Fundamentals of Convective Heat Transfer
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Module – 04
Convective Heat Transfer in External Flows – II
Lecture – 12
Laminar BL flow over flat plate: Uniform wall heat flux

Hello everyone. So, today we will consider Boundary Layer flow over a flat plate with Uniform wall heat flux condition. So, in last lecture we considered uniform wall temperature condition, but today we will consider uniform heat flux boundary condition. We wish to determine the wall temperature T_w, as a function of x and the local Nusselt number.

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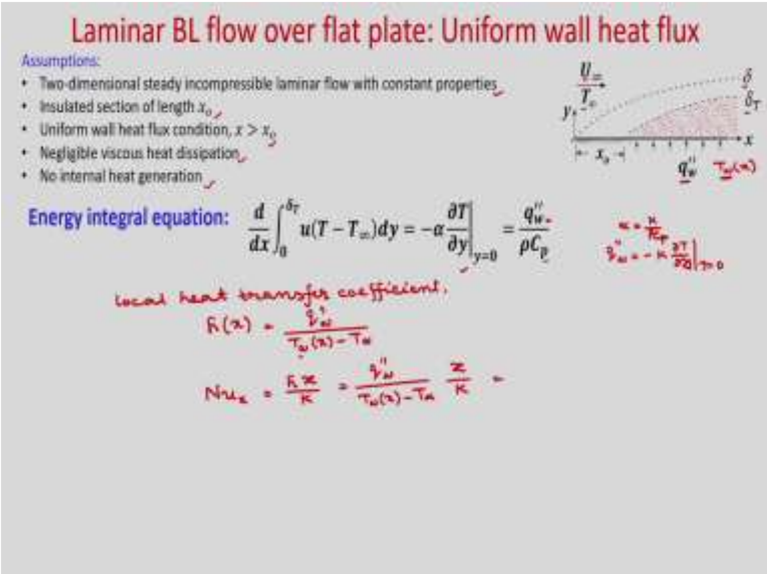

So, let us consider this flat plate, y is measured perpendicular to the flat plate, your free stream velocity is U_∞ and temperature is T_∞. Up to $x = x_0$ it is insulated, so it will be maintained at temperature T_∞ as there will be no heat transfer. From $x = x_0$, you can see this plate is maintained at uniform wall heat flux $q_w^{"}$.

So, your thermal boundary layer thickness will start developing from $x = x_0$ and hydrodynamic boundary layer thickness will start developing from $x = 0$. So, these are the assumptions; two dimensional steady incompressible laminar flow with constant properties, insulated section of length x_0, uniform wall heat flux condition $x > x_0$, negligible viscous heat dissipation and no internal heat generation.

So, in last class already we have derived the energy integral equation. So, that we will use. And, we will first find the temperature distribution using third degree polynomial. And, we will put the velocity distribution and temperature distribution in the energy integral equation and we will find the expression for thermal boundary layer thickness.

And, then we will find the wall temperature distribution as well as local Nusselt number. So, we can see this is the energy integral equation already we have derived. This right hand side $-\alpha \frac{\partial T}{\partial y}|_{y=0}$ this we can write $\frac{q_w^{"}}{\rho C_p}$, because $\alpha = \frac{K}{\rho C_p}$ and $q_w^{"} = -K \frac{\partial T}{\partial y}|_{y=0}$.

Hence, this right hand side can be written as $\frac{q_w^{"}}{\rho C_p}$. You know that in this particular case $q_w^{"}$ is constant and ρC_p are the properties and that also are constant. So, right hand side is a constant term.

However, you can see here temperature T_w, wall temperature T_w will be function of x; because along the x, your T_w will increase. So, as $q_w^{"}$ is constant, you can find the local heat transfer coefficient $h(x) = \frac{q_w^{"}}{T_w(x) - T_\infty}$. So, you can see T_∞ is your free stream temperature and T_w is function of x. And, $q_w^{"}$ is constant.

So, this is from Newton's law of cooling we have written. Now, Nusselt number $Nu_x = \frac{hx}{K}$. So, you can see, you can write $Nu_x = \frac{q_w^{"}}{T_w(x) - T_\infty} \frac{x}{K}$. So, this is the expression for Nusselt number. Now, first let us consider a third degree polynomial for temperature distribution and we will find the coefficient using the boundary conditions.

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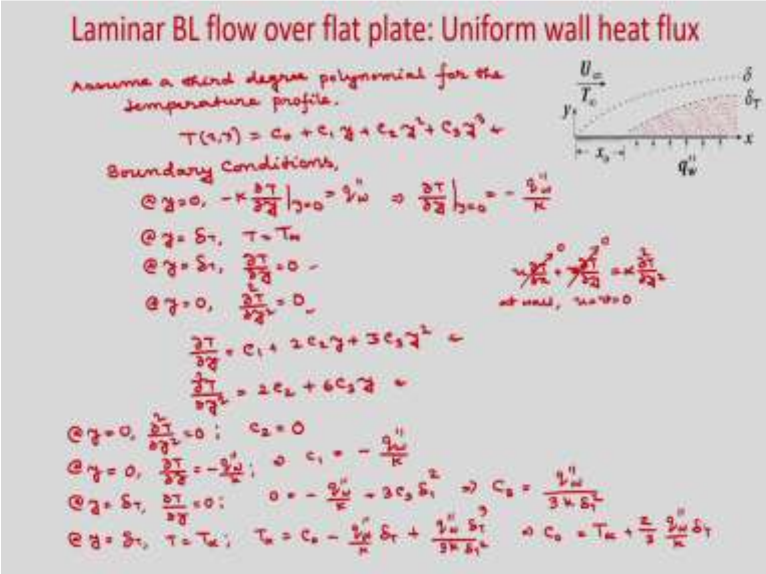

