrate this energy equation in this thermal boundary layer. So, we will integrate from y = 0 to δ_T, where δ_T is your thermal boundary layer thickness. So, integrating the above equation between 0 and δ_T, where δ_T is your thermal boundary layer thickness. So, you can see $\int_0^{\delta_T} u \frac{\partial T}{\partial x} dy + \int_0^{\delta_T} v \frac{\partial T}{\partial y} dy = \int_0^{\delta_T} \alpha \frac{\partial^2 T}{\partial y^2} dy$.

So, first the second term you consider in the left hand side and integrate using integration by parts. So, you will see integrating the second term by parts. So, first term you keep it as it is. So you will get, $\int_0^{\delta_T} u \frac{\partial T}{\partial x} dy + [vT]_0^{\delta_T} - \int_0^{\delta_T} T \frac{\partial v}{\partial y} dy = \alpha \int_0^{\delta_T} \frac{\partial}{\partial y}(\frac{\partial T}{\partial y}) dy$.

So, you can see we will get $\int_0^{\delta_T} u \frac{\partial T}{\partial x} dy$ plus you see at y =0. At y = 0, what is the velocity? Obviously, it is 0. So, you will get 0 v X T. And at y = δ_T, what is the temperature? It is free stream temperature, so free stream temperature is T_∞. And what is the velocity?

Velocity is v δ_T. So, you will get T_∞ v at y = δ_T that we need to find from the continuity equation and these $\frac{\partial v}{\partial y}$. So, what we will do now, you have continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$. So, this $\frac{\partial v}{\partial y}$ we can write $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$. So, that we can substitute.

So, you can write $\int_0^{\delta_T} u \frac{\partial T}{\partial x} dy + T_\infty v_{\delta_T} + \int_0^{\delta_T} T \frac{\partial u}{\partial x} dy = \alpha [\frac{\partial T}{\partial y}]_0^{\delta_T}$. So, now we need to find v_{δ_T} . So, to find the v_{δ_T} , we will use this continuity equation and we will integrate this continuity equation in the thermal boundary layer. So, if you see from this expression if you integrate it $\int_0^{\delta_T} \frac{\partial v}{\partial y} dy = -\int_0^{\delta_T} \frac{\partial u}{\partial x} dy$.

Now, you see at, so it will be v, right v at the limit 0 and δ_T. So, at y = 0, v = 0, so you will get $v_{\delta_T} = -- \int_0^{\delta_T} \frac{\partial u}{\partial x} dy$. So, these you substitute it here. So, what you will get now?

$$\int_0^{\delta_T} u \frac{\partial T}{\partial x} dy - \int_0^{\delta_T} T_\infty \frac{\partial u}{\partial x} dy + \int_0^{\delta_T} T \frac{\partial u}{\partial x} dy = \alpha [\frac{\partial T}{\partial y}|_{y=\delta_T} - \frac{\partial T}{\partial y}|_{y=0}] .$$

So, you can see, so $\frac{\partial T}{\partial y}$ at y = δ_T is 0. So, $\alpha[\frac{\partial T}{\partial y}|_{y=\delta_T} - \frac{\partial T}{\partial y}|_{y=0}]$. So, you can see this term will be 0 because at the edge of the thermal boundary layer this is 0.

So, now you can write this equation $\int_0^{\delta_T} \{(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}) - T_\infty \frac{\partial u}{\partial x}\}dy = -\alpha \frac{\partial T}{\partial y}|_{y=0}$.

So, what is this term? These two terms together you can write $\int_0^{\delta_T} \{\frac{\partial (uT)}{\partial x} - \frac{\partial (uT_\infty)}{\partial x}\}dy = -\alpha \frac{\partial T}{\partial y}|_{y=0}$.

So, if you take this $\frac{\partial}{\partial x}$ common, then you will get integral $\int_0^{\delta_T} \frac{\partial}{\partial x}\{u(T - T_\infty)\}dy = -\alpha \frac{\partial T}{\partial y}|_{y=0}$. So, now you can see that you have the derivative $\frac{\partial}{\partial x}$ inside the integral. So, now, using Leibniz rule we will take these $\frac{\partial}{\partial x}$ outside and we will write in terms of ordinary derivative.

