Advanced Thermodynamics and Combustion
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Module - 02
Entropy and Exergy
Lecture - 06
Entropy Analysis (Part III)

Dear learners, greetings from IIT Guwahati. I welcome you to this course Advanced Thermodynamics and Combustions we are in the module 2, Entropy and Exergy.

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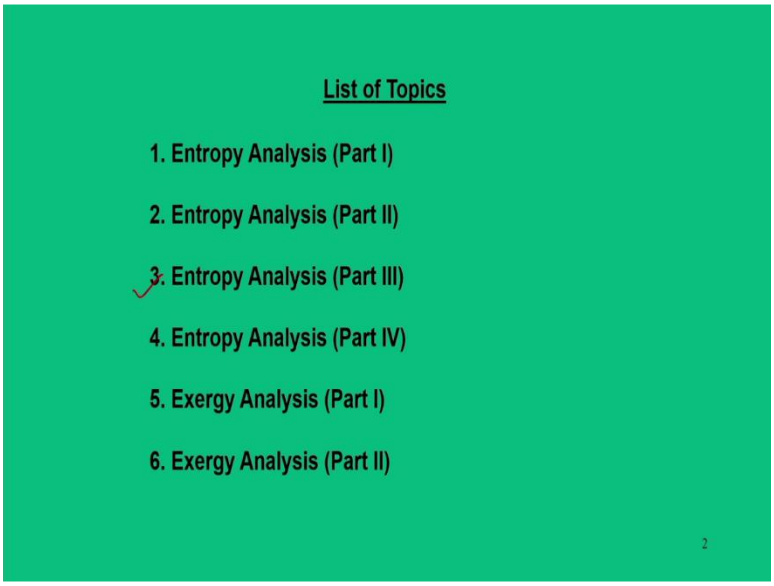

List of Topics

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In this module we have already covered two lectures on entropy. So, we are now moving for the 3rd lecture.

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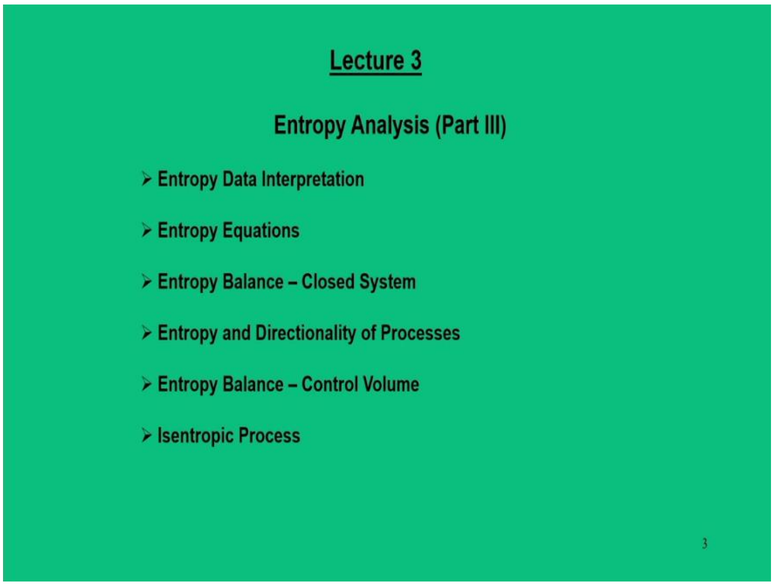

Lecture 3

Entropy Analysis (Part III)

- Entropy Data Interpretation
- Entropy Equations
- Entropy Balance – Closed System
- Entropy and Directionality of Processes
- Entropy Balance – Control Volume
- Isentropic Process

So, in this lecture we are going to discuss about the following topics that is entropy data interpretations, entropy equations, entropy balance for a closed systems of course, we will also discuss about entropy balance for the Control Volume then we will discuss something about entropy as a directionality measures for thermodynamic processes then something on isentropic processes. So, prior to this lectures we discussed exhaustively the entropy by definitions from the second law and its consequence analysis in variety of systems.

In fact, the concept of entropy was introduced through Clausius theorem which is commonly known as Clausius inequality. Now to make our measurement systems with the viewpoint of thermodynamic analysis for closed systems and control volume, in this lecture we are going to target on the entropy balance equations and how we can form the governing equations for the closed systems and control volume which is similar for energy analysis that we did using the first law.

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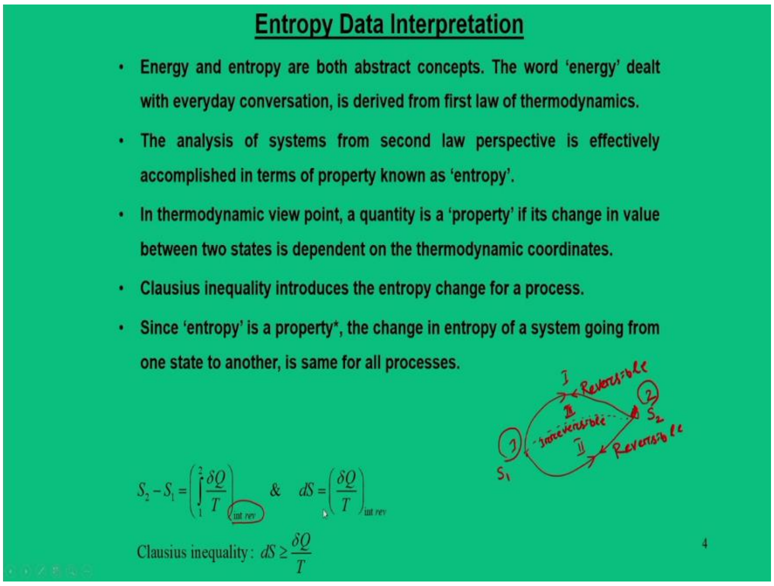

Entropy Data Interpretation

- Energy and entropy are both abstract concepts. The word 'energy' dealt with everyday conversation, is derived from first law of thermodynamics.
- The analysis of systems from second law perspective is effectively accomplished in terms of property known as 'entropy'.
- In thermodynamic view point, a quantity is a 'property' if its change in value between two states is dependent on the thermodynamic coordinates.
- Clausius inequality introduces the entropy change for a process.
- Since 'entropy' is a property*, the change in entropy of a system going from one state to another, is same for all processes.

$$S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T}\right)_{\text{int rev}} \quad \& \quad dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

Clausius inequality: $dS \geq \frac{\delta Q}{T}$

So, to start with the first thing that we are going to discuss is that we know what is entropy. So, it is a property of systems which was introduced by the second law, now once you know the entropy data then how you are going to interpret.

So, in fact, the word entropy talks about the disorderness of the systems, it allows us to know in a particular directions how a system is moving. So, we know that from the first law, energy was introduced and from the second law, entropy was introduced and both energy and entropy are the abstract concepts and we mainly dealt with energy in our day to day conversions and for which there is a balance equations or we call is a energy auditing in which we talk about the energy balance which comes into and out of the systems.

Now similar concept if you want to extend for the entropy how you are going to do that is our main viewpoint. So, now, in a thermodynamic sense that when you talk about a property, it depends on the states of the systems.

For example, if I say that system is at point 1 and another system is at point 2 and if you talk this entropy for the system 1 that is at state point S_1 and at the point 2 it is S_2. So, this property is coordinate dependent. So, now, by definition of a thermodynamic property if it changes value between these two states, is independent on the thermodynamic coordinates.

So, irrespective of the fact whether I take this path or another path when I move from system 1 to system 2. So, I say path 1, path 2. In fact, I can also target moving to path 2 in an irreversible path 3. So, path 1 and path 2 you can say they may be reversible, this dotted line shown here it talks about irreversible path.

So, why I am drawing here is that irrespective of whether we are at state 1 or state 2 whatever path it follows, if it is a thermodynamic property it is not going to change this is by definitions and this is also true for entropy as well. So, since entropy is a property the change of the entropy of the system going from one state to another is same for all the processes.

And by Clausius inequality we know that for an internal reversible processes the entropy change $S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T}\right)_{\text{int} \quad rev}$ and if we go for a infinitely small processes then we can write this $dS = \left(\frac{\delta Q}{T}\right)_{\text{int} \quad rev}$ and this is true when we are dealing with an internal reversible processes.

But what Clausius inequality says that always not all the processes will be internally reversible. So, in a general sense this says that this particular relations which is known as Clausius inequality which is always true that is $dS \geq \frac{\delta Q}{T}$ and when it is a reversible processes the first equation holds good.

