

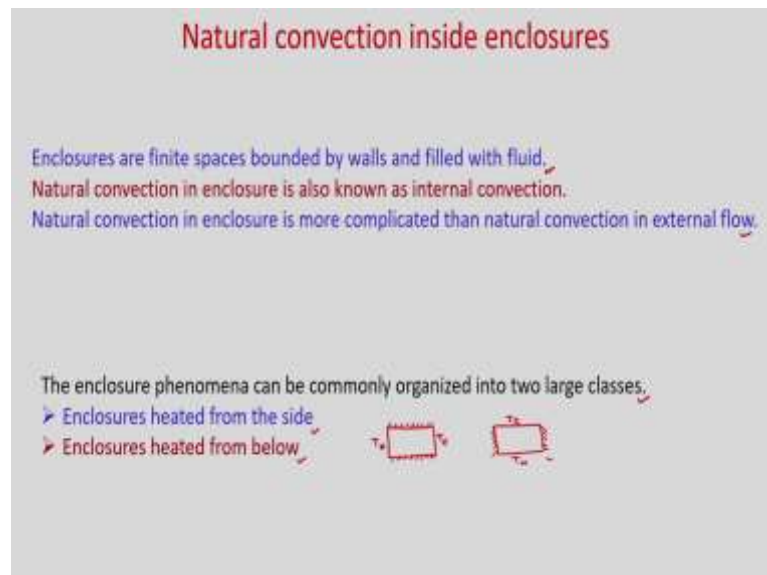
Fundamentals of Convective Heat Transfer
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Module – 09
Natural Convection - II
Lecture – 31

Natural convection inside enclosures

Hello everyone, so in today's class, we will study Natural convection in enclosures. Natural convection in enclosure is classical problem and it is also known as internal natural convection.

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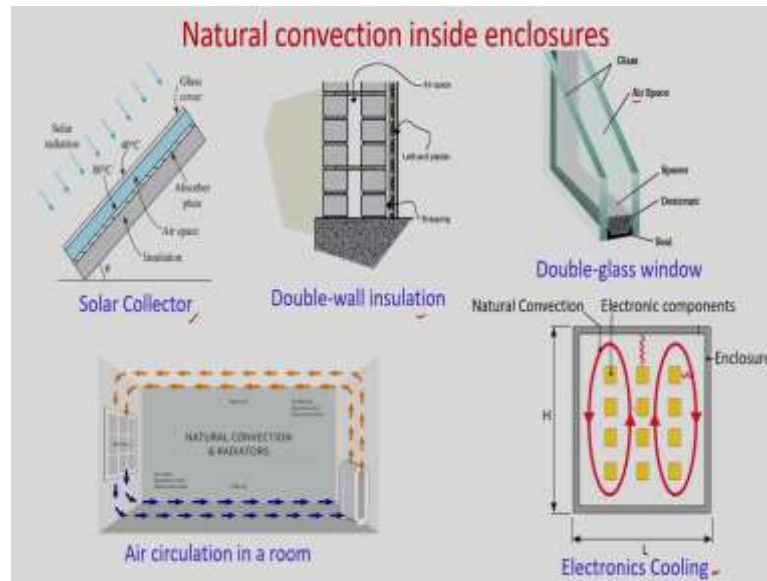


So, you can see what is enclosures? Enclosures are finite spaces bounded by walls and filled with fluid. And natural convection in enclosure is more complicated than natural convection in external flows. Because you know that natural convection in external flows this boundary layer will grow, and it will continue to grow, and it will go to turbulent flow regime. But, when we consider internal natural convection, then its growth will be limited due to the height or length of the enclosure.

The enclosure phenomena can be commonly organized into two large classes; enclosures heated from the side and enclosures heated from the below. So, you can see if you have enclosure and top and bottom walls are adiabatic and sidewalls are heated say let us say

it is maintained at hot temperature and it is your cold temperature, then these are you can see heated from the side. So, this enclosure is heated from the side. Another problem you can have, where enclosures heated from below. So, if you have the bottom wall is maintained at higher temperature than the top wall and side walls may be insulated or maybe it may be insulated, then this is another class of problem.

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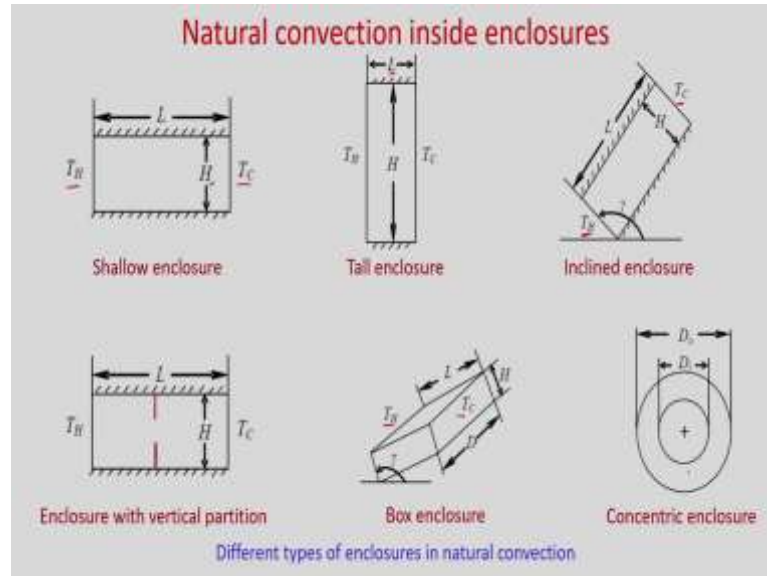
There are many applications of natural convection in enclosures; in solar collectors, then double wall insulator, also we can have application in some type of heat exchangers. So, you can see here. So, application of natural convection inside enclosure this is your solar collector, you have double wall insulation so, these are the bricks and inside you have air space, you know air is a common insulator. So, air is placed inside and double glass window so, you can see here.

So, these are double glass window inside air is placed, heat loss from the inside to outside can be prevented using these insulated air keeping in between the glasses. Also, we have air circulation in a room. So, you can see you have a radiator and obviously, from radiator hot air will go up, it will travel horizontally, then it will come to window, where it will be cooled and it will again travel like this.

So, these you can see that air circulation in a room it is also application of this natural convection of enclosure. Also, in electronics cooling, you will get application. So, these

chips will be mounted and if it is kept horizontally, then there will be natural circulation inside the enclosure.

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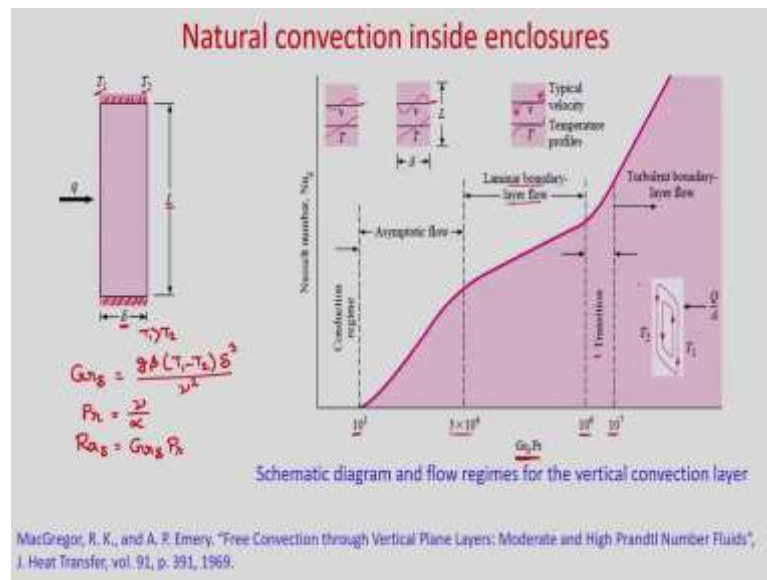


So, when we were talking about enclosures and with two different thermal boundary conditions so, you can see here we have shown different types of enclosures and different orientation. So, you can see this is known as shallow enclosure, where length is very large compared to the distance between these two plates, then this enclosure is known as shallow enclosure, and its sidewalls are heated; sidewalls are heated and top and bottom walls are adiabatic.

