Principles of Vibration Control
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Lecture-13
Modelling of Dynamic Vibration Absorber

Welcome to the course on principles of vibration control and today we will be talking about dynamic vibration absorber which is generalised case of the vibration absorber neutraliser or mass damper.

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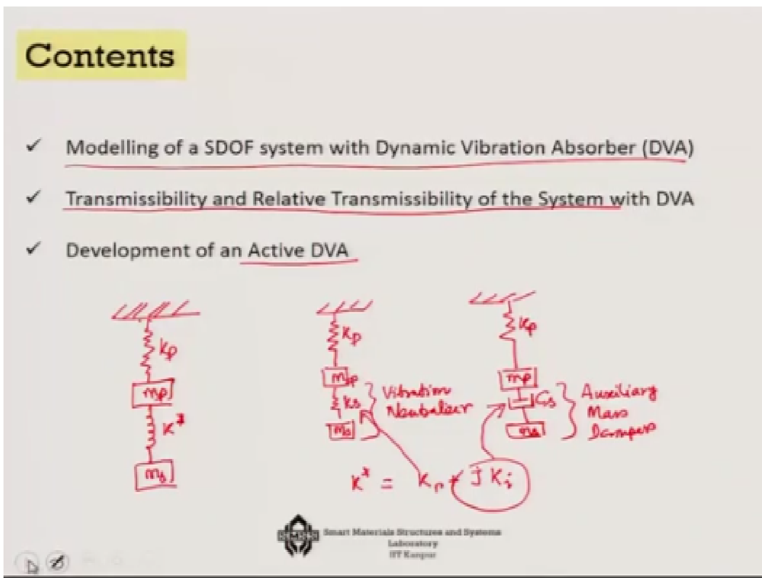

So we will talk about that how to model a single degree of freedom system with dynamic vibration absorber and then will talk about the transmissibility and relative transmissibility of such system and finally also will talk about an active dynamic vibration absorber. So just to keep yourself you know intact with the subject if you remember a warier said that suppose you have a simple spring and a mass system.

And then if I add another spring and another mass which is the secondary mass okay so this is a ms secondary and this is the primary okay stiffness primary case secondary then this system is a special case of DVA which is known as vibration, vibration neutraliser. Also there is another special case possible, that special case possible is that you have again the primary system and instead of a spring.

You have a damper here and then the secondary system so let's call it damping constant of the secondary system and then these systems AMD or auxiliary mass damper. The suppose we want to makes these 2 things together that means we want to have a handle not only on stiffness but also an damping then we will be getting the generalised case of DVA in which we will be having a spring okay KP.

Then the mass MP and followed by giving a different way of presentation of it something which is a stiffness which is complex in nature. Now complex stiffness is equivalent to having a spring and damper that is something that we have to keep in mind that K* means K real +J K imaginary and while K real is actually our secondary spring the K imaginary J K imaginary where J is the square root of -1 so imaginary thing.

This part the imaginary part is actually giving us the damping part. So that is how by denoting this secondary part as K*. We are taking care of both of such a system. Now let us look into the DVA system.

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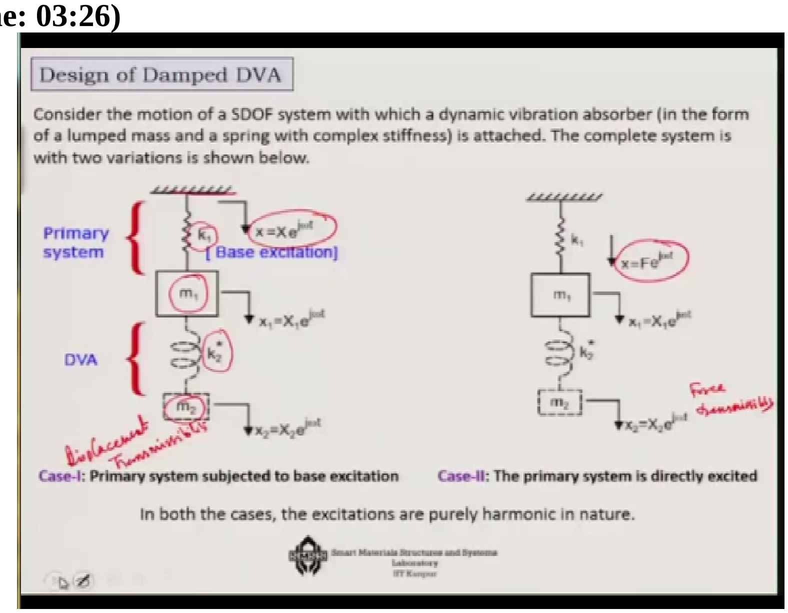

So here we have the primary system as has been shown in here K1 is the stiffness instead of Kp here we are denoting as k1, m1 is the mass and then k2* is the actually is the complex stiffness and m2 is the is the mass of these secondary system. Now such a system is useful for 2 different types one is called basic citation problem where the support is getting an excitation harmonic excitation.

