Muffler Acoustics - Application to Automotive Exhaust Noise Control

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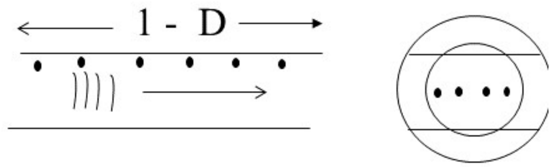
Lecture – 13
Lumped System Analysis: Inertance and Compliance

Welcome back to week 3of lecture3, in this week's lecture; in this lecture more precisely, we are going to study about **Lump System Analysis**.



So, what do you mean by lump system analysis? Let us consider this tube our famous our and our old friend the1D wave propagation system where you have waves that propagate in this direction and in this direction.

So, what I am trying to say by means of this figure is that you see here there are air molecules everywhere. So, we are basically there is an entire continuum that is what we are assuming, and these air molecules are oscillating about their mean positions if there is flow, they are also convicted. So, the point is that because of these continua of air molecules we are considering wave propagation in a system with almost infinite degrees of freedom, although it is a one-dimensional system it has infinite degrees of freedom.

So however, for low frequencies or if the length of the duct is very small, then it might be considered that the entire particles all the particles are oscillating with the same phase or they are behaving like one lumped unit that is to say they are no longer continua, but this entire suppose you have a duct like this. So, the entire mass in the duct is behaving like 1 1 integrated one common system all the air molecules can be lumped together the effect in one system.

So, such a special case arises under 2 circumstances, where we have the frequency f is very, very small that is f or f naught is verysmall. Low frequency approximation what we call or talking in terms of a more mathematically elegant manner or putting it in non-dimensional term we, $k_0 L \ll 1$; where wave number.

So, let me also as we go along let me also recall the units is hertz that is per second, the unit of k naught ismeter to the power minus 1 the unit of L, L need not be the length, but it could be any characteristic dimension. So, it could be the transverse dimension that is say cross sectional dimension of the length it is some measure of length it has unit of meter.

So, this entire $k_0 L$ it is non-dimensional that is non-dimensionalwave number that is what we call anon-dimensional wave number.Also known asa fewpeople have heard of it those who are acquainted with a little bit of acousticsHelmholtz number.

So, the information of frequency is really embedded within this Helmholtz number

$$k_0 L = \left(\frac{2\pi f}{C_0}\right) L$$

$$\left(\frac{\omega}{C_0} L\right)$$

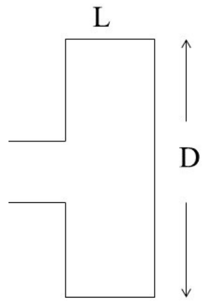
So, obviously when f is much smallyour k_0 Lor k_0 wave number is also small.

But you can also have situation in frequency is very small very low frequencies 10 hertz,20 hertz,50 hertz, less than 100 hertz in general, but length is such that it is large. So, the entire effect will be that the lumped approximation may not be valid; so, if the length or the characteristic dimension is very large, and you are yet in the low frequency domain.

So, the more fundamental definition is that the Helmholtz number k_0 Lshould be much,<<1. Similarly, you might also havethe other scenario where f_0 is need not be very small, but the length or the characteristic dimension is very short or very small.

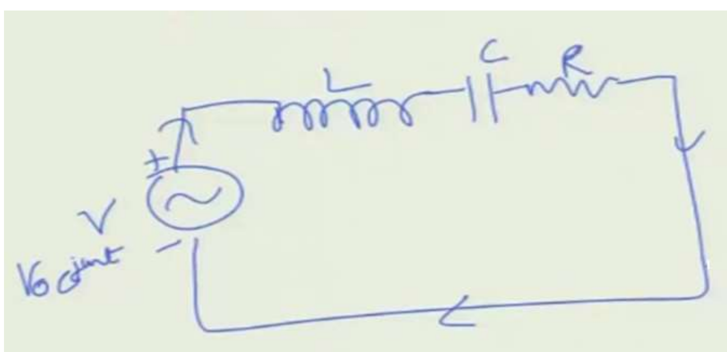
So, under that situation also the entire system behaves like a lumped system, like a very short cavity or something like the length can be considered this length is much much smaller is very small maybe you can actually have things like very flattened out duct something like this.

So, this is the length is the diameter. So, L /D<<1.



So, well the idea is when you have such a situation then how we go about doing the analysis of such a system. So, that is what we are going to study spend most of the time in this lecture.

