Principles of Vibration Control
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Lecture – 11
Complex Modulus & Applications Of VEM

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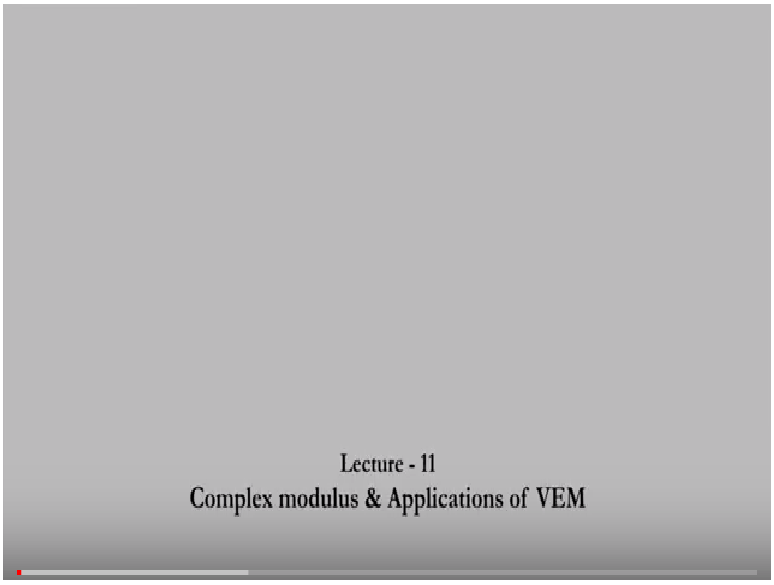

Good morning to all of you. In the last lecture we have talked about the viscoelastic materials and the modeling of the viscoelastic materials. In this lecture we will focus on the applications of Visco elastic materials because after all in this course we have to see how this will be important in terms of the basically vibration control of a system. Now, in order to do that there are certain some more things that we have to keep in our mind.

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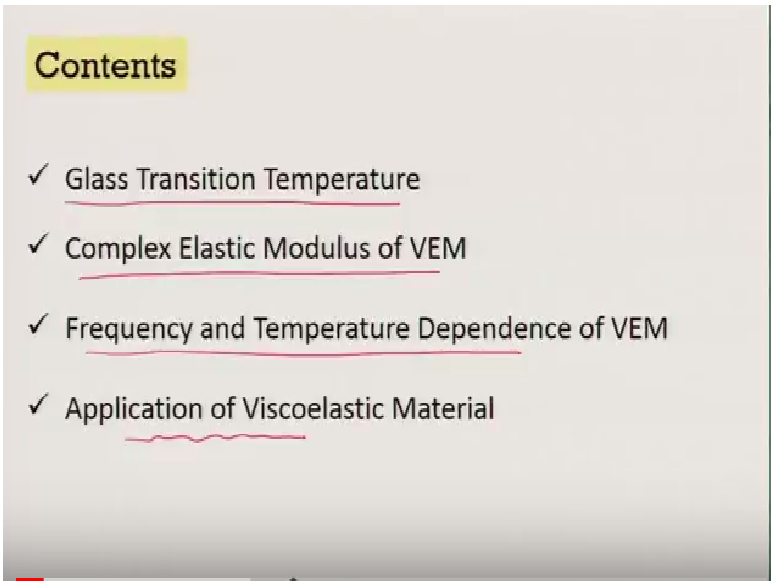

One is called the so called glass transition temperature of a viscoelastic material the complex elastic modulus of Visco elastic material and we have to also understand the frequency and temperature dependence of such material and finally once we have a good knowledge of these we will come to the point of application of viscoelastic material so that is the journey that we are going to make today.

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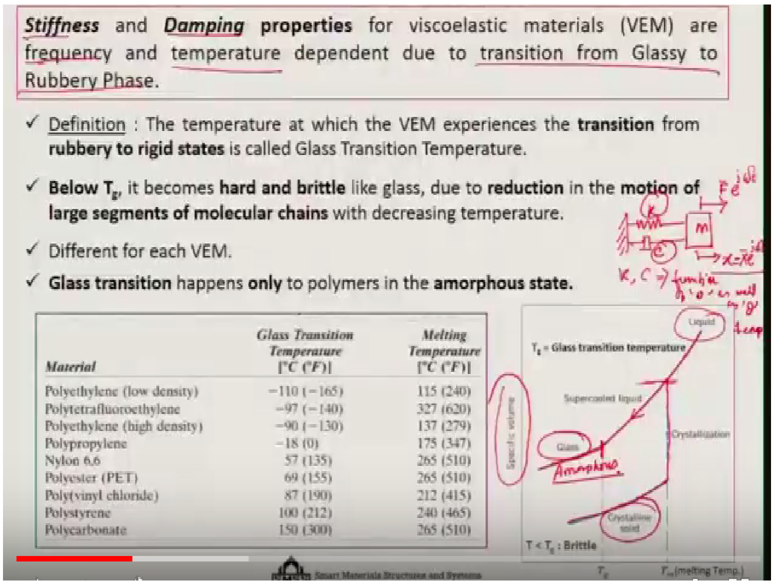

Now stiffness and damping in comparison to all other materials for viscoelastic materials both stiffness as well as damping they are actually frequency and temperature dependent. So which means, that when you are applying a dynamic load and if you remember that you know single degree of freedom system. Right. So if I just draw that single degree of freedom system for your reference this was the Kelvin Voigt model of the viscoelastic material.

