Principles of Vibration Control
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Lecture-17
Basics of Classical Control System

Welcome to the course of principles of vibration control. Today we are going to cover some of the basics of active vibration control let us discuss that in the board itself.

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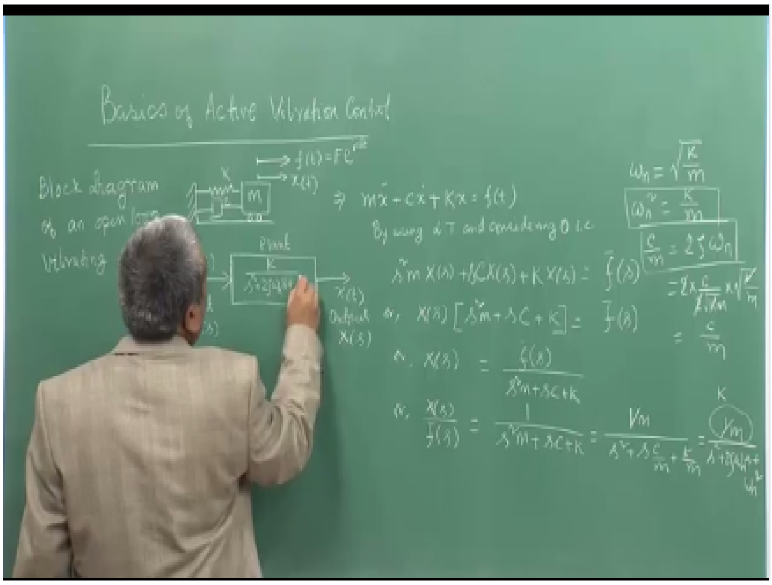

So we are going to talk about basics of active vibration control. To discuss this let us consider a simple block diagram of a vibrating system as a plant. So first we will take up block diagram of an open loop vibrating system. Let us see how we can translate the kind of the graphical representation of a open loop system in terms of a block diagram. So we have a mass M, K is the stiffness.

C is the damping and this is moving on a fraction thus will and let us say that I am applying a force ft to the mass and it is getting displaced towards this direction as xt. Also this Ft is actually periodic in nature and you can write is as something like Fe rest to the power a omega t where F is the amplitude and e rest to the power g omega t talks about the hormonic variation of the system.

Now these type of a system if I have to represent it in terms of a block diagram how will it come?. So we have to have a plant here this is also known as plant and that plant is actually the system and we have an excitation force Ft working on the plant and the plant is showing a response which is xt. Now the point is what goes inside this block diagram, to find that out we refer to first write the equation or motion of the system which can be written as I am not going into the POD diagram anymore.

We have discussed about it many times, so we can write is as Mx double dot+Cx+Kx equals to ft. Now this is a time domain ordinary differential equation, we need to convert it in to an algebraic equation by using laplase transformation by using laplase transformation and considering 0 initial conditions. We can write these in the form of s squared m Xs+**C** sCXs+xs+K xs equals to f bar s.

In other words we can also write it as Xs times S square n+Sc+K equals to F bar s or in other words Xs equals to F bar s divided by s square m+Sc+K. If I want to find out the transfer function which is output Xs invert f bar s which is the input to the system so this is input and this is output. Then this would become 1 over s square m+Sc+K. Now I can do a little more work on it.

I can actually write it in terms of the model coordinates of the system by dividing all these parameters with respect to say for example m if I divide it so it will be 1 over M and here it will be s square+Sc over M+K over M. And that would mean that we can actually try to find out that what are these other 2 things in terms of the model parameters. We know that the natural frequency of a single degree of freedom system omega n is square root of k by m.

In other words omega n squared is actually k by n, so this is known to us and also we know that C by M can be written in terms of 2geta omega n, you can cross check it, it is 2 times Cx over CC which is 2 root Km times square root of K by M which means this K and this k gets canceled, 2, 2 canceled and we are going to get C by M. And that means this is also acceptable for us along with omega M square equals to K by M.

