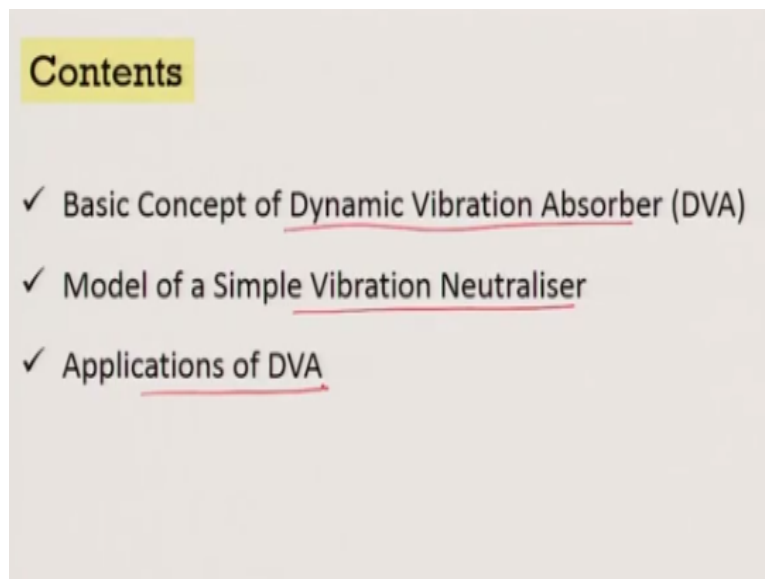


**Principles of Vibration Control**  
**Prof. Bishakh Bhattacharya**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology-Kanpur**

**Lecture-12**  
**Dynamic Vibration Absorber**

Welcome to the course on principles of vibration control and we are now nodding the application of the vibration control in terms of vibration absorber.

**(Refer Slide Time: 00:29)**



So today we will talk about the basic concept of dynamic vibration absorber and we will try to illustrate it to the help of a model simple vibration neutraliser. You have seen very similar thing in the beginning of the course and you can just watch that once more than, the stock bridge damper which is a kind of a very simple vibration neutralizer which is used in the power transmission. So, that kind of a systems the model of frequency and we will also see some of the applications of DVA or the dynamic vibration absorber.

So, let us first talk about what is the basic concept of a dynamic vibration absorber. Now basically when we talk about a dynamic vibration absorber, this is actually system modification, this will come to that category of vibration control and here the way we are model modifying the system is by attaching a secondary mass to the primary vibrating system.

**(Refer Slide Time: 01:28)**

## Dynamic Vibration Absorber (DVA)

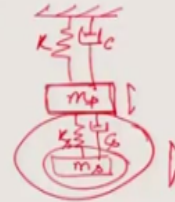
### Basic Concept

Attaching a secondary mass to a primary vibrating system such that:-

- Secondary mass dissipates the energy
- Thus reduce the amplitude of vibration of the primary system

There are many application of DVA, A few are noted below:

- ✓ Vibration control of transmission cables
- ✓ Control of torsional oscillation of crankshaft
- ✓ Control of rolling motion of ships
- ✓ Chatter control of cutting tools
- ✓ Control of noise in aircraft cabin
- ✓ Vibration control of hand held devices



The, what is our intention; the intention is that the secondary mass will vibrate and it will dissipate the energy while the amplitude of vibration of the primary mass will actually come down. So, that means you have say for example a primary mass step like this a spring and a damper may be you can also make it a Kelvin Voight model so, some K, some C very small may be and this is MP and what we are doing in the dynamic vibration absorption system.

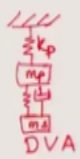
Is that we are actually adding a secondary mass along with a spring so this is MS and this is a secondary spring and there can be a secondary damper also, depending on that there are various classifications of such a system. So, this entire system is design so, that this system vibrates more and more and this system vibrates lesser less. So, you are sacrificing the vibration of the secondary system in order to save the primary system.