And another is a force excitation problem where the primary is actually subjected to a force excitation of the system. Now actually for many linear cases it can be shown that the transmissibility that we talk about is actually the same, for both that cases. One case this is called displacement transmissibility, so, this is displacement transmissibility, you can find out the transmissibility of such a system.

And another one is called force transmissibility in this case it is the force transmissibility. In fact both the transmissibility are actually equivalent, so, if you work either of the model it is actually good and have to talk about the other, in order to find out the govern equation, let us then *t with one of the model okay so,

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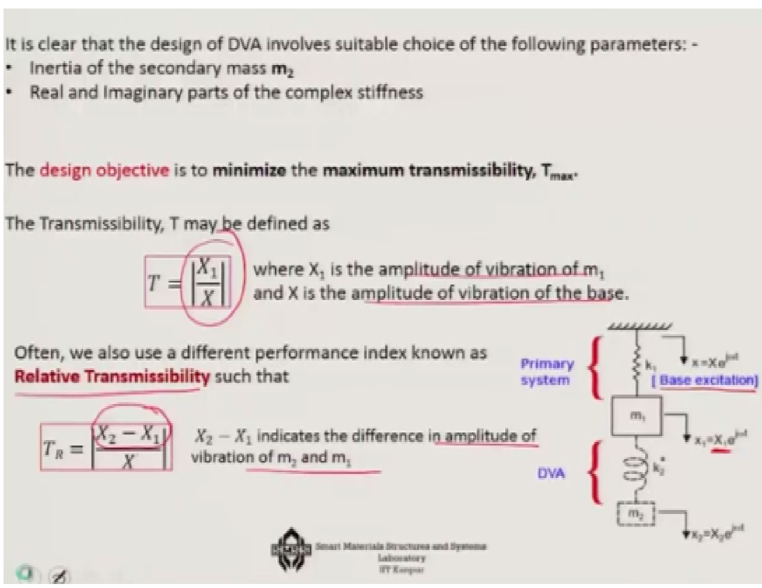

Let us say that we are taking the first case where we are having a transmissibility as X1 over X. Where x1 is the amplitude of vibration of m1, and X is the amplitude of vibration of the base, so, this the base excitation problem, ok, and x1 is you may also think of this way that X is the input to the system, and is the output response of system, so, this is something like an output over input.

Now if I have to do this one thing for the other case, ok, so, suppose for the force transmissibility what you would we have done the output in this case one output could have possibly that what is the force that is transmitted here ok. So, if it call it as Ft so, that Ft, in case the force transmissibility so, the transfer function in this case would have been the input here is F itself, so, just like here you have the input here.

As x here as the amplitude of vibration of the base, so, similarly here in this case the input is the force that you are giving so, that is F and the force you are transmitting and then amplitude of the 2, so, that is going to give us the transfer function in this case, now the other we will be just now going through in this case also you can actually find out that what is the force which is transmitted.

So, just try to find out that what is the force that is transmitted for the active of to looking to the equilibrium of first up all the mass m1, so, m1 if look at it then what are the forces working on a one if you look at it, then m1 is undergoing you know these forces that we can see that m1 x1 double dot, that is what you call inertia force, and then you also have the k1, k1, x1-that is the k1 x1 part itself.

This is fixed of course, so, this is k1 x1 part, and here you are also going to have k2* x1-x2 right, and of course you have the force which is working on the system that means it is F e rest to the power j omega t. or you can call it as F e rest to the power j omega t. So, these are the total forces that are working on the system, now if you look at the spring point of view, so, the point of view then the view the force that is transmitted is simply k1x1 right.

So, the force that you are getting here is actually k1 X1, that is what is transmitted through the spring to the base, so, you can actually by solving the governing equations of motions in this particular case, so, this is what and the other one is of course related to the m2 united to have both of them with you, so far, m2 you will be having k2* x2-x1 that is the way till work right and you will be having an inertia force right.

The inertia force is m2x2 double dot, so, that is for the m2, so you can get a governing equation corresponding equation to this diagram, I leave at as a exercise for you, so, and you can get one corresponding to this, and then should able to find out what is the transfer function for the force transmissibility and you will find out and you can confirm yourself that this will be equal to the displacement transmissibility of the system.

Now, as per as the displacement transmissibility force, which respects to basic citation, sometimes, you also use as a relative transmissibility term and in this case it is actually the denominated remains the same but the numerator because the difference between the amplitude of vibration of m2 and m1. And that is instead of x1 itself it is x2-x1 that comes into the picture. So sometimes this relative transmissibility is also used in certain applications.

