

Fundamentals of Industrial Oil Hydraulics and Pneumatics

By Professor R. Maiti

Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Module 5

Lecture 19

Analysis of an Axial Piston Swash Plate type Hydrostatic Pump (Discharge Flow Characteristics)



(Refer Slide Time: 0:24)

MODULE- 5

HYDROSTATIC UNITS (PUMPS AND MOTORS)

LECTURE- 19

Analysis of an Axial-Piston Swash-Plate type Hydrostatic Pump
(Discharge Flow Characteristics)



Welcome to today's lecture on analysis of an Axial Piston Swash Plate type Hydrostatic Pump (Discharge Flow Characteristics).

(Refer Slide Time: 0:32)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :

Description / Feature of the Pump :

Figure -1 shows a typical 'Axial Inline Linear Piston', also simply called as 'Linear Piston' swash-plate type hydrostatic pumps.

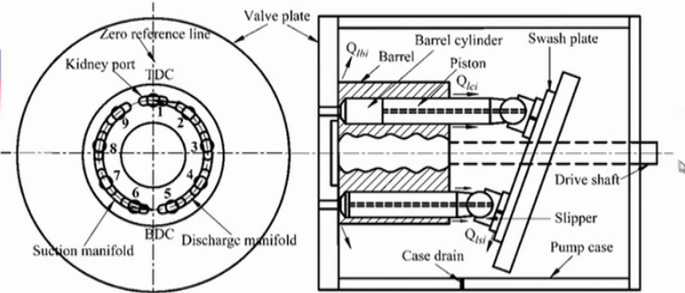




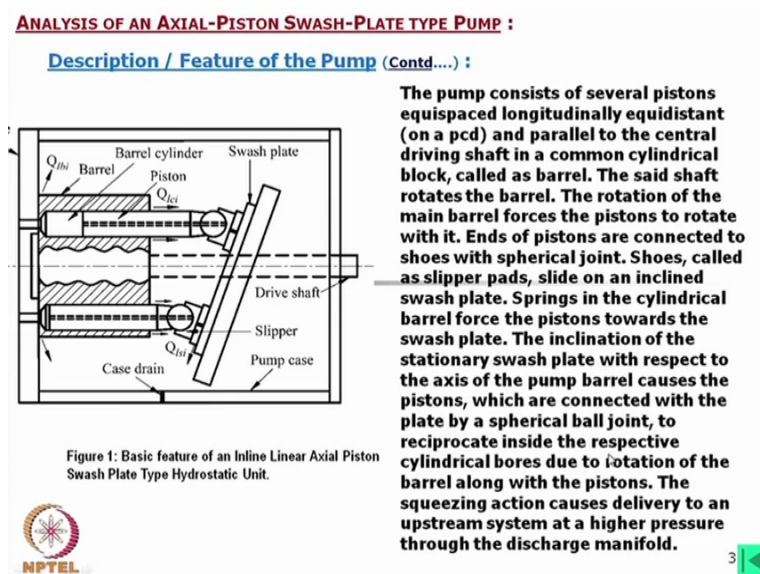
Figure 1: Basic feature of an Inline Linear Axial Piston Swash Plate Type Hydrostatic Unit.



Now I would like to mention that I have written here hydrostatic pumps I have mentioned hydrostatic pumps but basically we call it rotary hydrostatic units which includes pumps and motors, we shall describe a pump, we will analyse a pump here. However, the analysis for the motor will be more or less same considering their flow pressure, etc. Now here in figure 1, what I have shown a typical axial inline linear piston which is simply called as linear piston swash plate type hydrostatic pumps.

Now why it is axial inline linear pistons? The first of axial means you will find that the pistons are parallel to the axis of the shaft, inline means all pistons you can put if you develop that will be inline which is acting one after another. Linear means in that way it is we can call the motion is linear and the piston is for the pistons. Now swash plate means this one is the swash plate which I will describe in the next slides also, so that is why it is called swash plate type pump.