Let us say and we have a mass here and this is the viscoelastic material K and C which is subjected to some kind of a dynamic loading f bar e to the power J Omega T. Now while finding out the response of this system X which is also something like X Bar Y raised to the power J Omega T while finding out this response of the system we have to note down that this K and C both are actually function of Omega.

That is the excitation frequency as well as temperature as well as theta which is the temperature, theta is the temperature for us ok so our life will be more complicated because the constants you usually treat in Kelvin Voigt model these 2 as a parameter stiffness and damping as the constants but now they are no longer constant they will be function of temperature and the function of frequency

Now why does this happen. First of all, let us try to look into that this happens due to transition from glassy robbery phase is viscoelastic materials they have a certain kind of a configuration and that configuration is very sensitive to dynamic loading as well as temperature.

So glass transition temperature is something which actually works like a water shape that before glass transition there is some kind of a behavior and after glass transition there is some other kind of a behavior. In fact, if you look at that what happens for a liquid to crystal formation is that if you kind of you know keep a cooling process which is a control cooling process then most of the metals and metallic oxides go through this route that you will develop a crystalline solid.

Whereas if you have a very fast cooling process or you know if it is a viscoelastic materials etc then you will not see any remarkable change in terms of the specific volume at this point rather it will as if behave like a liquid and then beyond this point this solid like behavior will appear which is the glassy behavior and this is when the crystals are not forming.

This happens because this structure is amorphous in nature that means there is no well-defined crystal structure that will be able to be formed in this structure and that is true for the viscoelastic materials also that if you look at their even in a solid form you know we call the glassy form if you look at that face you will see that it has totally amorphous structure.

And from the glassy phase as we increase the temperature or as we decrease the frequency either of it we will go to the rubbery phase and while going these there will be a transition that transition is known as the glass transition temperature so below this crust transition temperature TG the material will be very much hard and brittle like glass and this is mostly due to the reduction in the motion of large segments of molecular chains.

Because those chain missions will be frozen they because they will not get enough activation energy and beyond these you get the rubbery behavior now this temperature is of course different for each viscoelastic material. For example, if you think of polyethylene your common plastics it is - 110 degree that means the way you use a plastic bag at that phase it is not at all in a glass state it will be in the rubbery state ok.

Now same thing is true for polyethylene high-density, polypropylene etc nylon on the other hand at room temperature is actually at the glassy stage because 57degree nylon gets changed polyester also 69 degrees so in room temperature they are also in the glassy stage same things true cross the PVC and all these materials.

So some materials at room temperature are actually at the glassy state this is the these are this and some material at the room temperature are actually and the rubbery stage so this is different for each viscoelastic material and this happens you know the glass transition only 2 polymers in the amorphous state. So the question now is that it this how can we define the modulus of elasticity of a material because it is undergoing.

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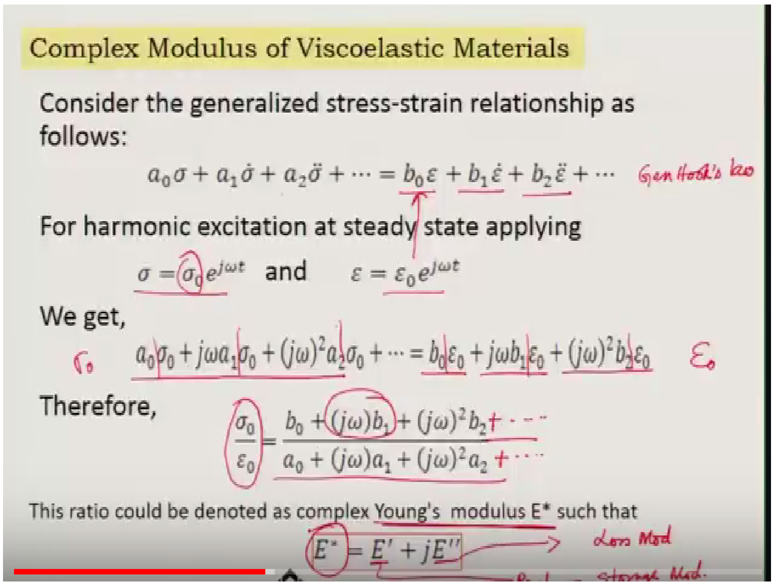

Such a change well we have the generalized stress-strain relationship which I earlier said as generalized Hooke's law we have that with us that model if you remember which we have used for obtaining the material parameter so in that model if I write it in long hand then it is a 0 Sigma a 1 Sigma dot a2 Sigma double dot etc that equals to B 0epsilon B 1 epsilon dot B 2 epsilon double dot etc now you imagine that I am giving either a forced harmonic excitation.