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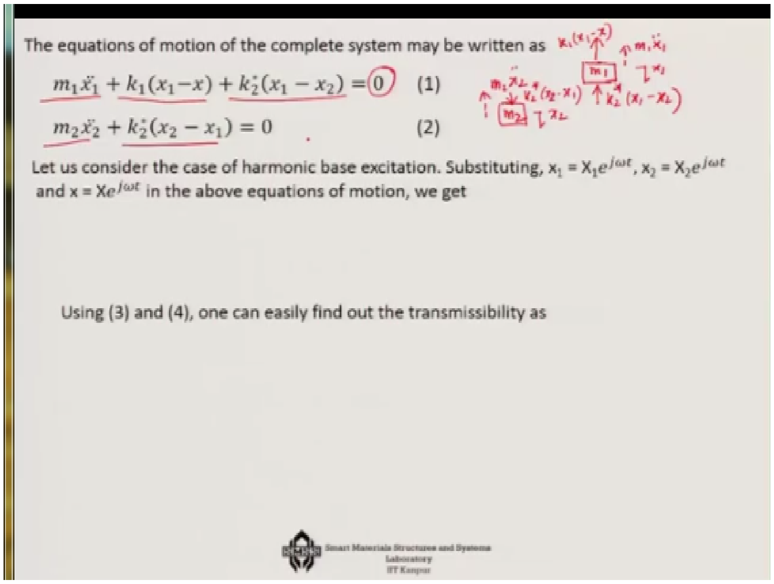

Now what will be the equations of motion of these systems. So in this case it is only subjected to base excitation. so that means if we try to draw this part if you remember what are the forces we are getting subjected to so m1 was undergoing a so there is a these direction of movement of m1 x1 so that means there will be an inertia force m1 x1double dot that is what is our faster here and then you also have in this case the spring k1 which is actually you know there is a base excitation there.

So it is k1 x1-x that’s what is the spring force and also you have an additional so that is second force and also you have an additional force which is k2* x1-x2 that is for the secondary system. So, that’s the equation of equilibrium corresponding to the fast case. And you see the right hand side is origenius there is nothing else because it is a base excitation problem so no other force is you know applied on the system.

On the other hand, for the second one also you can drawn very quickly that this is the secondary mass into so this is going down so, that what will see inertia force is in the that is into x2double dot. And there is a resistive force here that is from the k2 just opposite in direction which is k2 *t x2-x1. So this in this or that 2 components nothing else excites the secondary mass so thus you can get the second equation, so, now we have both the equations with us corresponding to the displacement transmissibility.

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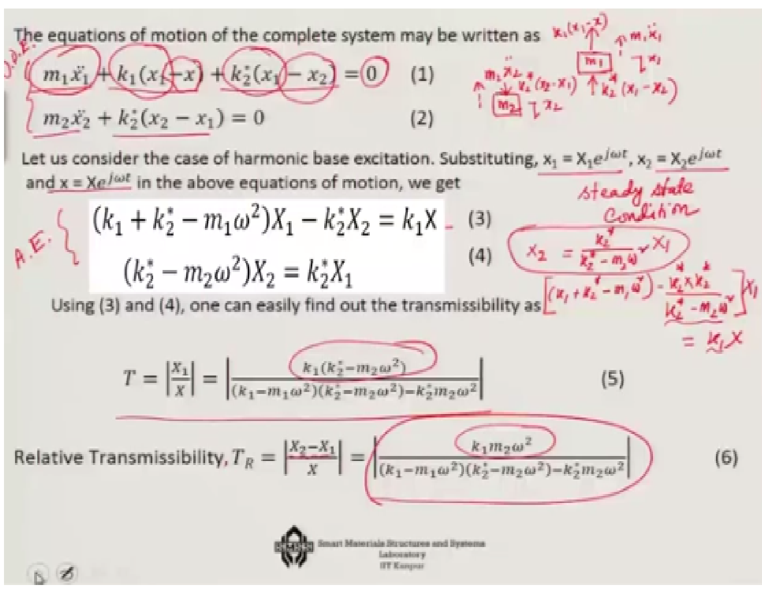

Let us now consider that what will happen if the x is x e rest to the power j omega t, let try to imagine and then x1 is the displacement of m1 let say this is also harmonic and that is x1 e rest to the power j omega t and at the same frequency an x2 as x e rest to the power j omega t. Now this will not happen immediately to give our basic citation, this **is** will happen when system will under steady state condition.