(Refer Slide Time: 2:16)



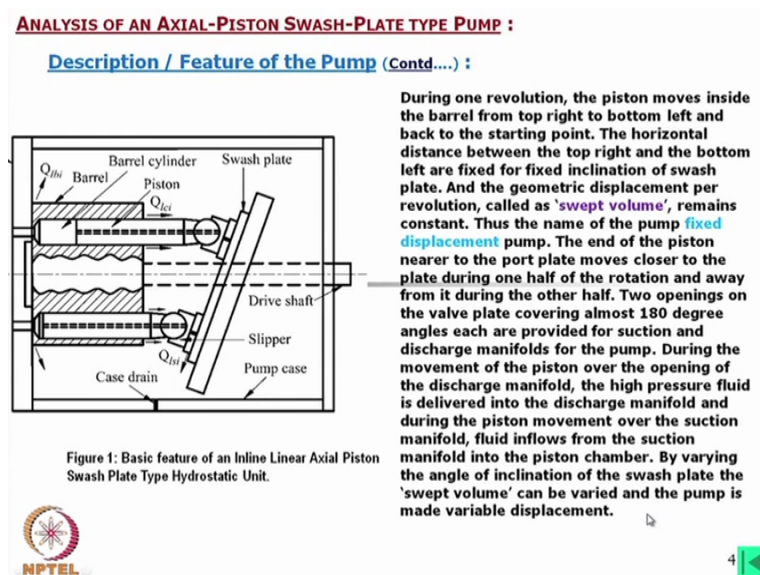
Now this pump such pumps it is widely used it consists of several pistons equispaced longitudinally equidistant and parallel to the central driving shaft in a common cylindrical block called as barrel. Now this one is the barrel, this is the drive shaft and these are the pistons, these pistons are led on a pitch circle diameter and angle between two pistons are constant also, okay.

Now this is the swash plate which is fixed positionally fixed means it is not rotating it can be only the tilting angle can be changed or may be also fixed for fixed displacement type. The rotational main barrel forces the piston to rotate with it. Ends of the pistons are connected to

shoe called the slipper pads and these shoes slides on this inclined plate which is swash plate. There is a spring inside the cylinder which forces this barrel to touch the valve plate in the other side.

Now while it is rotating the barrel will rotating the piston will reciprocate along the axial directions. However, we need another returner which holds all the slipper pads together so that it is not detached because there is no physical connections between the swash plate and the slipper pad, so while it is moving in this directions there is no pulling only by a spring arrangement this is kept contact with the valve plate as well as the swash plate. The squeezing action causes delivery to an upstream system at higher pressure through the discharge manifold. In case of the motor the phenomena is vice versa.

(Refer Slide Time: 4:56)



Now during one revolution, the piston moves inside the barrel from top right to bottom left, top right means this is as it is in the right side we are saying top right and to the left. The horizontal distance between the top right and the bottom left are fixed and fixed inclination of swash plate this means if we increase this inclination this length will increase that movement will increase which is called stroke length, the stroke length will increase.

The geometric displacement per revolution called the swept volume already defined that remains constant for a constant tilting angle of the swash plate. Thus the name of the pump fixed displacement when this remain fixed we call it fixed displacement. The end of the piston nearer to the port plate moves closer to the plate during one half of the rotation and

away from it during the other half, you can understand that for one half it is in the mode of compression, in other half it is in the mode of suction.

Two opening on the valve plate covering almost 180 degree angles each are provided for suction and discharge manifolds for the pump the details plate will be shown later. During the movement of the piston over the opening of discharge manifold the high pressure fluid is delivered into the discharge manifold and during the piston movement over the suction manifold fluid inflows from the suction manifold into the piston chamber. By varying the angle of inclination of swash plate the swept volume can be varied and then the pump is called variable displacement pump.