Which will produce the harmonic stress or I will give a harmonic strain either of them if I give a harmonic stress that let us assume that is resulting in a harmonic strain. So if I apply back these 2 in the equation above what I am going to get is that since yes to the power J Omega T will be cancelled from both the sides so we are going to get this kind of an expression ok in terms of stress you have AZ Sigma 0 is the amplitude of the stress ok.

So a 0 Sigma 0 + J Omega 1 Sigma zeroes etc and similarly here if you apply it here in epsilon you get V 0 epsilon 0 + J Omega B 1 epsilon 0 + J Omega Square B 2 epsilon 0 etc so you can take Sigma 0 common here and you can take epsilon 0 common in the other side and you can write the expression of Sigma 0over epsilon 0 which will be something like B 0 only this term then J Omega B 1then J Omega Square B 2.

So that is what will be our numerator + of course if you want to go this is a series and similarly here you have a 0 + J Omega 1 + G square a 2 that is what is our denominator + some additional terms if you wish to go so what does this means it means that our numerator is a complex quantity because you have this imaginary part in it so it is complex as well as our denominator is the complex one so this Sigma 0 over epsilon 0.

Let us say if it is after all it is stress over strain let us say I denote it with Young's modulus e star which now I know is complex because my numerator is complex my denominator is complex so let us say this e star is actually having e prime which is the real part and e double prime which is the imaginary part so e prime is the real part we often call it as the glass modulus or storage modulus storage modulus and e double prime.

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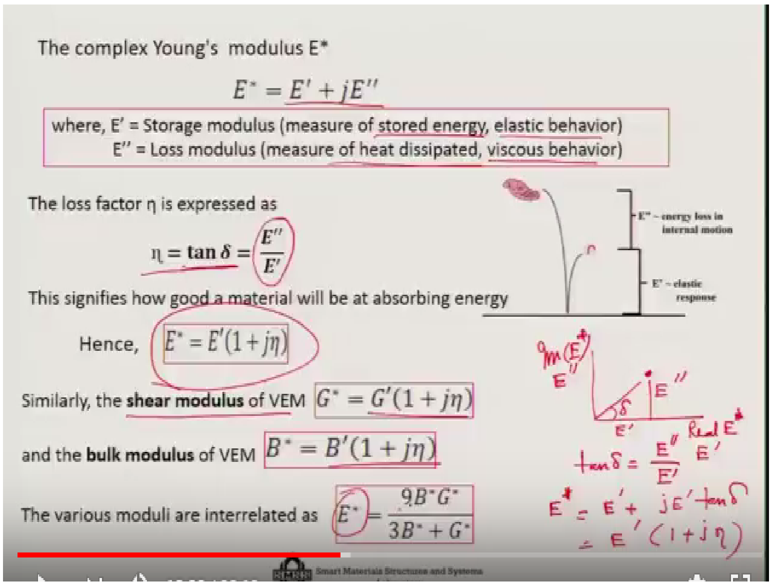

We call it as the loss modulus so these are the 2 parts that are there in the complex expression of the modulus of elasticity now we can also find our old friend Loss Factor now with this definition because e star is e prime + j e double prime e star Y prime I told you is the storage modulus which is the measure of the stored energy and it depicts the elastic behavior.

On the other hand, a double prime gives you measure of the heat or energy dissipated the viscous behavior the loss factor is defined as the ratio between these e double prime over u prime and it is also written in the form of a tangent because it is in the real versus imaginary plot right so you know you consider real of E or E prime that is what is our e prime and imaginary of E is star let's put it rather ok that is what is my E double prime.

So basically if you want to locate a particular point you have this pair of coordinates here ok so you have E prime as the real modulus and E double prime as the loss modulus and this angle here the Delta is expressed as tan Delta this is nothing but your E double prime over E Prime. That is what we have noted in this that is the kind of a geometric expression so this you know this heat loss factor here is something that we are getting back.

Or if you remember earlier we have discussed about Loss Factor and you can apply actually these equations to get the same Loss Factor back I will leave it as an exercise for you ok now if you actually look into the basic expression of e star now E star is e prime + J e double prime so what is our J so what is our J e double prime.

E double prime is nothing but e prime tan Delta right so that means e star is nothing but e prime into 1 + of course the J is here 1 + J tan Delta and tan Delta is ETA so that is what we have written here that E star is e prime into 1 + J now that is for the Young's modulus similarly the shear modulus can be derived as G prime into 1 + j eta assuming that the loss factor is same in all the cases.