Getting you know a large amplitude of vibration. There are many applications of this DVA concept. For example, I told you already that vibration control of transmission cables fall into this category. Then there are these control of torsional oscillation of crankshaft in automobiles, control of rolling motion of ships, control of cutting tools, control of noise in aircraft cabin and vibration control of hand held devices many, many abounded applications of the dynamic vibration absorber.

**(Refer Slide Time: 03:29)**

DVAs are generally of three types:-

- **Vibration Neutralizer:** Here the secondary mass is connected to the primary system using a spring element. TMD - Tuned Mass Damper
- **Auxiliary Mass Damper:** Here the secondary mass is connected to the primary system by a damper/dashpot.
- **Dynamic Vibration Absorber:** A general case where **both spring and damper** are used to connect the secondary mass with the primary system.



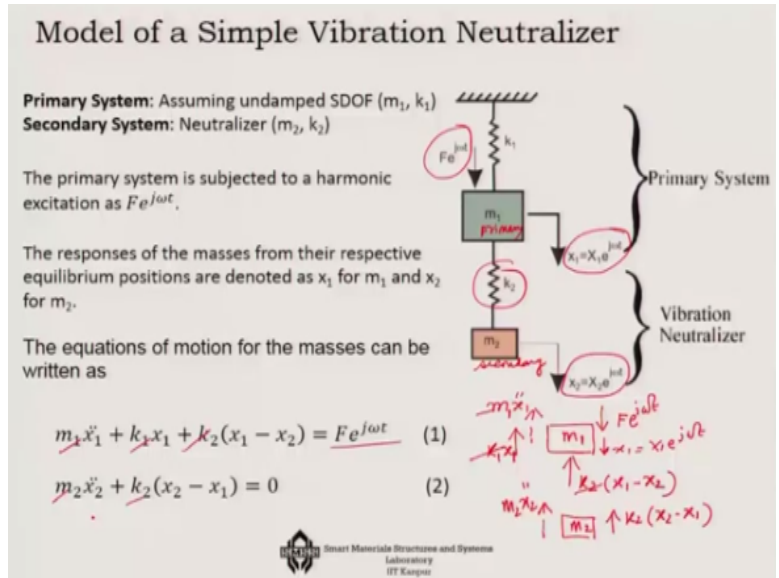


Now, we can classify all these applications to 3 basic categories. The first category is where you are adding the secondary mass to the primary with the help of only a spring element. Then it is called vibration neutralizer. So, we have once again a primary system. This is primary MP, this is K primary and I have only adding one spring and one secondary mass. So this is what is our neutralizer.

In the second case auxiliary mass damper, we are actually not giving these spring so, we will delete the spring and we are going to use instead of a spring a damper here. So, this will then become AMD, or auxiliary mass damper, if I replace the spring wire damper. In the third case we will have both spring as well as damper. So in the third case it is the most generalized case where you have both spring and damper and then we are going to call it as DVA or dynamic vibration absorber.

So DVA itself as 3 you know the first a special cases vibration neutralizer, second case vibration neutralizer by the by also known as tuned mass damper TMD or tuned mass damper that is also another name of the same system and then auxiliary mass damper and in the dynamic vibration absorbers.

**(Refer Slide Time: 05:34)**



Now let us look into the model of a simple vibration neutralizer or tune mass damper. Stock bridge damper is one such example, so here you have a primary system, so this is the primary one and this primary system is getting excited by harmonic excitation and let say the excitation response is also harmonic  $x_1$  which is capital  $X_1$  as the amplitude times  $e$  rest to the power  $j$  omega  $t$  and the secondary system we have added a stiffener.

And a spring and a mass and it is excitation  $x_2$  is  $X_2$  capital times  $e$  rest to the power  $j$  omega  $t$ , so, harmonic excitation is creating harmonic response in 2 places in the primary mass, this is primary and in the secondary mass this is secondary. Now with these let us try to formulate the equation of motion of the system first.