(Refer Slide Time: 7:02)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :

Analysis of the Idealized Pump Flow :

The flow rate Q_i generated by i^{th} piston is expressed as:



$$Q_i = -\frac{dV_i}{dt} \quad \dots (5.19-1)$$

Where, $\frac{dV_i}{dt}$ is the time (t) derivative of instantaneous volume (V_i) displacement of i^{th} piston .

Fig. 2: Schematic view showing geometry.

The maximum linear movement in one direction of the piston is called stroke (S_p) of the piston.

$$S_p = 2 \times \frac{d_p}{2} \times \tan(\alpha) = d_p \tan \alpha \quad \dots (5.19-2)$$

Now in this figure if we see that we have put we have shown this in this way and if we consider this plane then the plane behind that will be for the direction if we consider the direction of rotation, direction of rotation is the clockwise in that case. So this will be from this side to this side will be the compression phase and other side will be the suction phase and the manifold valve manifold we are talking about this is one and this is another, one is for suction and another is for discharge.

So in case of pump this will be for discharge because this is in the compression mode and this in the suction mode so this will be the suction manifold. Now to find out the flow rate for a piston we put Q_i , i is for the i^{th} pistons it is simply the time derivation of the volume here the negative sign has come due to the axis consider we will come later in that but the quantity is

that the volume which is interrupt the rate of change of volume will be the flow to a single chamber.

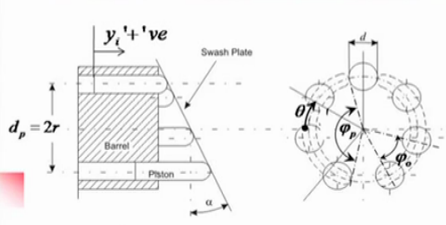
Now the maximum linear movement in one direction of the piston is called stroke, the stroke S_p can be given by if we consider the pcd on which the pistons are led is d_p which is equal to twice r then we can consider so this angle is α the angle of inclination of the swash plate is α . So d_p by 2 that is r by 2 r into $\tan \alpha$ is the stroke at one side and the other side also another d_p by 2 into $\tan \alpha$, so we can simply write 2 into d_p by 2 into $\tan \alpha$ is equal to $d_p \tan \alpha$.

That is the if we consider from this point to this point one piston will move in linear direction by this amount and it is called stroke. And this definitely will be fixed for a fixed swash plate angle that means $\tan \alpha$ itself is fixed. So when we are analysing even if for the variable displacement pump because this is our analysis normally it will be when at a particular angle it is fixed at a particular angle then we are analysing this means that for the variable piston pumps for a fixed swash plate angle we will consider the swept volume will be this amount for the angle where we have fixed α into the number of piston.

(Refer Slide Time: 10:36)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :

Analysis of the Idealized Pump Flow (Contd....) :



The maximum volume (V_{max}) the fluid can occupy in the piston chamber is the sum of clearance volume (V_c) and the product of the stroke (S_p) of the piston and the area (A_p) of the piston.



$$V_{\text{max}} = V_c + S_p A_p \quad \dots (5.19-3)$$

Fig. 2: Schematic view showing geometry.

If the displacement of the i^{th} piston inside the chamber is y_i , then from simple similar triangle considerations, we get:

$$y_i = r \tan(\alpha)(1 - \cos(\theta_i)) \quad \dots (5.19-4)$$

Where, θ_i is the angle covered by the i^{th} piston from the initial position.

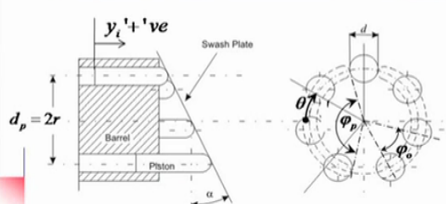
The maximum volume V_{max} the fluid can occupy in the piston chamber is the sum of clearance volume we will consider now there is a clearance volume and the product of stroke of the pistons and the area of the piston this means if A_p is considered as the area of this piston this diameter is d that means $\frac{\pi d^2}{4}$ is equal to A_p so A_p into stroke length

is the piston volume the fluid it is handling plus there will be some clearance volume which we give as a name it as V_c .

