Fundamentals of Convective Heat Transfer
Prof. Amaresh Dalal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module – 08
Natural Convection
Lecture - 26
Introduction and scale analysis

Hello everyone. So, till now we have considered forced convection. In forced convection the fluid is forced to flow over a surface or inside a tube by external means like using pump or blower. Today, we will start natural convection which is also known as free convection.

In natural convection fluid motion starts in natural way due to the temperature difference and hence, there will be density difference and in the presence of acceleration like gravitational acceleration.

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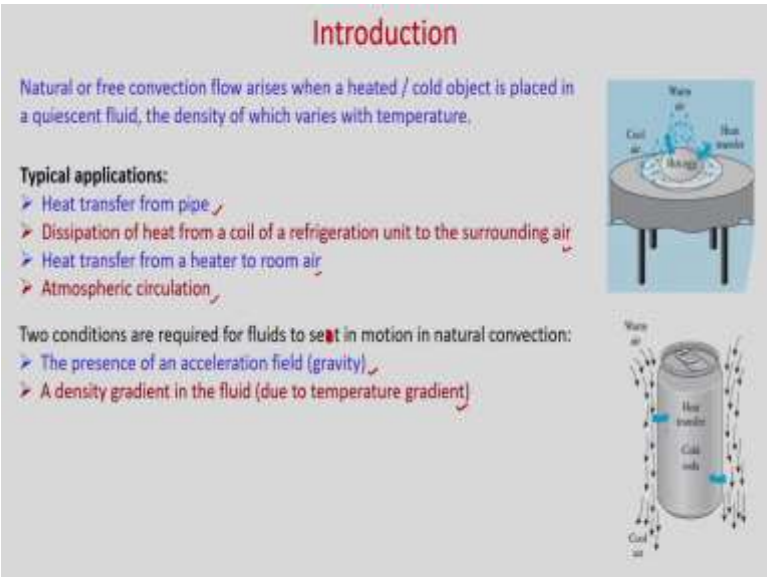

So, you can see natural or free convection flow arises when a heated or cold object is placed in a quiescent fluid. The density of which varies with temperature. So, these are some typical applications; heat transfer from pipe, dissipation of heat from a coil of a refrigeration unit to the surrounding air, heat transfer from a heater to room air, and atmospheric circulation.

So, as I told before the two conditions are required, for fluids to set in motion in natural convection, the presence of an acceleration field like gravity and a density gradient in the fluid which may occur due to temperature gradient. So, you can see here in the right hand side figure. So, if you keep one hot egg in a plate then; obviously, surrounding fluids temperature is lower than the hot egg.

So, the fluid which is coming into contact with hot egg will have higher temperature and it will have lower density and it will go up. So, warm air will go up and cool air will come to the hot egg. Again it will get heated and it will go up. So, in this way you can see that fluid motion starts.

Another example of this cold soda; so, it is actually warmed up in presence of the ambient air. So, you can see when you keep it in the ambient air, warm air will come into contact, there will be heat transfer its density will increase and it will go down.

So; obviously, warm air will come into contact in the upper half and there will be heat transfer due to that a density will increase and cool air will come down. So, these are some examples of natural convection. We can have simplest natural convection solution for flow over vertical plate.

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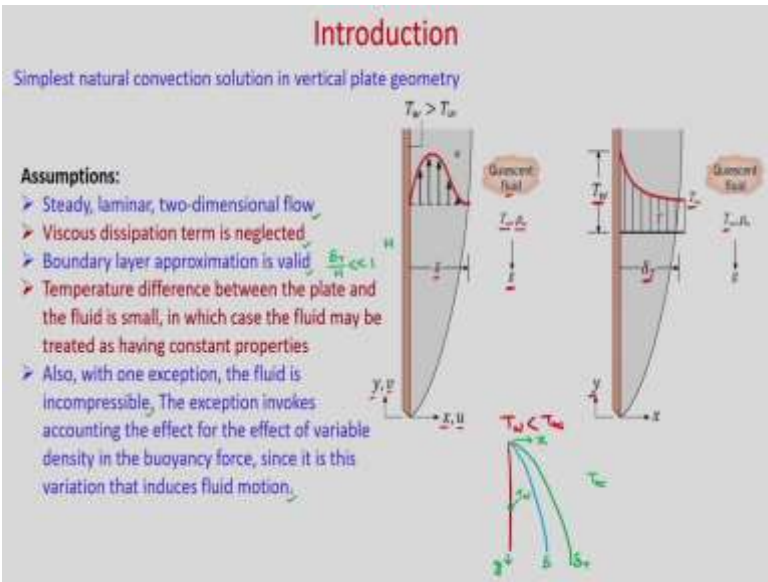

So, if you consider this vertical plate which is maintained at temperature T_w and the quiescent fluid is having temperature T_∞ and its density is $\rho\ \infty$ there will be acceleration field and that is your gravity.

So, we are taking x as normal to this vertical plate and velocity in that direction we will consider as u and along the plate we are taking y coordinate and velocity in that direction will be v. As $T_w > T_\infty$; obviously, your fluid will go up and thermal boundary layer and hydrodynamic boundary layer will form over this plate. So, here you can see in the first figure.

So, there will be a hydrodynamic boundary layer formation, whose hydrodynamic boundary layer thickness is δ and you can see the velocity distribution; obviously, for no slip condition, on the vertical wall velocity will be 0 and outside ambient is quiescent fluid so; obviously, at the edge of the boundary layer velocity will be almost 0. So, maximum velocity will occur inside this hydrodynamic boundary layer.