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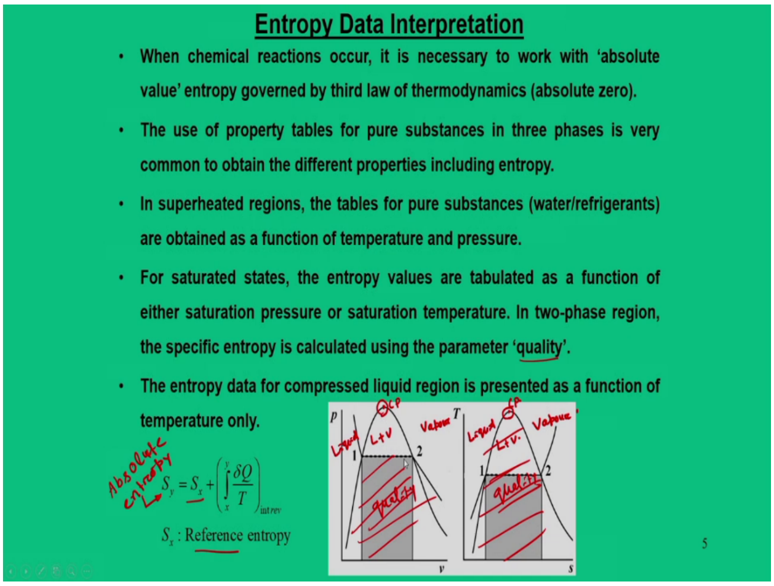

Entropy Data Interpretation

- When chemical reactions occur, it is necessary to work with 'absolute value' entropy governed by third law of thermodynamics (absolute zero).
- The use of property tables for pure substances in three phases is very common to obtain the different properties including entropy.
- In superheated regions, the tables for pure substances (water/refrigerants) are obtained as a function of temperature and pressure.
- For saturated states, the entropy values are tabulated as a function of either saturation pressure or saturation temperature. In two-phase region, the specific entropy is calculated using the parameter '<u>quality</u>'.
- The entropy data for compressed liquid region is presented as a function of temperature only.

Absolute entropy

$$S_y = \underline{S_x} + \left(\int_x^y \frac{\delta Q}{T}\right)_{\text{int}\,rev}$$

S_x : Reference entropy

Now moving further, there are another viewpoint of this entropy analysis that in some senses in a chemical engineering persons, they look the entropy evaluation in a different viewpoint because when we have the chemical reactions, it is always necessary to work with the absolute entropy. So, for that in order to get S_1 or S_2 we need to find out the entropy value at let us say state 1 will be addition of two parameters one is reference entropy at a given pressure and temperature or the given thermodynamic conditions plus the entropy transfer $\frac{\delta Q}{T}$.

So, this is how by definition what we called as absolute entropy. So, the concept of absolute entropy is given by $S_x + \left(\int_x^y \frac{\delta Q}{T}\right)_{\text{int}rev}$ where S_x is your reference entropy. Now in many situations when you deal with the pure substances and this pure substances have three different distinct phases that is solid, liquid and gas and their representation can be drawn in the property diagrams.

Typically, the property diagrams are pressure volume diagram as shown in this figure or the temperature volume diagrams. So, in both the diagrams you can see the dome that constitutes the distinction of three different phases for example, if you look at this dome on this dome there is a peak point and we call this as a critical point and this critical point essentially distinguishes the two different distinct phases.

And there are three different regions one is liquid, other is vapour here also we have liquid and we have vapour and in between within this dome we say liquid plus vapour. So, the important point to be noted here is that for a given coordinates; that means, for a given pressure volume or temperature entropy, we can find out the thermodynamic properties of all the parameters in the liquid regions or liquid plus vapour regions which is commonly known as two phase regions and only vapour regions we call this as a superheated regions and if you want to find out the property value and in particular in this case we talk entropy, we essentially look for which region we are putting our attentions.

So, for example, in this case if I locate a point 1 on this saturation curve another point 2 in this saturation curve and in this curve we can see the point 1 denotes to saturated liquid regions and point 2 denotes for the saturated vapour regions similarly, in the T S diagram here. And if you want to find out the property value we also define another parameter which is called as a quality. So, this quality within this dome; that means, any value within this dome is decided by its quality. So, we call this as either quality of steam or quality of any substance maybe any refrigerant. So, for that things the entropy data can be evaluated based on the thermodynamic coordinates.

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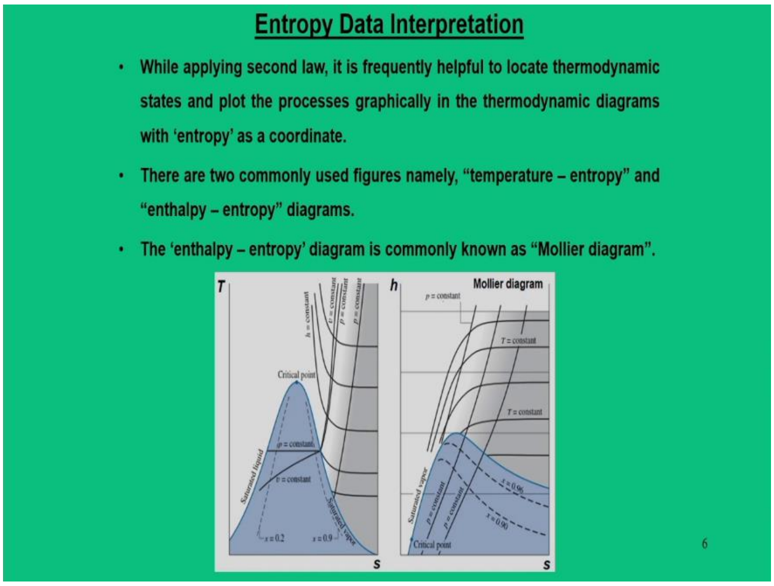

Entropy Data Interpretation

- While applying second law, it is frequently helpful to locate thermodynamic states and plot the processes graphically in the thermodynamic diagrams with 'entropy' as a coordinate.
- There are two commonly used figures namely, "temperature – entropy" and "enthalpy – entropy" diagrams.
- The 'enthalpy – entropy' diagram is commonly known as "Mollier diagram".

So, more clear pictures is shown here, the temperature entropy diagrams and enthalpy entropy diagrams. This enthalpy entropy diagrams we normally call this as a Mollier diagrams. So, you can see here, in this dome we have saturated liquid, we have saturated vapour and this is demarcated by the critical point $c_{p,}$ similarly in the enthalpy entropy diagram or Mollier diagram.

So, on this diagram various constant pressure lines, volume lines, enthalpy lines are plotted. So, the entire idea of showing this thing is that while evaluating the data for entropy, one can use this graphical representations to calculate the entropy as a thermodynamic property.

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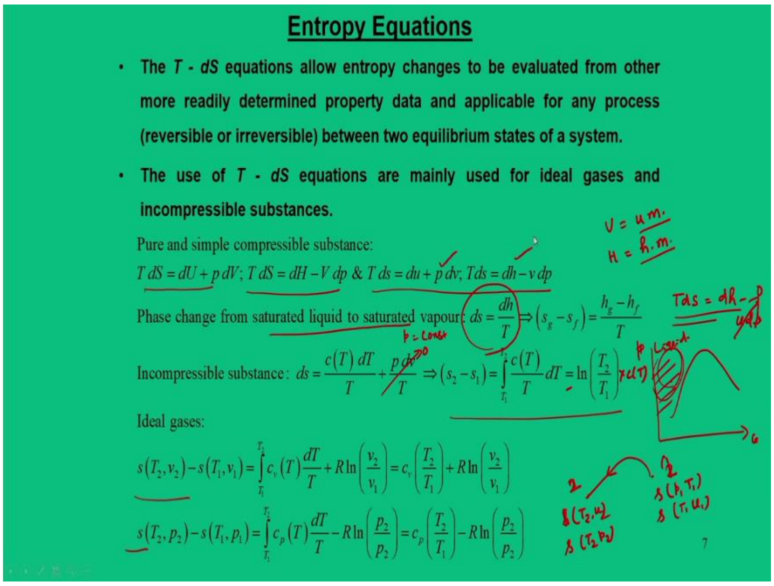

Entropy Equations

- The *T - dS* equations allow entropy changes to be evaluated from other more readily determined property data and applicable for any process (reversible or irreversible) between two equilibrium states of a system.
- The use of *T - dS* equations are mainly used for ideal gases and incompressible substances.