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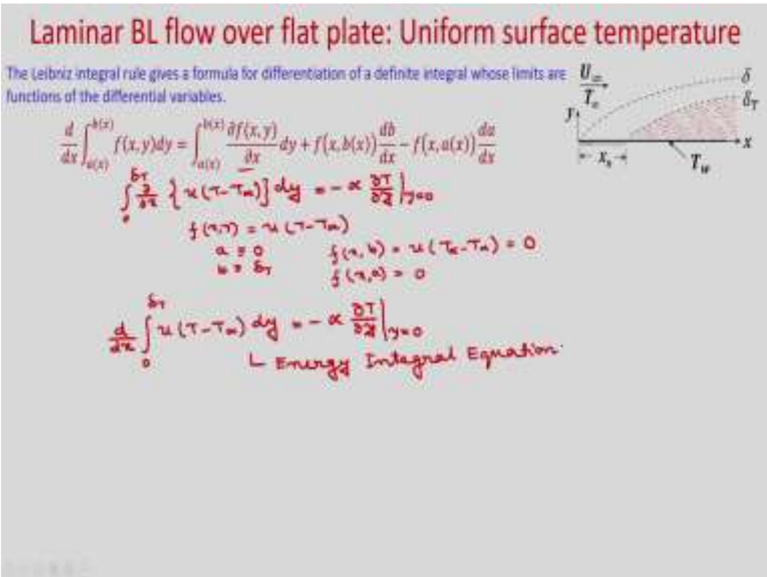

So, now, let us see what is our integral equation we have derived. So, this is your integral $\int_0^{\delta_T} \frac{\partial}{\partial x}\{u(T - T_\infty)\}dy = -\alpha \frac{\partial T}{\partial y}|_{y=0}$. So, if you compare these two terms then you will get $f = u(T - T_\infty)$. Your lower limit a, equivalent to here 0 and upper limit is equivalent to δ_T.

So, if you find f (x,b). So, this term, so f (x,b), so at y = δ_T. At y = δ_T if you find what is the value of f? So, it will be u. What is T? T will be T_∞. So, T_∞ - T_∞, so it will be 0. And what is f(x,a)? At y = a. Means y = 0. At y = 0, what is the velocity?

So, velocity is 0. So, this will be 0. So, these two terms will contribute 0. So, we can write directly these using this Leibniz integral rule we can write $\frac{d}{dx}\int_0^{\delta_T} u(T - T_\infty)dy = -\alpha \frac{\partial T}{\partial y}|_{y=0}$. So, this is known as energy integral equation.

So, we have completed the first step. What is the first step? That you integrate the governing equation and get the integral equation. So, we have got the energy integral equation. So, next step is to find the temperature distribution. So, we will assume polynomial, and in this case, we will consider third degree polynomial and with proper boundary conditions we will find the assume temperature profile. Once we find the assume temperature profile, then we will put that in the energy integral equation.

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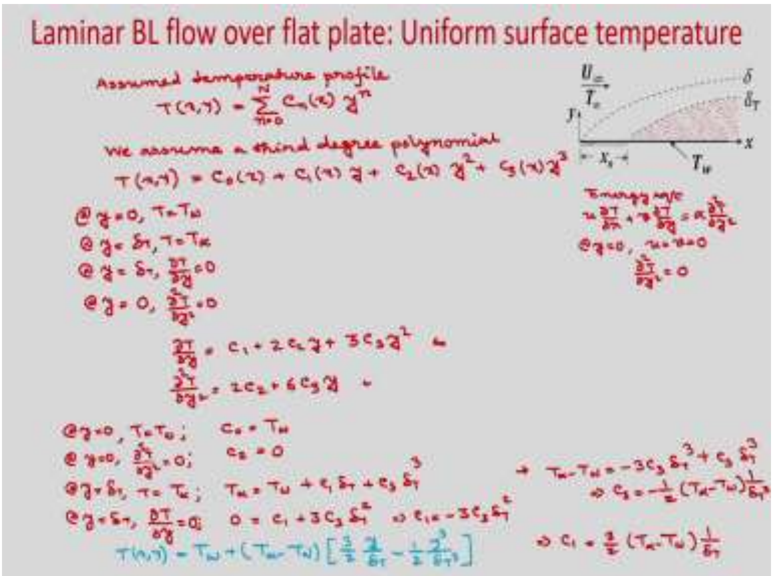

So, we will assume the temperature profile as, so $T(x, y) = \sum_{n=0}^{N} C_n(x) y^n$.

Now, we will consider third degree polynomial. So, we will take up to n = 3. So, we assume a third degree polynomial. So, you will get $T(x, y) = C_0 + C_1 y + C_2 y^2 + C_3 y^3$. So, how many unknown coefficients are there? C_0, C_1, C_2, and C_3. So, how many boundary conditions do we need?

We need 4 boundary conditions. Two boundary conditions you can easily find at y = 0 you have uniform wall temperature $T = T_w$, at $y = \delta_T$ you have $T = T_\infty$ which is your free stream temperature, and $y = \delta_T$ again your temperature gradient is 0. And another boundary condition will derive from the energy equation satisfying it at boundary.

So, you can see at y = 0, you have $T = T_w$, at $y = \delta_T$ you have $T = T_\infty$, at $y = \delta_T$ temperature gradient is also 0, so $\frac{\partial T}{\partial y} = 0$. Now, let us see at y = 0 if we satisfy the energy equation. So, what is your energy equation? Energy equation is $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$. So, at y = 0, u and v are 0. So, this left hand side terms will be 0. So, you can see it will be $\frac{\partial^2 T}{\partial y^2}$=0. So, it is derived boundary condition. So, you can write $\frac{\partial^2 T}{\partial y^2}$=0.