If you consider the thermal boundary layer; so, similarly it will also start from the bottom and δ T is the thermal boundary layer thickness. It will grow as you go up; that means, with increase in y δ T will also increase and $T_\infty < T_w$.

So, this will be the temperature distribution. So, at the wall you have T_w and at the edge of the thermal boundary layer you have temperature T_∞. If you consider $T_w < T_\infty$ so in that case your boundary layer will start forming from the top edge. So, that will be your δ and also you have thermal boundary layer δ_T. So, this is your δ_T and this is your δ and this is your direction y and this is your x this is your T_w and quiescent medium is having temperature T_∞.

So, first we will write down the governing equations for flow over vertical plate. Let us first make the assumptions. So, you can see the flow is steady laminar and two dimensional viscous. Dissipation term is neglected and; obviously, we will have boundary layer approximation is valid.

And if we consider the height of the vertical plate as H then we can consider that $\frac{\delta_T}{H}$ will be << 1. Temperature difference between the plate and the fluid is small, in which case the fluid maybe treated as having constant properties. So, we are considering the temperature difference as low and the thermal physical properties like viscosity, thermal conductivity, specific heat, we will assume that these to be constant.

Also, with one exception the fluid is incompressible the exception invokes accounting the effect for the effect of variable density in the buoyancy force since it is this variation that induces fluid motion. So, you can see another important assumptions we are taking that we are considering ρ to be constant.

So, that it will become incompressible, but as temperature difference will be there, there will be density difference and that density difference effect will take into account in buoyancy term only and rest other terms like continuity equation and the in inertia term of the momentum equation, we will take ρ as constant.

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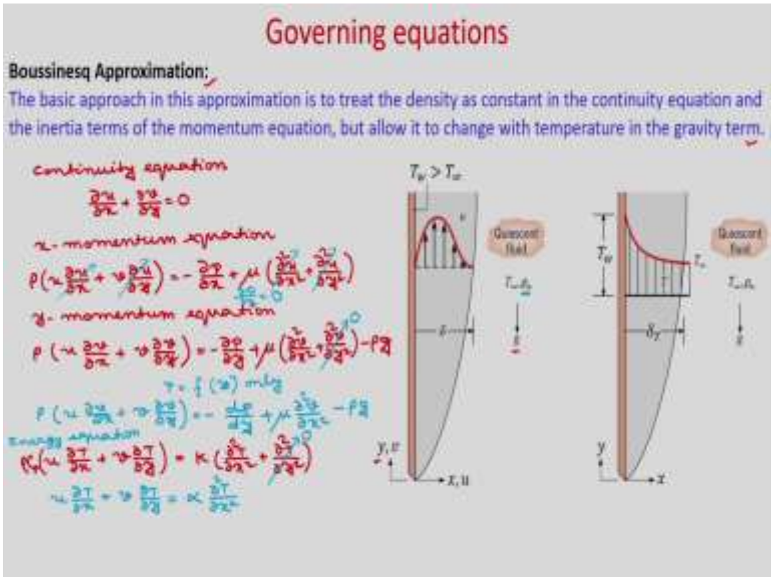

So, in natural convection whatever we discussed just now that is known as Boussinesq approximation. The basic approach in this approximation is to treat the density as constant in the continuity equation and the inertia terms of the momentum equation, but allow it to change with temperature in the gravity term. So, this is important assumptions we are taking which is known as Boussinesq approximation.

So, in this approximation what we are telling; that we will take density as constant in the continuity equation as well as in inertia terms of momentum equation, but we will take the effect of its change in the buoyancy term. So, first let us write the governing equations. So, what are the governing equations? First is continuity equation.

So, we consider two dimensional flow. So, it will be $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ and x momentum equation. So, $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ and y momentum equation $\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\rho g$.

Now, first write down the boundary layer equations. So, as we discussed in the forced convection in boundary layer approximation that δ_T << H k. So, in this particular case you can see that δ is the hydrodynamic boundary layer thickness and δ_T is your thermal boundary layer thickness and that will be much-much smaller than the height of the plate.

So, in this case now you can have the assumptions. So, if you have the similar way as we have done in the forced convection you can neglect $\frac{\partial^2 v}{\partial y^2}$ and here in x momentum equation all these terms will become negligible only from here so, all this terms will become negligible and you will have $\frac{\partial p}{\partial x}$ =0. And as $\frac{\partial p}{\partial x}$ is 0, p is function of y only, then y momentum equation you can write as $\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{dp}{dy}+\mu\frac{\partial^2 v}{\partial x^2}-\rho g$. So, this is your boundary layer equation for flow over vertical plate and this is your y momentum equation. In x momentum equation you will get $\frac{\partial p}{\partial x}$ =0. Now, let us write the energy equation.

So, in general your energy equation is $\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$. So, if you write the boundary layer equation then it will be 0. So, this is your energy equation.

So, you can write $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}$. Now, you see the y momentum equation. In the y momentum equation in the left hand side we have density ρ. So, invoking the Boussinesq approximation will take ρ as constant and it will be same as ρ_∞ .

So, in the first impression whatever you have ρ_∞ which is actually quiescent medium. So, that will be $\rho = \rho_\infty$. So, invoking the Boussinesq approximation will take the ρ in the left hand side of the y momentum equation as ρ_∞ which is your quiescent medium density ρ_∞. So, you can see this ρ_∞.