So, the thing is we must also draw, let me introduce something like simultaneously introduce,ElectricalorAnalogies an Electrical Analogies or ElectroAcousticAnalogies. So, what do I mean by that? So just like you have voltage in electrical circuits like this is the voltage and this is saying your inductor inductance L or a capacitance C, a resistance R and this is driven by a voltage say $V_0\, e^{j\omega t}$, drives a current alternating current.



So, the question that I am actually asking is that can we draw some kind of a parallel with the circuit presented here in acoustics. So, what is the equivalent of voltage V? So, voltage V in electrical circuits, what is the equivalent in acoustics and what is the equivalent of mechanical systems?

So, a voltage is something like a force electromotive force that drives a system. So, naturally one would be inclined to think that in mechanical systems V is replaced bythe force say V_0, F_0. And what is your force?

Force is in acoustics in the context of fluids. So, that is nothing but your pressure into the area. So, in acoustics it is$\tilde{p}$. So, that is what it is, so these 3 things are analogous. Similarly, what is the other thing that we areinterested in. So, let me sort ofrub this partand let me get some space here.

So, similarly what goes on is the current I, that is the flow of charged particles. So, in electrical system that isthe current I and in mechanical systems it is the velocity of the body say V_0 or maybe you should use some different thing U same thing in acoustics.

So, here you have obviously forced an electrical thing this is acoustics, acoustic pressure. So, electrical I will say this ascurrent, andthis ismechanical.Similarly, acoustics we also have particle velocity or volume velocity or mass velocity the idea is that it should have some sort of a velocity dependence.

So, I would say this is a convenience I would say this is mass velocity, which is what we discussed as small v it is nothing but rho naught times the cross-section area times U that is what it is. So, there is one to one correspondence and we will also see as we go deep in the discussion, how is the inductance L related to the acoustic inductance.What is the corresponding element, typically you in electrical circuits you have a, what is known as a solenoid and a current passing through thatcreates a magnetic field inside it?

Similarly, you have capacitance which is formed by 2 electrical plates and there is a vacuum, or a dielectric medium placed within that. So, what is the equivalent in acoustics a resistance and there are no prizes for guessing resistance is exactly equal to damping or the absorptivity in the air if we can model that. If you do not have damping, you will have perpetual oscillations.

So, let us worry about the more important thing which we will consider in acoustics that inductance or capacitance, because we largely will assume that the medium itself is lossless.Unless you of course use a dissipative material in which the physics will change, so we are not going to discuss that now.

Let us talk about inductance L. So, basically what happens is that we are trying to develop some sort of a relation between this thing. So, impedanceis defined as force by velocity in mechanical things or voltage by

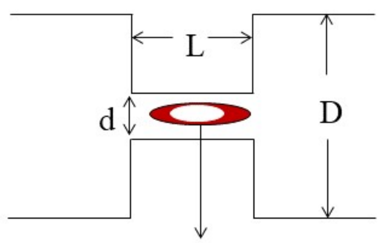
$$Z = \frac{V}{I} = \frac{F}{v} = \frac{p}{v}$$

So, what happens to the impedance in acoustics it is nothing but your pressure by mass velocity V.So, let,

$$Z = \frac{\tilde{p}}{\tilde{v}} = \frac{f_{mass}}{S} \ /\rho_0 SU \ = \frac{f_{mass}}{\rho_0 S^2 U}$$

$$L \ is \ small$$

So, if you have something a system such as abig cavity or a duct and you have very short thing short constriction, this you can consider this as construction or a small tube of a small length L whose diameter dis also much smaller than the big diameter between which the tube is sandwiched.



constriction

So, L is generally smallwe will be more precise what do you mean by smalland $\frac{d}{D} \ll I$, it is a short tubeconnecting 2 big cavities. So, what happens like we were discussing just a while back, the entire air molecules inside this duct can be considered as one lump sum or one particle. So, there are two ways that we will discuss how to go about doing the derivation;one is of course the simpler way a little bit mathematically more elegant ways to find out the mass.

The total pressure that is acting onthis lumped element, which is nothing, but fmass divided by the cross-sectionarea.