So if you look at the free body diagram of the system at each and every stage you should do that then it will be clear to you, than this is  $m_1$  that is the primary mass, it is subjected to a dynamic excitation lets put that force  $F e$  rest to the power  $j$  omega  $t$  and it is excitation is in this direction so this is your excitation  $x_1$  is equals to  $x_1 e$  rest to the power  $j$  omega  $t$  what it means is that there will be an inertia force which will be in the opposite direction.

And it will be  $m x_1$  double dot:  $m_1$  in this case and also it is going to have a spring resistance. So, let's start the other resistive force here, that is  $K_1 X_1$ , So, these are the total you know the forces working in the system, and of course there is another force that will be coming from this  $K_2$ . You cannot neglect that so, let's put us  $K_2 (X_1 - X_2)$ . That is the relative displacement between the 2.

So, now our things are complete, this is the total you know system, so, that means if I look at the equation on motion summing up all the forces  $m_1 \ddot{x}_1$  that this force then  $k_1 x_1$  that is this force. Last  $k_2 x_1 - x_2$ , that is this force, all this 3 should be equal to the force in the right hand side which is external force,  $F e^{j\omega t}$ . What is happening for  $m_2$ , for  $M_2$  does not have any external force.

$M_2$  only has the inertia force which will put us a dotted line,  $M_2 \ddot{x}_2$  and also the resistive force from the spring which is just equivalent opposite, so, it is  $k_2 x_2 - x_1$ , so that is how these expression is calling out, so this is the governing equation of motion of a simple vibration neutraliser.

**(Refer Slide Time: 08:48)**

In the steady state, the solutions of the governing equations are assumed to be  $x_1 = X_1 e^{j\omega t}$  and  $x_2 = X_2 e^{j\omega t}$ .

Substituting these in equations of motion, we get

$$(k_1 + k_2 - m_1 \omega^2) X_1 - k_2 X_2 = F \quad (3)$$

$$-k_2 X_1 + (k_2 - m_2 \omega^2) X_2 = 0 \quad (4)$$

Solving eqns. (3) and (4), we obtain

$$X_1 = \frac{F(k_2 - m_2 \omega^2)}{(k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2} = -\frac{0}{k_2} = 0 \quad (5)$$

$$X_2 = \frac{F k_2}{(k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2} \quad X_2 = -\frac{F}{k_2} \quad (6)$$

Handwritten notes:

$$k_2 = m_2 \omega^2$$

$$\omega^2 = \frac{k_2}{m_2}$$

$$\omega = \sqrt{\frac{k_2}{m_2}}$$

Smart Materials Structures and Systems Laboratory IIT Kanpur

Now in the steady state, let us  $X_1$  as  $X_1 e^{j\omega t}$  and  $x_2$  as capital  $X_2$  to the  $e^{j\omega t}$ , because, we are only looking for the steady state solution. If you do so, then  $e^{j\omega t}$  will be cancelled from both the sides. And we will get these 2 algebraic expressions in which you will have  $X_1$ ,  $X_2$  and  $F$ . So,  $X_1$ ,  $X_2$  will be the 2 unknowns and there are 2 equations we should be able to find them out.

So, if I solve it you will get  $X_1$  as  $F$  times  $k_2 - m_2 \omega^2$  and the denominator is  $k_1 + k_2 - m_1 \omega^2$  times  $k_2 - m_2 \omega^2 - k_2^2$  and in the second case for the secondary mass the denominator remains the same, so it is the same denominator the numerator becomes  $F$  times  $k_2$  so, what happens when for example your  $k_2$  will become equal to  $m_2 \omega^2$ .

Okay that means omega square will become k2 over m2 that means omega will be square root of k2 over m2 that is the natural frequency of the secondary system. So if I use the condition k2 equals to m2 omega square then what will see is that in this particular case corresponding to this condition x2 would become - F over k2 so that means so there is a displacement okay in the secondary mass x2.