So, now, you can see from here. So, if you find $\frac{\partial T}{\partial y}$. So, what you will get? $\frac{\partial T}{\partial y} = C_1 + 2C_2 y + 3C_3 y^2$ if you write. So, you will get $\frac{\partial^2 T}{\partial y^2} = 2C_2 + 6C_3 y$. So, now you put the boundary conditions. At y = 0, $T = T_w$. So, what you will get? So, if you put it here you will get $C_0 = T_w$. Then, at y = 0, $\frac{\partial^2 T}{\partial y^2}$=0. So, this you can see. So, at y = 0 you have $\frac{\partial^2 T}{\partial y^2}$=0, so you will get C_2 =0.

So, $y = \delta_T$, $T = T_\infty$. So, you can see from here this equation you will get $T_\infty = T_w + C_1\delta_T + C_3\delta_T^3$ and $y = \delta_T$ from here you can see $\frac{\partial T}{\partial y}$=0. So, $0 = C_1 + 3C_3\delta_T^2$.

So, you can see from here you will get $C_1 = -3C_3\delta_T^2$. If you put it here then you will get as $T_\infty - T_w = -3C_3\delta_T^3 + C_3\delta_T^3$. So, it will be - 2 $C_3\delta_T^3$. So, you will get $C_3 = -\frac{1}{2}(T_\infty - T_w)\frac{1}{\delta_T}$.

So, you are getting finally, the temperature profile $T(x,y) = T_w + (T_\infty - T_w)[\frac{3}{2}\frac{y}{\delta_T} - \frac{1}{2}\frac{y^3}{\delta_T^3}]$. So, now, we have found the temperature profile, using third degree polynomial.

So, if you see the energy integral equation in the energy integral equation you have velocity U and temperature T. Velocity profile already we have derived in the last class. So, that we need to substitute in the energy integral equation along with this temperature profile and we need to find what is the value of thermal boundary layer thickness.

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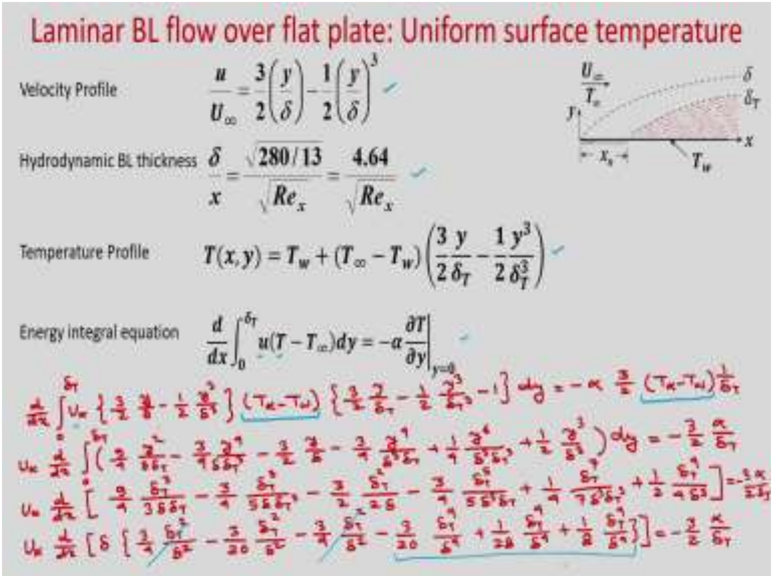

So, you can see this is your velocity profile we have derived in the last class. Hydrodynamic boundary layer thickness, this we have already derived in the last class and temperature profile today we have derived this, and this is your energy integral equation. So, in the energy integral equation now you put this u here and this T from this expression and you integrate it and find the thermal boundary layer thickness.

So, if you put it here you can see

$$\frac{d}{dx}\int_0^{\delta_T} U_\infty\{\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}\}(T_\infty - T_w)\{\frac{3}{2}\frac{y}{\delta_T} - \frac{1}{2}\frac{y^3}{\delta_T^3} - 1\}dy = -\alpha\frac{3}{2}(T_\infty - T_w)\frac{1}{\delta_T}.$$

So, now you integrate it. So, this you can see this term T_w -T_∞. And this is constant, so you can take it outside the integral and you can cancel it. So, we will not write this term in the next equation and U∞ is constant, so you can take it outside. So, now, $U_\infty \frac{d}{dx}\int_0^{\delta_T}(\frac{9}{4}\frac{y^2}{\delta\delta_T} - \frac{3}{4}\frac{y^4}{\delta\delta_T^3} - \frac{3}{2}\frac{y}{\delta} - \frac{3}{4}\frac{y^4}{\delta^3\delta_T} + \frac{1}{4}\frac{y^6}{\delta^3\delta_T^3} + \frac{1}{2}\frac{y^3}{\delta^3})dy = -\frac{3}{2}\frac{\alpha}{\delta_T}$