Pure and simple compressible substance:

$T\,dS = dU + p\,dV;\ T\,dS = dH - V\,dp\ \&\ T\,ds = du + p\,dv;\ T\,ds = dh - v\,dp$

Phase change from saturated liquid to saturated vapour: $ds = \frac{dh}{T} \Rightarrow (s_g - s_f) = \frac{h_g - h_f}{T}$

Incompressible substance: $ds = \frac{c(T)\,dT}{T} + \frac{p\,dv}{T} \Rightarrow (s_2 - s_1) = \int_{T_1}^{T_2} \frac{c(T)}{T}\,dT = \ln\left(\frac{T_2}{T_1}\right)$

Ideal gases:

$s(T_2,v_2) - s(T_1,v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R\ln\left(\frac{v_2}{v_1}\right) = c_v\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$

$s(T_2,p_2) - s(T_1,p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R\ln\left(\frac{p_2}{p_2}\right) = c_p\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_2}\right)$

And moreover there are other equations and we call them as entropy equations and this entropy equations can be effectively used. In fact, all the property datas were derived from this entropy equations and if you can effectively utilize them then we can find out various relations and in particular for entropy calculations the ideal choice is the temperature entropy equations and normally that equation we call as TdS equations. So, let us talk about what this TdS equation is all about and how it is derived for various pure substances.

So, let us recall that for a pure and simple compressible substances, we can write down the first law of thermodynamics that involves q that is nothing but TdS that is in terms of entropy, $TdS = dU + pdV$, we have dU that is change in the internal energy and we have pdV that is flow work or work transfer. Similarly, if you write this internal energy as a function of enthalpy then the first T-dS equations is derived in the form $TdS = dH - Vdp$.

So, these two are the fundamental TdS equations which are useful for evaluation of entropy. Now if I take on unit mass basis; that means, if I say $U = u \times m; H = h \times m$. So, m is nothing but mass of the substance. So, on unit mass basics the entropy equations are reduced in this form. Now if I want to apply this equations for a saturated liquid to vapour systems.

So, for this systems we know that is pressure is constant. So, from second TdS relation $ds = \frac{dh}{T}$ and dS is nothing but difference in between $s_g - s_f$.

So, from the enthalpy information it is possible to find out what is the entropy change between the saturated vapour to saturated liquid region and another expressions for an incompressible substance. So, now, when I say incompressible substance we are essentially looking the regions, which is typically liquid region. So, in this liquid regions entire property is governed by one parameters.

So, from this TdS equations, we can rewrite $ds = \frac{c(T)dT}{T} + \frac{pdv}{T}$; dU we can represent it as c_v times T that is specific in this case it is heat capacity and pdv and since this in the incompressible regions, there is no change in the density. So, this term vanishes. So, in such cases the entropy equations reduces to $(s_2 - s_1) = \int_{T_1}^{T_2} \frac{c(T)}{T} dT$.

And if you take specific heat as a constant you can say ln $\left(\frac{T_2}{T_1}\right)$. So, this is all about what you do it for the pure substance. Now we will move to ideal gas. So, for that case we have two states 1 and 2, now in this we say its property or entropy defined as $s(T_2, v_2)$ or $s(T_2, p_2)$.

And for point 1 we say $s(T_1, p_1)$and $s(T_1, v_1)$. So, a system goes from 1 to 2 the change of entropy is written by this two entropy equations and this is again derived from the two fundamental TdS equations.

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R\ln\left(\frac{v_2}{v_1}\right) = c_v\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R\ln\left(\frac{p_2}{p_2}\right) = c_p\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_2}\right)$$

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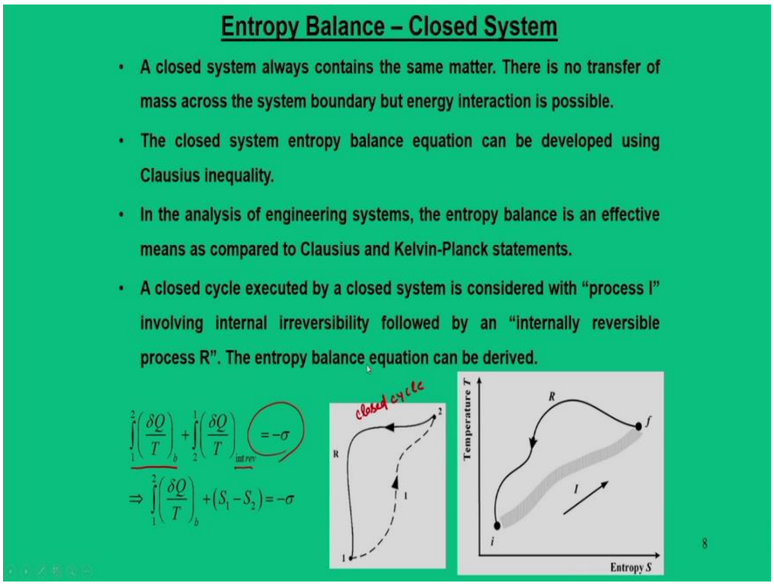

Entropy Balance – Closed System

- A closed system always contains the same matter. There is no transfer of mass across the system boundary but energy interaction is possible.
- The closed system entropy balance equation can be developed using Clausius inequality.
- In the analysis of engineering systems, the entropy balance is an effective means as compared to Clausius and Kelvin-Planck statements.
- A closed cycle executed by a closed system is considered with "process I" involving internal irreversibility followed by an "internally reversible process R". The entropy balance equation can be derived.

$$\underline{\int_1^2\left(\frac{\delta Q}{T}\right)_b}+\int_2^1\left(\frac{\delta Q}{T}\right)_{\underline{\text{int rev}}}=-\sigma$$

$$\Rightarrow \int_1^2\left(\frac{\delta Q}{T}\right)_b+(S_1-S_2)=-\sigma$$

So, this is all about how you are going to evaluate entropy. Now let us see that how we are going to frame entropy audit for a closed systems.

So, by closed system I mean that a system we consider as a closed systems because it contains the same matter and there is no transfer of matter across the system boundary, but the energy interactions is possible. So, there is no mass transfer, but energy interaction is possible. Now for such a systems we are going to find out what is the entropy balance equations.

So, for that we have the fundamental expressions which is called as Clausius inequality. In fact, prior to this we have analyzed this Kelvin Planck statements and Clausius statements. So, based on their analysis if you consider a closed systems which is executed by two cycles one is through a irreversible process other is through reversible process.

So, what you see here that a system undergoes the change of state from 1 to 2 first in an irreversible process, but it returns to the point 1 again through an reversible process. So, in this way it completes a closed cycle. Now for this closed cycle we can write this Clausius inequality in this manner that first one, system undergoes from 1 to 2 for which there is a boundary heat transfer $\frac{\delta Q}{T}$ and while return it goes from 2 to 1 in a reversible process.

And in this process there is an entropy productions because we need to equalize this Clausius inequality statement and from this we can write down $\int_1^2\left(\frac{\delta Q}{T}\right)_b+(S_1-S_2)=-\sigma$

and in fact, this is nothing but your entropy balance equation.

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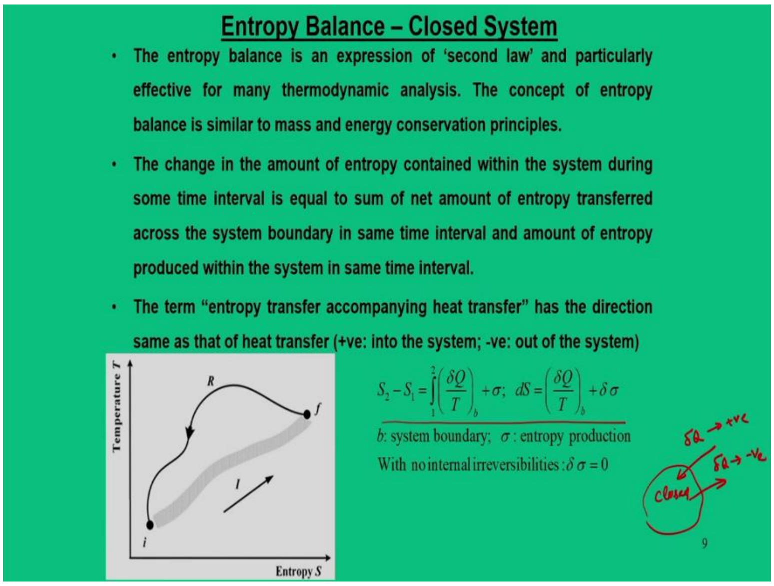

Entropy Balance – Closed System

- The entropy balance is an expression of 'second law' and particularly effective for many thermodynamic analysis. The concept of entropy balance is similar to mass and energy conservation principles.
- The change in the amount of entropy contained within the system during some time interval is equal to sum of net amount of entropy transferred across the system boundary in same time interval and amount of entropy produced within the system in same time interval.
- The term "entropy transfer accompanying heat transfer" has the direction same as that of heat transfer (+ve: into the system; -ve: out of the system)

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma; \; dS = \left(\frac{\delta Q}{T}\right)_b + \delta\sigma$$

b: system boundary; σ : entropy production

With no internal irreversibilities : $\delta\sigma = 0$

Now let us evaluate that particular equation in a more elaborate way; that means, here we have shown from initial state to final state it goes in an irreversible path and returns to a reversible path.