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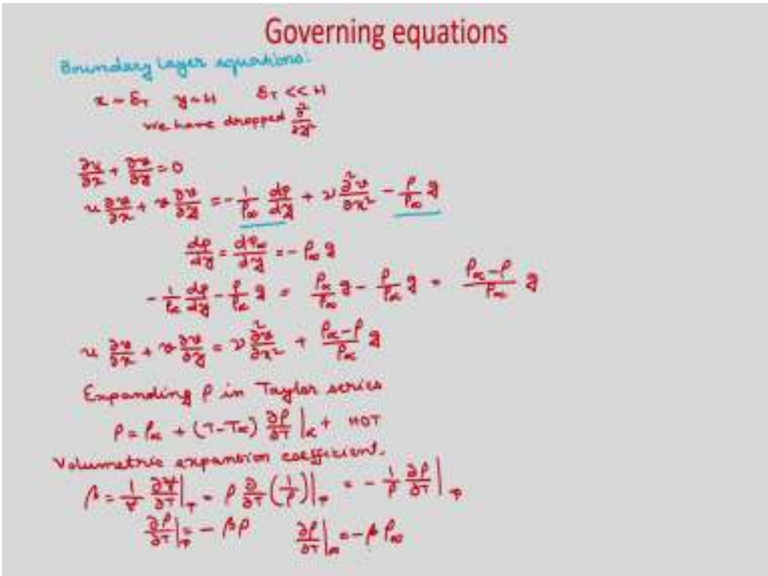

So, you can write down all these boundary layer equations now. So, you can write these boundary layer equations. So, whatever we invoked we have taken x ~ δ_T, y ~H and we have assumed δ_T << H and we have dropped $\frac{\partial^2}{\partial y^2}$ these terms.

So, your continuity equation is, your continuity equation will remain same $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ and invoking the Boussinesq approximation and rearranging you can write the y momentum equation as $u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_\infty}\frac{dp}{dy}+\nu\frac{\partial^2 v}{\partial x^2}-\frac{\rho}{\rho_\infty}g$.

Now, you have buoyancy term. So, what is your buoyancy term. So, buoyancy term will become $-\frac{\rho}{\rho_\infty}g$. So, now here you can see how you will calculate the $\frac{dp}{dy}$. So, $\frac{dp}{dy}$ you can write as $\frac{dp}{dy}=\frac{dp_\infty}{dy}$, because $\frac{\partial u}{\partial x}$ = 0. So, whatever pressure is there outside the boundary layer so that will be impressed inside the boundary layer. So, now, in the quiescent medium so you can write the hydrostatic pressure right. So, what will be the $\frac{dp_\infty}{dy}$?

So, this you can write as $-\rho_\infty g$. So, this is your from hydrostatic pressure distribution you can write it. So; obviously, you can see now these two terms together you can write this term and this term you can write as $-\frac{1}{\rho_\infty}\frac{dp}{dy}-\frac{\rho}{\rho_\infty}g=\frac{\rho_\infty}{\rho_\infty}g-\frac{\rho}{\rho_\infty}g$.

So, it will become $\frac{\rho_\infty-\rho}{\rho_\infty}g$. So, your momentum equation buoyancy term you can write as, $\frac{\rho_\infty-\rho}{\rho_\infty}g$ and now you can expand the ρ in Taylor series, expanding ρ in Taylor series. So, what you can write? $\rho=\rho_\infty+\left(T-T_\infty\right)\frac{\partial\rho}{\partial T}\Big|_\infty+HOT$.

So, neglect this high order term and you also you can have volumetric expansion coefficient as volumetric expansion coefficient, you can write $\beta=\frac{1}{\forall}\frac{\partial\forall}{\partial T}\Big|_p$.

So, here this is the $\forall$ volume at constant pressure. So, in terms of density if you write. So, it will be $\rho\frac{\partial}{\partial T}\left(\frac{1}{\rho}\right)\Big|_p$. So, it will be $-\frac{1}{\rho}\frac{\partial\rho}{\partial T}\Big|_p$. So, what you can write $\frac{\partial\rho}{\partial T}\Big|_p=-\beta\rho$.

So, similarly you can write $\frac{\partial\rho}{\partial T}\Big|_\infty=-\beta\rho_\infty$. So, if you invoke these in this equation so what you will get? You will get $\rho=\rho_\infty+\left(T-T_\infty\right)\left(-\beta\rho_\infty\right)$.

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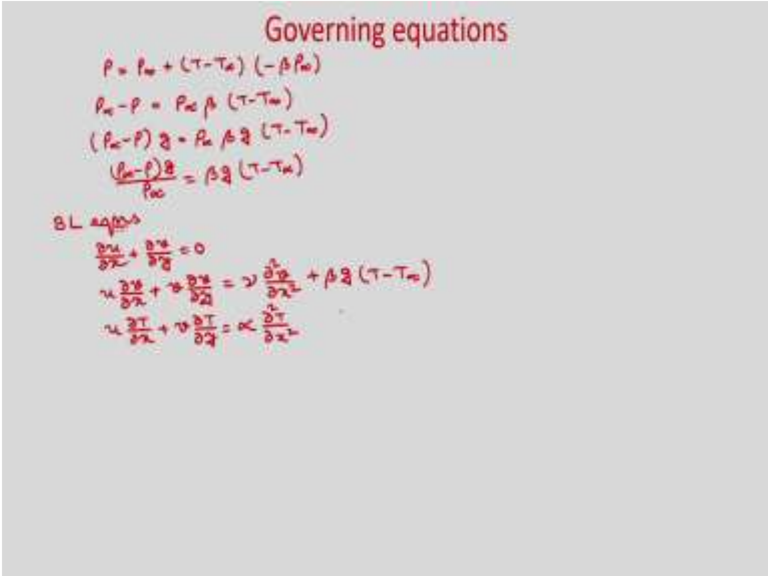

So, you can rearrange and you can write $\rho_\infty - \rho = \rho_\infty \beta \left(T - T_\infty\right)$ and $\left(\rho_\infty - \rho\right) g = \rho_\infty \beta g \left(T - T_\infty\right)$. So, now, you can see this term $\dfrac{\left(\rho_\infty - \rho\right) g}{\rho_\infty} = \beta g \left(T - T_\infty\right)$. So, this is your buoyancy term.