So we can very nicely substitute these things in terms of the model parameters the denominator s square+2 zeta omega ns+omega n square. So that is what we are going to get from these and hence I can actually write these as something like in terms of the frequency domain f bar s here and xt is xs here. And I can write these as the plant structure of a constant, let us call these 1 over m (()) (07:43) as a constant.

Some plant constant let us call it with kp or just simply K, so you can write it as K over s square + 2 zeta omega ns + omega n square. So that is that will call the transfer function representation of the block diagram of the system. If we look at this particular equation we focus on the denominator itself, there are several things that we can conclude from this denominator.

What we can conclude for example is that, if we equate this denominator s square 2 zeta omega ns+omega n squared equals to 0. Then this is what we give as the characteristic equation of the system. This is will be the characteristic equation. Now if I solve this characteristic equations I may get 2 roots s1 and s2 and these 2 roots can be something like in case of you know a stable system-alpha+-j omega t. We can try to get a graphical representation of the system now with respect to these 2 roots.

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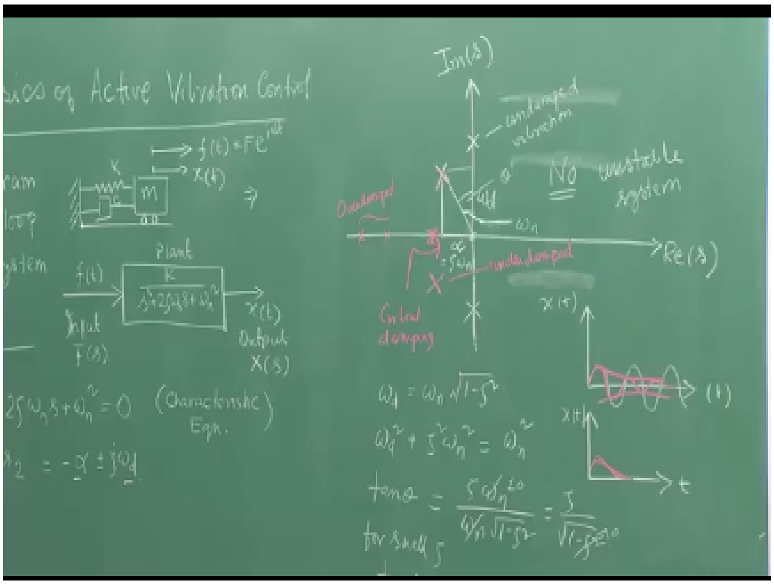

So let us try to clock these roots of the second order system and see what does it mean for us in terms of vibration control. So if clock this 2 roots what I am going to see is that we can start with an assumption that in a generalized case there is a real part and there is an imaginary part of these 2 roots. Now there are several possibilities of the roots. First of all can these roots being this side no if we consider then it will be an unstable system.

Now generally a vibrating system will not be unstable on its own. So hence when we consider the passive system the roots will not be on the right plane it may happen that the roots will be here, if it happens like that that means if the alpha part will be 0 and it will only have the imaginary part and in such a case the corresponding situation is called undamped vibration.

So that would mean that in terms of the 5 domain the response of the system xt so xt with respect to time itself, so this side is with respect to the time if I plot it, it will be typical of an undamped system, that is what it will be once the roots are on the imaginary axis. Now if the roots change from this location to let us say as there are complex poles in the left of plant, righter plane the roots are not there but these 2 roots are in the left of plane.

In that case what we will see is that these roots are going to show a different type of a response it is going to come down exponentially with respect to time. So there is a exponential profile and that is corresponding to when the roots are we will say under damped. So these are under damped. This is the case which we will frequently observe in terms of vibration control. Of course in extreme cases it can be that the roots are directly on the real axis itself.

And those type of responses in which it will be on the real axis itself t and xt and the vibration will essentially you know start and die down and it can be if it is say for example a pier in this real axis then it will be over dam system, so it can happened that the vibration is actually die down, so this cases are actually critical and so this is a case of let us say this is the case of over dam system.

And this is the case of critically damped system. These cases will not come across usually but in active control it can happen that post active control this case will happen to the system critical damping. Now considering that the most important case that you are going to come across is the case of an under dam situation, so there are certain things that we can try to see that what is this alpha and what is this omega d first of all.