Bulk modulus as B prime into 1 + J eta and the relationship between the Young's modulus and bulk modulus and shear modulus as Estar are as 9b star G Star over 3b star + G star. So thus we can actually get the modulus of elasticity of the system now once we know these you know Delta tan Delta etc this Delta value what is the range it ok it goes between zero to ninety degree and as Delta approaches zero degree.

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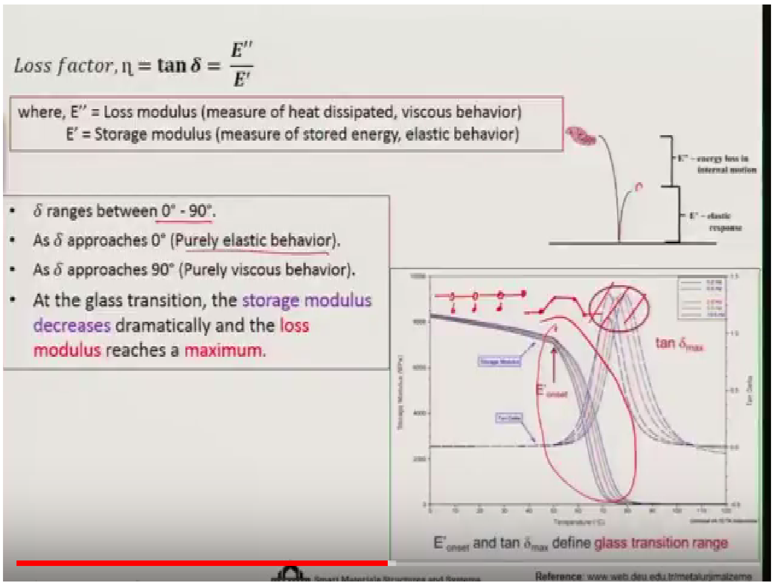

You have a purely elastic behavior so in such a case what you may see is that you know the material does not show this kind of phase transition at all ok if Delta becomes zero degree on the other hand if Delta approaches 90 degree then you get a purely viscous behavior and the glass transition is where this dramatic change is happening to the system now if you really look at it that a polymer at this stage it is like a long chain of the polymer.

And all this main chain of the polymer it may have some side chains also as I told you earlier so everything remains frozen because of the low temperature but around this point onwards the polymer may actually get itself mobilized in this kind of a form so you get a crank like motion and that means a lot of mobility comes into the polymer and that is why such a sharp decline in terms of the real modulus starts to take place.

And also because of this decline lot of energy is getting wasted so you get a sharp peak in terms of these you know the lost more modulus of the system so this is the kind of material explanation of what happens to the system now let us try to summarize the behavior of viscoelastic material with respect to frequency and with respect to temperature so with respect to frequency if we look at it we will see or maybe it will be easier first.

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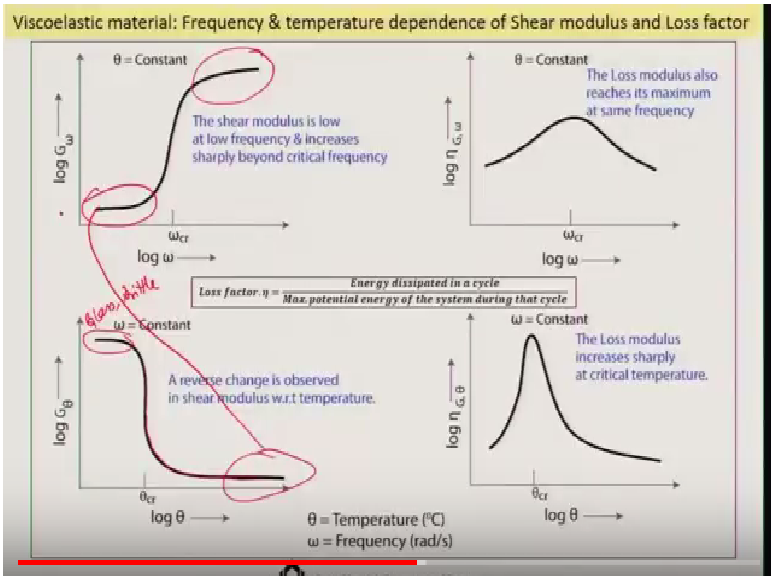

If we first look into it with respect to temperature that at the low temperature in this is your glassy phase and here it is brittle yet it is having a high modulus of elasticity but as the temperature increases this motion increases and there is a sharp fall in terms of the shear modulus now frequency behaves just the other way around that means whatever is good or whatever happens in a low temperature you will see that is going to happen at a high frequency level.

And whatever happens at a high temperature that is going to happen at a low frequency level what does it mean it actually means that if the dynamic excitation that you apply to a viscoelastic material if it is very low then the system actually gets enough time to self adjust itself so all the motions are possible and as a result of that that means in a no motion scenario.