So, this you have to keep in your mind, that this is the steady state condition, so, under steady state condition if I apply these things on the equations above what will be happens that we will be having now of course k1+k2*-m1omega square x1-k2*and then we will be having k2*-m2 omega square x2 equals to k2*x1. So, we will be getting 2 equations and in these 2 equations we will be having no forcing function part in it.

And these we are converting the ordinary differential equations by applying these conditions we are converting it to algebraic equations so, this is you know a conversion in the steady state, so, this is a ODE is begin with and this is a algebraic equation that we are getting, that is the advantage, so, it will be much easier for us to tackle the whole thing in the steady state ok.

Now using this 3 and 4 I can in suggest a matter of elimination, I can easily find out that what is x1 over x right, if we try to do it here, that first up all, x2 you need to note down that x2 is actually k2* over k2*-m2 omega square. And then you use it in the first equation, so, you should be able to get a relationship now in the first equation now in the first equation we have k1+k2*-m1omega square x1, that is considering these stand and k1 x1 term and m1x1 double dot.

Ok and k2*x2 we are getting that is considering these term, so, now let us looking to the third equation then, that means the third equation will now become k1+k2*-m1 omega square x1 so, this minus so, we can keep the k2* times this relationship so, this is x2 is these times x1 okay so, now x2 I replace this that means it will become k2* again k2 times k2* divided by k2*-m2 omega square and whole thing will be with respective x1.

That will be equal to k1x so, this thing if we will keep in our mind we should be able to find out what is x1 over x if you just continue in this direction, as you can see that you will be having this k2*-m2 omega square this one and k1 in the numerator back, so, that is what is happening here, and in the denominator so, little bit complex but you will be having k2*-m2 omega square, so, that term you will be getting k2*- m2 omega square term here and you will having k1-m1 omega square term here.

So, all this things you know after the algebraic manipulations will give you the final transmissibility function in this particular form, and similarly once you do it for one case you should able to do it for the Tr also, where which equals x2-x1 so, you already know the relationship between x2 and x1. So, that is why you will able to find out x2-x1. And then you can find out x2-x1 by x which will take out these forms.

One thing that we can see here is that denominator in both the cases remind unchanged, where as the numerator will be changing, in this case the numerator is k1 times k2*- into omega square but is here it is k1 m2 omega square so that numerator will be changed. So, thus we should be able to find out both transmissibility as well as Relative Transmissibility of the system.

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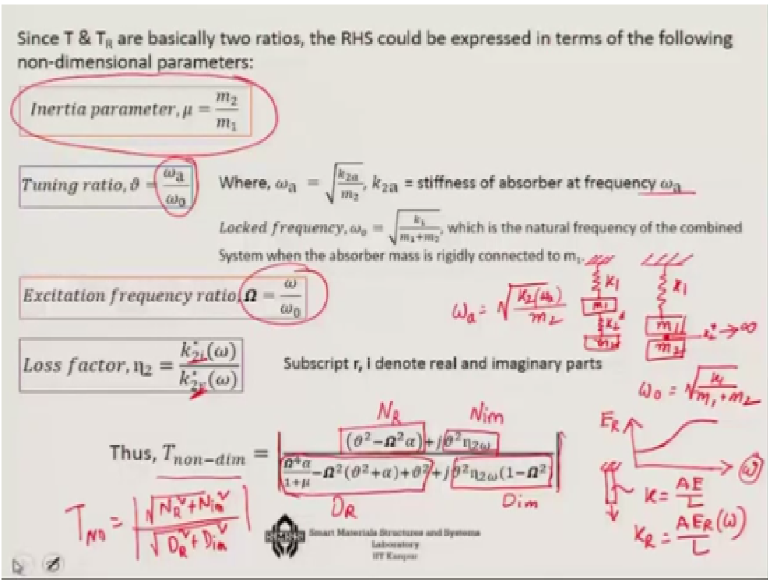

Now, now that we know this we need to do beta of you know non-dimensional (()) (17:32) of the problem. In order to do that some very important thing this is important from the design point of view at some very important non-dimensional parameters will introduced generally. The fast parameter is the inertia parameter or the mass ratio which is m2 over m1. That means the ratio of the secondary mass to the primary mass.

We generally do not allow this secondary mass to go beyond a particular level. Because then the mass penalty will be very high so this mew is 1 term that is needed which actually talks about the mass penalty. Then you have the tuning ratio, tuning ratio is having a frequency ratio omega a over omega0. Where omega 0 is actually a called the locked frequency of the system.

So that means omega0 is a non-dimensional with respective condition, where you have k1 m1 and then m2 is very, very rigidly connected with m1 there is spring stiffness here is actually k2*that tends to infinity that means it is rigidly connected. So that is why the natural frequency of this system will then become omega0 that is what square root of k1over m1+m2 that is the locked frequency.