So naturally this distance is alpha, we can see and this ordinate is actually omega t. So you can graphically place you know from the root where the system is and if I do like that then several things we can conclude from it. Say for example if you look at it then that what is these radial distance from the origin till this plant, we will see that this is nothing what omega A.

We can verify it because you know that omega d is equals to omega n into square root of 1-theta square. Now this alpha, this distance alpha equals to actually zeta omega n. So you can see that omega d square+zeta square omega n square equals to omega n square that is the Pythagoras law, so you can see that omega d square which is this distance+zeta square omega n square is going to give this radial distance, that is the omega n itself.

So that means the placement of this particular you know pole depends on the natural frequency, if the natural frequency is very high then it will be far away, if it is very close to 0 etc. then it will be closer to the origin. The second thing that we can also conclude from here is that what is this root theta. If you take a tan theta then as per our definition the tan theta is zeta omega n divided by omega n into square root of 1-zeta square which will be zeta because omega n is not equal to 0 you can cancel zeta over square root of 1-zeta square.

And that means that for small zeta, tan theta is approximately zeta itself. Thus you can neglect thing this zeta square, so it will be approximately zeta, so this slope actually gives us the measure the damping, that is why when the slope is 0 then you have non damped vibration and as it is rotating towards this direction and the roots are reaching here on the real axis.

You have you know critical damped and over damped conditions, so that is how this slope tells us about the damping and the radial distance takes tells about the natural frequency of the system. So, this several implications we have to keep in our mind because when we will be judging the performance of a system, then these things will be important for us.

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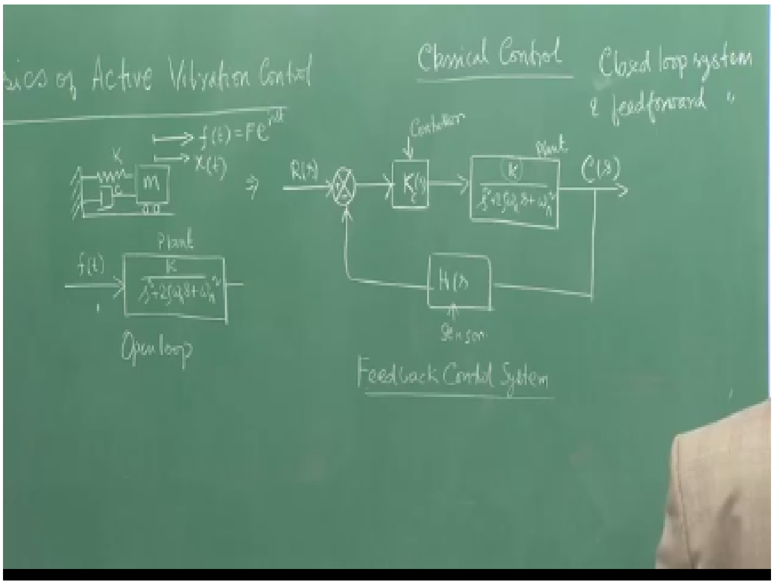

Now keeping these point in mind let us try to find out what are the possibilities for us in terms of developing different types of active controller under the domain of first of all classical control system. So first of all we will look into classical control systems and what we will see is that there are 3 or 4 possibilities for us corresponding to the classical control system.

So this is an open loop configuration. Now in the classical control we will go for close loop systems of course that is one system which is not totally close loop, so we will call that as feed forward system. What we will see is that there are 4 such possibilities to build up the control as far as. So one possibility is the one which will be discussing most that you have a referral signal Rs let us say positive.

And you have a controller here, so let us call it something like Kcs and then we have the plant here, that plant is our k over s square+2 zeta omega nx+omega n square and then that is the output we tell the output as Cs let us say and then we have a sensor here, that is what is Hs, and we are taking that sense signal back, that is with a negative feedback control. So keep in mind that this is the controller.