Now this is fixed for a construction this is fixed but this volume is needed when we analyse the fluid, okay. So $V_{i\max}$ is equal to V_c plus S_p into A_p remember this is for one piston. In the displacement of i th piston inside the chamber is y_i then from simple similar triangle considerations we get y_i is equal to $r \tan \alpha (1 - \cos \theta_1)$, now I think this θ_1 the angle covered by the i th piston from the initial position, okay sorry.

(Refer Slide Time: 12:15)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of the Idealized Pump Flow (Contd....) :



The volume of the piston chamber at any instant is given by:

$$V_i = V_c + A_p(S_p - y_i) \quad \dots (5.19-5)$$

Therefore,

$$\frac{dV_i}{dt} = -A_p \frac{dy_i}{dt} \quad \dots (5.19-6)$$



Fig. 2: Schematic view showing geometry.

or, $Q_i = \frac{-dV_i}{dt} = A_p \frac{dy_i}{dt} = A_p r \omega \sin(\theta_i) \tan(\alpha) \quad \dots (5.19-7)$

$$\theta_i = \theta_1 + (i-1)\phi_o \quad \dots (5.19-8)$$

Where, ϕ angle between two consecutive pistons in their laying plane (see Figure).

It is given by: $\phi_o = 360^\circ / Z \quad \dots (5.19-9)$

This is θ_1 must define that we are considering that this axis system we will show a little later, we consider that this θ_1 beginning from this point, okay this axis okay that means this we have looking into in this direction and we have drawn here that means if I look from the left side towards this barrel then left hand side this axis which is perhaps we have given y axis we will the θ_1 is beginning from that point.

So if I consider the i th piston then we have to consider the θ_1 and plus the total angle between the this pistons, suppose this is i th one so we will consider this angle is that this angle into the number of pistons in between say Z like that somewhere I have defined this we will come to that. Now the volume of piston chamber at any instant is V_i , so V_c plus $A_p S_p$ minus y_i , y_i now we have defined as a piston movement, so S_p if it is the total stroke that minus this y_i movement is the volume at that instant.

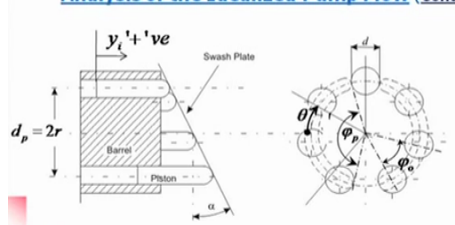
Therefore dV_i/dt is equal to $-A_p dy_i/dt$, now I think this minus sign has come over here due to this derivations that is why we put minus sign here so that ultimately it will again become

positive. So Q we can again define like this ya this is right this minus sign as it is coming over here so we originally load these equations and which is coming ultimately the positive signs.

So therefore this Q 1 for a particular pistons can be defined like this. Now here I have explained the θ I is equal to θ 1 for the one piston remember this is suppose this is the piston one this was here when θ is equal to 0 and then for i th positions it was the i minus 1 to the angle between two consecutive pistons and θ 1 is the rotation of the piston one, okay. Where, this here it will be subscript 0 will be there so this is the angle between two pistons. It is given by 360 degree divided by Z is the simple where Z is the number of pistons.

(Refer Slide Time: 15:52)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of the Idealized Pump Flow (Contd....) :



The total flow rate into the suction (for y_i^{+ve}) or discharge (for y_i^{-ve}) manifolds are given by:



$$Q = A_p r \tan(\alpha) \omega \sum_{i=1}^Z Q_i \quad \dots (5.19-10)$$

Fig. 2: Schematic view showing geometry.

Sign of y_i automatically comes from equation (5.19-4).

However, it is to be noted that for odd number of chambers, at any instant y_i^{+ve} either for $(Z+1)/2$ number of chambers or $(Z-1)/2$ number of chambers (including $y_i = +0$ or -0 , i.e. pistons at their dead centers).