Here, top and bottom walls are adiabatic, but height is much much greater than length. So, you can see it is known as a tall enclosure and it is the case where sidewalls are heated. Now, this shallow enclosure if you tilt it, then you will get inclined enclosure and obviously, you can see these are differentially heated. Enclosure with vertical partition. So, you can have these partitions you can see, this is the partition.

So, this is known as enclosure with vertical partition. In three-dimension if you consider this box, then this is known as box enclosure and you can see the side walls are heated T_H and T_C and other four walls are adiabatic. You can have concentric enclosure if two cylinders are there concentric cylinders or spheres so, inside you can have this natural convection so, which is your internal natural convection.

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So, in this figure, you can see the different regimes of natural convection in enclosures. So, here you can see, here this is the enclosure, where δ is the length and L is the height and differentially heated. So, top and bottom walls are adiabatic, there is no heat transfer across these walls and left wall is maintained at constant temperature T_1 and right wall is maintained at temperature T_2 and heat transfer will take place and $Ra_\delta = Gr_\delta Pr$.

So, $Gr_\delta = \frac{g\beta(T_1 - T_2)\delta^3}{\nu^2}$, if $T_1 > T_2$, then $\frac{g\beta\Delta T\delta^3}{\nu^2}$. So, this is your Grashof number and

$Pr = \frac{\nu}{\alpha}$ and we know that $Ra_\delta = Gr_\delta Pr$. So, in this particular case x axis is your Rayleigh number and y axis is a Nusselt number. So, you can see how the Nusselt number varies with Rayleigh number in an enclosure.

If $Ra < 10^3$, then it is almost conduction dominated. So, temperature will vary linearly and there will be less convection cell. So, velocity will be very very less, and heat transfer is dominated by conduction mode of heat transfer only. So, we can see velocity is very small and temperature is a linearly varying from T_1 to T_2 .

But, if you go above this Rayleigh number, then Rayleigh number 10^3 to 3×10^4 regime, then it is kind of asymptotic flow. Here also temperature you can see in the central region, it is almost linear, but near to the wall there is variation and velocity is comparatively a low. Now, in the range of Rayleigh number 3×10^4 and 10^6 , you will get

laminar boundary layer flow, and you can see gradually there will be increase in the Nusselt number, and you can see there will be a temperature profiles like this.

So, here you can see in central region, you have almost constant temperature, but maximum temperature variation you will get near to the wall, and similarly in the velocity. So, near to the central region, you will get very low velocity, but near to the wall, you will get very high velocity jet. Here, you can see this is the velocity jet. So, this is a laminar boundary layer flow. But, if you go in between 10^6 and 10^7 , it will be transition and then after Rayleigh number 10^7 , you will get turbulent boundary layer flow.

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Natural convection inside enclosures

Thermal boundary layer thickness
 $\delta_T \sim \sqrt{\alpha t}$
 $v \sim \frac{g \beta \Delta T \alpha t}{\nu}$

Final time
 $t_f \sim \left(\frac{2H}{g \beta \Delta T \alpha} \right)^{1/2}$

Thermal BL thickness at time t_f
 $\delta_{T,f} \sim \sqrt{\alpha t_f} \sim H Ra_b^{-1/4}$

Characteristics of natural convection in a rectangular enclosure heated from the side

Regime	I. Conduction	II. Tall Systems	III. Boundary layer	IV. Shallow systems
Condition of occurrence	$Ra_b < 1$	$H/L > Ra_b^{1/4}$	$Ra_b^{1/4} < H/L < Ra_b^{1/2}$	$H/L < Ra_b^{1/4}$
Flow pattern	Clockwise circulation	Distinct boundary layer on top and bottom walls	Boundary layer on all four walls. Core remains stagnant	Two horizontal wall jets flow in opposite directions
Effect of flow on heat transfer	Insignificant	Insignificant	Significant	Significant
Heat transfer mechanism	Conduction in horizontal direction	Conduction in horizontal direction	Boundary layer convection	Conduction in vertical direction
Heat transfer	$\dot{q} \sim k(T_1 - T_2)/L$	$\dot{q} \sim k(T_1 - T_2)/L$	$\dot{q} \sim (k \beta \Delta T)^{1/4} H$	$\dot{q} \sim (k \beta \Delta T)^{1/4} H$

So, we will not go into details in scale analysis derivation, but we will show; what is the scale of thermal boundary layer and the velocity as well as the heat flux. For details you can refer the “Heat Transfer” book by Adrian Bejan. Here, we will just show the scale of different parameters. So, you can see your thermal boundary layer thickness will be, $\delta_T \sim \sqrt{\alpha t}$, where t is the time and alpha is the thermal diffusivity. So, the velocity, $v \sim \frac{g \beta \Delta T \alpha t}{\nu}$.

Now, if you can see that in some cases, this natural convection will be steady state, but when it is starting the inside the enclosure, your fluid velocity will be 0, and temperature will be maintained at a film temperature or some temperature. Now, if you just start the

heating of these two walls, then there will be velocity generation due to the temperature difference, and there will be density difference and there will be recirculation and from there, you will get the velocity scale.

Now, in this case, you can see that after some time this will become steady state. There will be a recirculation and temperature control will not vary no more. So, in that case to reach from time 0 to sometime t_f final time to reach steady state that scale we can write

as; $t_f \sim \left(\frac{\nu H}{g \beta \Delta T \alpha} \right)^{1/2}$ and at this time, thermal boundary layer; thermal boundary layer

thickness at time t_f it will be $\delta_{T,f} \sim \sqrt{\alpha t_f}$ and it will be $\delta_{T,f} \sim \sqrt{\alpha t_f} \sim H Ra_H^{-1/4}$.

So, you can see at steady state, this thermal boundary layer thickness no longer increase with time. So, that is your $H Ra_H^{-1/4}$. So, this analysis is given in detail in the “Heat Transfer” book by Bejan. So, you can see here in this table. So, there are four different regimes you will get. So, in I regime we will get conduction; where obviously, $Ra < 1$ and there will be clockwise circulation, but velocity will be very low and effect of flow on heat transfer will be insignificant and heat transfer mechanism is conduction in horizontal direction.

At, if you consider the heat transfer, then from the scale analysis you can show that $q' \sim kH\Delta T/L$. Now, another regime we will get tall system, where $H/L > Ra_H^{1/4}$ and flow pattern will be distinct boundary layer on top and bottom walls.