So, now your boundary layer equations will become continuity $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$, your x momentum equation will become $u \dfrac{\partial v}{\partial x} + v \dfrac{\partial v}{\partial y} = \nu \dfrac{\partial^2 v}{\partial x^2} \beta g \left(T - T_\infty\right)$. Now, the buoyancy term $\beta g \left(T - T_\infty\right)$ and your energy equation $u \dfrac{\partial T}{\partial x} + v \dfrac{\partial T}{\partial y} = \alpha \dfrac{\partial^2 T}{\partial x^2}$.

So, these are the boundary layer equations for flow over vertical plate, for the case of natural convection and we have assumed density to be constant in continuity equation and in the inertia terms of y momentum equation and we have taken the change of density only in the buoyancy term. So, these are the equations.

Now, we will do the scale analysis and we will try to find what is the order of heat transfer coefficient and the Nusselt number and which are the forces is dominating for high panel number fluids and low panel number fluids for constant surface temperature. So, first we will consider uniform surface temperature case, then we will find the order of magnitude of this heat transfer coefficient and the Nusselt number.

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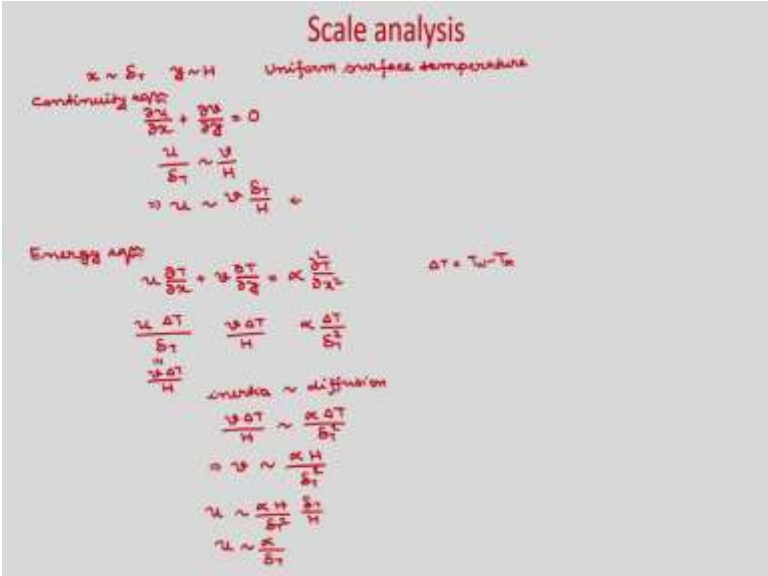

So, inside the thermal boundary layer your x ~ δ_T, y ~ H, where H is the height of the plate and we are assuming that uniform surface temperature and the continuity equation if you see that will be $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$. So, if you see the order of magnitude of u. So, $\frac{u}{\delta_T}\sim\frac{v}{H}$.

So, from here you can see $u\sim v\frac{\delta_T}{H}$, from energy equation. Now; so, this is your continuity equation now from energy equation $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}$. So, the order of $\Delta T=T_w-T_\infty$ and this Δ T ~δ $_T$. So, you can see here you can write $\frac{u\Delta T}{\delta_T}$.

So, this is the first inertia term the second inertia term. So, you can write $\frac{v\Delta T}{H}$ and the diffusion term will be $\alpha\frac{\Delta T}{\delta_T^2}$. So, now, you can see if you put $u\sim v\frac{\delta_T}{H}$. So, if you put it here you can see it will be equivalent to $\frac{v\Delta T}{H}$. So, you can see these inertia terms are comparable, because these are same $\frac{v\Delta T}{H}$ and this is your diffusion term.

So, now inertia should be comparable with the diffusion term. So, if you do that. So, you will get $\frac{v\Delta T}{H} \sim \alpha \frac{\Delta T}{\delta_T^2}$. So, from here you can see the, what is the scale of v. So, $v \sim \frac{\alpha H}{\delta_T^2}$. So, we have found the scale for v and similarly from this equation you can find the scale for u.

So, $u \sim \frac{\alpha H}{\delta_T^2} \frac{\delta_T}{H}$. So, $u \sim \frac{\alpha}{\delta_T}$. So, now, from using continuity equation and the energy equation, we have found the scale for velocity u and v. Now, let us consider momentum equation.