So, assume a third degree polynomial for the temperature profile. So, T which is function of (x, y) we can write $T(x,y) = C_0 + C_1 y + C_2 y^2 + C_3 y^3$. So, these coefficients Cc_0, C_1, C_2, C_3 are function of x. So, now, we have boundary conditions at y = 0; y = 0, you can see your heat flux is given. So, you can write $-K \frac{\partial T}{\partial y}|_{y=0} = q_w^{"}$.

So, you can write $\frac{\partial T}{\partial y}|_{y=0} = -\frac{q_w^{"}}{K}$, and at y = δ_T you have temperature T_∞ and also a temperature gradient $\frac{\partial T}{\partial y}$ = 0. So, at y = δ_T, you have T = T_∞ and also you have at y = δ_T, $\frac{\partial T}{\partial y}$ = 0; because, that is the free stream temperature, so there will be no gradient. And, another boundary condition we will derive from the energy equation satisfying it at the wall.

So, at y = 0, you can write $\frac{\partial^2 T}{\partial y^2} = 0$. So, you remember in last class we have done $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$. So, this is your boundary layer energy equation. So, at wall you have u = v = 0. So, if u and v are 0; so obviously, left hand side terms will be 0.

So, $\frac{\partial^2 T}{\partial y^2} = 0$. So, now, you have four boundary conditions and four coefficients. So, those four are known coefficients you can find satisfying these boundary conditions. So, $\frac{\partial T}{\partial y} = C_1 + 2C_2 y + 3C_3 y^2$; $\frac{\partial^2 T}{\partial y^2} = 2C_2 + 6C_3 y$.

So, we can see at y =0, you have $\frac{\partial^2 T}{\partial y^2} = 0$. So, if you satisfy this from this equation, you can see C_2 = 0. Then, at y = 0, you have $\frac{\partial T}{\partial y} = -\frac{q_w''}{K}$. So, this is your $\frac{\partial T}{\partial y}$. So, at y = 0 if you satisfy, so last two terms will become 0. So, that will give $C_1 = -\frac{q_w''}{K}$. And, now you see at y = δ_T, you have $\frac{\partial T}{\partial y}$ = 0.

So, if it is 0, so you can see from this equation, you will get $0 = -\frac{q_w''}{K} + 3C_3 \delta_T^2$. So, that means, from here you will get $C_3 = \frac{q_w''}{3K\delta_T^2}$. Now, another boundary condition you apply at y = δ_T is, you have T = T_∞.

So, from this equation you can see, you can write $T_\infty = C_0 - \frac{q_w''}{K}\delta_T + \frac{q_w''\delta_T^3}{3K\delta_T^2}$. So, you can see this will be $C_0 = T_\infty + \frac{2}{3}\frac{q_w''}{K}\delta_T$.

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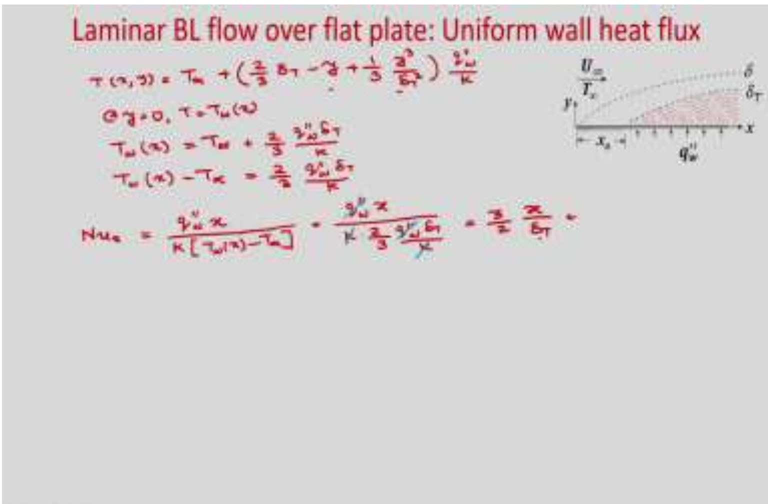

So, we have found four coefficients, now you put it in the temperature expression; then, your final temperature distribution will be for these boundary conditions, $T(x,y)=T_\infty+(\frac{2}{3}\delta_T-y+\frac{1}{3}\frac{y^3}{\delta_T^2})\frac{q_w^{"}}{K}$. So, this is the general temperature distribution. Now, at the wall if you want to find what is the temperature variation, then you put y = 0.