But what is going to happen to the primary mass if I applied k2-m2 omega square 0 this will be 0 this will also be 0. So that means it will become 0 - 0 bar k2 square what it means it will be 0 so that means if I can match the omega is nothing but the excitation frequency you know so if I can match k2 and m2 with the excitation frequency then we will have a case where we will have actually response of the secondary mass but, no response at the primary system that is the objective of such tune mass damper.

So that is that that we are also telling in the same case so the primary system comes to rest so that is the major conclusion that the primary system comes to rest after the tuning of the system provided you know what is the exact excitation frequency of the system. You need to have a sensing for that or somehow you need to estimate it properly.

(Refer Slide Time: 11:56)

### Design Considerations

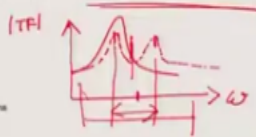
It should be noted that a tuned neutralizer makes the response of the primary system zero only at one frequency, namely,  $\omega_2$ . So, the **application** of such a neutralizer is **very much limited**.

Even though the tuned mass damping system could successfully neutralize the vibration response of the **primary system** when the excitation frequency is  $\omega_2$ ; it also **introduces two new resonating frequencies** to the original system. Hence, care should be taken such that the **two new frequencies are kept sufficiently away from the expected excitation frequency**.

From equation (6), one may note that at  $\omega = \omega_2$ , **displacement of the secondary mass,  $x_2 \neq 0$** . In fact in many system, there is a constraint on maximum permissible value of  $x_2$ . This is known as **rattle space**.

$$X_2 = \frac{F k_2}{(k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2}$$

$$\omega_2 = \sqrt{\frac{k_2}{m_2}}$$



Smart Materials Structures and Systems  
Laboratory  
IIT Kanpur

Now what are the design considerations for it, it should be noted that a tune neutralizer makes the response of the primary system zero only at one frequency that is at omega2, so the application of such a neutralizer is very much limited. There are some cases which will see where we can change this but in this case definitely it is very much limited and a even though

the tuned mass damping could successfully neutralise the vibration response of the primary system.

When, the excitation frequency  $\omega$  it also introduces 2 new resonating frequencies to the original system. So there should be care that the 2 new frequencies are actually sufficiently away from the expected excitation frequency. Otherwise, there will be high resonance in the system and finally one may note that even though  $x_1$  is 0 displacement of the primary system displacement of the secondary mass is not equal to 0.

So, there is a kind of a you know you need to give the space for the displacement and that you know space is also known as the rattle space for the system. So, these 3 conditions will be kept in mind that means if I look at the system so,  $\omega$  versus the transfer function then our initial system was something like these and once we have applied these kind of a in tune mass damper system.

Then our system's frequency response will have actually 2 peaks. And, if we are successful somewhere in between will be able to neutralize and damp down the vibration. But, these 2 frequencies' existence should be thought of so, that these distances must be kept as far away as possible so that your excitation frequency should come somewhere in between these are the tricks that we have to think in terms of the design consideration.

**(Refer Slide Time: 14:13)**



Now let us look at the application of the dynamic vibration absorbers well.

**(Refer Slide Time: 14:20)**

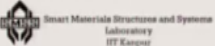
### Self-Tuned Pendulum Neutralizer

Self tunable neutralizers are attractive for applications where there is a **possibility of change of excitation frequency**.

It is known that in a multi-cylinder engine, the time period of the variation of the turning moment depends on the following parameters:

- the rotational speed of the engine
- the number of cylinders, and
- the nature of the operating cycle (i.e., two-stroke or four-stroke)

**This dynamic turning moment gives rise to torsional oscillations of the crank shaft**



The first one is because it has the frequency limitation we have seen now the self tuned type of pendulum neutraliser can actually take care of it. So it can deal with web, there is a possibility of change of excitation frequency. You know the example is that you can consider a case of multi-cylinder engine okay.