For even number of chambers, at any instant the numbers are always $Z/2$.

Now the total flow rate into the suction thus y_i is positive or discharge y_i is negative manifolds are given by the same equations will be same because this we are only this defining the quantity but we have to consider for y positive and y negative separately because if we integrate together ultimately it will become 0 because one is in positive and other is in negative so that consideration we have to make but if we do it then this will be the expression.

So while we are considering Q_i then we will consider for the pistons which are in the suction zone or which are in the compression zone ideally this should be equal but if we consider the leakages then definitely these two will be different. Sign of y_i automatically comes from equations that is okay but to quantify that how much it is pumping we have to take care of this.

However, it is to be noted that the odd number of chambers now here is the point that at an instant y_i is positive for Z plus 1 by 2 number of chambers or Z minus 1 by 2 number of chambers including y_i is equal to plus 0 and minus 0, what does it mean at an instance suppose one piston in the dead zone dead zone means which is in between these two manifold still we may write that Z plus 1 number of piston in suction and Z minus 1 number in suction mode suction and compression or it will be next movement it will be just opposite that we have to take care of that, that is again automatically we will come from the equations because the negative sign will appear for the negative direction motions.

But suppose if there are 7 pistons then what may happen the 4 pistons either in the suction side and 3 pistons in compression side or vice versa momentarily one piston is at the dead zone, 3 pistons in the suction and 3 pistons in the delivery. However, we need not take care of specially suppose we are working with 4 pistons in the suction zone the 4th piston when it will come in the dead zone automatically there y_1 will appear 0.

The for the even number of chambers it is just Z by 2, either the 3 pistons say for example 6 pistons are there 3 pistons in suction and 3 pistons in compression. Now what will happen for the dead zone when the pistons are dead zone you will find 2 pistons are in dead zone, so at that time 2 pistons are suction and 2 pistons in compression but again that 2 for that plus y_i is equal to 0 and minus y_i is equal to 0. Only you have to just take a little care while we are calculating this.

(Refer Slide Time: 19:48)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of the Idealized Pump Flow (Contd....) :

Fig. 2: Schematic view showing geometry.

Using the integral average of the above equation, the nominal discharge flow of the pump may be expressed as:

$$\bar{Q} = \frac{Z A_p r \tan(\alpha) \omega}{2\pi} \int_0^\pi \sin(\theta) d\theta = \frac{Z A_p r \tan(\alpha) \omega}{\pi} \quad \dots (5.19-11)$$

Dividing the general flow equation with the average flow equation derived above we get the normalized flow rate, which is given by:

$$\hat{Q} = \frac{Q}{\bar{Q}} = \frac{\pi}{Z} \sum_{i=1}^Z \sin(\theta_i) \quad \dots (5.19-12)$$

NPTEL 9

Now using the integral average of the above equation the nominal discharge flow of the pump may be expressed as we are using the summation signs instead of that if we integrate then what we find this is a constant part because we are considering on a fixed alpha and omega that is the speed we will consider that is also constant that is not varying and Z is the number of piston so this we take outside, only the integration parts inside remains sin theta 0 to Pi d theta and which gives this will be the average flow, okay we have integrated from half of the angle so 0 to Pi, so this will give us the nominal average flow.

During the general flow equation dividing the general flow equations with average flow which we have estimated like this the equation derived above we get the normalized flow rate which is given by this so this is the normalized flow rate that means if we plot this one this automatically it will give us the ripple.

(Refer Slide Time: 21:25)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of the Idealized Pump Flow (Contd....) :

Fig. 2: Schematic view showing geometry.

Now, let Z' be the number of pistons in same phase at an instant:

i.e., for odd Z :
$$\frac{Z+1}{2} \dots (5.19-13)$$

and for even Z :
$$Z' = \frac{Z}{2} \dots (5.19-14)$$

NPTEL 10

Now we consider Z' be the number of pistons in same phase at an instant that means Z' is equal to null, Z plus 1 by 2 or Z minus 1 by 2 in case of odd number, in case of even number this is simply Z by 2. Here I have expressed, okay so for even, odd number and even number we expressed in this way.