Now what is omega a, omega a is actually he apparent you know natural frequency of the secondary system that means in that case you have actually k1 you have m1 and you have this k2* and you have m2 and omega a talks about square root of this k2 at a particular frequency level that is why it is the apparent frequency level. The stiffness of absorber at frequency omega a.

So, once you know the frequency so, at that frequency level over m2. Now why do you think that omega A will actually vary and why do you need k2a as a functional of omega A it is because if you remember I told you long before that the viscoelastic materials if I plot their characteristic like you know omega versus their modular’s of elasticity which is actually somewhat reflected in terms of the stiffness.

So if you look at the real part of the modulus of a elasticity they low frequency is equivalent to high temperature behaviour and high frequency is equivalent to low temperature behaviour, that means the modules of velocity will be low initially and then it will increase, so, that means your modular's of velocity is a function of the frequency we are talking about, and since modular's of velocity is a function of frequency.

Hence k itself let us say if it is an actual member if you consider that this spring is a viscoelastic spring ok which an actual member then what is the k for this case it is A E over L, so, that means K real will be actually AE real over L, now since EDL itself its changing with respective frequency so, this will be a function of frequency, so, that is why your Kr is a function of the frequency and that's what the tuning ratio comes in to the picture.

The next is the excitation frequency ratio which is very simple it is actually the ratio of the actual frequency where I have exciting and normalised it respect to the lock frequency of the system and finally the last factor and that is define a standard the you know difference between the imaginary part that what is k2 i over k2 r that's what is the last factor. Now if you apply all these parameters it transmissibility is non dimensional representation will take this kind of a form.

Which is a complex form that's why you need these 2 things you know in order to get the amplitude of the system, so, as you can see here that in the complex form is having you know this is you may call it as N real and you may call it as N imaginary, so, this beat is N imaginary and this beat is N real, real part of the numerator similarly you have the real part of the denominated this is D real and the imaginary part of the D imaginary.

So, you can find out the non dimensional frequency very easily. Once you tars speciality where easily once you know what is Nr what N imaginary, what is Dr what is D im because all that will be between is actually taking the ratio of N r square + N imaginary square in the numerated and in the denominator Dr square + D imaginary square and that will be our the amplitude of these will be quart over T non dimensional you know will be. So, you can easily actually find out this whole non dimensional transmissibility.

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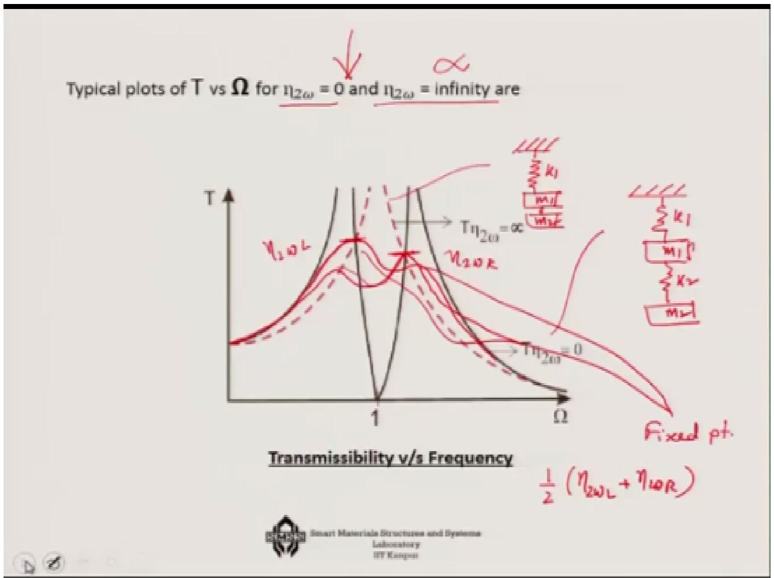

Now for certain case let us say you know for 2 extreme cases theta 2 omega is equals to 0 and theta 2 omega is equal to infinity if I try plot then so, how it will look like well for theta 2 omega equals to 0. It would mean actually that is like a vibration neutraliser problem, because there is no damping that is there so, that means theta 2 omega equals to 0 would behave something like K1 here, m1 here, K2 here, there is no damping there.

So, that is why K2* and neglecting it, so, this will be theta 2 omega is 0 and if theta 2 omega is equal to infinity then that means the other case, where the whole system behaves like a single degree of freedom system, thus the lot frequency system, so, in that case you have k1 here m1 here and you have m2 here. Rigidly connected system, so, naturally you can see that that is the case where there is a single degree of freedom.

So, that is only one peak where as in this extreme case you are getting 2 peaks to the system. Why we solve it for this 2, is because if you now look at these very carefully you can see that there is one point of intersection here and another point of intersection here and these 2 points are also called fixed points, and as you can see that irrespective of the frequency and the irrespective of the damping level.