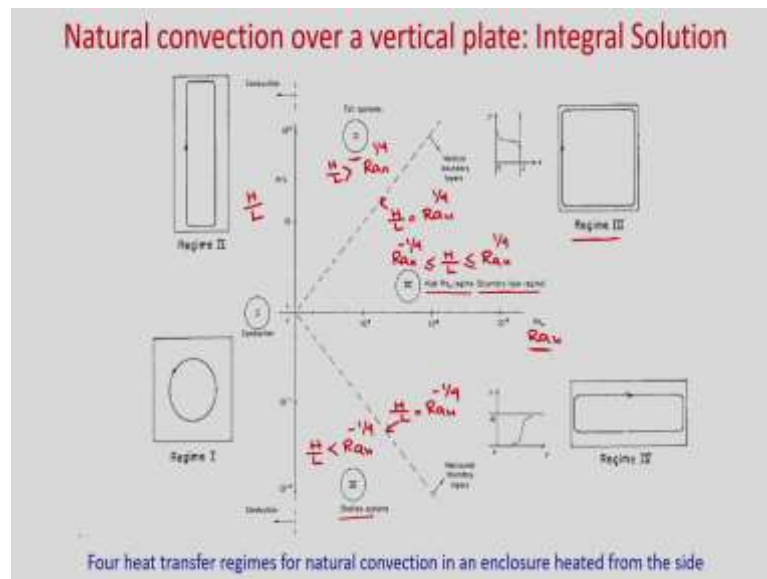
So, you will find distinct boundary layers on top and bottom walls, but effect of flow on heat transfer will be insignificant and conduction in horizontal direction and $q' \sim kH\Delta T/L$. So, this is the heat transfer rate per unit width.

Regime III you will get a boundary layer where $Ra_H^{-1/4} < H/L < Ra_H^{1/4}$. So, in this regime, you will get boundary layer on all four walls and core remains stagnant. Core remains stagnant and all four walls you will get. So, effect of flow on heat transfer it will be significant and you will get boundary layer convection and the heat transfer rate will be,

$$q' \sim \left(\frac{k}{\delta_{T,f}} \right) H \Delta T .$$

Now, the IVth regime you will get the shallow system, where $H/L < Ra_H^{-1/4}$. So, two horizontal wall jets flow in opposite direction. So, you will get that type of flow and effect of flow on heat transfer is significant and conduction in vertical direction you will get and the heat transfer rate you will get $q' \sim \left(\frac{k}{\delta_{T,f}} \right) H \Delta T$.

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So, schematically it is shown in the book of Adrian Bejan. You can see these are the four-regime shown. So, in this figure, you can see in the x axis, you have Rayleigh number and in y axis you have H/L . So, this is your y axis this is H/L and this is the x axis Rayleigh number and different regimes are shown here and you can see two dotted lines which you have vertical boundary layers and this is your $H/L = Ra_H^{1/4}$ and this is your horizontal boundary layer you will get and this is the line where $H/L = Ra_H^{-1/4}$.

So, you can see you will get at very low Rayleigh number, $Ra_H < 1$ you will get the conduction regime and here, you can see a velocity will be very low and conduction dominated heat transfer will get and temperature will vary from left to right wall linearly. And, in IInd regimes, you will get in this regime. So, if you see in the table where $H/L > Ra_H^{1/4}$. So, in this regime, you will get that tall system so, this is the tall system.

So, in this particular case, you will see that $H/L > Ra_H^{1/4}$ and we discussed here that distinct boundary layers on top and bottom wall you will get. Then, in the IVth regime which is your shallow system. So, you can see this is your shallow system and here H/L you will get; $H/L < Ra_H^{-1/4}$ so, this is your shallow system. In shallow system, you can see two horizontal wall jets flow in opposite direction you will get and conduction in vertical direction you will get.

And in this regime, this is the high Rayleigh number regime, which is known as boundary layer regime. So, in a horizontal walls and vertical walls in both directions you will get boundary layer. So, here boundary layer heat transfer will take place and it is in the regime of $Ra_H^{-1/4} \leq H/L \leq Ra_H^{1/4}$.

So, you can see this regime is your boundary layer regime and $Ra_H^{-1/4} \leq H/L \leq Ra_H^{1/4}$. So, you can see there are distinct four regimes based on the length and height ratio and in this regime, in the IIIrd regime which is your boundary layer regime, here you will get a significant heat transfer, because you will have the boundary layer heat transfer. So, you will get a boundary layer convection.

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Natural convection over a vertical plate: Integral Solution

Correlations:

Tall Enclosure $\overline{Nu}_H = 0.369 \frac{L}{H} Ra_H^{1/4} \quad \frac{H}{L} > 1$
 $\frac{H}{L} Ra_H^{1/4} > 5$

Shallow Enclosure $\overline{Nu}_H = 1 + \frac{1}{362.880} \left[\frac{H}{L} Ra_H \right]^2$
 which is valid when $\left(\frac{H}{L}\right)^2 Ra_H \rightarrow 0$ and $\frac{H}{L} < 1$

Rectangular Enclosure

$1 < \frac{H}{L} < 10$
 $\overline{Nu}_H = 0.22 \left[\frac{Pr Ra_H}{0.2 + Pr} \right]^{0.28} \left(\frac{L}{H} \right)^{0.05} \quad 1 \leq \frac{H}{L} \leq 10$
 $Pr \leq 10^5$
 $10^3 \leq Ra_H \leq 10^{13}$

$\overline{Nu}_H = 0.18 \left[\frac{Pr Ra_H}{0.2 + Pr} \right]^{0.29} \left(\frac{L}{H} \right)^{-0.13} \quad 1 \leq \frac{H}{L} \leq 10$
 $10^3 \leq Pr \leq 10^6$
 $\frac{Pr}{0.2 + Pr} Ra_H \left(\frac{L}{H} \right)^3 > 10^3$

With large aspect ratio, $\alpha = 0.02$

$\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr \left(\frac{H}{L} \right)^{-0.3} \quad 10 \leq \frac{H}{L} \leq 40$
 $1 \leq Pr \leq 2 \times 10^3$
 $10^3 \leq Ra_L \leq 10^7$

$\overline{Nu}_L = 0.046 Ra_L^{1/3} \quad 10 \leq \frac{H}{L} \leq 40$
 $1 \leq Pr \leq 20$
 $Ra_L > 10^6$

So, based on some empirical relations for different type of configuration, let us write the expression for a Nusselt number. So, these are the correlations. So, for tall enclosure, the

Nusselt number you will get $\overline{Nu}_H = 0.364 \frac{L}{H} Ra_H^{1/4}$, and it is valid for $H/L > 1$ and $\frac{L}{H} Ra_H^{1/4} > 5$.

Then, for shallow enclosure, the Nusselt number expression you will get as $\overline{Nu}_H = 1 + \frac{1}{362880} \left[\frac{H}{L} Ra_H \right]^2$ and this is valid; which is valid when $\left(\frac{H}{L} \right)^2 Ra_H \rightarrow 0$, and $H/L < 1$.

Now, let us consider rectangular enclosure, where we can have $1 < \frac{H}{L} < 10$, you can see

Nusselt number you can write as, $\overline{Nu}_H = 0.22 \left[\frac{Pr}{0.2 + Pr} Ra_H \right]^{0.28} \left(\frac{L}{H} \right)^{0.09}$. And, it is valid for $1 \leq \frac{H}{L} \leq 10$, $Pr \leq 10^5$ and $10^3 \leq Ra_H \leq 10^{13}$.