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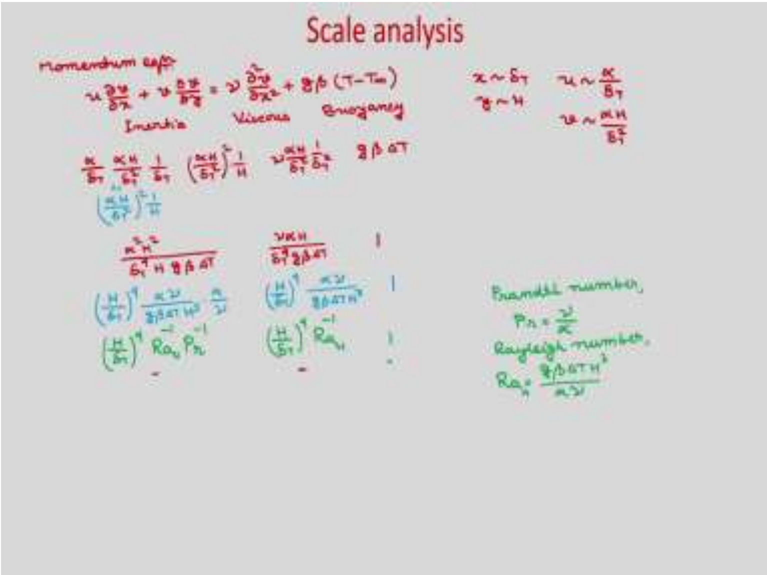

So, momentum equation we have $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2} + g\beta\left(T - T_\infty\right)$. So, these are inertia terms, this is your viscous term, and this is your buoyancy term and what is the order we have? x ~ δ_T, y ~H, $u \sim \frac{\alpha}{\delta_T}$, and $v \sim \frac{\alpha H}{\delta_T^2}$. So, using this scale now let us see each term.

So, the first inertia term if you see so, if we put $\frac{\alpha}{\delta_T} \frac{\alpha H}{\delta_T^2} \frac{1}{\delta_T}$. So, this is your first inertia term. Second inertia term now v so, it will be $\left(\frac{\alpha H}{\delta_T^2}\right)^2 \frac{1}{H}$. So, first look at this two terms. So, this is equivalent to this. So, you can see this two inertia terms are of same order, because both are having the same order $\left(\frac{\alpha H}{\delta_T^2}\right)^2 \frac{1}{H}$. now, let us see the viscous term. So, in the viscous term now we have ν. So, $v \sim \frac{\alpha H}{\delta_T^2}$ and x ~ δ_T and this buoyancy term, so $g\beta\Delta T$.

So, now what we will do we will just divide this terms with $g\beta\Delta T$. So, that it will become 1 buoyancy. So, we are dividing by $g\beta\Delta T$. So, you will get so, this term if you see. So, it will be $\frac{\nu\alpha H}{\delta_T^3 g\beta\Delta T}$. So, this is your viscous term and now you divide this term. So, these two terms are same. So, I will write only one term.

So, it will be $\frac{\alpha^2 H^2}{\delta_T^4 H g\beta\Delta T}$. Now, we will rearrange it. So, if you rearrange it so, you can see the viscous term . We will write it as so this will be 1 buoyancy term, it will be $\frac{\alpha\nu}{g\beta\Delta T H^3}$. So, now, H cube we have just divided. So, we will multiply in the numerator H.

So, it will be $\left(\frac{H}{\delta_T}\right)^4 \frac{\alpha\nu}{g\beta\Delta T H^3}\frac{\alpha}{\nu}$.

So, you can see, you can write it as now Prandtl number, you know right what is the Prandtl number? So, $\mathrm{Pr} = \frac{\nu}{\alpha}$ and we will define another non dimensional number that is known as Rayleigh number. So, Rayleigh number we are defining as $\mathrm{Ra}_H = \frac{g\beta\Delta T H^3}{\alpha\nu}$.

So, what you can do now, this you can write as $\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1}\,\mathrm{Pr}^{-1}$. This is your $\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1}$ and Rayleigh number we have defined based on the height of the plate. So, this is you can write H, here also you can write H and this is the buoyancy term this is 1.

Now, we will consider two different cases one is high Prandtl number fluids and low Prandtl number fluids. So, for high Prandtl number fluids, Prandtl number will be >>1 and you can see in this particular case. So, it is $\frac{1}{\Pr}$. So, if it is $\frac{1}{\Pr}$ and Prandtl number is very high. So, this term will be dominant term .

So, this term will be dominant term for the high Prandtl number fluids and when you have low Prandtl number fluids so; obviously, this will be your dominant term, but in each cases buoyancy term should be present.

So, either your inertia term will be comparable with buoyancy term or your viscous term will be comparable buoyancy term, because to have the natural convection you should have the buoyancy term present.

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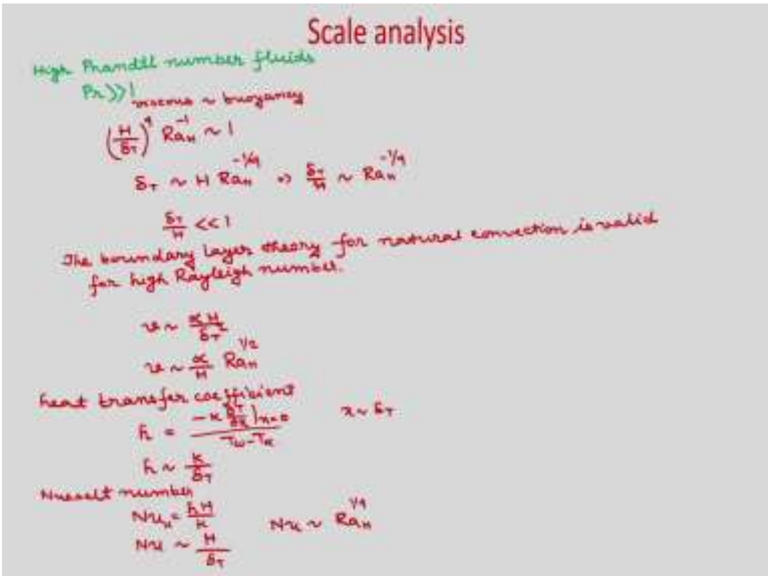

So, in two different cases now we will consider high Prandtl number fluids and low Prandtl number fluids. So, first we will consider high Prandtl number fluids. So, Prandtl number >> 1. So, you can see your $\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1} \sim 1$.