Where is my actuator, the actuator can be actually integrated with this plant itself, so this is the plant and this is what is our sensor. The actuator can be in terms of a simple gain here it can be adjusted or it can have a complete transfer function of the system, but this is one system which we will call it as a feedback control system. This is the lowest system that means you feel it there is some disturbance in the system it can be prove that this works much better than any other possible systems.

However, there are several varieties of the for example you can have a feed forward system and the feed forward system works very well when you know what is the disturbance the system will be subjected to.

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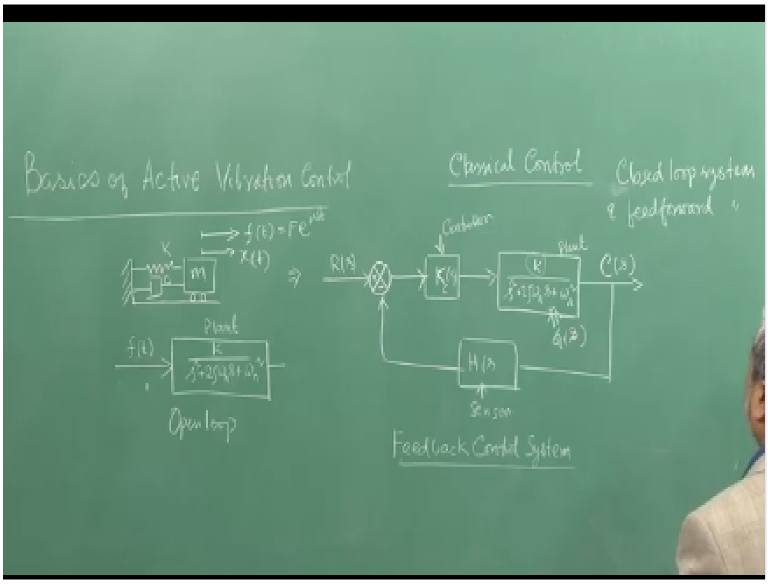

So for this kind of a system we have the reference signal here and then suppose it has there is a disturbance here Ds and that disturbance is coming to the I mean junction here. So that is the positive part of it and then the get output, we are going to call these many times this transfer function Gs. So this going to the Gs and the Gs is giving me an output Cs of the system.

Now this disturbance is coming so I need to reject it and if I know the model of the system I can actually develop a transfer function which will exactly replicate the disturbance by the phase and the gain and it is going to work on it. So that effect of the disturbance is gone and then you know this system is actually free from the disturbance, so this is what is known as a feed forward system.

However, the condition is that Cfs Ffs that is the gain for the feed forward system must match the gain and the phase of the Ds, so you must have the complete knowledge of it. Sometimes you may not have that, in that case we would rather go for a system which is a hybrid control system, so that would look something like this let us try to draw that kind of a system, so you have a Rs coming of reference signal positive and you have a feed forward here.

So let us call this to be Cffs that is what is our feed forward loop, that is going and working so this is a positive part and let us say this is how my disturbance is coming and we have a forming junction here, now on root we have these feedback gain, that is what is our Kcs, so Kcs is coming here, this is positive, this is positive in the forming junction, okay and if we consider this to be positive then this will be negative.

That is what is our Ds Cffs cancelling each other and then we have a you know Kcs coming up and this system is going to my plant which is Gs and then that I am going to take out that Cs, I am going to put a hs here and then I am going to put it back here with a negative sign, so here we get the advantage of both feed forward, so the tough part is the feed forward loop and the bottom is the feedback loop.

This is the hybrid control system. We can have one more variation of the system which is actually known as a Notch filter of a system. So which is something like an extension of the feedback control system?