So, this then becomes what is the mass?

$$= \frac{M\dot{U}}{\rho_0 S^2 U}$$

And what is force mass times particle velocity, time derivative of that, that is mass,

$$U = f\omega U(x)$$

Newton's second law that is what applies here, mass is not getting changed you just have this thing we have this square and remember, we are assuming time harmonicity. So, one thing that we will do in muffler acoustics is basically always insist time harmonicity.

$$U\ (x,t) = U(x)e^{j\omega t}$$

$$\frac{dV}{dt}$$

So, when you differentiate U dot means with respect to time. So, once you do this thing you are going to get j omega times x, but x again it is not changing over this length. So, we will just get rid of this U=jωU.

So, the reason we assume harmonicity is because all you know IC engine noise that occurs at the first most of the noise occurs at the first firing frequency and multiples of that and all machinery, they have a certain frequency of operation like;reciprocating engines they operate at certain frequencies and machine tools and so many things.

So, for all engineering signals it is advisable that we work in the frequency domain and the standard practice to assume time harmonicity and look at different parts of the spectra that is a different frequency range we will analyze the frequency response.

$$= \frac{M\dot{\tilde{U}}}{\rho_0 S^2 \tilde{U}}$$

$$\tilde{U} = j\omega\tilde{U}$$

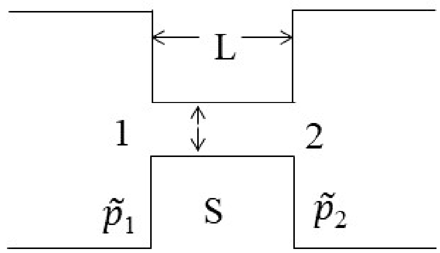
$$U\ (x,t) = U(x)e^{j\omega t}$$

So, basically once you substitute this here, so what does it become?

$$Z = \frac{M\ j\omega\tilde{U}}{\rho_0 S^2 \tilde{U}} = \frac{j\omega M}{\rho_0 S^2} = \frac{j\omega\rho_0 SL}{\rho_0 S^2} = j\omega\frac{L}{S}$$

So, that is what we are going to get. So, what does it mean then? So, if we do the math's further, we will figure out that.

And the mass within the element within the circled element where I have shown it with red color.



So, then the impedance of across 0.1 and 0.2 can be simply,

$$Z = j\omega\frac{L}{S} = j\omega L_{ind}$$

So, clearly this has the formjωL inductance, where so this is clearly the effect of sandwiching a small tube of a very short length L whose cross dimensions or diameter is much smaller than the two flanking cavities.

Where $L_{ind} = \frac{L}{S}$

Such a constriction acts like an inheritance at low frequencies it is idea is basic effect is to act like an inheritance and we will see why it is important in when we will discuss things like Helmholtz resonator. So, far we have seen quarter wave resonators, but we will soon see Helmholtz resonator, also you have to wait for a while, and then you know before actually we begin doing that this yet another way in which we can tackle the system.

So, we consider let us say the force is p_1 force is p_1 here and at this point it is p_2. So, the difference between the force isp_1 minus p_2, you can consider this as acoustic pressure. So, this is what it isand cross section area gives you the force that is acting on this element and using of course Newton's law it

$$(\tilde{p}_1 - \tilde{p}_2)S = \rho_0 SL\frac{du}{dt}$$

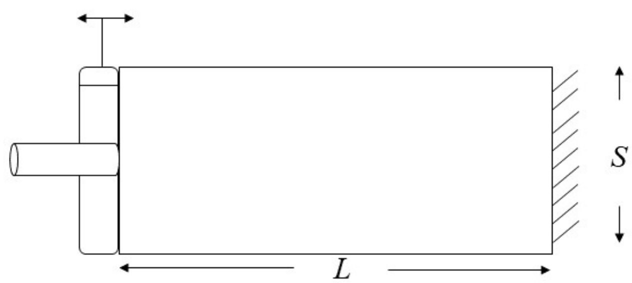
$$= \Delta pS = \rho_0 SL\, j\omega U$$

That will give you the mass and d u d tmass times acceleration.

$$= \frac{\Delta p}{SU\rho_0} = \frac{Lj\omega}{S}$$

$$= \frac{Dp}{\tilde{U}} = j\omega\frac{L}{S}$$

So, what we are going to do is that nowwe need to analyzethe Compliance C of aShort, Closedcavity.



So, let us consider a tube like thiswhose length is L and we mean precisely what we mean by short obviously intuitively we can say k_0 L <<1 cross section area is S. So, if you have a piston that is doingto and for motion here and here we will seethat your impedance that at this point.