We write this entropy balance equations as $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$, σ is entropy production. So, this is the fundamental equation what we call as entropy balance equation for a closed systems. Now if you look at this two terms here what it says is that the change in the amount of entropy contained within the systems during some time interval is equal to sum of the net amount of entropy transferred across the system boundary in same time interval and the amount of entropy produced within the system in same time interval. And in fact, mathematically when we write these equations, the value of entropy if it is positive which is into the systems.

That means, in a closed system we say when a heat is added into the system then we say dQ is positive and when heat is taken out from the systems dQ is negative. So, accordingly the change in the entropy is governed by this way whether it is added into the system or taken out from the system.

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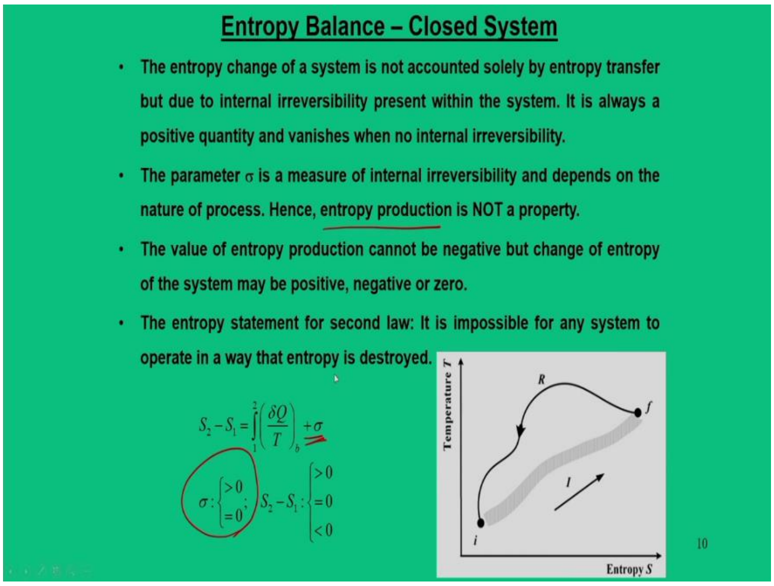

Entropy Balance – Closed System

- The entropy change of a system is not accounted solely by entropy transfer but due to internal irreversibility present within the system. It is always a positive quantity and vanishes when no internal irreversibility.
- The parameter σ is a measure of internal irreversibility and depends on the nature of process. Hence, entropy production is NOT a property.
- The value of entropy production cannot be negative but change of entropy of the system may be positive, negative or zero.
- The entropy statement for second law: It is impossible for any system to operate in a way that entropy is destroyed.

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

$$\sigma : \begin{cases} >0 \\ =0 \end{cases}; \quad S_2 - S_1 : \begin{cases} >0 \\ =0 \\ <0 \end{cases}$$

Now, we will now look into another viewpoint for the same equations. So, we derived this equations like $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$. Let us talk about the significance of this sigma what is this sigma? The sigma is nothing but what we call as entropy production. So, what does this mean that if a process becomes irreversible then the entropy is produced within the systems and it cannot be a negative quantity, that is the most important inferences from this entropy equations.

So, for an irreversible processes there is a entropy productions and it is always greater than 0 and if there is a possibility that entropy production can be equal to 0 if the system undergoes a reversible process. Now based on this the change of entropy from system 1 and 2 either it can be equal to 0 or it has to be greater than 0 there are possibility it can be less than 0.

So, based on this we have these three possibilities. So, the value of entropy production cannot be negative, but the change of entropy of the systems may be positive, may be negative or maybe zero. So, we have seen that how it can be positive, how it can be negative that all depends whether heat is added into the systems or heat is taken out of the systems.

So, from this analysis we can now frame the entropy statement of the second law which says that it is impossible for any system to operate in a way that entropy is destroyed; that means, entropy cannot be destroyed it has to be produced always. Now let us evaluate this entropy balance equation with another viewpoint; that means, in terms of solving the problems.

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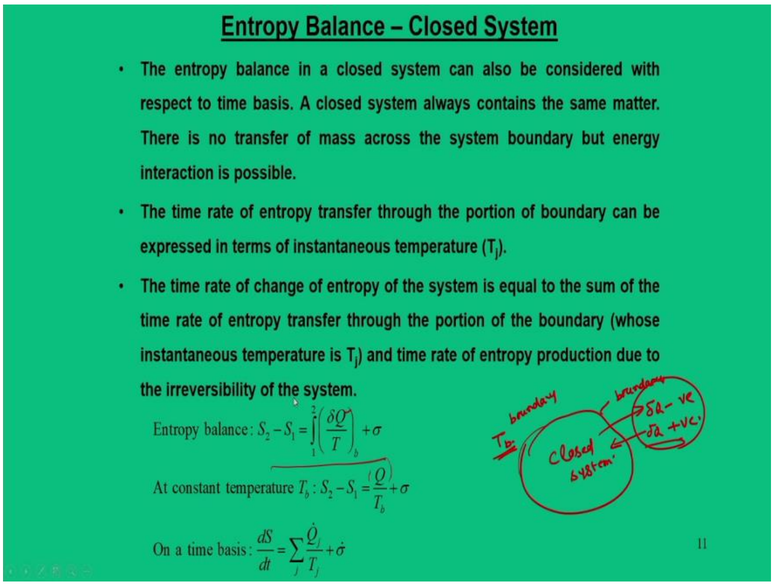

Entropy Balance – Closed System

- The entropy balance in a closed system can also be considered with respect to time basis. A closed system always contains the same matter. There is no transfer of mass across the system boundary but energy interaction is possible.
- The time rate of entropy transfer through the portion of boundary can be expressed in terms of instantaneous temperature (T_j).
- The time rate of change of entropy of the system is equal to the sum of the time rate of entropy transfer through the portion of the boundary (whose instantaneous temperature is T_j) and time rate of entropy production due to the irreversibility of the system.

Entropy balance: $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$

At constant temperature T_b: $S_2 - S_1 = \frac{Q}{T_b} + \sigma$

On a time basis: $\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$

One thing is that when you say productions let us bring the time scale into considerations. What is this time scale into considerations? Now if you say it is a closed system and this closed systems has a boundary and across which this dQ can be positive or negative. So, based on this we have this entropy balance equations.

Now, if this particular systems which has a boundary and this boundary is as constant temperature T_b. So, this equations can be rewritten in this manner because since the temperature is constants we can bring this out of the integral. So, at constant temperature T_b these entropy equations can now be written as $S_2 - S_1 = \frac{Q}{T_b} + \sigma$. Now on time basis; if you want to calculate what is the rate at which entropy is changing.

So, you can write in the form $\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$. This summation means there are various ways that this q can come into the systems. So, entire Q is governed with summation of all the number of sources that heat enters or comes out of the systems.

So, based on that the statement of entropy balance equation can be written as the time rate of change of entropy of the system is equal to sum of the time rate of entropy to the portion of the boundary whose instantaneous temperature is T_j and the time rate of entropy productions due to the irreversibility of the systems.