So; that means, viscous term will be comparable with buoyancy term, because Prandtl number is very high. So, the inertia term you can see you have $\frac{1}{\Pr}$. So, and Prandtl number is very high. So, that will be negligible compared to the viscous term and for high Prandtl number fluids you can compare the viscous force with the buoyancy force. So, that we are doing.

So, with this now you can find what is δ_T. So, $\delta_T \sim H\,\mathrm{Ra}_H^{-1/4}$ and we know that $\frac{\delta_T}{H}$ we are assumed that it is >> 1. From here you can see $\frac{\delta_T}{H} \sim \mathrm{Ra}_H^{-1/4}$.

If $\frac{\delta_T}{H} << 1$ then this Rayleigh number should be very high, because it is of the order of $\mathrm{Ra}_H^{-1/4}$. So, it is minus is there. So, you can write the boundary layer theory for natural convection is valid for high Rayleigh number fluids. So, now, you can find what is the scale of velocity v. So, we know $v \sim \frac{\alpha H}{\delta_T^2}$. So, $\frac{\delta_T}{H} \sim \mathrm{Ra}_H^{-1/4}$.

So, from here you can write $v \sim \frac{\alpha}{H}\mathrm{Ra}_H^{1/2}$. So, this is the scale for velocity v for high Prandtl number fluids. Now, let us find what is the heat transfer coefficient. So, for heat transfer coefficient so, you know $h = \frac{-K \frac{\partial T}{\partial x}\big|_{x=0}}{T_w - T_\infty}$.

So, as you know $x \sim \delta_T$. So, you can write $h \sim \frac{K}{\delta_T}$. So, now, Nusselt number. So, it will be $Nu_H = \frac{hH}{K}$. So, based on height H so Nusselt number will be order of; so you can see, $Nu_H \sim \frac{H}{\delta_T}$. So, $Nu_H \sim \mathrm{Ra}_H^{1/4}$.

So, you can see from the scale analysis we have found the scale for δ_T, thermal boundary layer thickness, your velocity v, and heat transfer coefficient and Nusselt number. You can see $Nu_H \sim \mathrm{Ra}_H^{1/4}$. So, later when we will do the analytical solution for the boundary layer equations for natural convection you will find that $Nu_H \sim \mathrm{Ra}_H^{1/4}$.

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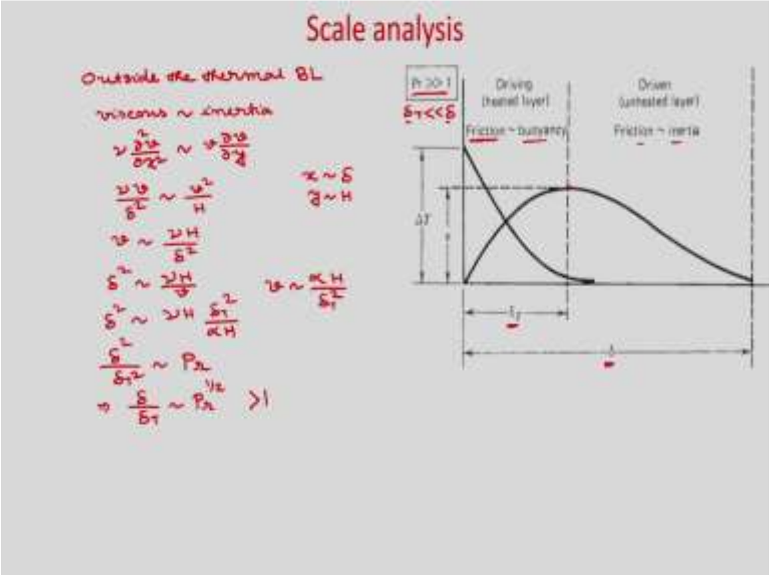

Now, let us see that for high Prandtl number fluids so; obviously, for high Prandtl number fluids $\delta_T << \delta$ right, where δ_T is your thermal boundary layer thickness and δ is your hydrodynamic boundary layer thickness. So, in this case you can see that this is your thermal boundary layer thickness.

So, temperature will vary. It will be high at the wall and gradually it will decrease and in the quiescent medium; obviously, there will be temperature is T_∞ and δ_T will almost will become 0 . So, this is the variation of temperature inside the thermal boundary layer, but as $\delta >> \delta_T$.

So, the effect of velocity will be still there outside this thermal boundary layer and you can see in the outside there will be effect of this velocity and you will get maximum velocity; obviously, it will be inside the thermal boundary layer. So, maximum velocity you will get inside the thermal boundary layer. So, inside the thermally boundary layer if you consider so; obviously, your viscous force will be order of buoyancy force; that means, friction force.

So, because you it is near to the solid wall. So, the viscous effect will be there and viscous effect will be comparable to the buoyancy force, but in the unheated layer. So, you can see; obviously, your buoyancy will be absent, because there is no temperature difference outside this thermal boundary layer. So, your buoyancy force will be absent and your viscous force will be comparable with the inertia force.