So, at y = 0, T = $T_w(x)$. So, if you put y = 0; so these two terms will become 0, so you can write $T_w(x)=T_\infty+\frac{2}{3}\frac{q_w^{"}\delta_T}{K}$. And, also you can write $T(x)-T_\infty=\frac{2}{3}\frac{q_w^{"}\delta_T}{K}$. So, now, you can put it in the Nusselt number distribution whatever we have found.

So, Nusselt number we have written, $Nu_x=\frac{q_w^{"}x}{K[T(x)-T_\infty]}$. So, if you put $T(x)-T_\infty$ this expression, so what you will get; $\frac{q_w^{"}x}{K\frac{2}{3}\frac{q_w^{"}\delta_T}{K}}$. So, finally, you can write this $Nu_x=\frac{3}{2}\frac{x}{\delta_T}$.

So, you can see in this expression Nusselt number now you can find, once you find the thermal boundary layer thickness δ_T. So, once we find $\frac{\delta_T}{x}$, then you will be able to find the local Nusselt number. So, what we will do now? We know the velocity profile, we know the temperature profile, we have the energy integral equation; so, you put it in the energy integral equation, then you will be able to find the thermal boundary layer thickness δ_T.

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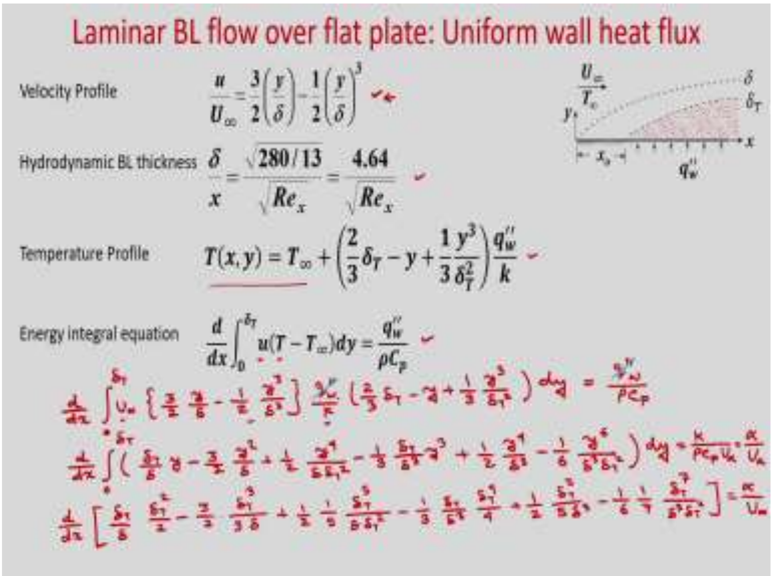

So, we can see we have the velocity profile already we have derived using third degree polynomial and from that solution, hydrodynamic boundary layer thickness we have found, $\frac{\delta_T}{x}$ as this. In today's class we have found the temperature distribution for the given boundary conditions and this is the energy integral equation. So, now, you put the value of u here and value of T here; so you will be able to find, what is the thermal boundary layer thickness?

So, you can write $\frac{d}{dx}\int_0^{\delta_T} U_\infty\{\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\frac{y^3}{\delta^3}\}\frac{q_w^{"}}{K}(\frac{2}{3}\delta_T - y+\frac{1}{3}\frac{y^3}{\delta_T^2})dy=\frac{q_w^{"}}{\rho C_p}$.

So, in this expression you can see $q_w^{"}$ is constant. So, these $q_w^{"}$ you can take it outside the integral and you can cancel, right. So, this $q_w^{"}$ can cancel and this k you can take in the right hand side. So, in the next step you can see, we can write $\frac{d}{dx}\int_0^{\delta_T}(\frac{\delta_T}{\delta}y-\frac{3}{2}\frac{y^2}{\delta}+\frac{1}{2}\frac{y^4}{\delta\delta_T^2}-\frac{1}{3}\frac{\delta_T}{\delta^3}y^3+\frac{1}{2}\frac{y^4}{\delta^3}-\frac{1}{6}\frac{y^6}{\delta^3\delta_T^2})dy=\frac{K}{\rho C_p U_\infty}$.

Because, these are constant, so you can take it outside the integral and you take in the right hand side. So, you can write $\frac{K}{\rho C_p U_\infty}$. And, $\frac{K}{\rho C_p}=\alpha$ right, thermal diffusivity; so, $\frac{\alpha}{U_\infty}$. So, now, you integrate it. So, if you integrate it. So, you can see we can find. So, at y =0, this will become 0 anyway and y $=\delta_T$, we will put after the integration.

So, we can write, $\frac{d}{dx}[\frac{\delta_T}{\delta}\frac{\delta_T^2}{2}-\frac{3}{2}\frac{\delta_T^3}{3\delta}+\frac{1}{2}\frac{1}{5}\frac{\delta_T^5}{\delta\delta_T^2}-\frac{1}{3}\frac{\delta_T}{\delta^3}\frac{\delta_T^4}{4}+\frac{1}{2}\frac{\delta_T^5}{5\delta^3}-\frac{1}{6}\frac{1}{7}\frac{\delta_T^7}{\delta^3\delta_T^2}]=\frac{\alpha}{U_\infty}$. So, now you simplify it, you cancel some terms.