Another Nusselt number correlation you will get, $\overline{Nu}_H = 0.18 \left[\frac{Pr}{0.2 + Pr} Ra_H \right]^{0.29} \left(\frac{L}{H} \right)^{-0.13}$, and this is valid in the range of $1 \leq \frac{H}{L} \leq 10$, $10^{-3} \leq Pr \leq 10^5$ and $\frac{Pr}{0.2 + Pr} Ra_H \left(\frac{L}{H} \right)^3 > 10^3$. So, it will give regime with good results.

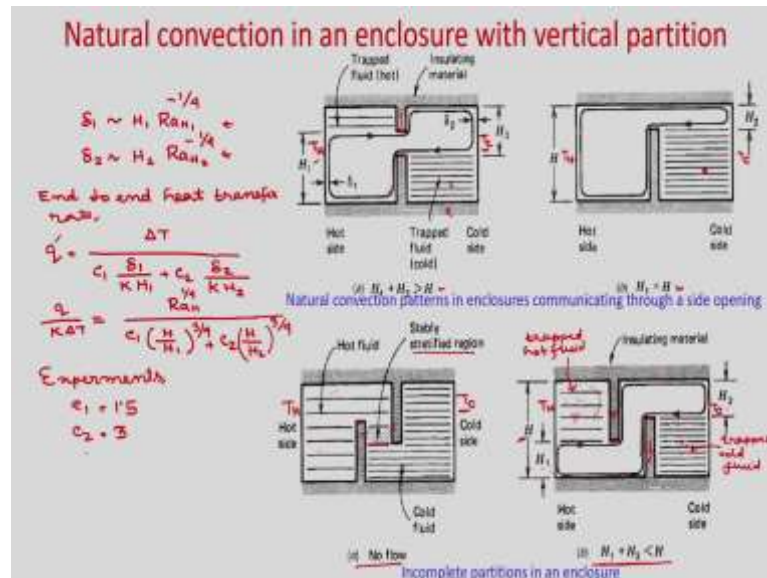
And, with large aspect ratio, Nusselt number these are all average Nusselt number and this is you can see it is based on the length L. Earlier, all these expression we have written based on the height, but here we are writing based on the length.

So, $\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012} \left(\frac{H}{L} \right)^{-0.3}$ and this is valid for $10 \leq \frac{H}{L} \leq 40$, $1 \leq Pr \leq 2 \times 10^4$ and $10^4 \leq Ra_L \leq 10^7$.

And, another expression you will get for Nusselt number for large aspect ratio so, here $\overline{Nu}_L = 0.046 Ra_L^{1/3}$ and it is valid in the range of $10 \leq \frac{H}{L} \leq 40$, $1 \leq Pr \leq 20$ and $Ra_L > 10^6$.

So, in this cases, you have seen that you have either shallow enclosure or tall enclosure or rectangular enclosure.

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Now, you can have partition in this enclosure. So, you can see here how the natural convection takes place, in an enclosure with vertical partition. So, here you can see this is differentially heated, top and bottom walls are insulated and you can see here left side is hot wall T_H ; left wall is hot wall T_H and right side is cold wall. So, what will happen? If it is hot wall, then obviously, the fluid will go up and it will try to move in this direction and when it till come into contact with the wall whose temperature is T_C , so this fluid will become heavier, density will be larger and it will come down.

But you can see as there are partitions, so in here, there will be trapped fluid which will be cold because it is a cold side so, these are a heavier fluid and it will sit just on the bottom, because you have this is adiabatic wall and it is cold wall. So, here cold fluid will be trapped and this heavier fluid will go this way and it will come here and again it will come into contact with the hot wall and again it will go up.

But here, you can see this is also trapped hot fluid, because its density is lighter because of this partition, it will be trapped here and there will be a hot fluid. So, you can see for this partition where this is your height here it is H_1 and this side from the top is H_2 and for this case, you will get a trapped fluid in here which is hot and here trapped fluid you will get which is a cold, and there will be circulation of the fluid in this region and δ_1 is the boundary layer thickness on the hot wall and here boundary layer thickness δ_2 in cold wall. Here, $H_1 + H_2 > H$, H is the distance between these two walls.

Now, another configuration you can see where H_1 is H ; that means, there is no partition from the top. So, here you will get and this is your H_2 and in this particular case; obviously, there will be no trap fluid, because it will go straight away like this and it will come into contact with the cold fluid and it will be heavier and cold fluid will be trapped here, here cold fluid will be trapped and it will go come this way, it will go down as it is heavier it will come down and again when it will come into contact with the hot wall, then it will go up.

So, you can see how the natural convection is taking place inside the enclosure. So, in these two cases, you can see natural convection patterns in enclosure communicating through a side opening.

Now, you see in these two cases. In these case, now partitions are not in line. So, they are staggered. So, here this is the partition, here it is this is the partition, and this is your hot side, and this is your cold side. So, what will happen? You can see the cold side, there will be heavier fluid due to the cold wall so, here cold fluid will be trapped in this region, cold fluid will be trapped.

In the hot side, you can see the although initially the fluid will go up, but it cannot go this way, because this is hot fluid so, it will be trapped from the top side and gradually it will be stratified, and you can see hot fluid will trapped in the left side and here, you will get stably stratified region. So, here there is no flow is occurring due to the orientation of these partitions.

Now, if you change the orientation of this partition. So, now, you have brought these this side and now this you have brought in the right side. So, you can see this is the partition, and this is the partition, from the top this is the partition attached with the bottom, this temperature is T_H and right wall temperature is T_c . So, in this particular case, $H_1 + H_2 < H$. So, this is the H_1 the distance from the bottom to this and this is from top to this it is H_2 , then $H_1 + H_2 < H$.

In this particular case, top and bottom walls are insulated, then the fluid which is in touch with the hot side; obviously, this hot fluid will be trapped here. So, it will be a stratified fluid and here also, the cold fluid will be trapped. So, this is cold fluid, trapped cold fluid and this is your trapped hot fluid.

And, natural convection now will take place so, it will go up and it will travel horizontally again as its density is low, it will go up, then again it will travel horizontally and as it will come into contact with the cold wall so, its density will be larger. So, heavier fluid will come down and as trapped cold fluid is here so, it will flow horizontally, again it will come down, as it is heavier it will flow like this, again it will go up. So, this way your flow will take place.

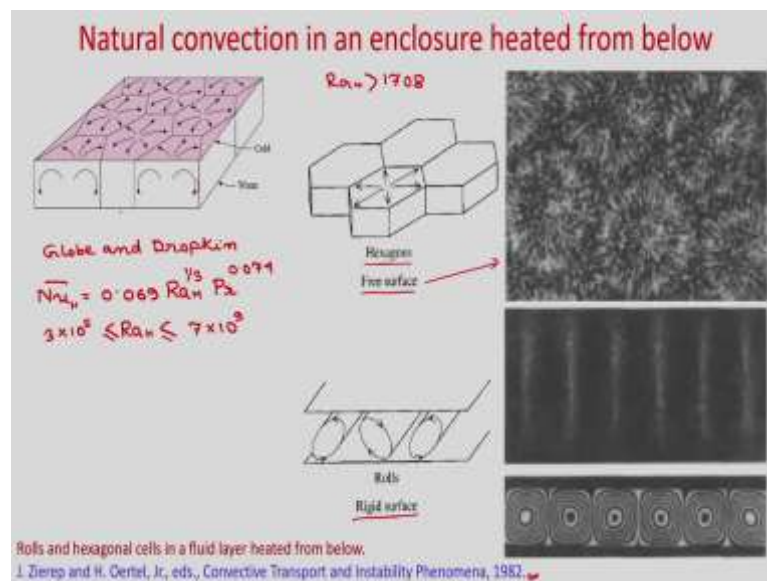
So, you see the orientation of partition how it works. In this case, there is no flow, but if you change the orientation, then you will get the natural convection flow. So, in this particular case if you see so, if $\delta_1 \sim H_1 Ra_{H_1}^{-1/4}$ and $\delta_2 \sim H_2 Ra_{H_2}^{-1/4}$ and end to end heat transfer rate will get here; end to end heat transfer rate you will get as,

$$q' = \frac{\Delta T}{C_1 \frac{\delta_1}{KH_1} + C_2 \frac{\delta_2}{KH_2}}$$

So, here, if you rearrange, you will get $\frac{q'}{K\Delta T} = \frac{Ra_H^{1/4}}{C_1 \left(\frac{H}{H_1}\right)^{3/4} + C_2 \left(\frac{H}{H_2}\right)^{3/4}}$ and from

experiments, these constants are found as $C_1 = 1.5$ and $C_2 = 3$.