Than that the time period of the variation of these (()) (14:54) turning moment depends on these 3 parameters. The rotational speed of the engine and the number of cylinders and the nature of the operating cycle whether that is 2 stroke or four stroke so these dynamic turning movement that will come out will give to the torsional oscillations of the crank shaft. And, let us see how we can control it.

**(Refer Slide Time: 15:18)**

### Self Tuning

The most dominant or the primary exciting frequency of a multi-cylinder engine is given by,

$$\omega = \alpha N n_0 \quad (1)$$

Where,  $\alpha=1$  for two stroke cycle and  $\frac{1}{2}$  for a four stroke cycle  
 $N$ = number of cylinders  
 $n_0$ = rotational speed of the engine, neglecting all the higher harmonics of the system.

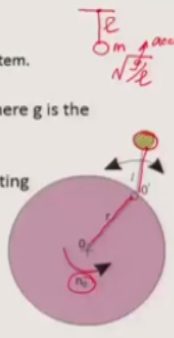
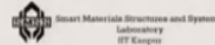
Now, the natural frequency of a gravity pendulum of length "l" is  $\sqrt{\frac{g}{l}}$ , where g is the acceleration due to gravity.

If the pendulum moves in a horizontal plane with its hinge point O' rotating at a speed  $n_0$  with radius r, then the gravity field (g) is replaced by the centrifugal field ( $n_0^2 r$ ).

The natural frequency ( $\omega_n$ ) of this centrifugal pendulum becomes

$$\omega_n = n_0 \sqrt{\frac{r}{l}} \quad (2)$$

Comparing Eqns. (1) and (2), we see that the necessary condition for self-tuning is

$$\frac{P}{l} = (\alpha N)^2$$





Now we will discuss the case of self tuning. In this case so the excitation frequency depending on whether it is a 2 stroke cycle or a four stroke cycle it will have an alpha that alpha equals to one or half and then you have N which is the number of cylinders and then you have  $n_0$  which is the rotational speed of the engine. If, you neglect all the higher harmonics that is the very simplified version of the primary excitation frequency that can come to a multi-cylinder engine.

Now you know that for a gravity pendulum, so suppose you have a gravity pendulum like these and it has length  $l$  then its natural frequency is square root of  $g$  by  $l$  which it actually signifies is that the numerator is nothing but the auxilaration so, you compare that with this particular case where there is a you know a kind of a rotational thing happening in the system with a rotational speed of  $m_0$ , as we have shown here.

So, then this acceleration  $G$  will be replaced by  $m_0$  square  $r$  and the natural frequency of these centrifugal pendulum would then become  $n_0$ . If I just apply this  $n_0$  square  $r$  here they need to become  $n_0$  square root of  $r$  by  $l$  and then comparing this equation with our equation 1, because this is what is the natural frequency of one such system if I add it here, is a pendulum except that the gravitational acceleration I have replacing it by this you know  $n_0$  square  $r$  and then the natural frequency of this system is coming out as  $n_0$  square root  $r$  by  $l$ .

And if I equate 1 and 2 then I am going to get a condition for which you can see that the condition for which is happening when  $r$  by  $l$  will be alpha N square. So, that means alpha for example for 2 stroke cycle alpha is one known to you, number of cylinder is also known to you, so, that means  $r$  by  $l$  is actually known to you and if you choose also you see  $r$  is known to you of you know where this thing is happening.

This radius of this cylinder is known to you so, is only choice of the length and you can choose find out the length from this equation and the irrespective of whatever is the frequency of excitation this system will always revertude. That is the beauty of self tuning system.

**(Refer Slide Time: 18:06)**

Tuned mass dampers are largely used in vibration control of crankshafts, hand-held devices and transmission cables. A few such applications are discussed here:

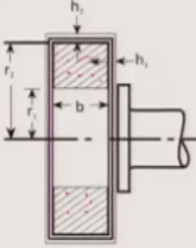
### Houdaille Damper

In this type of dynamic vibration absorber, a flywheel (as secondary mass) is coupled to the primary crankshaft with fluids.