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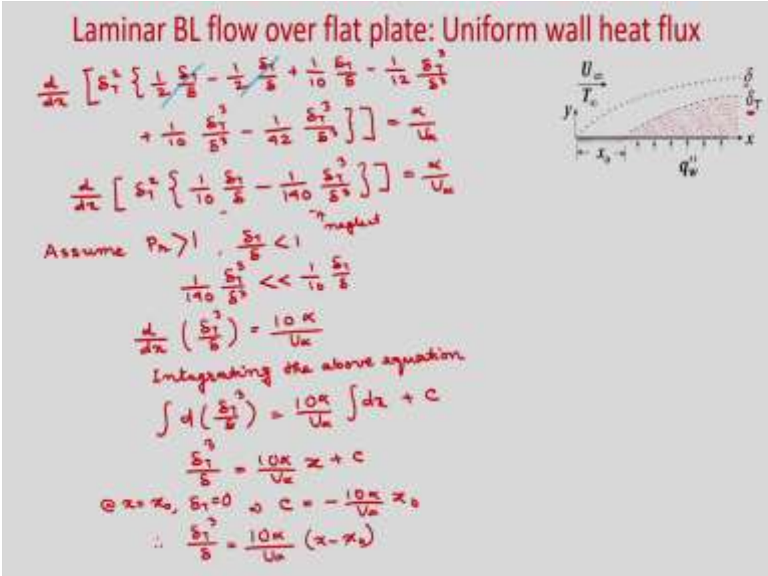

So, you can see here these 3, 3 will get cancel and you can write finally, $\frac{d}{dx}[\delta_T^2\{\frac{1}{2}\frac{\delta_T}{\delta}-\frac{1}{2}\frac{\delta_T}{\delta}+\frac{1}{10}\frac{\delta_T}{\delta}-\frac{1}{12}\frac{\delta_T^3}{\delta^3}+\frac{1}{10}\frac{\delta_T^3}{\delta^3}-\frac{1}{42}\frac{\delta_T^3}{\delta^3}\}]=\frac{\alpha}{U_\infty}$. So, you can see these first two terms. So, you will it will get cancel.

So, $\frac{d}{dx}[\delta_T^2\{\frac{1}{10}\frac{\delta_T}{\delta}-\frac{1}{140}\frac{\delta_T^3}{\delta^3}\}]=\frac{\alpha}{U_\infty}$. So, now, we will assume that Prandtl number is > 1. So, if Prandtl number > 1, then you know $\delta_T < \delta$. And, from this expression, now we will neglect the second term in the left hand side. So, you can see we are assuming, assume Prandtl number > 1 and for Prandtl number > 1, you know $\frac{\delta_T}{\delta} < 1$.

So, in this particular case, you can see that your thermal boundary layer thickness will be less than the hydrodynamic boundary layer thickness. So, if it is so, if you compare these two terms; then you can see $\frac{1}{140}\frac{\delta_T^3}{\delta^3} << \frac{1}{10}\frac{\delta_T}{\delta}$.

So, neglect this term, this term you neglect. So, you can write $\frac{d}{dx}(\frac{\delta_T^3}{\delta^3})=\frac{10\alpha}{U_\infty}$. Let us integrate this. So, this is ordinary differential equation. So, you can integrate it and you know that at $x = x_0$, you have thermal boundary layer thickness δ_T as 0.

So, if you integrate it, so you will get integrating the above equation $\int d(\frac{\delta_T^3}{\delta^3}) = \frac{10\alpha}{U_\infty}\int dx + C$. So, you will get $\frac{\delta_T^3}{\delta} = \frac{10\alpha}{U_\infty}x + C$. And, we know at x=$x_0$, you have $\delta_T = 0$.

So, your thermal boundary layer thickness starts from x=x_0; so here at x=x_0, $\delta_T = 0$. So, from here you can see, $C = -\frac{10\alpha}{U_\infty}x_0$. Hence, you can see that, $\frac{\delta_T^3}{\delta} = \frac{10\alpha}{U_\infty}(x - x_0)$.

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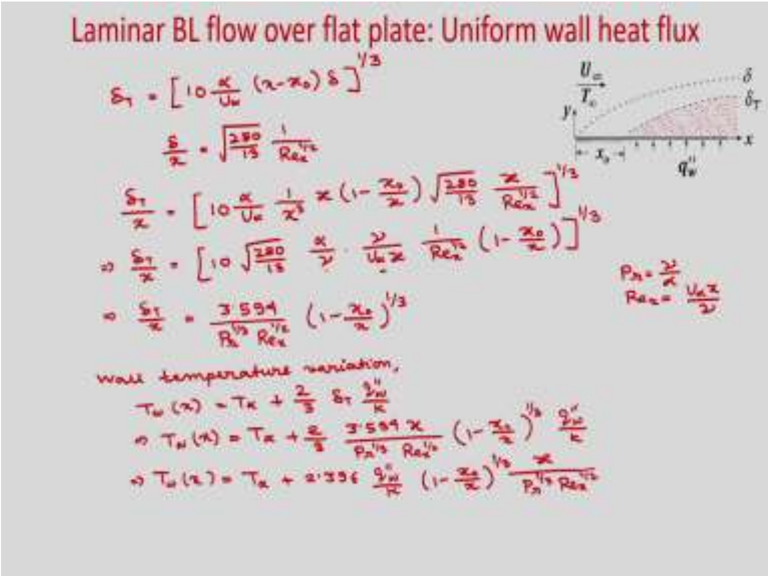

So, you can write $\delta_T = [10\frac{\alpha}{U_\infty}(x - x_0)\delta]^{1/3}$. Now, let us put the expression for hydrodynamic boundary layer thickness δ that we have already found from solving the momentum integral equation. So, you can write $\frac{\delta}{x} = \sqrt{\frac{280}{13}}\frac{1}{\mathrm{Re}_x^{1/2}}$. So, this is the expression we have.