So, now this we integrate between 0 and δ_T. So, you can see, at y = 0 anyway you every term will become 0. So, what you can do? At y = δ_T you can put, so after taking the integration. So, you can see you will get $U_\infty \frac{d}{dx}[\frac{9}{4}\frac{\delta_T^3}{3\delta\delta_T} - \frac{3}{4}\frac{\delta_T^5}{5\delta\delta_T^3} - \frac{3}{2}\frac{\delta_T^2}{2\delta} - \frac{3}{4}\frac{\delta_T^5}{5\delta^3\delta_T} + \frac{1}{4}\frac{\delta_T^7}{7\delta^3\delta_T^3} + \frac{1}{2}\frac{\delta_T^4}{4\delta^3}] = -\frac{3}{2}\frac{\alpha}{\delta_T}$

So, you can see we will get $U_\infty \frac{d}{dx}$. You take δ outside. So, what you will get? So, you can see it will be $U_\infty \frac{d}{dx}[\delta\{\frac{3}{4}\frac{\delta_T^2}{\delta^2} - \frac{3}{20}\frac{\delta_T^2}{\delta^2} - \frac{3}{4}\frac{\delta_T^2}{\delta^2} - \frac{3}{20}\frac{\delta_T^4}{\delta^4} + \frac{1}{28}\frac{\delta_T^4}{\delta^4} + \frac{1}{8}\frac{\delta_T^4}{\delta^4}] = -\frac{3}{2}\frac{\alpha}{\delta_T}$.

So, you can see here. you have $\frac{3}{4}\frac{\delta_T^2}{\delta^2}$ and here -$\frac{3}{4}\frac{\delta_T^2}{\delta^2}$, so it will cancel. And $\frac{\delta_T^4}{\delta^4}$. So, this we can together write. So, if you write it together, then you can see we will get $U_\infty \frac{d}{dx}[\delta\{-\frac{3}{20}\frac{\delta_T^2}{\delta^2} + \frac{-42+10+35}{280}\frac{\delta_T^4}{\delta^4}\}] = -\frac{3}{2}\frac{\alpha}{\delta_T}$.

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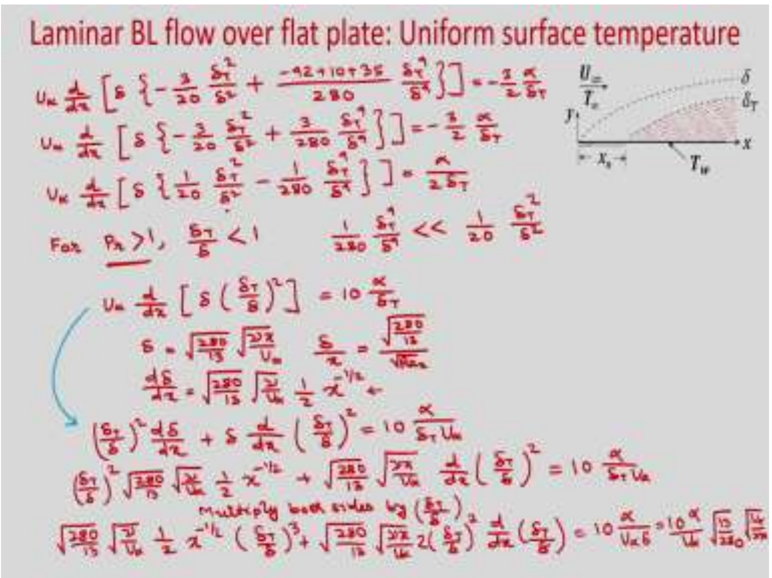

So, you can see these all terms, these 3 terms we have written together here. So, you can see finally, you can write it as $U_\infty \frac{d}{dx}[\delta\{-\frac{3}{20}\frac{\delta_T^2}{\delta^2}+\frac{3}{280}\frac{\delta_T^4}{\delta^4}\}]=-\frac{3}{2}\frac{\alpha}{\delta_T}$.

So, here you can see here 3 is there, 3 is there, 3 is there. So, if you cancel it you can write, $U_\infty \frac{d}{dx}[\delta\{\frac{1}{20}\frac{\delta_T^2}{\delta^2}-\frac{1}{280}\frac{\delta_T^4}{\delta^4}\}]=\frac{\alpha}{2\delta_T}$. So, this is the expression we have derived till now, now we will make one assumptions.

So, let us consider that your $\delta_T < \delta$. So, if $\delta_T < \delta$ that means, Prandtl number > 1. So, for Prandtl number > 1, $\frac{\delta_T}{\delta}<1$. So, we are considering for the cases where Prandtl number > 1.