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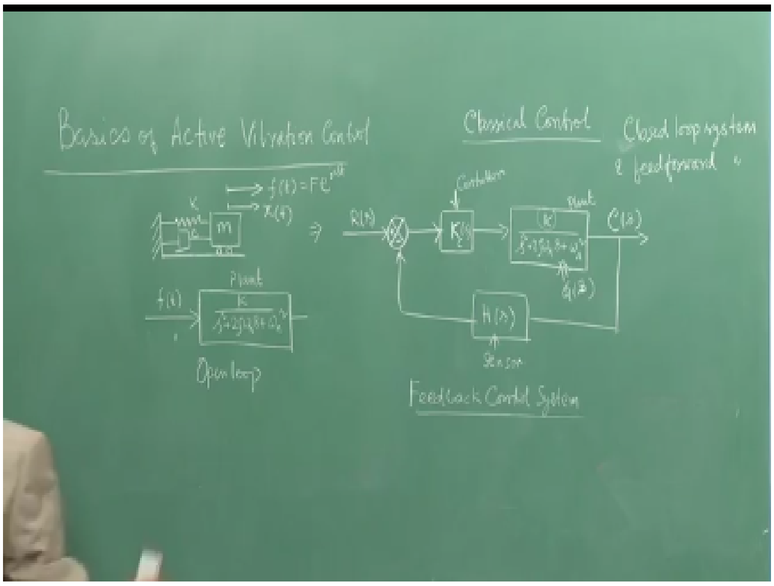

So let us try to dray this last system and for that we have Rs as the reference signal, we have a forming junction as usual, we have error which fast goes through the notch filter, so C notch s, then it goes to the controller, feedback controller, Kcs, then it goes to the plant that is Gs and we have the output of the plant Cs, that comes back, we have a gain here for the sensor Hs the transfer function and that closes the loop.

So this is feedback control with notch filter, so thus there are this 1, 2, 3, and 4. A 4 basic types of classical active vibration control system, that is possible you know for us to design. No we will however, look into only one of this system that is this particular system and let us try to see that what happens to this particular system, if I choose, if I make certain simplified assumptions and then try to see what will happen to the response of this particular system.

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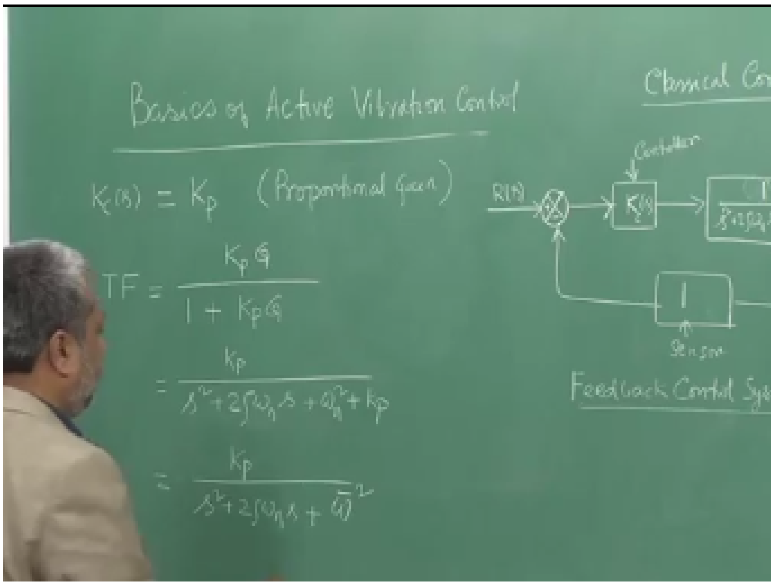

So in this case we will make 2 assumptions one is that this Hs will consider it to be unity, that means the sensor will have 2 dynamics, it will be proportionately feedback the output and compare to the Rs and then the whole system will run. So that is one thing and secondly this is tabulated in such a manner that this simply became unity for us calibrated in such a manner and that is what let us say is the very simplified system for us.

Now for this kind of a system let us first consider the Kcs as a proportional gainer, so just a constant gain which is known as the proportional gain. That means Kcs as simply a constant Kp. Under such conditions what will be close look transfer function CLTF which we have discussed in the last class, that it is actually KpG divided by 1 + KpG, that is what is by close root transfer function.

Remember that the H is unity here, that is why it is KpG over 1+KpG. So if I write we know the structure of G, so that means it will become Kp and below what we are going to see is S square+2 zeta omega ns+omega n square+Kp. In other words the close loop transfer function will be simply Kp over S square+2 zeta omega Ns+something which we may call it as the changed national frequency omega bar square where omega bar square is actually omega n square+Kp.