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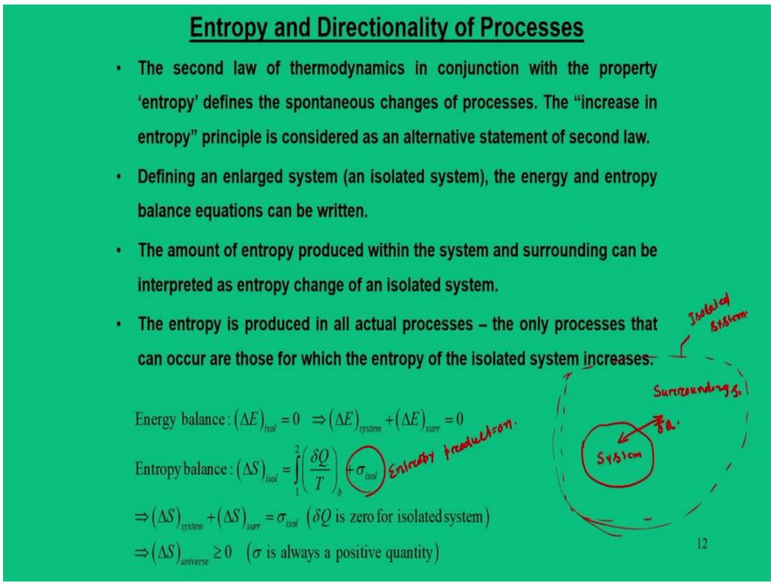

Entropy and Directionality of Processes

- The second law of thermodynamics in conjunction with the property 'entropy' defines the spontaneous changes of processes. The "increase in entropy" principle is considered as an alternative statement of second law.
- Defining an enlarged system (an isolated system), the energy and entropy balance equations can be written.
- The amount of entropy produced within the system and surrounding can be interpreted as entropy change of an isolated system.
- The entropy is produced in all actual processes – the only processes that can occur are those for which the entropy of the isolated system increases.

Energy balance : $(\Delta E)_{isol} = 0 \Rightarrow (\Delta E)_{system} + (\Delta E)_{surr} = 0$

Entropy balance : $(\Delta S)_{isol} = \int_1^2 \left(\frac{\delta Q}{T}\right)_b \boxed{+\sigma_{isol}}$ Entropy production.

$\Rightarrow (\Delta S)_{system} + (\Delta S)_{surr} = \sigma_{isol}$ (δQ is zero for isolated system)

$\Rightarrow (\Delta S)_{universe} \geq 0$ (σ is always a positive quantity)

Now having said this entropy balance, we are now able to think about an extended systems or enlarged systems what does this mean?

So, we already say that we have some system and which is not included in the systems we say its surroundings. Now this heat interactions is always there between the system and surroundings, what I am looking at is that let us bring entire system and surrounding into one closed loop or closed platform and we call this as a isolated system. Now when I say isolated systems it involves the systems plus surroundings.

So, for that energy balance equations we can write for isolated system is equal to 0 total change of energy for system and surrounding is 0, similarly we can write the entropy balance equations in a different way; that means, we can write delta S of isolated systems is equal to delta Q by T plus delta isolated and this is nothing but delta isolated is nothing but entropy productions.

So, by looking this equation closely we can say that the since the isolated systems involve system and surroundings, we can write $(\Delta E)_{isol} = 0 \Rightarrow (\Delta E)_{system} + (\Delta E)_{surr} = 0$ and from this equation we can see that this particular term is always greater than 0 because we have already proved that ΔS is always greater than 0 or only possibilities that it can be equal to 0.

So, this system and surroundings and in particular we call that as universe and for this we can frame this law that $(\Delta S)_{universe} \geq 0$. So, which says that the entropy is always produced by all actual processes and for which the entropy of the universe is always greater than or equal to 0 or in other words we can say entropy of an isolated systems increases. So, in this case the isolated system is nothing but the universe.

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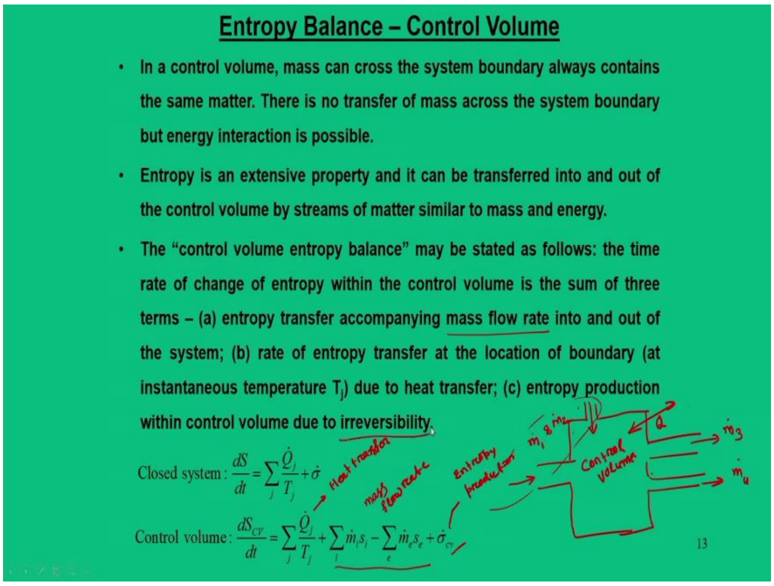

Entropy Balance – Control Volume

- In a control volume, mass can cross the system boundary always contains the same matter. There is no transfer of mass across the system boundary but energy interaction is possible.
- Entropy is an extensive property and it can be transferred into and out of the control volume by streams of matter similar to mass and energy.
- The "control volume entropy balance" may be stated as follows: the time rate of change of entropy within the control volume is the sum of three terms – (a) entropy transfer accompanying mass flow rate into and out of the system; (b) rate of entropy transfer at the location of boundary (at instantaneous temperature T_j) due to heat transfer; (c) entropy production within control volume due to irreversibility.

$$\text{Closed system}: \frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$$

$$\text{Control volume}: \frac{dS_{CV}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \underbrace{\sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e}_{} + \dot{\sigma}_{cv}$$

Now, let us move to entropy balance equations for the control volume. So, basic difference between a closed systems and control volume is that in a control volume, both mass and energy interaction is possible.

So, what we have is that in a control volume typically represented by multiple inlets or multiple outlet as you can see here. So, we may have something entering into the systems something leaving out of the systems. So, this entering we say some mass that is entering into the systems m_1 and m_2 and some mass which is m_3 and m_4 that comes out and we call this as a control volume.

Apart from this there is energy and there is heat interactions Q from this control volume to the surroundings. So, all these things are possible and for these things if you want to write down the entropy balance equations, it says that the time rate of change of entropy within the control volume is the sum of three terms first term is entropy transfer accompanying the by mass flow rate into and out of the systems.

Second term is rate of entropy transfer at the location of the boundary which is located at instantaneous temperature T_j and that entropy transfer is due to heat transfer and third term involves entropy productions within the control volume due to irreversibility.

$$\frac{dS_{CV}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

So, from the analysis of closed systems if you enlarge it or elaborate it for control volume, we can write down the entropy change for the control volume involving three important terms, first term we see accompanying due to the mass flow rate. So, this two terms represent due to mass flow rate. And this particular first term refers to entropy change due to heat transfer across the system boundary and the last term is nothing but production and this production is mainly due to irreversibility.

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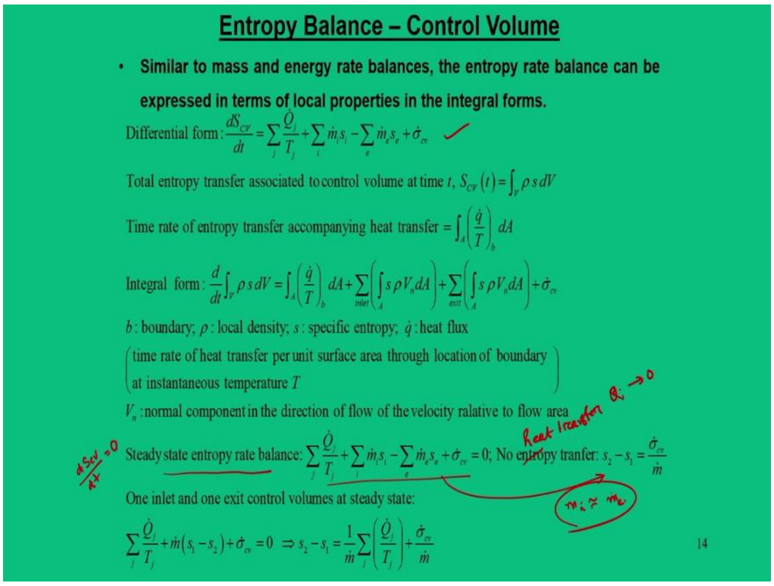

Entropy Balance – Control Volume

- **Similar to mass and energy rate balances, the entropy rate balance can be expressed in terms of local properties in the integral forms.**

Differential form: $\frac{dS_{CV}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$ ✓

Total entropy transfer associated to control volume at time t, $S_{CV}(t) = \int_V \rho s \, dV$

Time rate of entropy transfer accompanying heat transfer $= \int_A \left(\frac{\dot{q}}{T}\right)_b dA$