So, $\frac{\delta_T}{\delta}<1$ and if $\frac{\delta_T}{\delta}<1$, so you can see the last term $\frac{\delta_T^4}{\delta^4}$ will be much much less than this term. So, we can neglect this second term in the left hand side. So, we can write if $\frac{\delta_T}{\delta}<1$ then $\frac{1}{280}\frac{\delta_T^4}{\delta^4}<<\frac{1}{20}\frac{\delta_T^2}{\delta^2}$.

So, what we will do as we are assuming that Prandtl number >1 so obviously, $\delta_T < \delta$ and the second term in the left hand side now we can neglect for simple calculation. So, if you neglect it then it will be easy to integrate. So, you can write it as, $U_\infty \frac{d}{dx}[\delta(\frac{\delta_T}{\delta})^2]=10\frac{\alpha}{\delta_T}$.

So, now from the last class you have derived $\frac{\delta}{x}$ which is your hydrodynamic boundary layer thickness. So, $\frac{\delta}{x}$ we have already derived as $\delta=\sqrt{\frac{280}{13}}\sqrt{\frac{\nu x}{U_\infty}}$ or $\frac{\delta}{x}=\frac{\sqrt{\frac{280}{13}}}{\sqrt{\mathrm{Re}_x}}$. And $\frac{d\delta}{dx}=\sqrt{\frac{280}{13}}\sqrt{\frac{\nu}{U_\infty}}\frac{1}{2}x^{-1/2}$, because $x^{1/2}$ is there. So, $\frac{1}{2}x^{-1/2}$. So, this we need it in this calculation.

So, from here now you derive U_∞ take in the right hand side, so you can see that it will be $(\frac{\delta_T}{\delta})^2\frac{d\delta}{dx}+\delta\frac{d}{dx}(\frac{\delta_T}{\delta})^2=10\frac{\alpha}{\delta_T U_\infty}$. So, now $\frac{d\delta}{dx}$ we have already found here, so you put it here. So, if you put it here, so you will get

$$(\frac{\delta_T}{\delta})^2\sqrt{\frac{280}{13}}\sqrt{\frac{\nu}{U_\infty}}\frac{1}{2}x^{-1/2}+\sqrt{\frac{280}{13}}\sqrt{\frac{\nu x}{U_\infty}}\frac{d}{dx}(\frac{\delta_T}{\delta})^2=10\frac{\alpha}{\delta_T U_\infty}.$$

So, now multiply both side, multiply both sides by $\frac{\delta_T}{\delta}$. So, $\frac{\delta_T}{\delta}$ we are multiplying in both sides. So, what you will get? So, here you will get $\sqrt{\frac{280}{13}}\sqrt{\frac{\nu}{U_\infty}}\frac{1}{2}x^{-1/2}(\frac{\delta_T}{\delta})^3+\sqrt{\frac{280}{13}}\sqrt{\frac{\nu x}{U_\infty}}2(\frac{\delta_T}{\delta})^2\frac{d}{dx}(\frac{\delta_T}{\delta})=10\frac{\alpha}{U_\infty\delta}$.

So, now you can see $10\frac{\alpha}{U_\infty}$. And what is δ? So, δ is this one. So, you can see it will be $\frac{1}{\delta}$, right.

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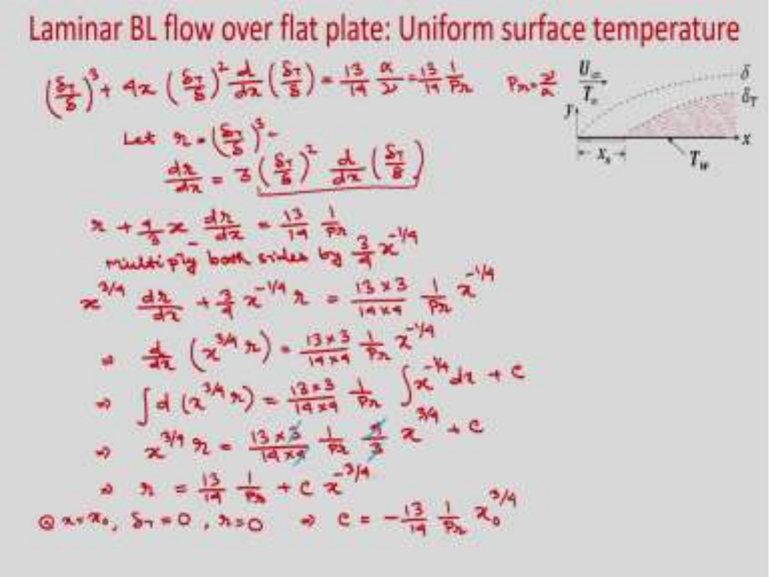

If you simplify it, what you will get? You will get $(\frac{\delta_T}{\delta})^3+4x(\frac{\delta_T}{\delta})^2\frac{d}{dx}(\frac{\delta_T}{\delta})=\frac{13}{14}\frac{\alpha}{\nu}$, so you can write $\frac{13}{14}\frac{1}{\Pr}$. So, after simplification you will get this expression.