Now, you can see that outside the thermal boundary layer. So, you will have viscous force will be comparable with inertia force. So, you can see viscous force, viscous force will be comparable with the so this is your viscous and this is your inertia.

So, from here you can see. So, viscous term will be your $\nu \dfrac{\partial^2 v}{\partial x^2}$ will be comparable with the inertia term. So, that will be any term you can say. So, it will be let us say $v\dfrac{\partial v}{\partial y}$. So, in this case now it will be $\dfrac{\nu v}{\delta^2} \sim \dfrac{v^2}{H}$. So, from here you can see the velocity $v \sim \dfrac{\nu H}{\delta^2}$ and $\delta^2 \sim \dfrac{\nu H}{v}$ and v we have already found, it will be $v \sim \dfrac{\nu H}{\delta^2}$. So, if you put the value of v. So, it will be $\delta^2 \sim \nu H \dfrac{\delta^2}{\alpha H}$. So, you can see $\dfrac{\delta^2}{\delta_T^2}$.

So, this H H will get cancelled and $\Pr = \dfrac{\nu}{\alpha}$. So, you can see $\dfrac{\delta}{\delta_T} \sim \Pr^{1/2}$ which will be >> 1 right, because Prandtl number > 1 and from here you can see that $\delta > \delta_T$.

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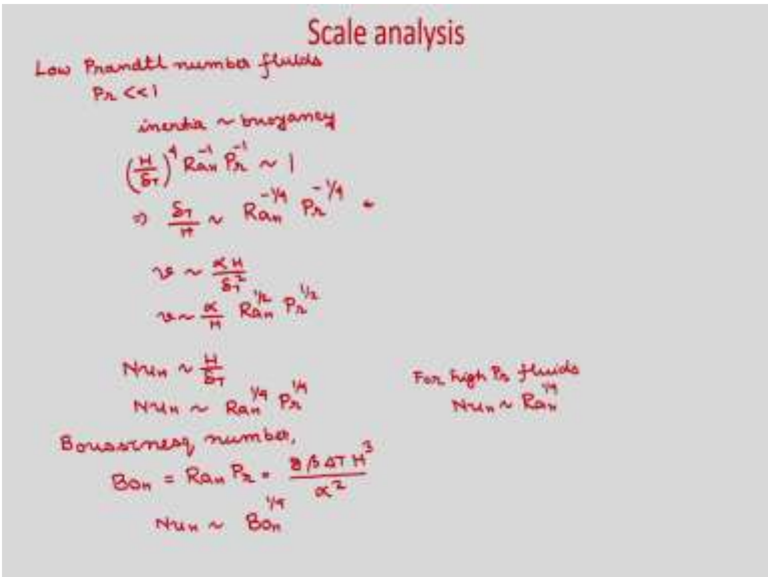

Now, let us consider low Prandtl number fluids; that means, Prandtl numbers << 1, low Prandtl number fluids. So, in this particular case now as Prandtl number is low so inertia force will be dominant . So, you can neglect the viscous force. So, inertia force will be comparable with the buoyancy force right. So, in this case inertia force will be comparable with buoyancy force so; obviously, it will be $\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1}\,\mathrm{Pr}^{-1} \sim 1$.

So, from here you can see $\frac{\delta_T}{H} \sim \mathrm{Ra}_H^{-1/4}\,\mathrm{Pr}^{-1/4}$. So, this is your $\frac{\delta_T}{H}$ and what will be the scale for velocity v; $v \sim \frac{\alpha H}{\delta_T^2}$. So, $v \sim \frac{\alpha}{H}\mathrm{Ra}_H^{1/2}\,\mathrm{Pr}^{1/2}$.

So, now you can see your Nusselt number. So, $Nu_H \sim \frac{H}{\delta_T}$, we have already shown. So, Nusselt number H from here you can see it will be $Nu_H \sim \mathrm{Ra}_H^{1/4}\,\mathrm{Pr}^{1/4}$.

So, you can see, we can conclude Rayleigh number into Prandtl number plays the same rule for low Prandtl number fluids as Rayleigh number plays for high Prandtl number fluids, because for high Prandtl number fluids, we have shown that $Nu_H \sim \mathrm{Ra}_H^{1/4}$ and this is for low Prandtl number fluids and from here we will define another non dimensional number that is known as Boussinesq number.

So, this is $Bo_H = Ra_H\,\mathrm{Pr}$. So, you can write it as $\frac{g\beta\Delta T H^3}{\alpha^2}$. So, $Nu_H \sim Bo_H^{1/4}$.

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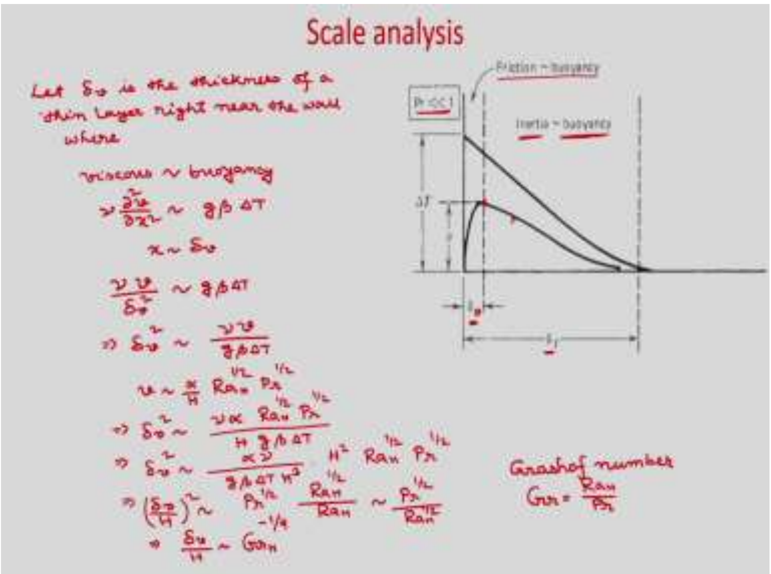

So, now, in this case you see for low Prandtl number fluids so; obviously, your this is your thermal boundary layer thickness δ_T. So, at the wall you will have high temperature difference and at the edge of the boundary layer; obviously, the $\delta_T \rightarrow 0$.