So, this we saw from the last few lectures where we first talked about wave propagation effect, we talked about reflections from one end and so on. So, when we put the impedance condition at the other end. So, this is precisely what it is, p by v here if you measure p and you measure the acoustic velocity,

$$\frac{\tilde{p}}{\tilde{U}} = Z_{in} = -jY_0 \cot k_0L = -\frac{jY_0}{tank_0L} = -\frac{jY_0}{k_0L}$$

$$\tan k_0L = k_0L + \frac{1}{3}(k_0L)^3 + \cdots$$

So, this can further be written as$\simeq k_0 L \quad k_0 L \ll I$, that is your low frequency approximation. So, you eventually get this sort of a thing tan k naught L is approximately equal to k naught L. So, once we substitute this approximation here, we will get the denominator to bea simple looking monomial term let us do the simplification on the next page.

$$\frac{\tilde{p}}{\tilde{V}} = -\frac{jY_0}{k_0 L} = -\frac{jC_0}{S\frac{\omega}{C_0}L} = \frac{C_0^2}{jS\omega L}$$

Well, it turns out that we probably can do a bit better in terms of mass velocity.

$$=> \frac{\tilde{p}}{\rho_0 S\tilde{V}} = \frac{C_0^2}{jS\omega L} \qquad => \frac{\tilde{p}}{\tilde{V}} = \frac{\rho_0 C_0^2}{j\omega L}$$

$$=> \frac{\tilde{p}}{S\tilde{V}} = \frac{\rho_0 C_0^2}{j\omega LS} \qquad => \frac{\tilde{p}}{S\tilde{V}} = \frac{\rho_0 C_0^2}{j\omega V_c}$$

$$\frac{\tilde{p}}{\tilde{U}S} = \frac{\rho_0 C_0^2}{j\omega V_c} = \frac{1}{j\omega C} C_0^2 = \frac{\gamma p_0}{\rho_0}$$

So, capacitance

$$C = \frac{V_c}{\rho_0 C_0^2} \qquad => \quad \rho_0 C_0^2 = \gamma p_0$$

$$C = \frac{V_c}{\gamma p_0}$$

So, what does this mean is that if we have our ambient pressure p naught is your ambient pressure it can be 1 atmosphere and gamma is your adiabatic constant, V c is the volume of the cavity.

So, basically what it means is that pushing a piston into a long cylinder is much easier than pushing into a shorter one, because clearly if the volume Vc is less that means, you have a shorter cylinder. So, basically it offers much more impedance compared to a cylinder that is much longer or has a lower volume.

So, in other words capacitance or C, C part this thing this is proportional to the volume. So, the impedance that you see Z is equal to 1 by j omega C that is inversely proportional to the capacitance. So, naturally if the volume goes up itis such a cavity would offer less impedance to the flow of current or to the basically to the acoustic velocity and a much shorter cavity will impose a much greater impedance.

So, typically when we have such a system, so if we although we will do the Helmholtz resonator in a greater detail as we go. So, it is like a mushroom kind of a thing that I am making, so here we have something like a irregular mass and here and to the neck of length L is attached, whose cross section area is S and this can be really or any arbitrary mass we all we care about is the volume, what is the volume?

Vc it should be much larger than the volume of this neck and at low frequencies of course, it has a lumped analysis as we saw now. So, suppose if the wave is coming at this point here and it is going to see some impedance at this point and some part will be going in the downward or the downstream direction.

So, the question that we are asking is that what is a net impedance seen. So, this clearly this j omega L and 1 divided by j omega C that will act in series. So, the net impedance is as simple effective impedance is as simple as adding up the two individual impedances. So, you will get something like this.

So, in a sense so suppose your length,

$$Z_y = j\omega L_{ind} + \frac{1}{j\omega C}$$

$$Z_y = j\omega\left(\frac{L}{S}\right) + \frac{\gamma\rho_0}{j\omega V_c}$$

So obviously there needs to be things like end correction effects and all those kinds of things and we can simplify this thing further to find out the precise frequencies, where the impedance will tend to 0. All those data and analysis even a better approximation of the Helmholtz resonator or our short cavity that we will consider in the next lecture, where we will first discuss about the end correction effects and how does it happen.

And then probably develop an even better expression for a short end cavity by considering a higher order term and relating both things together. And then probably going into more complicated elements as we and discuss more about Electro Acoustic Circuits. So, that is all for this lecture.

Thanks a lot.