So, now, you can see, you can write from this expression $\frac{\delta_T}{x}$; so we are dividing by x. So, if you divide the right hand side by x and if you take inside this power; so you will get x^3, right. So, you can write $\frac{\delta_T}{x} = [10\frac{\alpha}{U_\infty}\frac{1}{x^3}x(1 - \frac{x_0}{x})\sqrt{\frac{280}{13}}\frac{x}{\mathrm{Re}_x^{1/2}}]^{1/3}$.

So, you can see, you can find $\frac{\delta_T}{x}$ as so, here you have x^2 and so, if you rearrange it; so you can see, you will get, $\frac{\delta_T}{x} = [10\sqrt{\frac{280}{13}} \frac{\alpha}{\nu} \frac{\nu}{U_\infty x} \frac{1}{\text{Re}_x^{1/2}} (1 - \frac{x_0}{x})]^{1/3}$. So, you can write $\frac{\delta_T}{x}$.

So, you can see here; what is this expression? This is here $\frac{1}{\sqrt{\text{Re}_x}}$.

So, $\frac{1}{\sqrt{\text{Re}_x}}$ and here $\text{Re}_x^{1/2}$; so it will be 3/2. So, it will be 3/2 and here $\frac{\nu}{\alpha} = \text{Pr}$. So, you have $Pr = \frac{\nu}{\alpha}$. So, and $\text{Re}_x = \frac{U_\infty x}{\nu}$. So, you will get here $\frac{1}{\text{Pr}}$, here you will get $\frac{1}{\text{Re}_x}$ and $\text{Re}_x^{1/2}$ you have.

So, it will be 3 /2, and outside this bracket if you take, then it will become $\sqrt{\text{Re}_x}$. So, you can see, you can write $\frac{\delta_T}{x} = \frac{3.594}{\text{Pr}^{1/3}\,\text{Re}_x^{1/2}} (1 - \frac{x_0}{x})^{1/3}$. So, after simplification, we have now derive the expression for $\frac{\delta_T}{x}$.

Now, once you know $\frac{\delta_T}{x}$, now you will be able to find, what is the temperature distribution and what is the Nusselt number? So, if you put this $\frac{\delta_T}{x}$, then you can get your wall temperature distribution as; wall temperature variation as $T_w(x) = T_\infty + \frac{2}{3}\delta_T \frac{q_w^{"}}{K}$.

So, if you put this expression, then you will get, $T_w(x) = T_\infty + \frac{2}{3} \frac{3.594x}{\text{Pr}^{1/3}\,\text{Re}_x^{1/2}} (1 - \frac{x_0}{x})^{1/3} \frac{q_w^{"}}{K}$.

So, hence you will get $T_w(x) = T_\infty + 2.396 \frac{q_w^{"}}{K} (1 - \frac{x_0}{x})^{1/3} \frac{x}{\text{Pr}^{1/3}\,\text{Re}_x^{1/2}}$. So, this is the wall temperature variation. So, you can see from this expression that it is function of x, right. Now let us find, what is that local Nusselt number? Already, we have written local Nusselt number in terms of the thermal boundary layer thickness.

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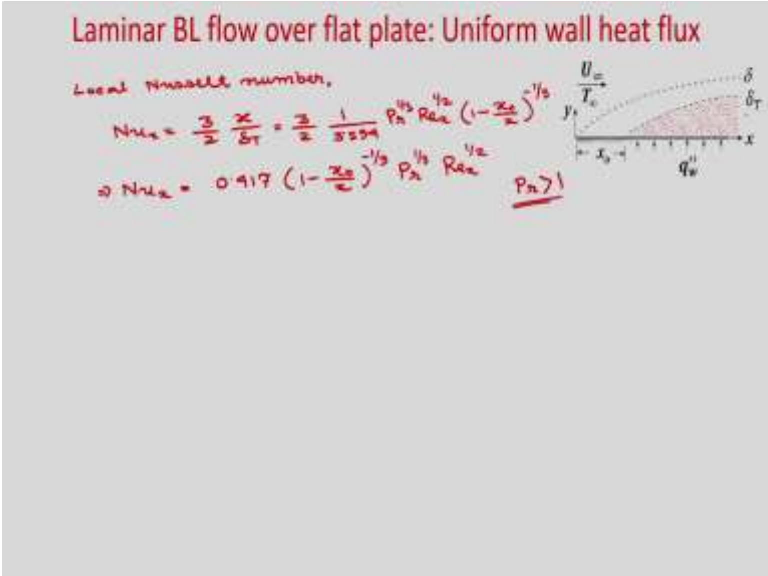

So, if you remember, we have already derived this expression local Nusselt number $Nu_x = \frac{3}{2}\frac{x}{\delta_T}$. And now, we know the expression of $\frac{\delta_T}{x}$. So, we can write $\frac{3}{2}\frac{1}{3.594}\Pr^{1/3}\mathrm{Re}_x^{1/2}(1-\frac{x_0}{x})^{-1/3}$. So, if you rearrange, you will get Nusselt number as, $Nu_x = 0.417(1-\frac{x_0}{x})^{-1/3}\Pr^{1/3}\mathrm{Re}_x^{1/2}$.