Now because this is identity nature so we can see that with the help of the proportional gain we can actually increase the new close loop natural frequency of the system which means we can actually the proportional gain can actually increase the stiffness of the system. So whenever we have a vibrating condition where we do not go up to resonance that means it is control by stiffness we can use the proportional gain technique very well under such a case.

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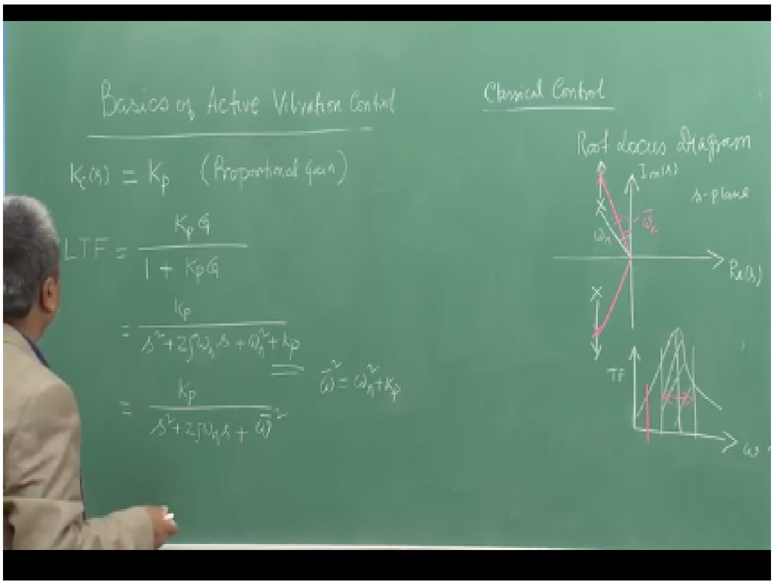

So if I try to draw this in terms of something which we controller designers very routinely do that call it to be root locus diagram. Then this is our real of s and this is what is our imaginary of s and we will call this as a S plane and then both of our roots will be initially placed somewhere like these and as you are increasing the gain they are going this is the close loop direction.

So that means for a particular gain what you were going to get is may be the close loop pole locations will be like this and what it means is that this distance form the origin is increasing. If you remember this is what we call a natural frequency of the system, correct. So this distance the omega M bar the new natural frequency with respect to the earlier natural frequency, which was this one?

You can see that that is increasing at the cost of what at the cost of damping of course because the damping is more earlier and the damping is less here. So proportional but does not matter because if you think of the vibration control system the frequency was this the transfer function and I told you that you know we are this is the region which is actually damping controlled.

And if you are somewhere here if you excitation frequency is away from the natural frequency then damping you do not bother about and what you are bothered about is that you got a good stiffness in the bargain and hence you have a good vibration control. But let us say there is a situation where I am somewhere here so I am going to do in that case I will definitely go for the you know velocity feedback in the system and in that case I will definitely go for the gain which is not simply proportional but something more than that.

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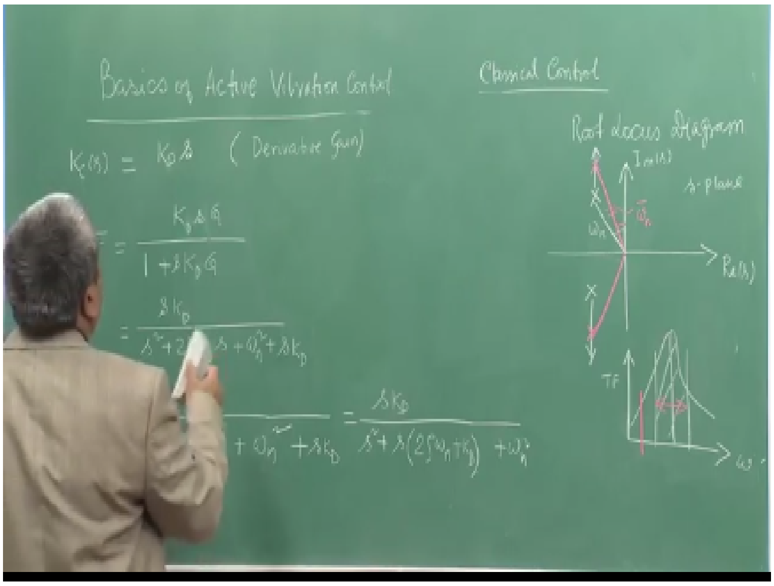

So we can change that gain in terms of something like a Kds is actually our differential gain system, so we have proportional integrate like derivative control, so let us call it the derivative gain and hence the Kds if I get in this manner then this is slightly change it will become Kds G here also this will change it will become SKdG, so here also it will change this fellow would now become + Skd.