(Refer Slide Time: 21:56)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of the Idealized Pump Flow (Contd....) :

It can be shown that,

$$\sum_{i=1}^{Z'} \sin(\theta_n) = \sin\left(\frac{\pi Z'}{Z}\right) \operatorname{cosec}\left(\frac{\pi}{Z}\right) \sin\left(\theta_1 + \pi \frac{Z'-1}{Z}\right) \quad \dots (5.19-15)$$

From simple trigonometric considerations, this reduces for even and odd cases as follows:

For odd number of pistons:


$$\hat{Q}_o = \frac{\pi}{Z} \operatorname{cosec}\left(\frac{\pi}{2Z}\right) \cos\left(\theta_1 - \frac{\pi}{2Z}\right) \quad \dots (5.19-16)$$

Where, $0 \leq \theta_1 \leq \frac{\pi}{N}$

For even number of pistons:

$$\hat{Q}_e = \frac{\pi}{Z} \operatorname{cosec}\left(\frac{\pi}{Z}\right) \cos\left(\theta_1 - \frac{\pi}{Z}\right) \quad \dots (5.19-17)$$

Where, $0 \leq \theta_1 \leq 2\frac{\pi}{N}$



Now we can show that this expression because now we are considering for one side only, so we will take here Z dash and for that this equation can be expanded in this form this is the matter of mathematics so you can expand in this form and then for simple trigonometric consideration this reduces for even and odd cases as follows. For odd number of pistons the Q 0 will be expressed by this Pi by Z cosec Pi by 2Z cos theta 1 minus Pi by 2Z, for theta is equal to theta 1 Pi by N sorry this will be again Z not N this is Z, Z is the number of pistons. Here also this is expressed for even number of becomes like this.

(Refer Slide Time: 23:08)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of the Idealized Pump Flow (Contd....) :

For odd number of pistons, the normalized amplitudes are given by:

$$\lambda_{Qo} = \frac{\pi}{2Z_o} \tan\left(\frac{\pi}{4Z_o}\right) \quad \dots (5.19-18)$$


and the period of pulse is:

$$t_{Qo} = \frac{\pi}{Z_o} \quad \dots (5.19-19)$$

Similarly for even number of pistons, the normalized amplitudes are given by:

$$\lambda_{Qe} = \frac{\pi}{Z_e} \tan\left(\frac{\pi}{2Z_e}\right) \quad \dots (5.19-20)$$

and the period of pulse is:

$$t_e = \frac{2\pi}{Z_e} \quad \dots (5.19-21)$$


Now this two expression here the amplitude similar way we can define the amplitude in this way and the period of pulse is Pi by Z 0 this is ofcourse : for the odd number of pistons. For

even number of pistons you can see the differences we have now used subscript e and o for odd and even, okay so you can find this amplitude and the period, here period is higher and amplitude is also greater than this as we have already reviled from our phasor analysis and this earlier two equations that is that if you I would not say the simplified but I would say that in phasor analysis we have used a simplified formula, here this formula is coming exactly fitting to this linear piston pump but you will find this value will be very close.

(Refer Slide Time: 24:22)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of Actual Pump Flow :

In the analysis of the actual pump flow, various leakages and also the compressibility of the fluid is considered.

The flow rate from the orifice equation is given by:

$$Q_i = [\text{sign}(P_i - P_d)] C_{di} A_{oi} \sqrt{\frac{2}{\rho} |P_i - P_d|} \quad \dots (5.19-22)$$

Where, P_i is the instantaneous fluid pressure within the i^{th} Chamber.



P_d is the discharge pressure of the pump,

A_{oi} is the instantaneous lowest cross-sectional area through which oil is being discharged from i^{th} Chamber,

C_{di} is the discharge coefficient correspond to A_{oi} ,

and ρ is the fluid density.