So, this is the Nusselt number expression we have found for Prandtl number > 1 using the approximate method; because we have approximated the velocity profile as well as the temperature profile. So, this is valid for Prandtl number >1, because we have assumed $\delta_T < \delta$.

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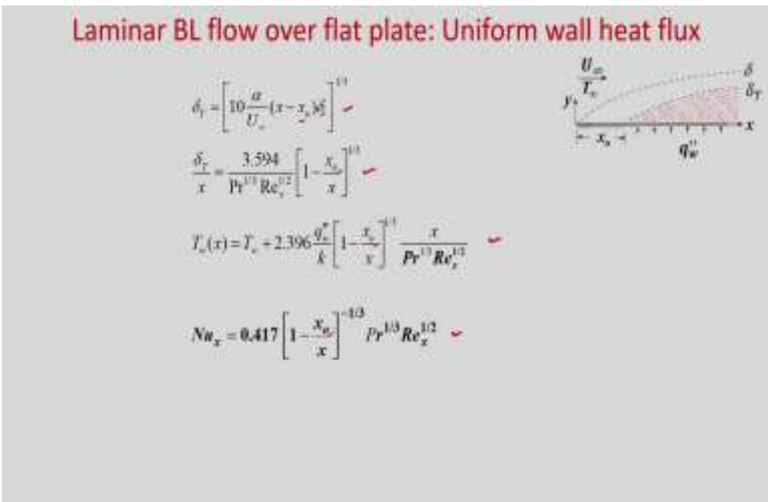

So, you can see, we have finally derived in today's class this δ_T in terms of hydrodynamic boundary layer thickness δ; then putting the value of δ, we have found $\frac{\delta_T}{x}$. And, you can see it is also function of Prandtl number and Reynolds number. And, putting this expression in the wall temperature variation, we found this is the wall temperature variation and then, we have found the local Nusselt number as this.

Now, let us consider a special situation when there is no insulated region; so that means $x_0 = 0$. So, in this expression you can see, if you put $x_0 = 0$; then, you will get the expression for thermal boundary layer thickness, wall temperature variation and local Nusselt number for the unheated region as 0.

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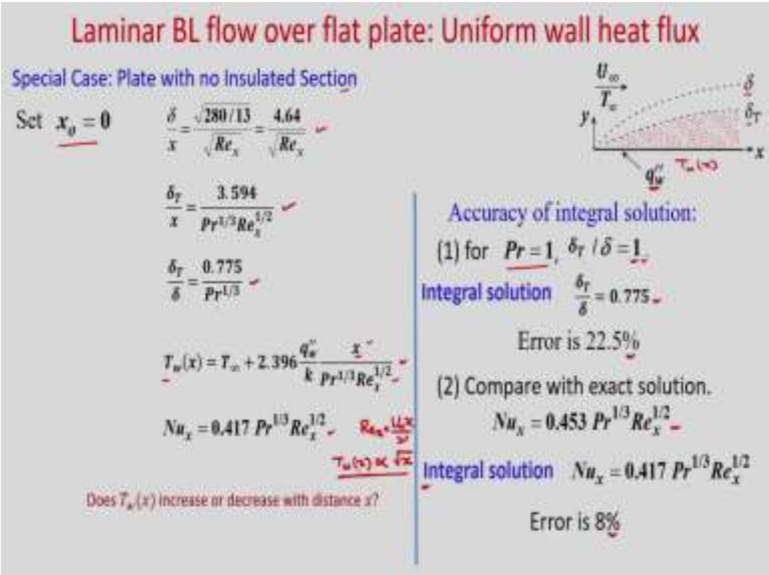

So, you can see in this particular case $x_0 = 0$; so, thermal boundary layer thickness and hydrodynamic boundary layer thickness starts developing from x = 0. So, in for the special case, in earlier expression if you put $x_0 = 0$, where you have plate with no insulated section; then, we have already found $\frac{\delta}{x}$, then this is your $\frac{\delta_T}{x}$ putting x_0=0and $\frac{\delta_T}{\delta}$.

If you can see that $\frac{\delta_T}{\delta}$ if you put; then, you will get as $\frac{0.775}{\Pr^{1/3}}$. And, wall temperature variation you can see here you will get as $T_w(x) = T_\infty + 2.396 \frac{q_w''}{K} (1 - \frac{x_0}{x})^{1/3} \frac{x}{\Pr^{1/3} \mathrm{Re}_x^{1/2}}$.