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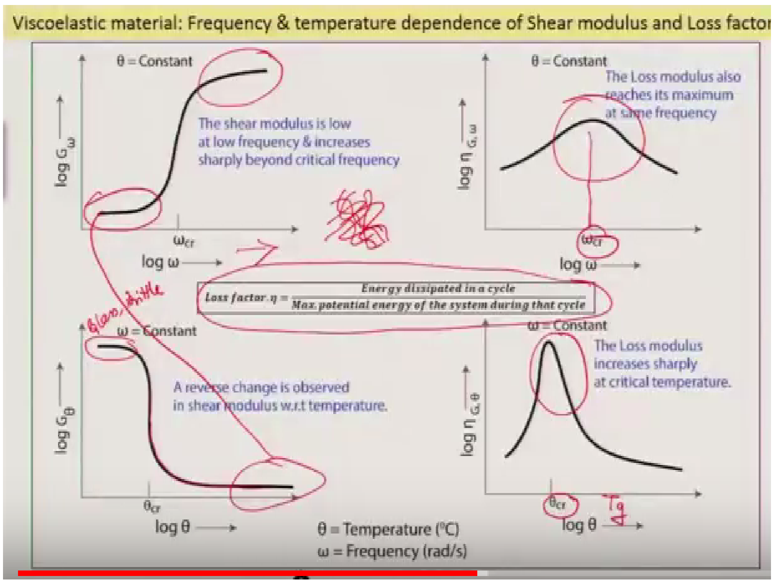

You get a low modulus of elasticity because it behaves like a flexible material but if the frequency increases like in this direction then it the chains do not get enough time they say something like you know noodles they get actually confused they get jumbled up and as a result the modulus of elasticity increases so that is what happens in this particular case as you have seen the behavior of the shear modulus with respect to temperature and frequency.

Now as far as the loss module is concerned it always happens at the transition period so that means nothing much before that or after that but at the transition period where suddenly you have lot of energy dissipation possible then these things happen and this happens not only for frequency but also for temperature.

For temperature, we call it to be theta critical which is your glass transition temperature some people all it TG and here you call it acritical frequency Omega CR where this kind of a phenomena you will be able to see. So this exercise you can do that with this model you try to use these your earlier definition of ETA which is energy dissipated in a cycle over maximum potential energy of the system during the cycle.

And then you will see that you will get a kind of a you know expression of e star which will conform to this expression as well. So now we know how this material behaves in this manner.

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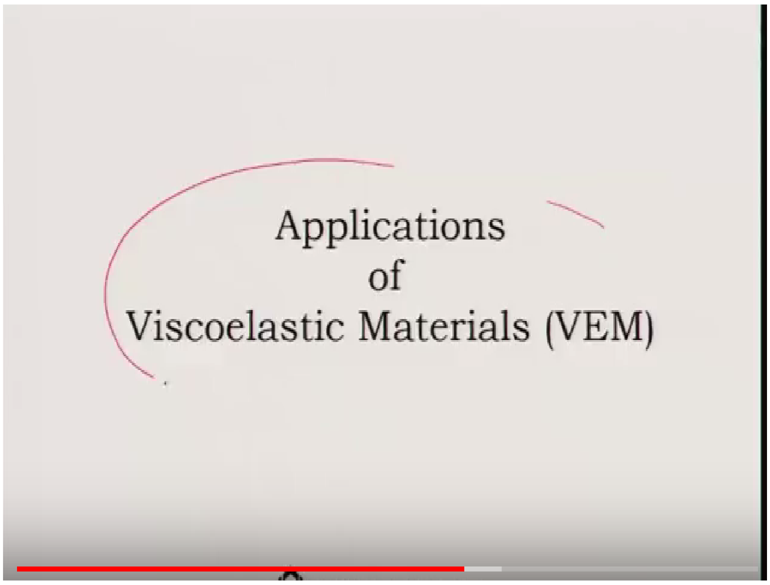

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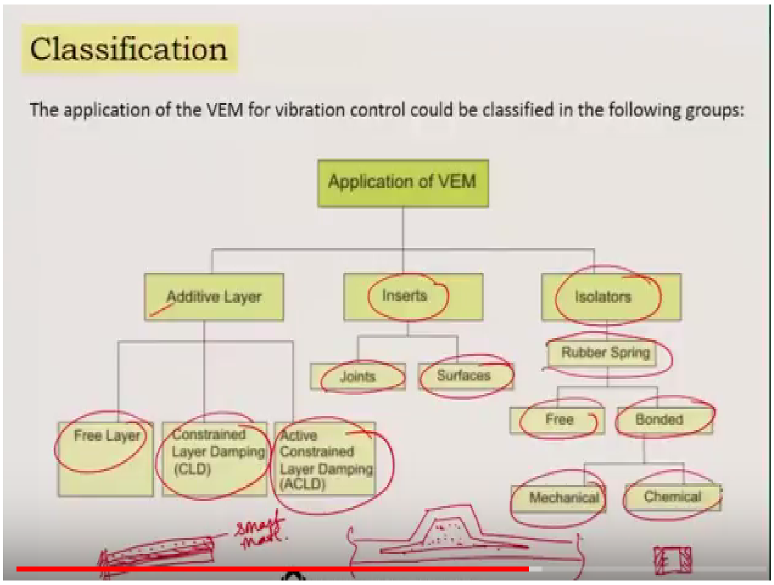