The damping constant is given by:

$$c = 2\pi\mu \left[ \frac{r_2^2 b}{h_2} + \frac{1}{2} \frac{r_2^4 - r_1^4}{h_1} \right]$$

where,  $\mu$  is the viscosity of the fluid



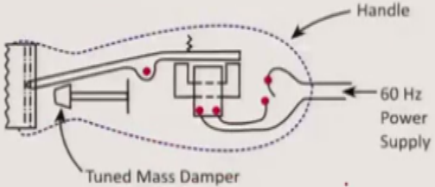
Smart Materials Structures and Systems  
Laboratory  
IIT Kanpur

Is the Houdaille damper and in this of course there is damping constant that is involved in it, because these part in a Houdaille damper is filled up by viscoelastic fluid. So, if  $\mu$  is the viscosity of the fluid, then we get this type of a damping constant for the system. Of course strictly speaking this is a case of what you can say is a case of auxiliary damper because it is damping that is coming in to the picture.

**(Refer Slide Time: 18:36)**

### Damping of hand-held devices

Electromagnetic Motors are used extensively to power hand held devices such as Hair clipper, Dry Shaver and similar instruments. Usually, the motors operate at a fixed frequency such as 60 Hz.



**Electric Hair clipper**

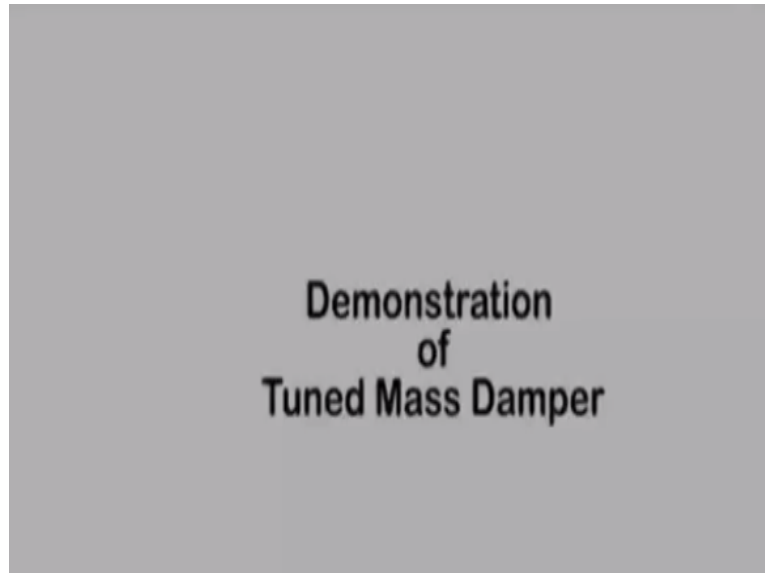
In such hair-clippers, an electro-magnet is used to develop vibrating force for cutting. However, this also generates an unpleasant vibration of the housing. This vibration is neutralized by the application of a pair of mass dampers fixed to the housing at two different points.