So, you can see it varies with x. And, $Nu_x = 0.417 (1 - \frac{x_0}{x})^{-1/3} \Pr^{1/3} \mathrm{Re}_x^{1/2}$. In this expression you can see, does $T_w(x)$ increase or decrease with distance x? You can see that, you have here Re_x,one x is there and also here x is there; so you can see that your wall temperature will increase along x. So, if you although in this particular case your; you have this plate with uniform wall heat flux; but T_w which is function of x will increase along x.

Now, let us see, what is the accuracy compared to the exact solution? Because, in this particular case, we have used approximate method where we have approximated the velocity profile as well as the temperature profile as third degree polynomial; so, we have found what is the thermal boundary layer thickness as well as the Nusselt number. Now, let us compare this with the exact solution. So, you can see for Prandtl number= 1 exact solution $\frac{\delta_T}{\delta}$ should be 1; because $\delta_T = \delta$ for Prandtl number= 1. But, from the integral solution you can see, for Prandtl number =1 , $\frac{\delta_T}{\delta}$ = 0.775. So, error is much in predicting the thermal boundary layer thickness, it is 22.5 %.

Now, if you compare the Nusselt number with the exact solution. So, this is the exact solution, you can see $Nu_x = 0.453 \Pr^{1/3} \mathrm{Re}_x^{1/2}$. So, this is your follows a solution with unheated region. So, you can see this is the expression; but from the approximate solution, we have found $0.417 \Pr^{1/3} \mathrm{Re}_x^{1/2}$.

So, you can see error is almost 8 %, but it is Nusselt number is predicting well right; but here δ_T is having much difference with the exact solution. So, in this particular expression you can see, your $\mathrm{Re}_x = \frac{U_\infty x}{\nu}$. So, you have; so that means your, in the denominator you have $\sqrt{x}$ and this is your x, so that means T_w varies with $\sqrt{x}$, you can see.

So, this is in the numerator we have x and in the denominator you have $\sqrt{x}$. So, in the $\frac{x}{\sqrt{x}} = \sqrt{x}$. So, T_w varies $\sqrt{x}$. Now, if we assume a variable temperature profile in the flat plate; then, can we get back the same expression of Nusselt number whatever we have got assuming the constant wall heat flux boundary condition. So, let us see that.

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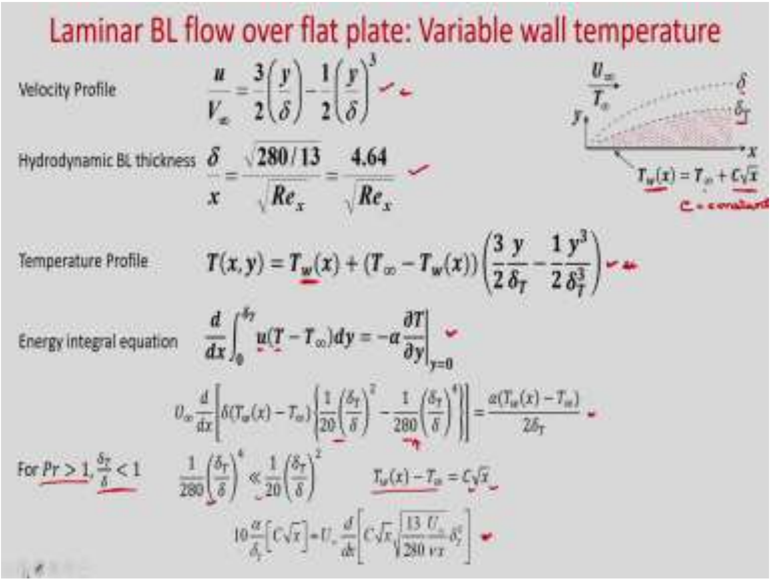

So, now we are considering laminar boundary layer flow over flat plate with variable wall temperature. So, you can see that your wall temperature varies with $\sqrt{x}$. So, we have taken this flat plate where temperature varies as $T_\infty + C\sqrt{x}$, where C is your constant.

And, in last slide we have seen that, generally for constant wall heat flux condition T_w varies as $\sqrt{x}$. So, we have taken $T_x(x) = T_\infty + C\sqrt{x}$. So, you have free stream temperature T_∞ and Prandtl number > 1, so that $\delta_T < \delta$.

So, for this expression if you use the third degree polynomial for velocity profile; so, we have already derived this, $\frac{\delta}{x}$ we have derived this, temperature profile. Now, with these boundary conditions if you see that we have, in the last class we have used uniform wall temperature boundary condition and for that, we have found the temperature profile. So, same temperature profile we can put it, where T_w is function of x.

So, we can see this is the same expression what we have derived already for uniform wall temperature boundary condition and this is the T (x, y); but here T_w is function of x. So, $T(x,y) = T_w(x) + (T_\infty - T_w(x))(\frac{3}{2}\frac{y}{\delta_T} - \frac{1}{2}\frac{y^3}{\delta_T^3})$. And, this is the energy integral equation, right. So, now, in this expression you put u and T.