Integral form: $\frac{d}{dt}\int_V \rho s \, dV = \int_A \left(\frac{\dot{q}}{T}\right)_b dA + \sum_{inlet}\left(\int_A s \rho V_n dA\right) + \sum_{exit}\left(\int_A s \rho V_n dA\right) + \dot{\sigma}_{cv}$

b: boundary; ρ: local density; s: specific entropy; $\dot{q}$: heat flux

(time rate of heat transfer per unit surface area through location of boundary at instantaneous temperature T)

V_n: normal component in the direction of flow of the velocity ralative to flow area

$\frac{dS_{cv}}{dt} = 0$

Steady state entropy rate balance: $\sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv} = 0$; No entropy tranfer: $s_2 - s_1 = \frac{\dot{\sigma}_{cv}}{\dot{m}}$

Heat transfer $Q_i \to 0$

$m_i \approx m_e$

One inlet and one exit control volumes at steady state:

$\sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} = 0 \Rightarrow s_2 - s_1 = \frac{1}{\dot{m}}\sum_j\left(\frac{\dot{Q}_j}{T_j}\right) + \frac{\dot{\sigma}_{cv}}{\dot{m}}$

Now let us move more closely and this particular slide shows the elaborate version of this differential form of equations for the control volume and the simplified sense that this particular equation gets simplified for a steady state entropy rate balance.

So, what does it mean is that is for a steady state entropy balance we say $\frac{dS_{CV}}{dt} = 0$. So, based on this entropy balance equation reduces to this form $\sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv} = 0$. Now again when I say that there is no heat transfer so; that means, we say Q_j goes to 0 this entire equation reduces to $s_2 - s_1 = \frac{\dot{\sigma}_{cv}}{\dot{m}}$ here we have assumed that mass flow rate inlet and mass flow rate exit, they are same.

Further simplifications can be also done for single inlet and single exit control volume at steady state. So, in fact, all these equations are relevant or will be required when you solve the problems.

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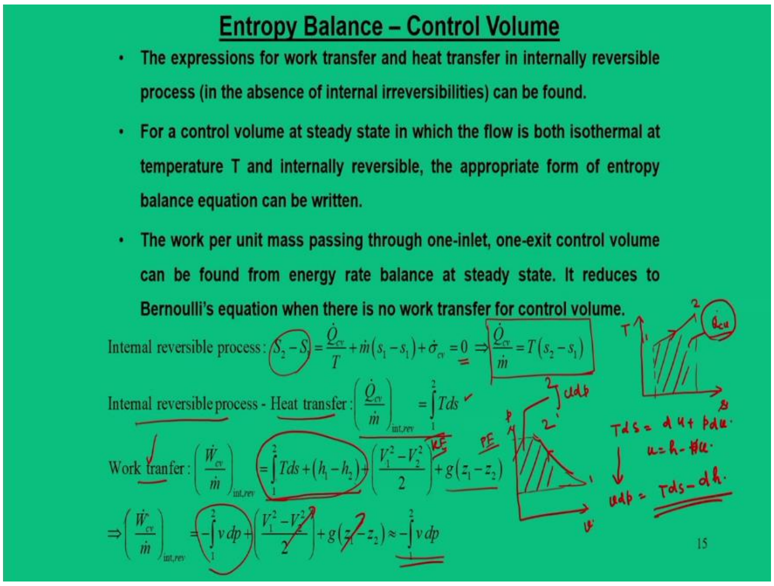

Entropy Balance – Control Volume

- The expressions for work transfer and heat transfer in internally reversible process (in the absence of internal irreversibilities) can be found.
- For a control volume at steady state in which the flow is both isothermal at temperature T and internally reversible, the appropriate form of entropy balance equation can be written.
- The work per unit mass passing through one-inlet, one-exit control volume can be found from energy rate balance at steady state. It reduces to Bernoulli's equation when there is no work transfer for control volume.

Internal reversible process: $(S_2 - S_1) = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_1) + \dot{\sigma}_{cv} = 0 \Rightarrow \boxed{\frac{\dot{Q}_{cv}}{\dot{m}} = T(s_2 - s_1)}$

Internal reversible process - Heat transfer: $\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{int,rev} = \int_1^2 T ds$ ✓

Work tranfer: $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{int,rev} = \int_1^2 T ds + (h_1 - h_2) + \underbrace{\left(\frac{V_1^2 - V_2^2}{2}\right)}_{KE} + \underbrace{g(z_1 - z_2)}_{PE}$

$\Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{int,rev} = -\int_1^2 v\,dp + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g(\cancel{z_1} - z_2) \approx -\int_1^2 v\,dp$

$Tds = du + pdv$

$u = h - \cancel{p}v$

$vdp = Tds - dh$

Now another aspect of entropy balance for the control volume is to elaborate the expressions of work transfer and heat transfer for the internal reversible processes.

And when you say internal reversible processes, it means the internally irreversibilities are absent. So, for an internal reversible processes this entropy equations we can say this is $S_2 - S_1 = 0$. So, we start with the first basic expressions $\frac{\dot{Q}_{cv}}{\dot{m}} = T(s_2 - s_1)$ and this particular term $\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\text{int},rev} = \int_1^2 Tds$.

So, in turn it gives an impression that if you draw a temperature entropy diagram when the system undergoes from change of state from 1 to 2 and this area under that diagram is nothing but $\dot{Q}_{cv}$. Now, so this is what we do if it is an internal reversible process for heat transfer. Now if this internal reversible process, there is work transfer expressions we can rewrite that particular equations.

That if system does not have the heat transfer, but only work transfer is possible we can rewrite that equations in this form, so $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\text{int},rev} = \int_1^2 Tds + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$ and here we can say it involves the kinetic energy change and also potential energy change.

And important point that I need to emphasize here that if you recall this $Tds = dU + pdv$ and $u = h - pv$ and when you simplify this we will arrive at $vdp = Tds - dh$ and this is nothing but this particular term $\int_1^2 Tds + (h_1 - h_2) = -\int_1^2 vdp$.

And with no kinetic energy transfer and potential energy transfer, the internal $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\text{int},rev} = -\int_1^2 vdp$ which means if you can draw on a thermodynamic pressure volume diagram. So, the process undergoes a change of state from the 1 to 2 so in fact, this is a compression process and the area under that diagram that is vdp is shown here.

So, this is the significance of the entropy balance equation for control volume how you can represent them in the thermodynamic diagrams.

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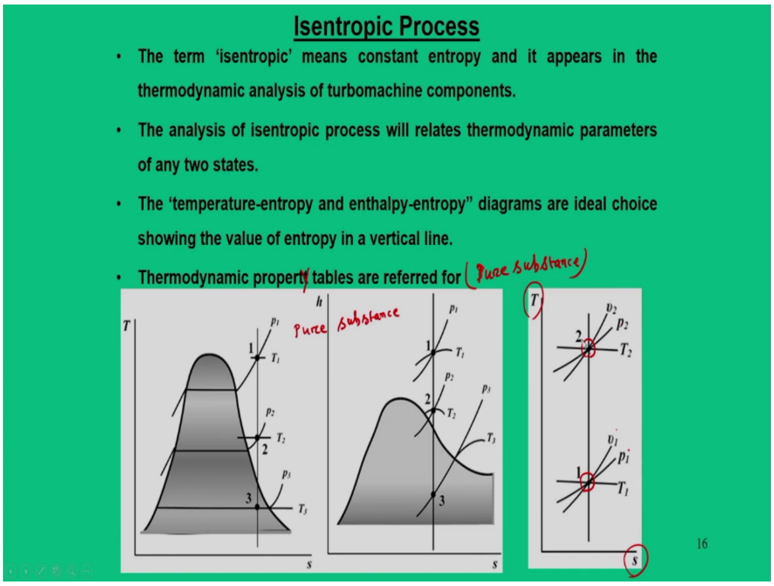

Isentropic Process

- The term 'isentropic' means constant entropy and it appears in the thermodynamic analysis of turbomachine components.
- The analysis of isentropic process will relates thermodynamic parameters of any two states.
- The 'temperature-entropy and enthalpy-entropy" diagrams are ideal choice showing the value of entropy in a vertical line.
- Thermodynamic property tables are referred for (Pure substance)

So, last segment of our discussion is about isentropic processes. So, the word isentropic process we mean it is a constant entropy process; that means, system undergoes change of state from one to two without involving in change of entropy.