The cross sectional area A_{oi} is to be calculated knowing the instantaneous port geometry as shown in figures -3 , 4 and 5 shown in next slides. Calculation of such areas also follows.

Now we will consider an actual pump which we would like to analyse for the ripple. Now what we should consider when we will go for actual analysis of a pump our main purpose definitely we should find out the flow there and ripples pressure pressure ripple all such parameters related to the performance of the pump. In that case the main important thing we must consider the leakage also.

Now the flow rate from the orifice equations that is already known we can write this equation considering this this sign depending on this sign this will be this will indicate the which direction it is flowing then this is the coefficient of discharge and this is the area general orifice area of i^{th} piston that orifice area we need to calculate, okay the C_d coefficient of discharge that also we should verify for a depending on the size of the orifice, shape and size of the orifice.

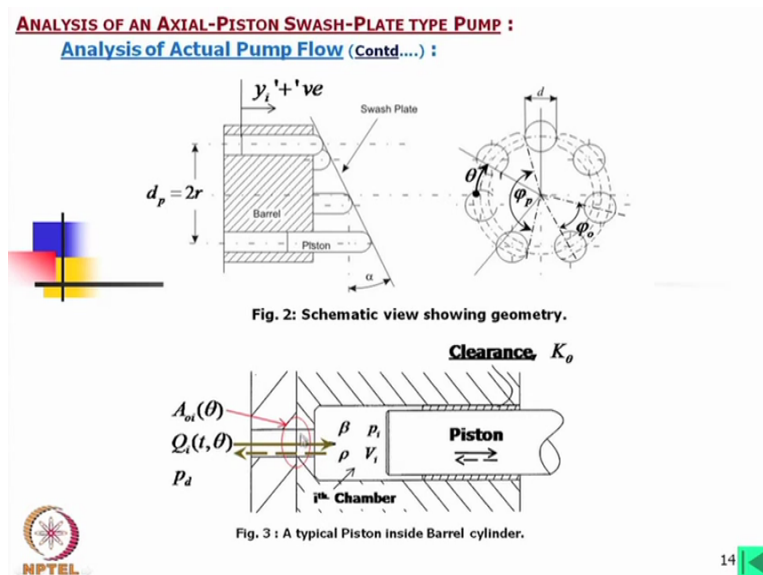
Now this is usually there is no way you have to go for the experiment but what is done normally depending on the orifice geometries irrespective of their physical sizes, depending on the shape at different opening of such orifices the C_d will be fixed. Say for example in

one case the size of this total maximum area through this orifice is say 1 centimetre square, in other case it is 5 centimetre square but their shape are same, so when that particular orifice opening for whether it is 1 maximum 1 centimetre or 5 centimetre coefficient of discharge is same when in one case say half centimetre square, in other case 2.5 centimetre square then is also C_d is same. And also it is found that this C_d remains more or less constant from the minimum opening to maximum opening, so for a particular type of orifice these are fixed.

Now here the P_i is the instantaneous fluid pressure within the chamber, P_d is the discharge pressure, A_{oi} already I have described is the cross-sectional area through which the discharge is taking place we will explain further C_d I have already explained and ρ is the fluid density. Sometime this ρ we have defined as ρ_o for oil, the cross-sectional area A_{oi} is to be calculated knowing the instantaneous port geometry as shown in the figure 3, 4, 5 we have we will come next to that in the next slides and how to calculate the area.