Most importantly theta 2 omega whether it is 0 or infinity, the transmissibility called has to pass through this 2 points, because all the extreme cases are covered, so, what we generally try to do is that, we try to put one of the peaks to pass through one of these peaks points, anyway the fixed part like a hurdle that you have to cross, so, we try to put our curves like these, this is one option, or other option is that it will go small here but it will touch this one here.

So, that means any way you have to cross the fix point, so, you try to place the peaks corresponding to either of the transmissibility through this fix points and then under such condition you will able to get that what is the optimal theta 2 omega because that has somewhere between the 2, so, it is neither 0 nor infinity, for which theta 2 omega you will either get this configuration or get that configuration.

Now, suppose I get 1 theta 2 omega, theta 2 omega and I get left, another as theta 2 omega right, so, that means my best condition will be average of that one, that is half of theta 2 omega left + theta 2 omega right, also I can find out the corresponding to what tuning ratio that etc., this particular situation comes out and that's we can find out all the optimal values of it, I leave it o you as an exercise.

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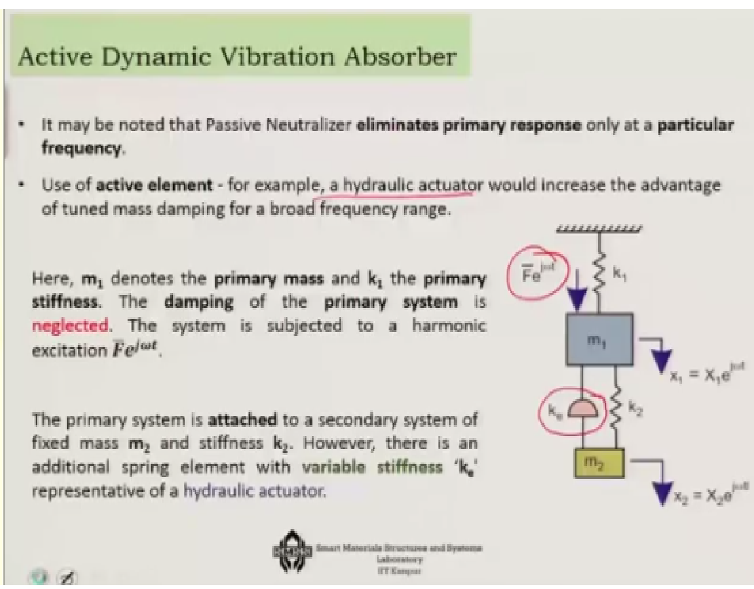

Now from here we need go to the case which is known as active dynamic vibration absorber, so, in the passive case, at least the dynamic vibration absorber case the most generalised case is better, because you have additional point with you pointer with you, which is the damping part, that is why instead getting a good transmissibility only at a particular frequency level, to reduce that, you can actually get it for a range of frequency so, that is there in your hand.

However, it is passing nature that is once you decided that transmissibility called is fixed forever, now suppose I do not want to do that, I want to have a more dynamic control of the transmissibility. In such a case instead of adding damper passive, I will be adding a feedback element like a hydraulic actuator. And if I do like that so, what will happen is that I have a k2 I have an additional contributor here which can work like a stiffness contributor or is can like a damping contributor.

So, by varying by tuning this Ke I can actually cancel this effect of a bar is rest to the power j omega t, let us looking to the formulation of such a system.

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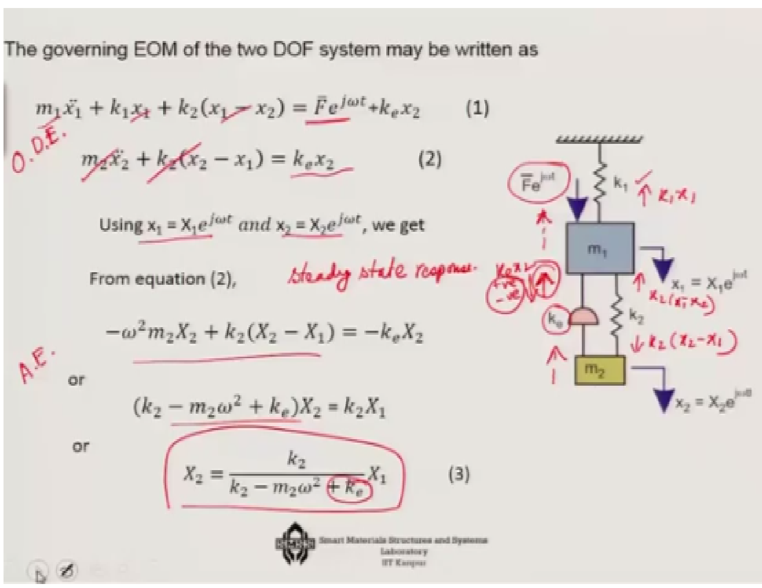

So, as you can see here that under such a condition now the right hand side is not 0, so, we have m1x1 double dot, which is the inertia force here, if you remember, that this is what we have and k1 x1 is also for here, that’s the spring resistance k1 x1 and then k2 this part k2 x1-x2 that is the resistance that is coming from the bottoms that is this part and then we have the force excitation which is a bar e rest to the j omega t, in the right hand side here.