And such a process in a T-s diagram and h-s diagram is represented here. So, these two are for pure substance and it is governed by whether this initial state is in which region whether it is liquid state or it is maybe mixture of liquid and vapour state for example, state 1 here it is shown as only vapour state even 2 also is a vapour state, but 3 is in the liquid plus vapour state.

But all of them have same entropy. So, ideal choice of temperature entropy diagram or enthalpy entropy diagram is that the vertical line indicates the constant entropy axis. Now for other situations we can refer the thermodynamic property tables for pure substances.

So, for pure substances entropy calculation is governed through the property tables. Now for ideal gases, instead of enthalpy entropy diagram our ideal choice is the temperature entropy diagrams and in this case we can see that we can have the coordinates of thermodynamic processes 1 and 2, but within this 1 and 2 process we can draw constant volume line, constant pressure line, constant temperature line; constant temperature line is a horizontal straight line.

So, similarly for state point 2 we will have constant volume lines, constant pressure lines, but one thing is that this vertical line talk about the constant entropy line.

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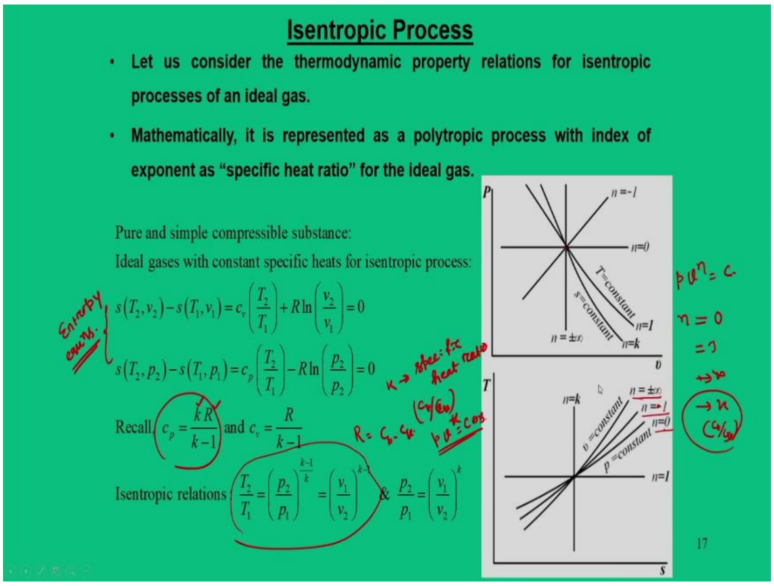

Isentropic Process

- Let us consider the thermodynamic property relations for isentropic processes of an ideal gas.
- Mathematically, it is represented as a polytropic process with index of exponent as "specific heat ratio" for the ideal gas.

Pure and simple compressible substance:

Ideal gases with constant specific heats for isentropic process:

Entropy eqns.

$$s(T_2,v_2)-s(T_1,v_1)=c_v\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{v_2}{v_1}\right)=0$$

$$s(T_2,p_2)-s(T_1,p_1)=c_p\left(\frac{T_2}{T_1}\right)-R\ln\left(\frac{p_2}{p_2}\right)=0$$

Recall $c_p=\frac{kR}{k-1}$ and $c_v=\frac{R}{k-1}$

k → specific heat ratio (C_p/C_v)

$R = C_p - C_v$

$pv^k = \text{cons.}$

Isentropic relations $\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}=\left(\frac{v_1}{v_2}\right)^{k-1}$ & $\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^{k}$

$pv^n = c$

$n = 0$

$= 1$

$\to \infty$

$\to n$ (C_p/C_v)

And here in more elaborate expressions for isentropic process is shown with a classical viewpoint of thermodynamics where we find the entropy equations.

And this entropy equations we can start with situations for an ideal gas gases with constant specific heat for an isentropic process we can write down two fundamental equations of entropy equations $s(T_2,v_2)-s(T_1,v_1)=c_v\left(\frac{T_2}{T_1}\right)+R\ln\left(\frac{v_2}{v_1}\right)=0$; $s(T_2,p_2)-s(T_1,p_1)=c_p\left(\frac{T_2}{T_1}\right)-R\ln\left(\frac{p_2}{p_2}\right)=0$ and from this equations we can recall the expressions for specific heat as a function of $c_p=\frac{kR}{k-1}$. So, k is specific heat ratio which is $k=\frac{C_p}{C_v}$; $R=C_p-C_v$.

By putting this equations in this entropy equations we are able to derive the isentropic relations that $\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}=\left(\frac{v_1}{v_2}\right)^{k-1}$ and fundamental equation that we still use in our classical things which is $pv^k=C$.

So, thermodynamically we represent in this classical diagram pressure and volume, and temperature and entropy and here we draw different constant lines for entropy. So, here the exponent is represented for a constant volume process, n goes to infinity, n=1, n=0.

So, from this equations we say $pv^n = C$. So, n can be 0, n can be 1, n can goes to infinity or n can go to k, $k = \frac{C_p}{C_v}$. So, this is nothing but your isentropic equations and which you we use in a classical way.

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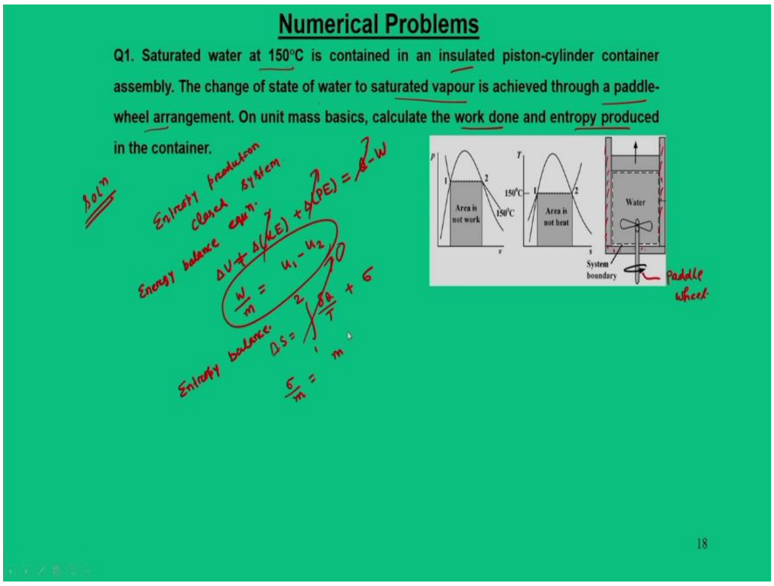

Numerical Problems

Q1. Saturated water at 150°C is contained in an insulated piston-cylinder container assembly. The change of state of water to saturated vapour is achieved through a paddle-wheel arrangement. On unit mass basics, calculate the work done and entropy produced in the container.

Sol^n

Entropy production closed system

Energy balance eq^n.

$\Delta U + \cancel{\Delta(KE)} + \cancel{\Delta(PE)} = \cancel{Q} - W$

$\boxed{\frac{W}{m} = u_1 - u_2}$

Entropy balance.

$\Delta S = \cancel{\int_1^2 \frac{\delta Q}{T}} + \sigma$

m

$\frac{\sigma}{m} =$

So, with this we come to the end of the entropy analysis. So, before you close this lectures let us solve some simple problems.

So, this first problem is about entropy production for closed system. We have saturated water at 150 C which is contained in an insulated piston cylinder container assembly, this change of state of water goes from saturated liquid state to saturated vapour state and change of state is achieved through a paddle wheel arrangements.

So, schematically we can draw a piston cylinder assembly which is coupled with a paddle wheel arrangement. So, basically we are stirring this water, as a result we are introducing entropy into the systems and as you are entropy introducing the systems, there is some work transfer which is done into the systems by virtue of which the change of state has happened from saturated water to saturated vapour.

So, first thing that we are going to calculate the work done and entropy produced. To calculate the work done we have to recall energy balance equation from the first law that is $\Delta U + \Delta(KE) + \Delta(PE) = Q - W$. Now here one assumption that we have, it is an insulated container.

So, that is Q is 0, but there is work transfer and there is no kinetic energy, no potential energy. So, the specific work can be now written as $\frac{w}{m} = u_1 - u_2$. Similarly, we can get the second expression for entropy balance. So, we start with this expression as $\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right) + \sigma$.

Here there is no heat transfer into and out of the systems. So, it goes to 0. So, we can write $m(s_2 - s_1) = \sigma; \frac{\sigma}{m} = s_2 - s_1$.