And here also it is Skd. That is what will happen to the system, so we are going to get no change in terms of the natural frequency, it will be omega M square. However, we are going to get 1 new term here which is SKd and here we will be having SKd. Now if I focus once again at the denominator here then this new denominator is telling us that we are going to have a system which is something like which has a changed damping. So it will be +S 2 zeta omega n+Kd. And then we have omega n square.

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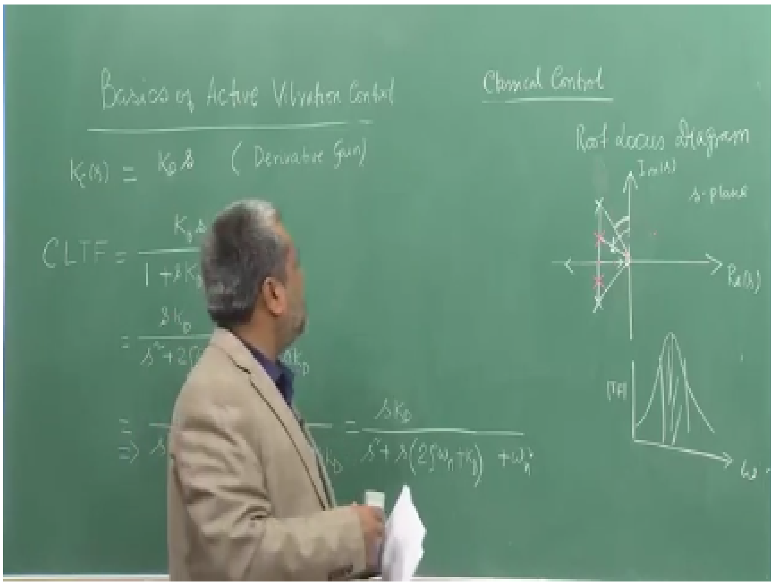

Now that means if I have a derivative gain then instead of this type of a change the change that may happen to the system let us try to figure it out that how it would look like so we have the initial placement of the system, but now we are going to have a 0 here, so let us say we have a 0 here and this were the 2 poles systems and as we increase the gain K this 2 poles will very quickly come towards the real axis.

And one will go to meet this 0 and the other will go away. So what it will mean is that with new system I can have many different types of close loop performance, I can have critical damping when the poles will be here, I can have you over damp system with the poles will be like this. I can have higher damping which is now generally will try to do where will sacrifice to some the natural frequency of the new natural frequency of the system.

And that we will do in terms of the damping because the damping has now increased from this much of an angle to this angle. So we are increasing the damping, we are giving up the stiffness and it will be does not matter because we are once again, if you figure it out that we are in such a case we are mostly in this domain, in the damping control to omega of the system. So we can do that and then we can actually design our system accordingly.