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Now, let us discuss the enclosure when the bottom wall is heated. So, you can see when the bottom wall is heated, then the fluid obviously, will go up and there will be formation

of cells which are known as Benard convection cell. It is very well-known problem and very complicated.

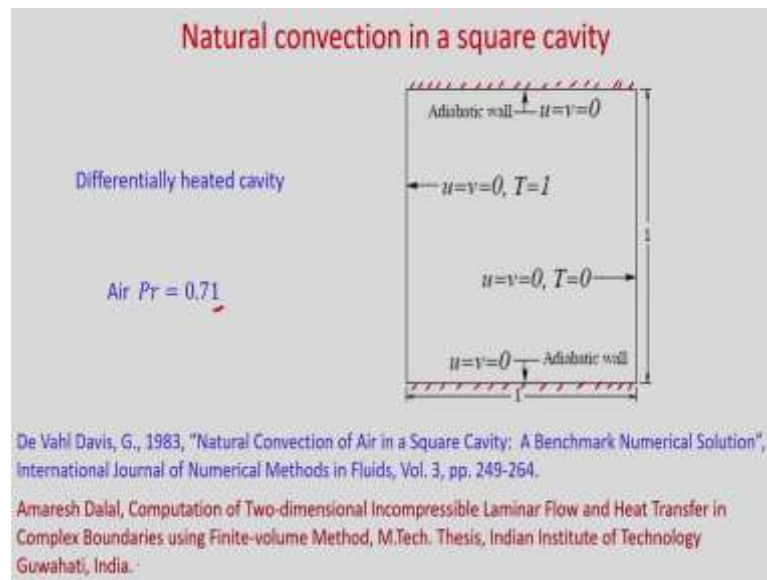
And, you can see in this case so, these are the in three-dimension if you see so, there will be formation of these as cells. So, we will go up, it will go again it will go down so, these were different cells you will get. You can see there will be roles also for the rigid surface and if it is a free surface, then you will get hexagon type cells. So, you can see the experimental visualization.

So, this is kind of free for free surface; So, you can see in this particular case, there will be kind of hexagon kind cells, here you can see and for rigid surface top and bottom walls are rigid, then you can see these are the cells you will get. So, these are experimental visualization and these are the Rayleigh-Benard cells and from the top if you see so, these are the rolls you can see; these are the roles. So, it is continuous in the other direction. So, this is from the top view and this is from the side view and you can see the rolls.

So, obviously, you know that this you will get when the $Ra_H > 1708$ and in this case, Globe and Dropkin proposed this empirical relation of Nusselt number.

So, $\overline{Nu}_H = 0.069 Ra_H^{1/3} Pr^{0.074}$ and it is valid in the range of $3 \times 10^5 \leq Ra_H \leq 7 \times 10^9$. So, this $\frac{H}{L}$ should be sufficiently large, so that the effect of the side walls can be negligible, because there will be side walls so, that effect you can neglect. So, these are some correlations, based on the experiments, we have written the Nusselt number expression.

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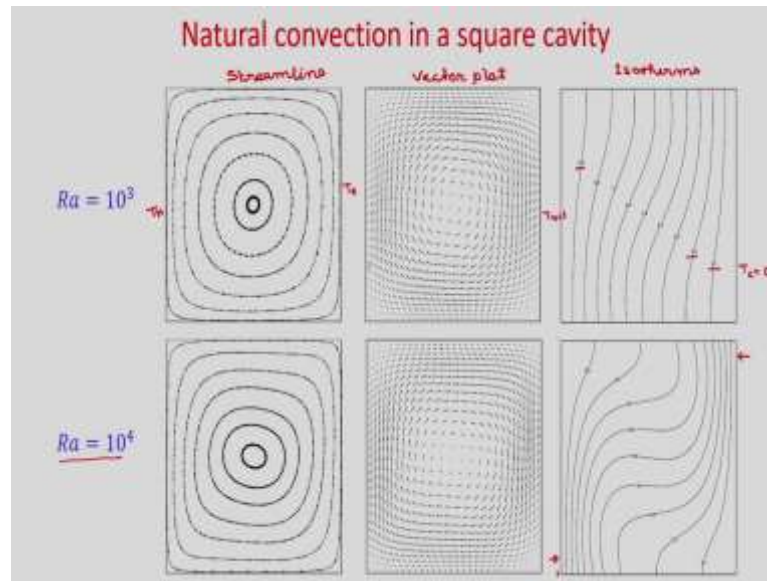


Now, we will show some visualization from the numerical simulations for different cases. One well-known problem is the differential heated cavity. So, natural convection taking place in a square cavity and these are sidewalls are heated. So, this is known as differential heated cavity. So, De Vahl Davis is having a well-known paper on this problem where he considered air where $Pr = 0.71$. So, this is the well-known paper.

De Vahl Davis natural convection of air in a square cavity; a benchmark numerical solution. So, what is this problem? Top and bottom walls are adiabatic; top and bottom walls are adiabatic and in non-dimensionally these are solved. So, you can see this is a square cavity so, length and height is 1, velocity is obviously, you have 0 in all walls, because it is cavity and these are differentially heated so, $T_H = 1$ and $T_C = 0$. So, 0 and 1. So, numerically, it is solved and now I will show some results from my M.Tech thesis.

So, you can see that for low Rayleigh number range in the range of $Ra = 10^3$ or less than that, you will get conduction dominated flow. So, fluid velocity will be very low and heat transfer will take place mainly by conduction, and if you plot the temperature variation, you will get almost linear. But if it is higher Rayleigh number, then obviously, convection cell will be formed and heat transfer you will get by boundary layer convection.