Smart Materials Structures and Systems  
Laboratory  
IIT Kanpur

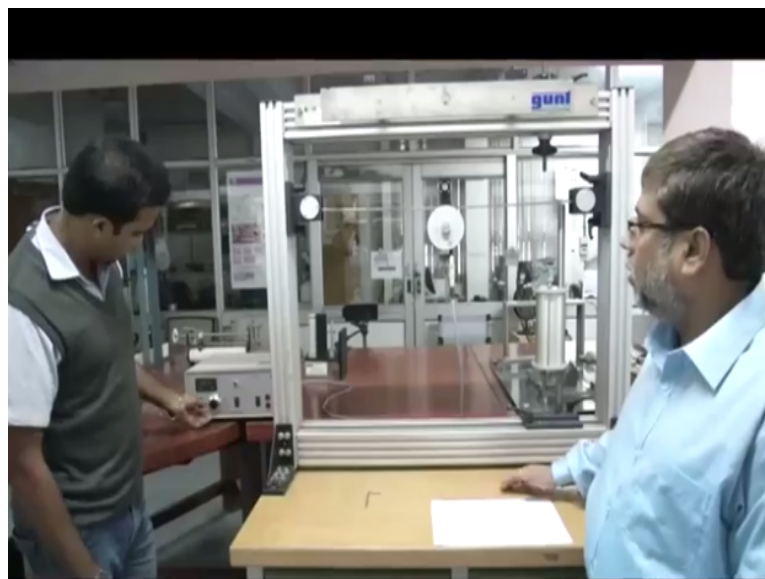
In here it another case we have the damping of hand held devices like say for example electric hair clipper, in this case the excitation is coming mainly because of the power supply which is above 60Hz, so, as this excitation is happening you can add a tune mass damper you can actually see that almost all such hair clippers will be coming up with the some more other form of the tuned mass damper.

Which can actually take care of this particular excitation frequency, the driving frequency here is fixed, so, design of a tune mass damper or vibration neutraliser, is very much possible in this type of a case.

**(Refer Slide Time: 19:19)**



**(Refer Slide Time: 19:20)**



What we are going to show now is that once you have a beam and it is mostly and undamped you, we are going to show that how it can get affected by harmonic excitation, particularly what you will see that once the harmonic excitation, which is close to the natural frequency of this particular beam we will see that the resonance is going to according to the system.

Very much similar what we will also see that happens in practicing very much in various types of other structures like power transmission cables, or like machines etc., now we first

show that how this excitation is affecting the performance of the beam how the resonance is happen and then we will show that once we attach this special system to it, which is called tuned mass damper, we can also this excitation.

We can control the response of the system and bring it down, so, believe me when I request Mr.RPC is to remove this additional mass now from the system, he is removing and you will see that what is the natural excitation of this beam read out in additional mass which we called tuned mass damper read out the addition of that. We are going to see this, now we have to remove this mass.

Let us now we can start to the system and you can see what is happening, as we can see here, the piece photon is actually giving an eccentric harmonic excitation to the system, and due to that mode we can see that the beam is getting gradually excited, and the frequency we are controlling from here, and we will be gradually raising a frequency which is close to 7Hz, 7Hz is the fast natural frequency, the fundamental frequency of the system.

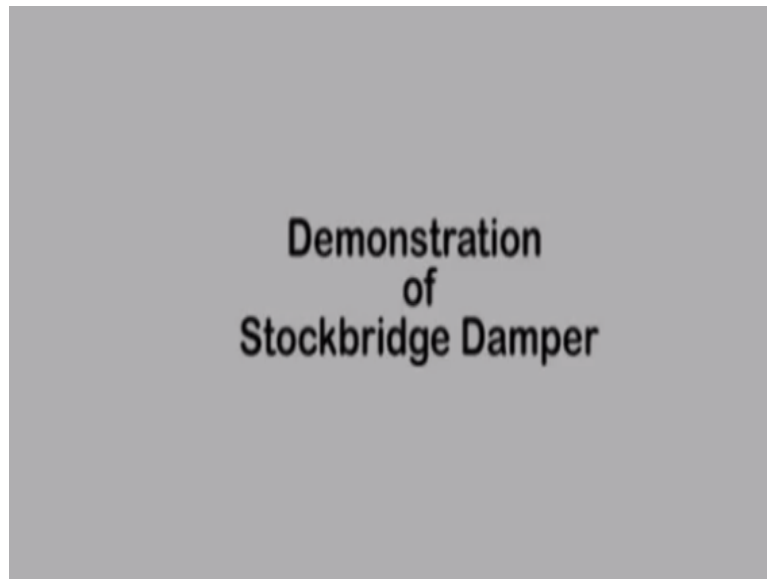
When will see that resonance you have, yeah now you can see it is a sewing dot machines. What is the frequency right now? 6.98. so we are right now 6.98 actually 7hour there as you can see that the fast mode of the weave vibration is become very much apparent and you can easily imagine that means this kind of often excitation happensly structure.