So, now, what we will do we will just put $r=(\frac{\delta_T}{\delta})^3$. So, what will be, $\frac{dr}{dx}$? $\frac{dr}{dx}=3(\frac{\delta_T}{\delta})^2\frac{d}{dx}(\frac{\delta_T}{\delta})$. So, if you put it here you can see you will get, so it will be $r+\frac{4}{3}x\frac{dr}{dx}=\frac{13}{14}\frac{1}{\Pr}$.

So, multiply both sides by $\frac{3}{4}x^{-1/4}$. So, if you put it here and let us write this as a first term. So, you will get $x^{3/4}\frac{dr}{dx}+\frac{3}{4}x^{-1/4}r=\frac{13\times 3}{14\times 4}\frac{1}{\Pr}x^{-1/4}$.

So, you can see these two terms together you can write $\frac{d}{dx}(x^{3/4}r)=\frac{13\times 3}{14\times 4}\frac{1}{\Pr}x^{-1/4}$.

Now, if you integrate it, $\int d(x^{3/4}r)=\frac{13\times 3}{14\times 4}\frac{1}{\Pr}\int x^{-1/4}dx+C$. So, if you integrate it, it will get $x^{3/4}r=\frac{13\times 3}{14\times 4}\frac{1}{\Pr}\frac{4}{3}x^{3/4}+C$. So, now these 3, these 3, these 4, these 4, will cancel. So, you will get $r=\frac{13}{14}\frac{1}{\Pr}+Cx^{-3/4}$.

So, now, let us put what is the value of δ_T at x = x_0. At x = x_0 you have δ_T as 0, right. So, at x=x_0 you have δ_T = 0. And x = x_0 you can see here δ_T is 0 that means, r = 0 because we are starting your thermal boundary layer thickness is starting from x = x_0, where the value of δ_T is 0. And if δ_T is 0 then r will become 0. So, that will give $C=-\frac{13}{14}\frac{1}{\Pr}x_0^{3/4}$.

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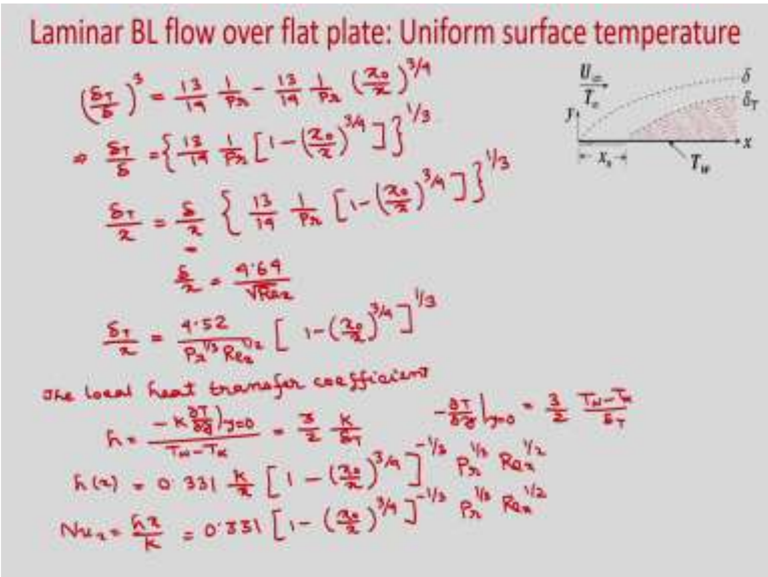

So, if you put it in this expression what you will get? $(\frac{\delta_T}{\delta})^3 = \frac{13}{14}\frac{1}{\Pr} - \frac{13}{14}\frac{1}{\Pr}(\frac{x_0}{x})^{3/4}$. So, that means, you can see $\frac{\delta_T}{\delta} = \{\frac{13}{14}\frac{1}{\Pr}[1-(\frac{x_0}{x})^{3/4}]\}^{1/3}$.

So, $\frac{\delta_T}{\delta}$ now we have found. And we know what is the value of δ, right. So, $\frac{\delta_T}{x} = \frac{\delta}{x}\{\frac{13}{14}\frac{1}{\Pr}[1-(\frac{x_0}{x})^{3/4}]\}^{1/3}$. So, $\frac{\delta}{x}$ we know, right. So, $\frac{\delta}{x} = \frac{4.64}{\sqrt{\mathrm{Re}_x}}$. So, if you put it here, so you will get $\frac{\delta}{x} = \frac{4.52}{\Pr^{1/3}\mathrm{Re}_x^{1/2}}[1-(\frac{x_0}{x})^{3/4}]^{1/3}$. So, once you know the thermal boundary layer thickness you will be able to find what is the heat transfer coefficient, right.