In addition to that we have this effect Ke and this Ke effect is in my hand I can choose it this way or I can choose it that way, by actually I the actual expression is Ke x2. But this Ke can be positive or it can be negative. In fact see that what will happen to positive or a negative feedback, so, thus we can vary this direction of this additional force that will be there in our hand.

Now as per as m2 x2 goes once again you have m2 x2 double dot that is the inertia force and then you have k2 you know k2 x2-x1 that's this force that's the spring force and an additional force which is in the right hand side that is ke x2 okay so that also control the direction of now we can put x1 as x1x2 the power J omega T x2 as x2x2 the power J omega T just to get the steady state response.

Because we are converting the ODEs just like last time we are converting the ODES into algebraic is these are odes right are ordinary differential equations. We are converting the to the algebraic equations and also do that in the steady state again this relationships working on which will get an relationship between x2 and x1 in which you can actually control x1 or x2((refer 30:24)) by controlling this game ke and can make it positive make it negative. We can actually play with it that's the advantage of active vibration control in this case.

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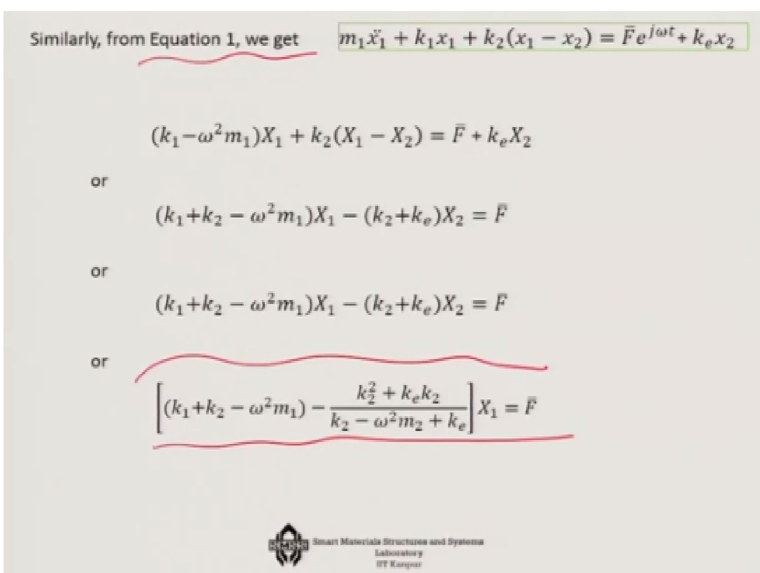

Now in this case then we can get the first equation of motion work on it and this will be final form of the first equation of motion so it this is the steady state form of the first equation on motion so, in the last slide you have seen in the steady state form in the second equation on motion, and this is the equation 1, the first equation of motion we can also get the steady state representation of the first equation of motion.

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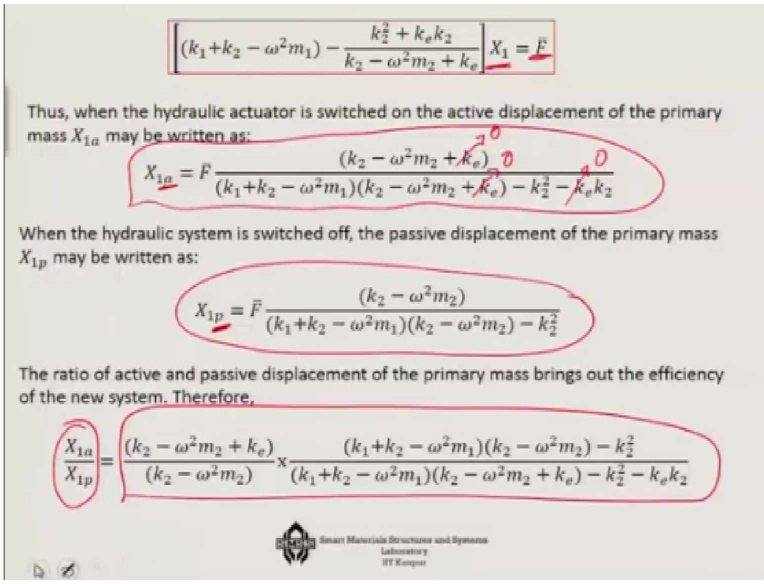

With that now we will be able to find out that what is x1 that is the response of the primary, and subjected to a force amplitude F bar, so, this is what the expression that will come out you can try using these last 2 steady state equation and this what is x1 a and if I put these K to the 0 this k 0 this k 0 this k 0 then I get a simplified form where that active part is not that, the hydraulic control is not there.