Any bridge any machine get naturally the machine would not able to resist this vibration. So our motto will be how resist this vibration amplitude. So, now what we are going to do a going to excite this simple sting, but with the help of this additional massage and will sure that how we can control the vibration. We are attached these 2 additional masses and we are going to excite the system.

As you can see initially both the system is vibrating because these type of arrangement is only effective very close to the fundamental frequency. Here we are having 6.98 so we are 6.98hertz and a frequency there is no vibration apparenting this mode beam why because the entire energy called primary beam as been transfer to the secondary and the secondary beam to these masses that is vibrating when primary wall is set.

This is what strategy of vibration control which will teach in much more details in this course and this known as tuned mass damper best vibration control.

**(Refer Slide Time: 22:24)**



**(Refer Slide Time: 23:26)**



What are going to show you today is the fantastic application of vibration control which has been originally designed in 1920 by stock bridge and it is named after him as the stock bridge damper. Now these fantastic damper if you can see that this is a busy highway and there is a high voltage power line which has just crossed in and on these high voltage power line you can see that the each of these high voltage line.

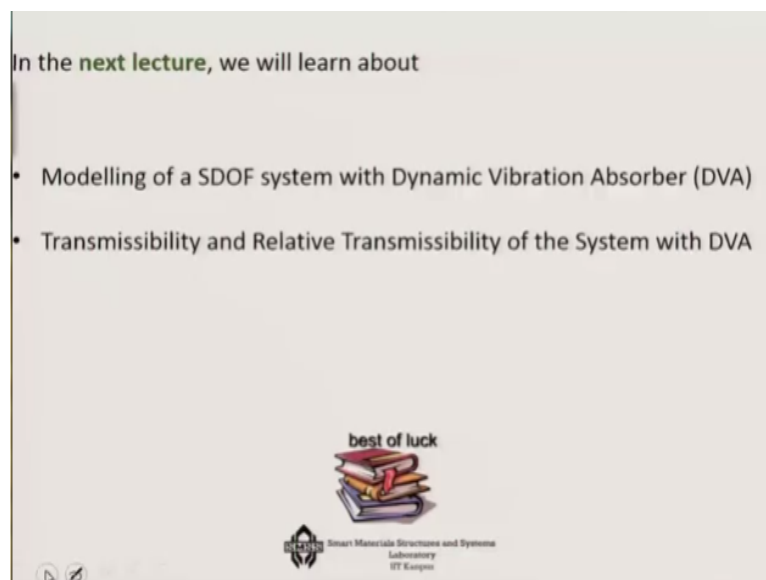
There are pair of this stock bridge damper which is attached to it. The way this stock bridge damper helps to control the vibration is whenever there is a what it is mean used vibration

that comes into the wires which happens in a pretty big frequency range from about 5 hertz to 150 hertz. Then this, what is induced vibration creates a huge you know change of a amplitude in the wires and you can actually destroy the wire it can create a huge power problem.

Now these stock bridge damper which it essentially has 2 additional masses hanging with the power line with the help of a messenger cable this what exactly like a tune mass damper that I have shown you in the laboratory and these tuned mass damper system absorbs this hevalion vibration the what is induce vibration of this wires and keep this wires very steady.

So that they do not give up and as a result continuous power supply is possibly the system. So, this is a fantastic application of the vibration control with the help of tuned mass damper system.

**(Refer Slide Time: 25:23)**



So, this is where we will comfort an entry into this lecture. In the next lecture we will talk about modelling of a system with a DVA and will also talk about transmissibility and relative transmissibility of a system with DVA. Thank you.