So, you can see, so now, the local heat transfer coefficient. So, $h = \frac{-K\frac{\partial T}{\partial y}|_{y=0}}{T_w - T_\infty}$. So, if you put the value you will get $\frac{3}{2}\frac{K}{\delta_T}$. So, because $-\frac{\partial T}{\partial y}|_{y=0} = \frac{3}{2}\frac{T_w - T_\infty}{\delta_T}$. So, if you put this, so now, you know the value of δ_T, right. If you put the value you will get $h(x) = 0.331\frac{K}{x}[1-(\frac{x_0}{x})^{3/4}]^{-1/3}\Pr^{1/3}\mathrm{Re}_x^{1/2}$. So, now $Nu_x = \frac{hx}{K}$. So, $\frac{hx}{K}$ if you put it here, so you will get $Nu_x = 0.331[1-(\frac{x_0}{x})^{3/4}]^{-1/3}\Pr^{1/3}\mathrm{Re}_x^{1/2}$.

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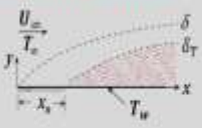

$$\frac{\delta_T}{\delta} = \left\{\frac{13}{14}\frac{1}{Pr}\left[1-\left(\frac{x_o}{x}\right)^{3/4}\right]\right\}^{1/3}$$

$$\frac{\delta_T}{x} = \frac{4.528}{Pr^{1/3}Re_x^{1/2}}\left\{\left[1-\left(\frac{x_o}{x}\right)^{3/4}\right]\right\}^{1/3}$$

$$h(x) = 0.331\frac{k}{x}\left\{1-\left(\frac{x_o}{x}\right)^{3/4}\right\}^{-1/3} Pr^{1/3}Re_x^{1/2}$$

$$Nu_x = 0.331\left\{1-\left(\frac{x_o}{x}\right)^{3/4}\right\}^{-1/3} Pr^{1/3}Re_x^{1/2}$$

So, you can see that in today's class now starting from the energy equation we have found the thermal boundary layer thickness $\frac{\delta_T}{\delta}$ as this one. Then, putting the value of $\frac{\delta}{x}$ we have found $\frac{\delta_T}{x}$ as this. Then, we have found h(x) which is your local heat transfer coefficient, then local Nusselt number.

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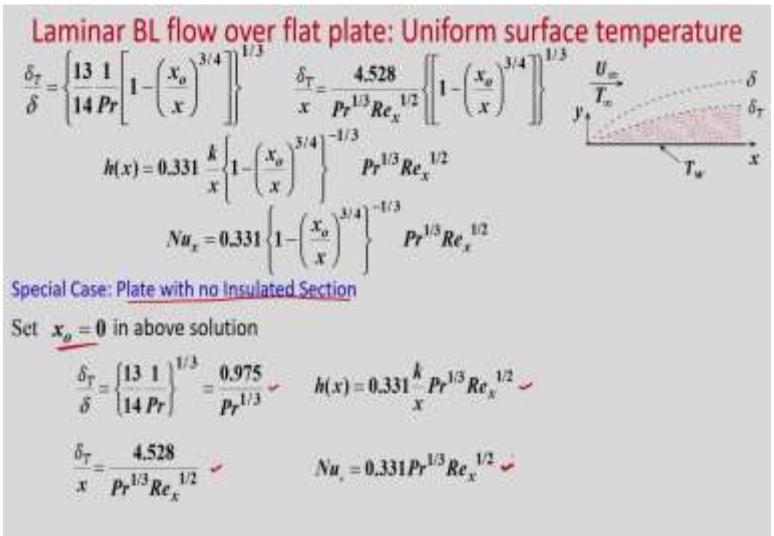

Now, let us consider a special case, plate with no insulated section. So, there is no insulated section so that means, $x_0 = 0$. If you put $x_0 = 0$ then your thermal boundary layer and hydrodynamic boundary layer will start going from x = 0. So, if $x_0 = 0$ if you put in the above equation, so you will get $\frac{\delta_T}{\delta}=\frac{0.975}{Pr^{1/3}}$ and $\frac{\delta_T}{x}=\frac{4.528}{Pr^{1/3}Re_x^{1/2}}$ and local heat transfer coefficient, and local Nusselt number.