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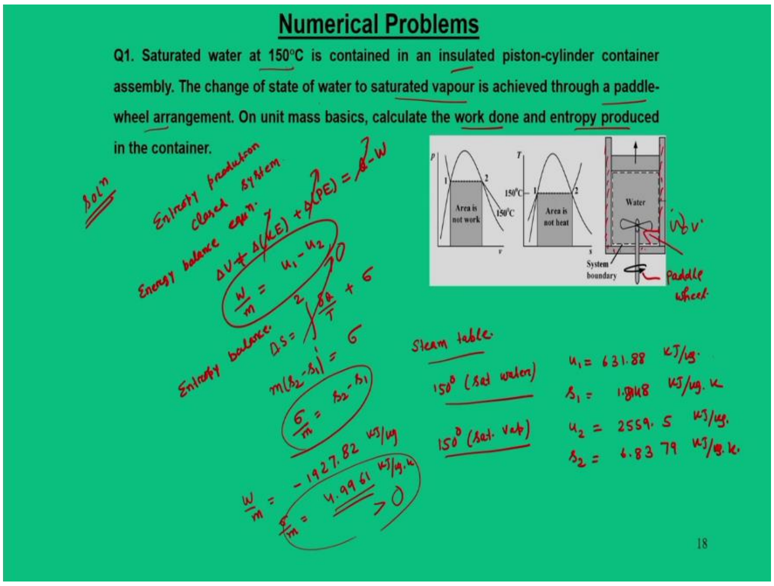

Numerical Problems

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Sol^n

Entropy production
closed system
Energy balance eq^n.

$\Delta U + \cancel{\Delta(KE)} + \cancel{\Delta(PE)} = \cancel{Q} - W$

$\frac{W}{m} = u_1 - u_2$

Entropy balance.

$\Delta S = \cancel{\int_1^2 \frac{\delta Q}{T}} + \sigma$

$m(s_2 - s_1) = \sigma$

$\frac{\sigma}{m} = s_2 - s_1$

$\frac{W}{m} = -1927.82$ kJ/kg

$\frac{\sigma}{m} = 4.9961$ kJ/kg.K > 0

Steam table.

150° (Sat water)

$u_1 = 631.88$ kJ/kg.

$s_1 = 1.8418$ kJ/kg. K

150° (Sat. Vap)

$u_2 = 2559.5$ kJ/kg.

$s_2 = 6.8379$ kJ/kg. K.

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Now our main intention is to calculate what u_1, u_2, s_1, s_2. So, for that we have to use steam table data.

So, in the steam table so, we can say 150 C saturated water we can find out $u_1 = 631.88 \frac{kJ}{kg}; s_1 = 1.848 \frac{kJ}{kg-K}$, then we have 150 C saturated vapour.

So, at this state we can find we can find the data from the steam table which is $u_2 = 2559.5 \frac{kJ}{kg}; s_2 = 6.8379 \frac{kJ}{kg-K}$. So, from this we can now find out $\frac{w}{m} = u_1 - u_2 = -927.82\ kJ/kg$.

And $\frac{\sigma}{m} = s_2 - s_1 = 4.9961\,\frac{kJ}{kg-K}$. So, what we see from this data is that this work transfer is negative. So, which means that work is being fed into the systems, but the entropy is positive so; that means, entropy production is always greater than 0.

So, this satisfies this fact that through a stirring wheel mechanism, it is possible to change the state of water which is initially saturated at 150 C and we can move this to saturated vapour through a paddle wheel arrangement.

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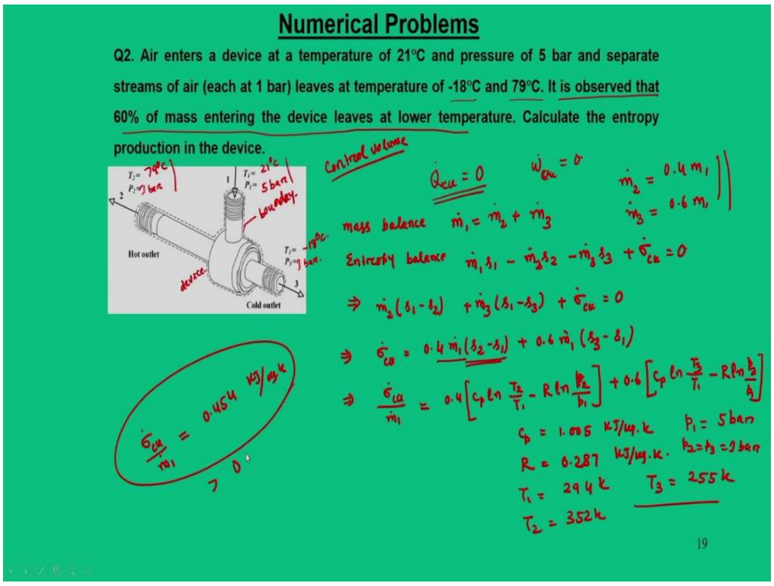

Numerical Problems

Q2. Air enters a device at a temperature of 21°C and pressure of 5 bar and separate streams of air (each at 1 bar) leaves at temperature of -18°C and 79°C. It is observed that 60% of mass entering the device leaves at lower temperature. Calculate the entropy production in the device.

Control volume

$\dot{Q}_{cv} = 0$ $\quad \dot{W}_{cv} = 0$

$\dot{m}_2 = 0.4\,\dot{m}_1$

$\dot{m}_3 = 0.6\,\dot{m}_1$

mass balance $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Entropy balance $\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv} = 0$

$\Rightarrow \dot{m}_2 (s_1 - s_2) + \dot{m}_3 (s_1 - s_3) + \dot{\sigma}_{cv} = 0$

$\Rightarrow \dot{\sigma}_{cv} = 0.4\,\dot{m}_1 (s_2 - s_1) + 0.6\,\dot{m}_1 (s_3 - s_1)$

$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}_1} = 0.4\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right] + 0.6\left[c_p \ln\frac{T_3}{T_1} - R\ln\frac{p_3}{p_1}\right]$

$c_p = 1.005$ kJ/kg.K $\quad p_1 = 5$ bar

$R = 0.287$ kJ/kg.K. $\quad p_2 = p_3 = 1$ bar

$T_1 = 294$ K $\quad T_3 = 255$ K

$T_2 = 352$ K

$\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = 0.454$ kJ/kg.K > 0

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Now, the next problem is about the entropy productions for a control volume.

So, the data which is given that air enters a device at a temperature 21 C, pressure 5 bar and separate streams of air at 1 bar leaves the device at temperature -18 C, other is at 79 C. So, schematically if you represent this is the device, there is one inlet for which temperature enters at 21 C, pressure at 5 bar.

But the air leaves in both the states 2 and 3 with pressure 1 bar, but temperatures are different, one is at hot outlet at 79 C and cold outlet is at -18 C. So, since some mass is entering and leaving the systems we say it is a steady flow device and we can say make system boundary as shown in the dotted lines.

So, what assumption that we are going to say is that there is no heat transfer and there is no work transfer through this control volume. So, two fundamental equations is required one is mass balance which is $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$, and entropy balance equation can be written as $\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv} = 0$

Because we have through boundary work $Q_{CV} = 0$. Now also there is another data, it is observed that 60 percent of mass entering the device leaves at lower temperatures. So, from this we can say $\dot{m}_2 = \dot{}0.4\, m_1, \dot{m}_3 = \dot{}0.6\, m_1$. Now when I put these equations here this equations can now be simplified $\dot{m}_2(s_1 - s_2) + \dot{m}_3(s_1 - s_3) + \dot{\sigma}_{cv} = 0$.

By putting these two numbers we finally, get the expressions $0.4\, \dot{m}_1(s_2 - s_1) + 0.6\, \dot{m}_1(s_3 - s_1) = \dot{\sigma}_{cv}$

$$\begin{aligned}\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = 0.4\left(C_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right) + 0.6\left(C_p \ln\frac{T_3}{T_1} - R\ln\frac{p_3}{p_1}\right)\\ = 0.4\left(1.005\ln\frac{352}{294} - 0.287\ln\frac{1}{5}\right) + 0.6\left(1.005\ln\frac{255}{294} - 0.287\ln\frac{1}{5}\right)\\ = 0.454\frac{kJ}{kg-K}\end{aligned}$$

So, by putting all the data we can find out this as $0.454\ \frac{kJ}{kg-K}$. So, this also says that this term is greater than 0. So, we can say entropy productions from this device is a positive quantity. So, in other words it gives the feasibility of existence purchase for such a device. So, we have solved two problems one on entropy balance for closed systems other is for control volume. So, with this we come to the end of entropy.

Thank you for your attention.