Now let us see that what are the applications of this viscoelastic material for particularly keeping vibration control in mind well there are huge applications I have sub divided them into 3 parts additive layer where viscoelastic material is applied as an additional layer so let us say this is what is my structure which is vibrating and I am applying a small layer of viscoelastic material so this is an additive layer and this additive layer.

In this case is a free layer that is I have just added it on top of it may happen that on top of these I actually give another layer which is constraining the viscoelastic material then it is constrained layer damping CLD it may happen that on top of these I am actually applying another layer which is actually a smart material layer so this is a smart material in that case we will get something which is known as active constraint layer damping or ACLT but these are all layer wise treatments viscoelastic materials can also be used as inserts.

For example, in joints or in surfaces one example could be that you have one you know part of the system and then you are actually having another part of the system coming up so you are joining the 2 so you have something like what we call it off at stiffener composition so this is where the viscoelastic materials will be given ok because this is a joint of the 2 layers so you are going to get viscoelastic materials here in this part so as inserts.

Similarly, it is also used in surfaces now viscoelastic materials are also used in a large way for the Isolators like rubber Springs it can be a free spring it can be a bonded spring in case of a bonded spring you have you can have something like a viscoelastic material and in 2 sides you have the metal casing which is bonded with it so you can have this kind of a situation and the bonding between the 2 could be either mechanical or it could be chemical.

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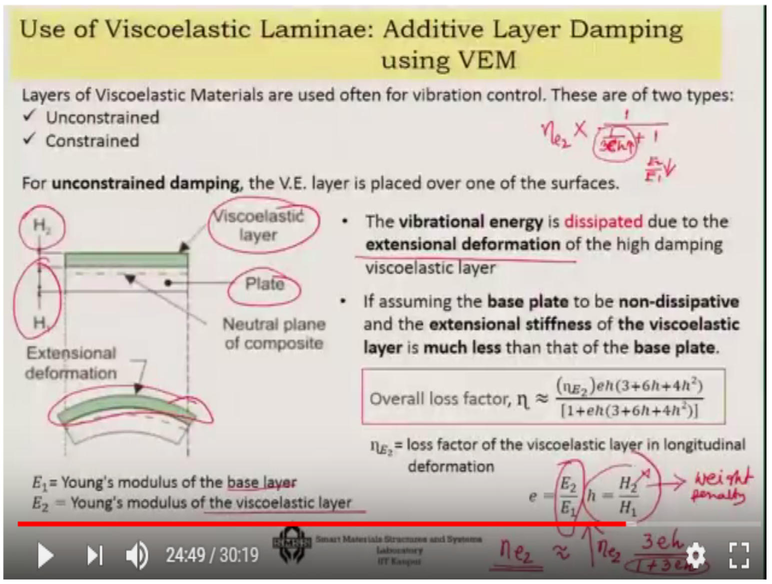

So there are so many applications of his viscoelastic material that is possible you know so it is really very good in terms of vibration control now let's look into a few of them I just give you the overall results of them one is as I told you that unconstraint viscoelastic material and the other one is constrained viscoelastic material so these are all damping layer type so for unconstrained viscoelastic material say for example you have a plate here and your viscoelastic material completely unconstrained free to deform.

In this case the energy is generally dissipated due to the extensional deformation of this viscoelastic material which is undergoing an extensional deformation the expression for this Loss Factor can be written in terms of the height ratio and the modulus ratio height ratio is H2 over H 1 that is these ratio of the thickness of the base layer and the viscoelastic layer and also the corresponding modulus of elasticity ratio.

You will see that this is the way it actually you will get the expression in fact if you neglect all higher order terms then what will happen your ETA e 2will be approximately ETA e 2 times let us say I neglected all the higher dot terms. So 3 eh over one + 3 EH that is what it will be so essentially what it means is that this is one factor which will be less than you know because this one + 3 H is there your denominator is more unless E2 over E 1 ratio is actually very high you will not get you will not gain much out of it so in order to get a high Loss Factor.

Suppose in terms of this kind of a simple application either you should have a large ETA e2 that is one thing that you can have and the other possibility is that your e 2 over H 1 of course H 2over H 1 you will never try to increase this because then it there will be a weight penalty ok. So you will actually not be able to do that but e 2 over e1where e2 is the viscoelastic layer modulus of elasticity and E 1 is the modulus of elasticity of the base layer now let us say that your E is very small.