So, the first case is equal to the active second case equal to the passive, now I can make a ratio of passive and active it will be a nice non dimensional (())(31:52) also it will tell us that you know how much effective with the active response with respect to the passive one, so, we can find out it by comparing this 2 equations we have a pretty complex equations, and that you can find out the expression of it. So, we can have x1a over x1 b here with us and.

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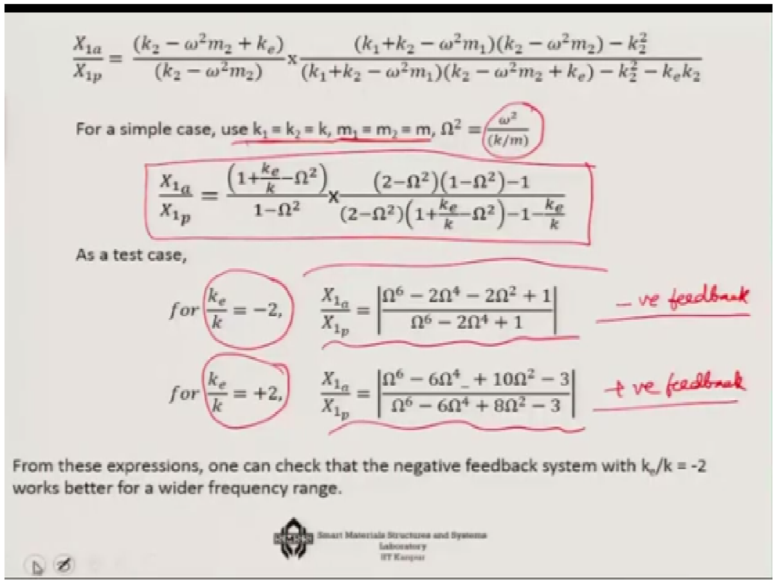

We can take a simple case lets say primary and secondary stiffness both are same, primary and secondary mass both are same, is will not happen, but in terms of a kind of a simple comparison and we put omega square, as omega square working by in, then I can non-dimensionlised and I can get this falling expression over x1a over by x1p, now with this expression in hand I can try various k u r k, so, for example I can try k u r k-2, then I will get this expression with us.

I can try K u R k +2 then I get this expression with us, so, this -2 actually shows that I have a in a negative feedback system, and this one actually is a positive feedback system, so, what you will see is that this expression if you now plot this transfer function with respective omega you will see that much better solution in comparison to the positive feedback, so, you can verify yourself that active vibration control with negative feedback of the displacement of the system, would work much better in terms of vibration control.

So, this is a special case and when we will control I will talk more about it.

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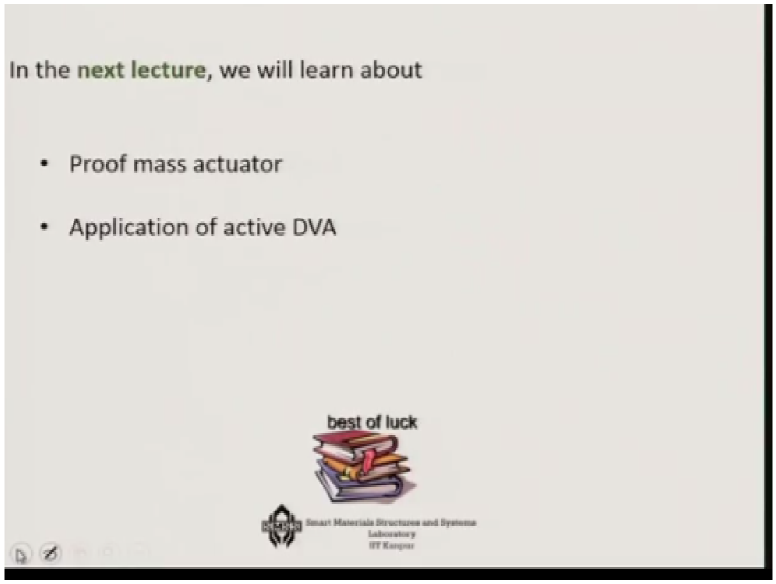

This is where I put an end, in a next lecture we will learn about proof mass actuator, and application of active dynamic vibration absorbance. Thank you.