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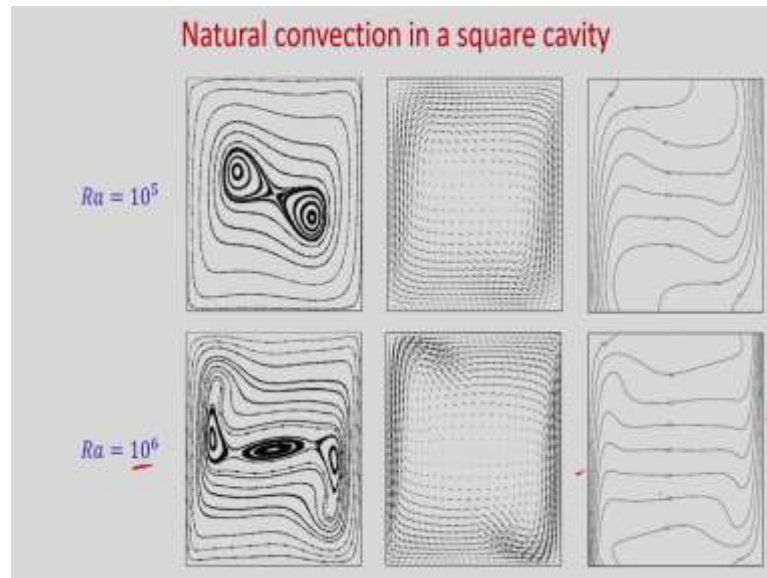


So, you can see $Ra = 10^3$, fluid velocity is just having this like this and it is very low velocity and this is the streamline plot at $Ra = 10^3$ and this is your vector plot. So, velocity vector you can see how it is rolling. So, as you have left wall is T_H and right wall is T_C , you can see these fluids are going up and it is moving horizontally, then again it is coming down, because these fluids are getting heavier, again it is flowing horizontally and going up. So, this way you can see the convection is taking place.

And, the temperature profile you can see how it is varying from $T_C = 0$ to $T_H = 1$. So, you can see it is 0.1, 0.2, 0.3 and point this is your 0.9. So, you can see these are some kinks there, but almost it is linear. But as you increase Rayleigh number, then fluid velocity increases. So, and you can see the isotherms, these are isotherms that means, temperature control. Isotherms means temperature control.

So, you can see how the temperature control looks like. So, there will be clustering of the isotherms near right top wall and left bottom wall and if you see the Nusselt number variation, if you see the local Nusselt number variation along this wall on the hot wall from bottom to top will be similar to the local Nusselt numbers distribution in the right wall from top to bottom, because you can see it is symmetric profile. So, you will get similar local Nusselt number variation.

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Now, if you further increase the Rayleigh number, then $Ra = 10^5$ you can see there is some cells formed near to the central region and it is very low velocity you can see and this is magnitude of the velocity vector so, it is very low velocity, but some circulations are there and near to the wall, you can see now velocity jet is there, you can see higher velocity is coming near to the wall.

And, you will can see the isotherms so, the isotherms are more clustered near to the top right wall and left bottom wall. And, if you further increase a $Ra = 10^6$, you can see from two cells now you got three cells in central region and these are two in near to the side walls. So, you can see it is almost near to the central region, it is stagnant fluid and more velocities yet you are getting.

You see velocity, vector, magnitude is increasing and more velocity you are getting and how the flow physics is happening you can see and here also, you can see clustering of the isotherms near to the cold wall and hot wall. And, in the central region, you can see it almost remain same, the temperature almost remains same in the central region you can see here, it is almost horizontal. So, near to the central region, you can see the temperature this is the constant temperature region it is 0.5, 0.6, 0.7 so, you can see vertically it is almost linear in the central region.

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$Ra = 10^3$ Natural convection in a square cavity $Ra = 10^4$					
	a	b	c	d	$\frac{a-d}{d} \times 100$
u_{max}	3.089	3.544	3.544	3.600	-0.3014
v	0.813	0.832	0.814	0.725	
u_{max}	3.097	3.593	3.586	3.706	-0.2111
v	0.178	0.168	0.186	0.258	
\bar{Nu}	1.118	1.108	1.111	1.105	1.102
Nu_{min}	1.505	1.496	1.540	1.491	0.0302
y	0.092	0.0925	0.112	0.125	
Nu_{min}	0.092	0.730	0.727	0.679	0.1179
y	1.0	0.9925	0.991	0.991	

$Ra = 10^5$					
	a	b	c	d	$\frac{a-d}{d} \times 100$
u_{max}	34.73	35.73	37.141	34.992	-0.7544
v	0.855	0.857	0.855	0.775	
u_{max}	68.30	69.08	68.91	68.79	-0.2916
v	0.066	0.067	0.061	0.125	
\bar{Nu}	4.519	4.430	4.904	4.461	0.775
Nu_{min}	7.117	7.626	8.93	7.582	-6.334
y	0.081	0.0825	0.080	0.091	
Nu_{min}	0.729	0.821	1.01	0.701	3.843
y	1.0	0.9925	1.0	0.991	

$Ra = 10^6$					
	a	b	c	d	$\frac{a-d}{d} \times 100$
u_{max}	64.03	68.81	66.42	64.09	-0.3605
v	0.830	0.872	0.807	0.775	
u_{max}	217.36	221.8	226.4	221.32	-1.7888
v	0.0379	0.0375	0.0390	0.035	
\bar{Nu}	8.209	8.754	10.39	8.912	-1.284
Nu_{min}	17.025	17.872	21.41	20.04	-11.7392
y	0.0378	0.0375	0.030	0.025	
Nu_{min}	0.989	1.232	1.58	0.916	-7.3811
y	1.0	0.9925	1.0	0.991	

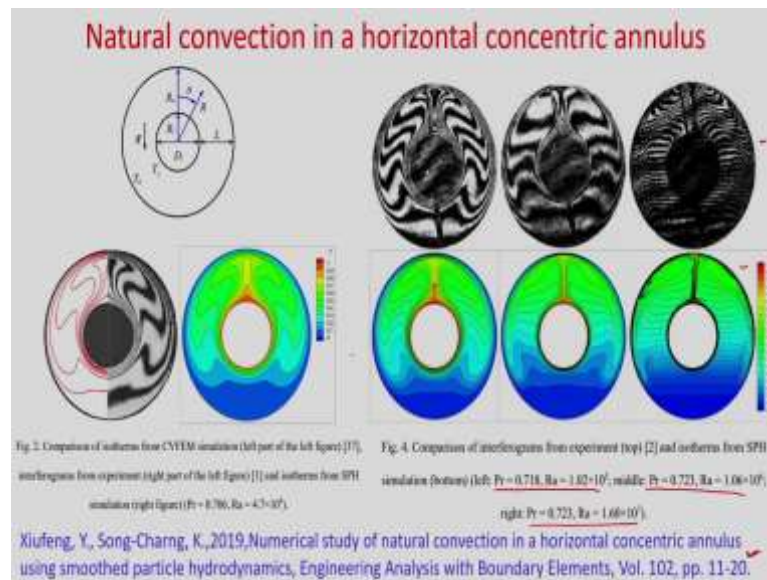
a, solution of de Vahl Davis; b, solution of Markatos and Perikleous; c, solution of Hadjisophocleous et al.; d, present solution on 61x61 grid

Now, if you see the average Nusselt number it will be same in cold wall and hot wall and maximum velocity and in these tables, the results are compared with the solution of De Vahl Davis and Markatos and Perikleous and Hadjisophocleous. So, a is the solution of the De Vahl Davis.

So, you can see $Ra = 10^3$, u_{max} and location of u_{max} , v_{max} , location of v_{max} , aboriginal number, maximum Nusselt number, location of maximum Nusselt number, minimum Nusselt number and max a location of minimum Nusselt number these are plotted and d, d is the present solution; that means, from my M.Tech thesis, whatever solution I got this is your d and a, b, c are from literature and you can see the last column is the percentage difference.

So, in this case, you can see obviously, Nusselt number, average Nusselt number is increasing as Rayleigh number is increasing. So, you can see average Nusselt number is increasing as Rayleigh number is increasing and also the velocity. So, if you see v_{max} it is also increasing. So, you can see as Rayleigh number increases, your vertical jet velocity increases and average Nusselt number also increases.