So you can actually write this expression in a slightly neat manner suppose if I write it here that it is ETA e 2 times let us say I divide top and bottom so it will be by 3 H so it will be 1 over reciprocal of 3eh + unity so if I get small eh yeah so if I get a large value of e 2 over e 1 so if this part increases the numerator will increase so if the numerator increases then the denominator will actually the overall reciprocal will actually comedown.

So that will not have a good effect for us but if e 2 over y 1 difference is actually very small ok so in that case what will happen is that this spall ratio will be much larger in the reciprocal party in comparison to unity so then this will be the dominating part so if there is a large difference of modulus of elasticity and then if this becomes the dominating part then with ETA e 2 it will have a reciprocal effect that means the overall damping will get sacrificed very much.

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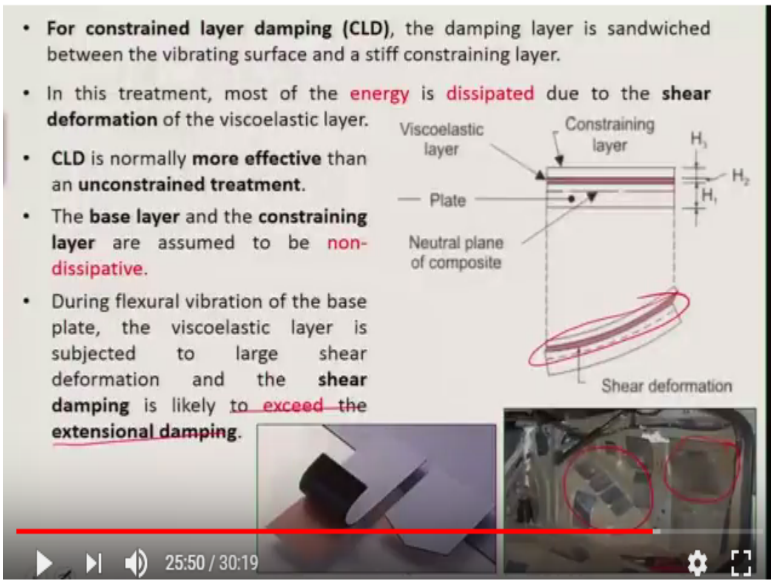

So we have to judiciously choose the modulus of elasticity ratio of the system now constrained layer damping is little bit different from the free layer damping as you can see this any aircraft where this constrained layer dampers area actually utilized so here you have a constraining layer and hence that bending of the Flex or so to say the you know flexural expansion of the viscoelastic material is not happening.

Rather what is happening is a kind of a shear that will happen viscoelastic materials are generally very much responsive to shear kind of a thing so hence more energy dissipation is possible in this manner and that is why constantly a damp parts are generally much better in comparison to the free layer damping so they actually exceed the extensional damping of that you find for the constraint 3-layer damping situation.

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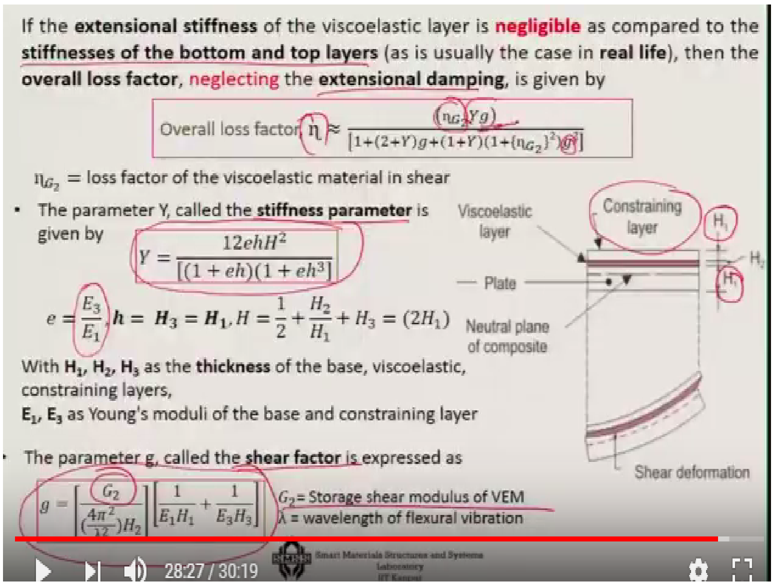

Now if the extensional stiffness of the viscoelastic layer is negligible in compared to the stiffness of the bottom and the top layers and that is what usually you will find in the real life that viscoelastic material would not have much of an extensional stiffness but we will have much of a shear you know is kind of sensitive to here so then the overall loss factor you can neglect the extensional damping and then it will be you can you can neglect the damping due to extension of stiffness that is what is potential damping would mean.