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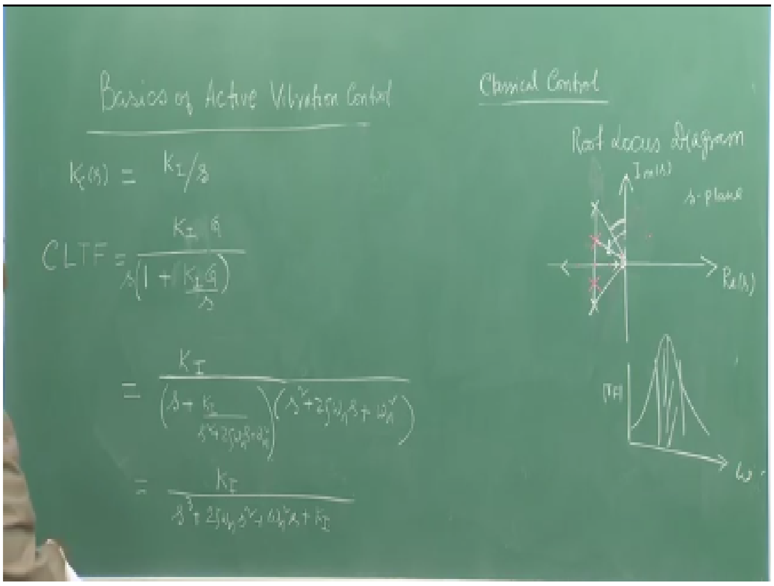

Now what happens is that sometimes this kind of systems may also become very fluffy system you know they do not stabilize when if asked etc. So we can try for another system which is known as Ki over s and we call that to be an integral control system. So they it will be Ki over s, so Kg Ki over s times 1+ki over s. This type of system you may get and if we try to work out for this system then it would look something like we have the G as 1 over s square so that will be there.

So let us say we put that in the G part so we have this complete formation with us as an additional term 2 zeta omega n s + omega n square and here we have this s coming up here so that means there will be this s and this s will actually cancel so this will be s+Ki. So this ss are going to cancel which is on each other, so we have s+Ki and over s+Ki over s square + 2 zeta omega ns+omega n square.

And this whole thing multiplied by these and as a result we are going to get a slightly higher order system because these 2 are going to cancel and we are going to get a system which will be like sq+2 zeta omega ns square+omega n square s+Ki. So this is no longer second order system that we are going to get a higher order system here.

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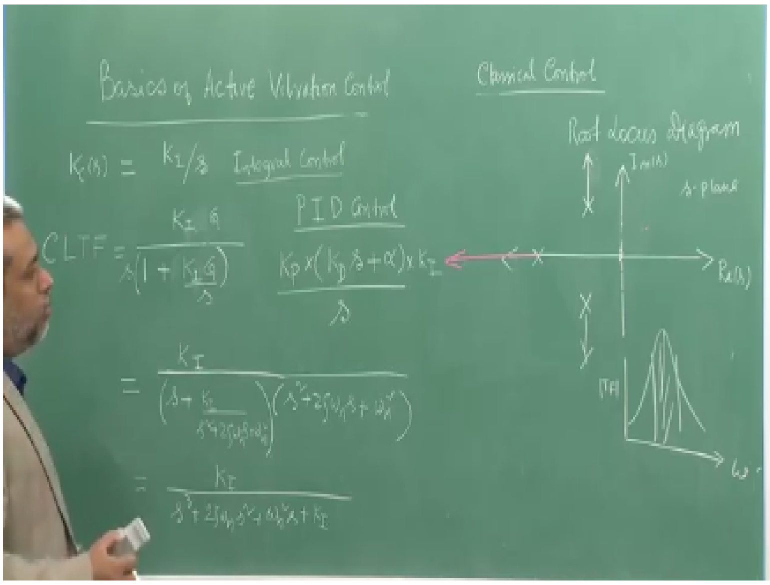

And that sometimes helps us in terms of various types of you now design of a system where the study state performance for example can we actually improve our if there is a input which is having something like higher order things like parabolic inputs etc. this system will be able to follow such a case, so in this case depending on various types of respect of the systems you may have 3 poles like this and hence you can have something like this where the 2 poles are going on these line.

And the third pole will be going in this particular direction. So this kind of systems will actually help in terms of the study state response of the system. This is known as our integral control. Now many times we can actually have a kind of us combination of all of them together, so we can have a PID control and I have already shown you 1 example where we have shown that how such a PID controller system would work where you have both Kp and times you have a Kds.

And let us say sometimes we add a constant need and we have a Ki, we have a Ki gain here and that gain supported by s itself. So we can get a complete you now proportional integral and derivative controller of the system. So that is all we can actually build up the classical control systems. So this is where we will cross this particular discussion and in the next dissuasion we will talk about modern control systems. Thank you.