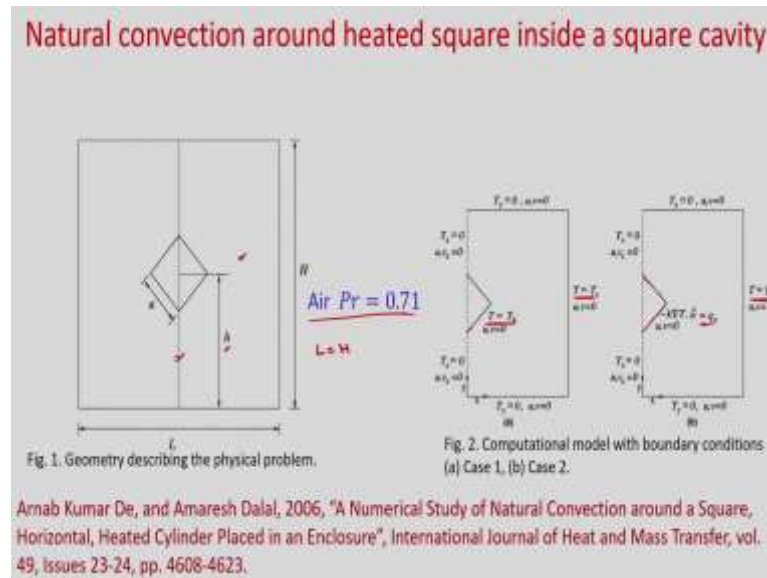
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Now, you see natural convection in a horizontal concentric annulus. So, these are concentric annulus, and in this case, you can see these are the experimental result right-hand side and left side is the numerical result of this paper. So, these are isotherms you can see so, almost it is looking same and you can see these are the isotherms at different Prandtl and Rayleigh number. So, these are experimental results, and this is your numerical results.

So, what will happen? As it is a hot inner cylinder so, the fluid will go up and there will be a band flow in this direction and it will go up and again as you have outer cylinder is cold so, the fluid will come down; so, we will come down. So, there will be a convection like this, and it is symmetric about the vertical centerline up to certain Rayleigh number.

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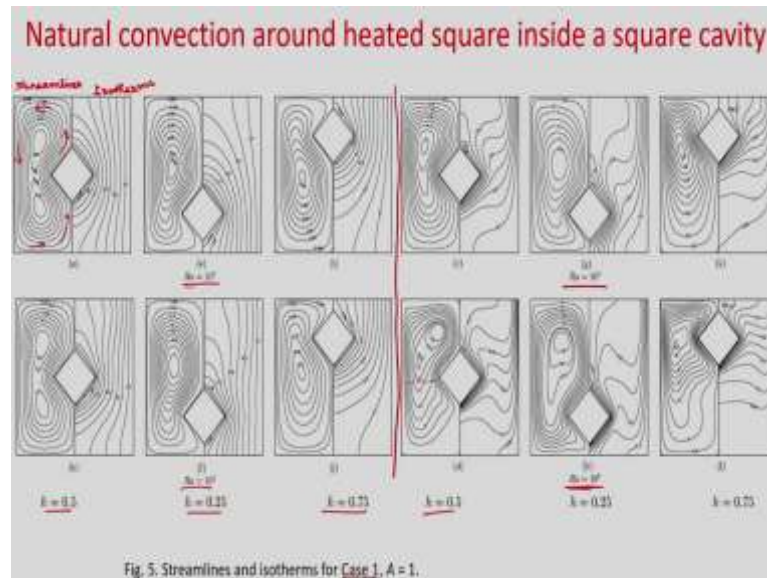


Now, let us consider a hot square cylinder inside and square enclosure. So, here also, air is considered and this results we are showing from this paper Arnab Kumar De and Amaresh Dalal, "A Numerical Study of Natural Convection around a Square, Horizontal, Heated Cylinder Placed in an Enclosure". So, you can see this is one square enclosure so, in this way, it is kept inside the enclosure and inside you have air, ok. So, $Pr = 0.71$. So, it is a square enclosure.

So, $L = H$ and this H is varied from the bottom. So, this is solved as about the vertical centerline, it is symmetric so, half of the domain is solved with two different boundary conditions, one is this wall is maintained at a constant wall temperature T_H and this wall is maintained at a cold temperature T_C .

And another boundary condition is this hot cylinder is kept with uniform wall heat flux ; uniform wall heat flux and this is cold wall at with constant temperature T_C . So, these are the two cases. So, this is your case 1 with a where the heated square cylinder is maintained at a constant wall temperature and this heated square cylinder is maintained at a constant wall heat flux.

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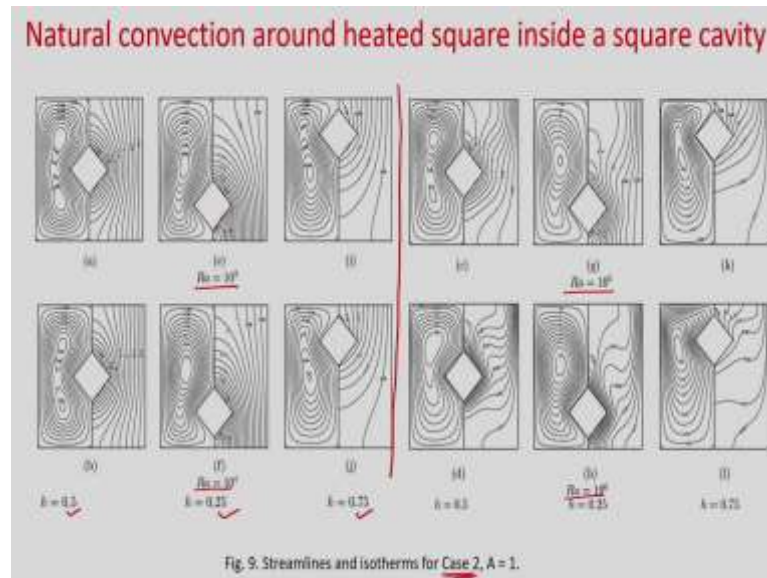


So, now, you can see left-hand side these are streamlines and right side it is isotherms, for a different Rayleigh number, so and with a different height. So, you can see height how it is varied $h = 0.5, 0.25, 0.75$ for these if you see the left half so, for $Ra = 10^3$ and 10^4 , it is shown and it is $Ra = 10^5$ and $Ra = 10^6$ with different height.

You can see how the convection S L is formed, because left hand side is you can see the streamlines; so, obviously, this is hot wall so, it will go up, then it will flow horizontally, it will come down, again it will flow horizontally and it will go up. So, this way your flow is taking place and these are the isotherms. As $Ra = 10^3$ so, you can see it is almost conduction dominated, but higher Rayleigh number it is convection dominated.

And now, how the cells are formed you can see and when you have $Ra = 10^6$ for $h = 0.5$, you can see you got only the two cells in the upper half ; in the upper half, but here it is you can see one is on the top half, one is in the lower half. So, obviously, with increase of Rayleigh number, you can see that your heat transfer also will increase and these are shown for the case 1.