Then you will get it in such a manner that the overall loss factor will depend on ETA g2 will depend something called the Y G which is actually a kind of you know parameter is called the stiffness parameter ok that you will get and also you will get a shear factor G so there is a shear factor G which will come into picture and ETA G and the parameter Y which is the stiffness parameter so this is the definition of the stiffness parameter that you will find ok and the other the shear parameter is actually defined by this relationship.

So here once again you will see for example for the stiffness parameter you will see that the ratios that are important here is the stiffness ratio of E3 over E1that means the modulus of elasticity of the constraint layer with respect to the modulus of elasticity of the base layer so naturally if the higher is the value of e you will see that it may increase the value of y and as a result you may get a higher loss factor.

So we try to always get a kind of a constraining layer which has a very high modulus of elasticity and something like people say jokingly that Diamond is very good as a constraining layer as a theme diamond layer because you always look for a very high stiffness material now if you look at the other factor that is the G parameter then you will find that here the storage modulus of the viscoelastic material plays the major important role and the mode is the storage modulus G then that is going to affect your overall Loss Factor.

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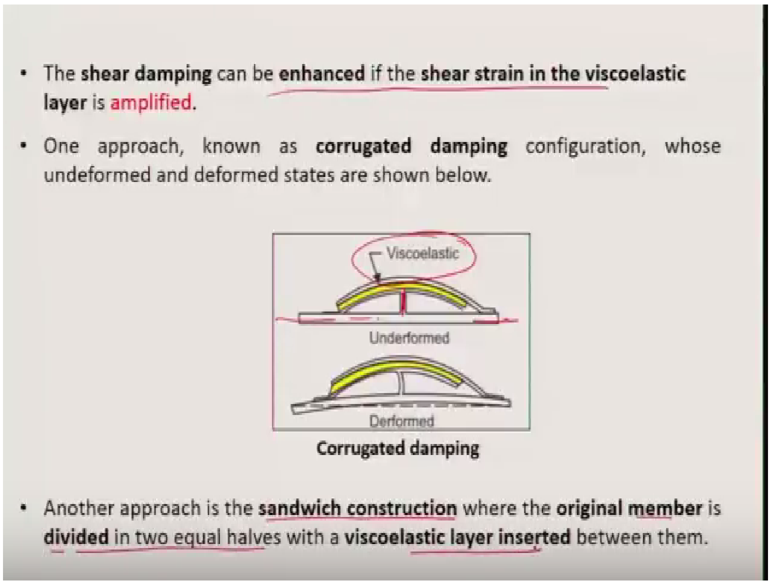

So this parameter G itself so this is how this whole thing behaves in terms of a constraint layer system depth now you can also make because the sheer damping is actually much higher than the flexural damping so you can try to enhance these shear strain by actually artificially constructing a system in which you keep this entire viscoelastic layer and the constraint layer away from the neutral axis of the system by developing you know a kind of a extension corrugated kind of a configuration.

so this is one way of doing and another approach is of course to make a sandwich construction where the original member is divided in 2 equal halves with a viscoelastic layer inserted between them so we try all these different types of technologies in terms of having more shear in the viscoelastic material and there by generating more loss into the system.

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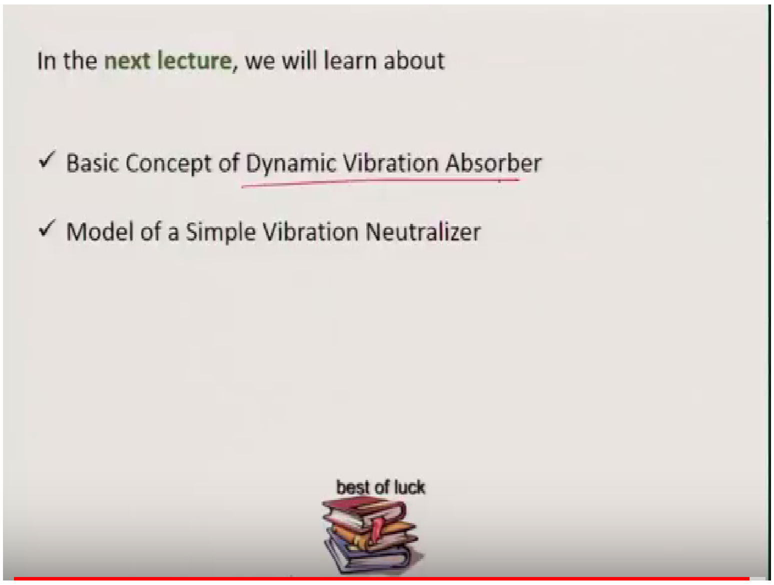

So this is where will come to an end as far as the application of viscoelastic material is concerned in the next lecture we will talk about dynamic vibration observers and a model of a simple vibration neutralizer. Thank you.