So, already this we have derived and here already we have derived this; but here T_w is function of x, because your wall temperature varies like this. So, if you put it and you will get U_∞ is constant. So, we have taken outside $\frac{d}{dx}$. So, you will get from this you can see, it will be $\frac{d}{dx}\int_0^{\delta_T} u(T - T_\infty)dy = -\alpha \frac{\partial T}{\partial y}|_{y=0}$.

So, you will get, $U_\infty \frac{d}{dx}[\delta(T_w(x) - T_\infty)\{\frac{1}{20}(\frac{\delta_T}{\delta})^2 - \frac{1}{280}(\frac{\delta_T}{\delta})^4\}] = \frac{\alpha(T_w(x) - T_\infty)}{2\delta_T}$.

So, if you see these two terms and we have used Prandtl number > 1; so that means $\frac{\delta_T}{\delta}$<1. So, in this particular case you can see, you can neglect this term; because, this term will be much much less than the this term. And, $T_x(x) = T_\infty + C\sqrt{x}$. So, these if you put it here, you can see we will get this expression. And, it is easy to integrate, because you can see here you can put the expression for $T_x(x) - T_\infty = C\sqrt{x}$.

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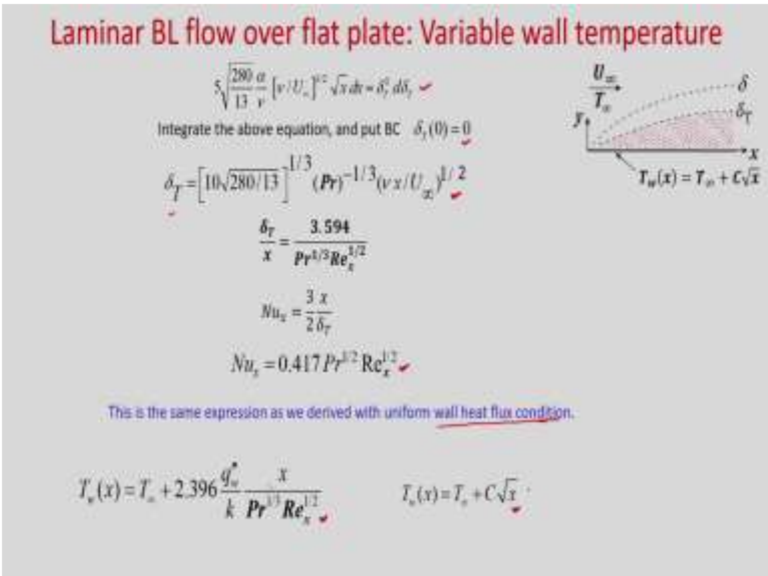

And, after simplification you will get this, and integrate this above equation and put the boundary condition that at x = 0, you have δ_T = 0. So, you will get a δ_T like this expression. And, if you rearrange this you will get $\frac{\delta_T}{x} = \frac{3.594}{\Pr^{1/3} \mathrm{Re}_x^{1/2}}$ and $Nu_x = \frac{3}{2}\frac{x}{\delta_T}$ and Nusselt number x you will get this. And, you can see that this is the same expression as we derived for uniform wall heat flux condition. And, you can see the temperature profile whatever we have got it from the uniform wall heat flux condition. So, here we can see, if you take from Re_x this $\sqrt{x}$ outside; then, you will get $\frac{x}{\sqrt{x}}$ and it will be $\sqrt{x}$ and all other terms are constant, because q_w'' is constant, k is constant, Prandtl number is constant and here free properties and velocity are constant.

So, all these will be constant. So, you can write $T_x(x) - T_\infty = C\sqrt{x}$. So, you can see that, keeping the flat plate at uniform wall heat flux condition or keeping the flat plate as variable wall temperature where wall temperature varies as $\sqrt{x}$, both will give the same result; because, you have seen that Nusselt number expression and these $\frac{\delta_T}{x}$ expressions are same in both the cases. So, in today's lecture, we considered laminar flow over a flat plate with uniform wall heat flux boundary condition.

So, q_w'' is constant on the flat plate; however, you have T_w which is your wall temperature varies with x. We considered initially up to x = x_0 as a unheated region, and from x > x_0, it is maintained at a uniform wall heat flux boundary condition.

Then, we found the temperature profile using third degree polynomial; applying four boundary conditions, we found the four coefficients. And finally, these velocity profile as well as the temperature profile, we put it in the energy integral equation. And, integrating that equation we got finally the expression for $\frac{\delta_T}{x}$, which is your δ_T is your thermal boundary layer thickness.

Once you got the expression for $\frac{\delta_T}{x}$; then, we found the wall temperature variation T_w and local Nusselt number Nu_x. And, putting the x_0 = 0; that means there is no unheated region, then we found the as a special condition what are the expression for δ_T as well as the wall temperature and Nusselt number. Next, we have considered variable wall temperature boundary conditions.

So, we have taken the wall temperature variation $T_x(x) = T_\infty + C\sqrt{x}$. And, putting that wall temperature condition and using third degree polynomial of velocity profile and temperature profile, we have found the same thermal boundary layer thickness as well as same Nusselt number. So, we have seen that both conditions are same; however, if you maintain the variable wall temperature $T_x(x) = T_\infty + C\sqrt{x}$, it is equivalent to maintaining the flat plate as uniform wall heat flux condition.

Thank you.