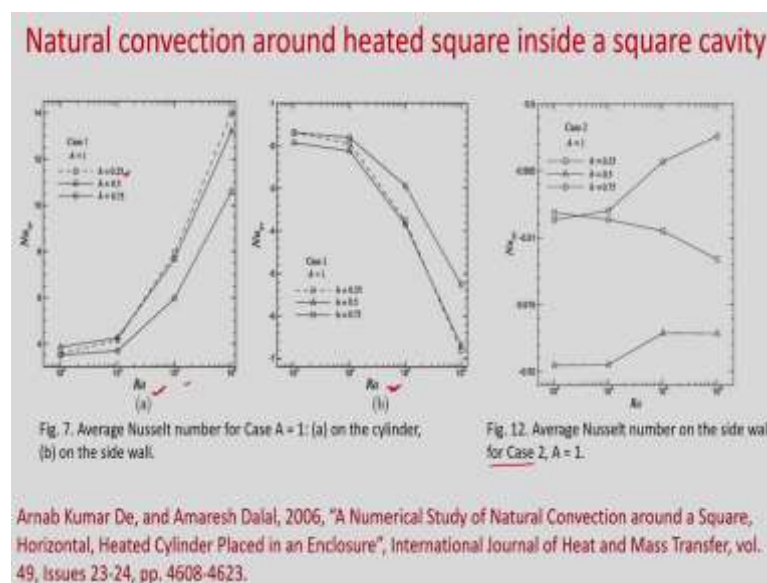
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And this is for case 2, where heated square cylinder is maintained at constant heat flux. So, you can see this is for $Ra = 10^3$, $Ra = 10^4$, $Ra = 10^5$ and $Ra = 10^6$ for different h .

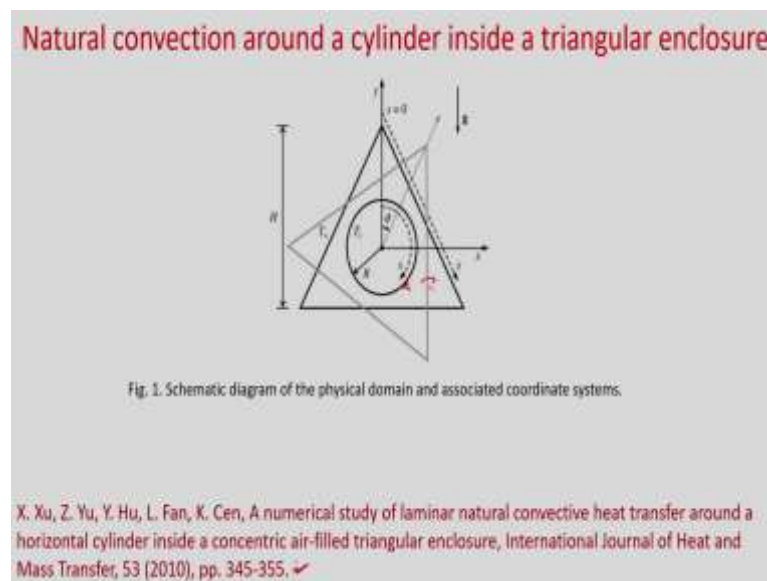
So, here, you can see the temperature ; temperature will vary along the surface, you can see temperature is varying along the surface, because it is maintained at a constant wall temperature as it is maintained as a constant wall heat flux, then obviously, temperature will vary along the surface.

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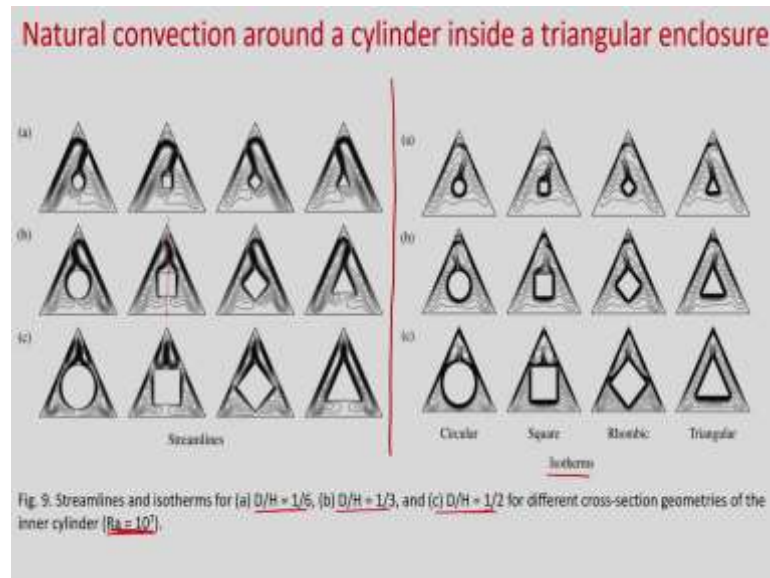
Now, you can see the average Nusselt number for case A this is and on the cylinder and on the side walls, on the cold wall. So, obviously, with increase of Rayleigh number, you can see your heat transfer is increasing and maximum heat transfer you are getting for $h=0.25$. And in the cold wall, this is the Nusselt number, average Nusselt number variation with Rayleigh number and this is average Nusselt number on the side wall for case 2 how it is varying.

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Another configuration you see where you have a triangular enclosure, and one cylinder is kept inside this triangular enclosure with different shape. So, in this case, you can see this is the triangular enclosure and here, you have inside you have some cylinder of different shapes and inside you have fluid. So, these results are shown from this paper.

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So, you can see these are left-hand sides, you have streamlines plot and right-side you have isotherms, ok. For different configurations you can see it is circular cylinder, it is square cylinder, it is rhombus, this is your triangular cylinder and D by H ratio are shown, ok. So, (a) is $\frac{D}{H} = \frac{1}{5}$, (b) $\frac{D}{H} = \frac{1}{3}$ and (c) $\frac{D}{H} = \frac{1}{2}$.

So, now you can see that, this is for this case a, b, it is not symmetric about the vertical centerline ; it is not symmetric about the vertical centerline, in these cases also isotherms you can see. So, how the flow physics is happening you can see so, it is going up, it is coming down, but however, it is not symmetric and $Ra = 10^7$. So, that is why you can see you have lost the symmetry.

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Natural convection around a cylinder inside a triangular enclosure

Nusselt

Table 4. Comparison of the average Nusselt number for various cross-section geometries of the inner cylinder at $Ra = 10^7$.

D/H	Circular	Square	Rhombic	Triangular
1/6	17.714	16.206	17.327	16.634
1/3	26.416	24.070	26.309	24.877
1/2	32.286	32.826	34.697	31.708

X. Xu, Z. Yu, Y. Hu, L. Fan, K. Cen, A numerical study of laminar natural convective heat transfer around a horizontal cylinder inside a concentric air-filled triangular enclosure, International Journal of Heat and Mass Transfer, 53 (2010), pp. 345-355.

So, these are comparison of average Nusselt number for various cross section geometries of the inner cylinder at $Ra = 10^7$. So, $\frac{D}{H} = \frac{1}{6}$, $\frac{D}{H} = \frac{1}{3}$ and $\frac{D}{H} = \frac{1}{2}$ and these are circular square rhombic and triangular. So, these are the average Nusselt number.

So, in today's class, we have studied four different regimes for the case of enclosures with side heated walls. So, those four regimes are tall regime, shallow regime, then you have conduction regime as well as you have high Rayleigh number regime. Then, we have written down the empirical relations for these four cases.

Then, we have also discussed about the case, where for the enclosure heated from the below and that is known as Rayleigh Benard convection. After that, we have shown some numerical results of differentially heated cavity and also we have discussed about the variation of Nusselt number. Then, we discussed about some heated cylinder kept inside an enclosure, and for different configurations, we have shown the streamlines and isotherms and also we discussed about the Nusselt number variations.

Thank you.