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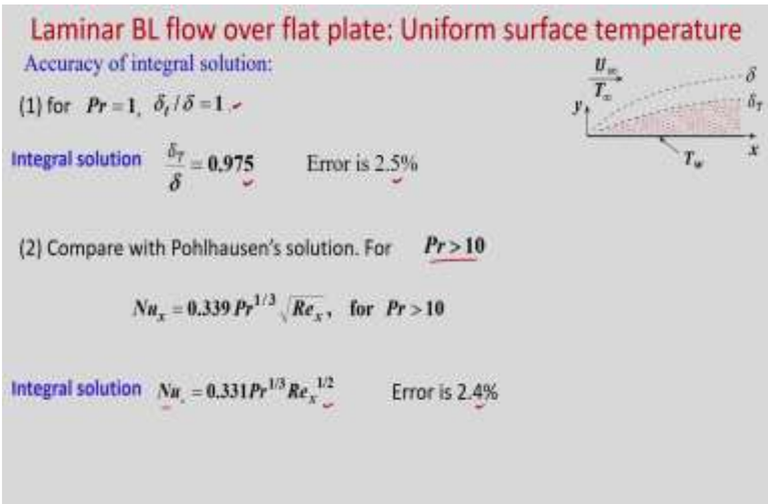

So, now what is the accuracy of this integral solution? If you see for Prandtl number = 1, $\frac{\delta_T}{\delta}$ = 1. So, that is the exact solution. But from the integral solution $\frac{\delta_T}{\delta}$, for Prandtl number=1 we got $\frac{\delta_T}{\delta} = 0.975$. You can see if you put it in this expression Prandtl number =1, then $\frac{\delta_T}{\delta} = 0.975$. So, you can see that error is 2.5% using integral solution.

And, if you see the exact solution from the Pohlhausen solution for Prandtl number >10, $Nu_x = 0.339\,\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}$, but in today's class we have found from the integral solution $Nu_x = 0.331\,\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}$. So, you can if you compare it you can see error is 2.4 %.

So, you can see we have assumed that Prandtl number > 1 that means, your $\delta_T < \delta$ and we have found the expression power heat transfer coefficient, Nusselt number and thermal boundary layer thickness. If Prandtl number < 1, where $\delta_T > \delta$ then you have a rigorous derivation. So, that we will not derive, but just I will give the expression, final expression for the Nusselt number.

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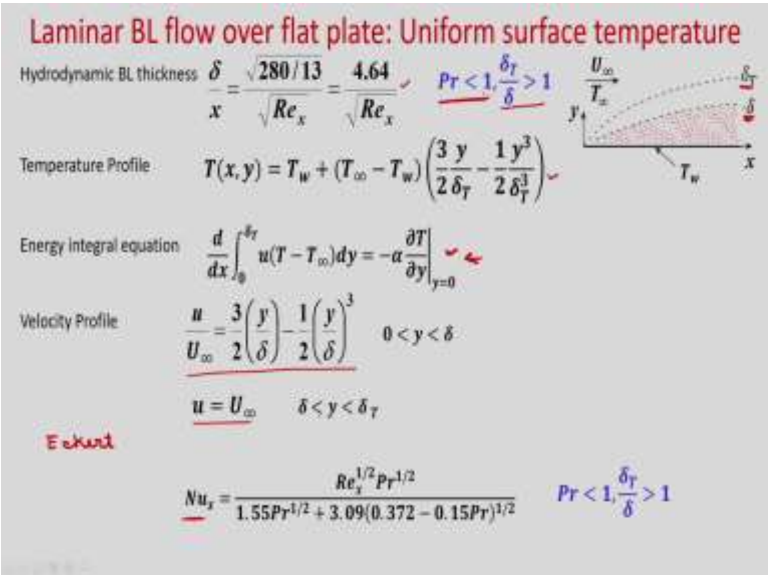

So, we can see in this particular case we are considering Prandtl number < 1. So, $\frac{\delta_T}{\delta} > 1$. So, that means, δ_T is higher than the δ. So, in this particular case, you can see $\frac{\delta}{x}$ already we have found, this is your T(x, y) this is your energy integral equation.

So, in the energy integral equation, you can see 0 to δ. 0 to δ you have this velocity distribution, but δ to δ_T here you will have $u = \infty$. So, if you put it these two expression in this energy integral equation then you can find $Nu_x = \frac{\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/2}}{1.55\,\mathrm{Pr}^{1/2} + 3.09(0.372 - 0.15\,\mathrm{Pr})^{1/2}}$. So, this solution actually scientists Eckert found analytically for Prandtl number <1.

So, in today's class, we considered the laminar boundary layer flow over a flat plate and we considered uniform wall temperature T_w from $x = x_0$, and from $x = 0$ to $x = x_0$ it was adiabatic. So, your thermal boundary layer thickness starts developing from $x = x_0$.

And, first we derived the energy integral equation, then assume the third degree polynomial we derived the temperature profile, and we put the velocity profile and temperature profile in the energy integral equation. Then, we found the thermal boundary layer thickness δ_T, and then we found the local heat transfer coefficient and local Nusselt number and putting $x_0 = 0$ we found a special case where there is no heat unheated region. And finally, for Prandtl number < 1 we have discussed that how we can find the Nusselt number expression.

Thank you.