So, the δ_T variation and the velocity so; obviously, as inside the thermal boundary layer, buoyancy effect is there and there will be fluid motion and as fluid motion will be there. So, you can see your velocity effect. So, this is the velocity.

So, this is the velocities v. So, velocity effect will continue till the thermal boundary layer thickness. So, near to the thermal boundary layer thickness so; obviously, it effects will extend to the edge of thermal boundary layer.

So, the effect of velocity or the fluid motion effect will extend to the edge of the thermal boundary layer so; obviously, it is due to the buoyancy effect inside the thermal boundary layer. So, you cannot say that your hydrodynamic boundary layer thickness will be much-much smaller than δ_T for this low Prandtl number fluids, but you can see you have the maximum velocity near to the wall.

So, this is this will be your maximum velocity and if this thickness you can see, this is δ_v. So, viscous effect will be acting only near to the solid wall and your viscous force will be balancing to the buoyancy force inside this layer δ_v, but outside you can see viscous force will be less and here inertia will be comparable with the buoyancy force.

So, whatever we are introducing 1 thickness δ_v you have maximum velocity. So, inside this your viscous force will balance with the buoyancy force, but outside it your viscous force will be less and you can compare inertia force with the buoyancy force.

So, now, you can see that let δ_v is the thickness of a thin layer right near to the wall, where viscous force will be comparable with the buoyancy force, because already we have shown that for low Prandtl number fluids inertia force will be comparable with the buoyancy force and we have derived the thermal boundary layer thickness and the Nusselt number.

But near to this wall now, viscous force will be balancing with the buoyancy force and from here you can see $\nu \frac{\partial^2 v}{\partial x^2} \sim g\beta\Delta T$. So, we know that $x \sim \delta_v$.

So, you can write $\frac{\nu v}{\delta_v^2} \sim g\beta\Delta T$. So, you can write $\delta_v^2 \sim \frac{\nu v}{g\beta\Delta T}$. So, what is the order of v?

So, now, $v \sim \frac{\alpha}{H} Ra_H^{1/2} \Pr^{1/2}$. So, if you put this v here so you will get $\delta_v^2 \sim \frac{\nu\alpha Ra_H^{1/2} \Pr^{1/2}}{Hg\beta\Delta T}$.

So, now, we will rearrange it. So, if you rearrange. So, you will get $\delta_v^2 \sim \frac{\alpha\nu}{g\beta\Delta T H^3} H^2 Ra_H^{1/2} \Pr^{1/2}$.

So, here you can see this is you can write as Rayleigh number. So, $\left(\frac{\delta_v}{H}\right)^2 \sim \Pr^{1/2} \frac{Ra_H^{1/2}}{Ra_H} \sim \frac{\Pr^{1/2}}{Ra_H^{1/2}}$. So, now, we will define another non dimensional number that is known as Grashof number.

$Gr = \frac{Ra_H}{\Pr}$. So, now, we are defining another non dimensional number that is your Grashof number. So, $Gr = \frac{Ra_H}{\Pr}$. So, from here so, you can see, you can write $\frac{\delta_v}{H} \sim Gr_H^{-1/4}$.

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And from here now we have $\frac{\delta_v}{H} \sim Gr_H^{-1/4}$ and also we have $\frac{\delta_T}{H} \sim Ra_H^{-1/4} \Pr^{-1/4}$.

So, from here you can see $\frac{\delta_v}{\delta_T} \sim Pr^{1/2}$ and; obviously, it will be less than 1. So, $\delta_v << \delta_T$.

So, you can see that it has to be noted that δ_v is not same as the hydrodynamic boundary layer thickness δ . So, δ_v is the thickness where your viscous force is comparable with the buoyancy force very near to the wall and for low Prandtl number fluids this $\delta_v << \delta_T$. So, in today's class we have introduced the natural convection.

So, natural convection will occur in presence of some acceleration like gravity and density change and density change may occur due to the temperature difference. So, in the beginning we have shown some applications of this natural convection, then we considered the simplest case flow over vertical wall and we considered uniform wall temperature case starting from the Navier-Stokes equation and using the boundary layer approximation we have written down the boundary layer equations and from there we invoked the Boussinesq approximation.

So, what is Boussinesq approximation? In Boussinesq approximation we assume that density to be constant in continuity equation as well as in the inertia terms of the momentum equations, but the change of density effect we take into account in the buoyancy term.

From there we have used the scale analysis and using scale analysis for two different cases low Prandtl number fluids and high Prandtl number fluids, we have shown the scale for thermal boundary layer thickness, velocity v, heat transfer coefficient and the Nusselt number. And for low Prandtl number fluids we have also defined 1 thickness δ_v which is very near to the wall where viscous force is comparable with the buoyancy force.

Thank you.