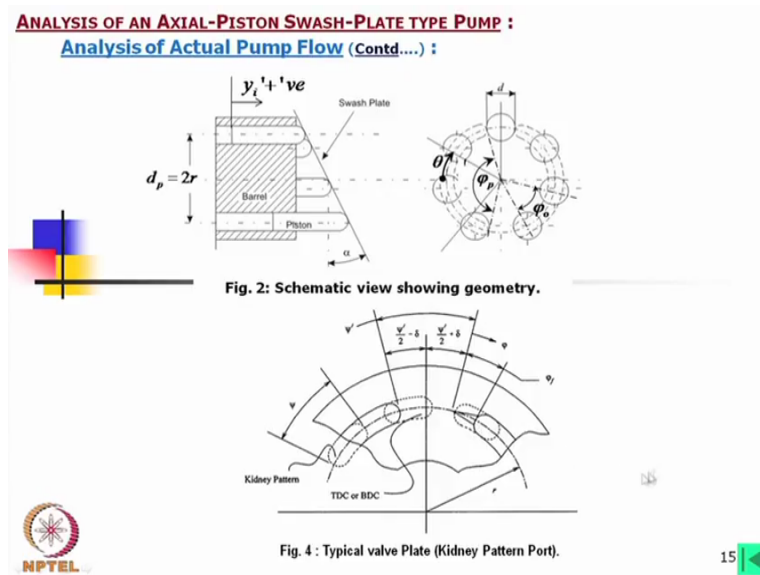
Now as I told that C_d we can judge from the shape but every instant we have to calculate this orifice area. How it is calculated?

(Refer Slide Time: 28:18)



Now this figure only shows that different positions and this is the pistons and we are we can say this is the nomenclature here the A_{oi} apparently here we are looking that this is a constant but this is not depending on the position this is varying and we have to calculate this area.

(Refer Slide Time: 28:42)



Now how it is varying, here this is the port we have shown the manifold this is actually continuous, on the barrel we could leave this hole as a bore equal to the diameter of the piston that means this could be made through hole but you will find normally this hole the cylindrical hole upto almost upto the end but end is not the cylindrical hole instead of that there will be a small kidney port, this is kidney pattern it is written kidney pattern usually it will be of this shape.

Now interestingly say this is included angle (θ)(29:38) this angle is actually you may consider ideally it should be equal that means this is the dead bend zone from this point to this point is the dead bend zone this is almost equal ideally it should be equal but you may find that it is at one side it is less, other side it is slightly more this is due to the fact that depending on the direction of rotations in case of pump one if this side is taken less and this side is taken slightly more the performance will be better than ideally if it is taken exactly equal, even this total angle sometimes taken a slightly larger than this angle, okay.

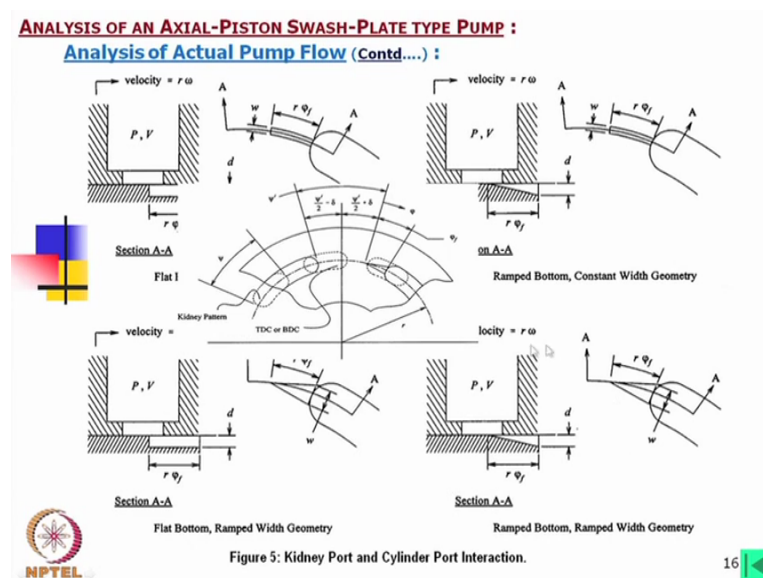
But this needs a dynamic analysis of this pump from the static analysis we will not be able to understand why this angle should be taken more if you go for the dynamic analysis if we look into the pressure build up for of the oil interrupt inside the piston during this dead zone we will be able to understand.

However, while we are going for the static analysis still for the opening of the orifice we have to consider these angles and here what we find in this port say this is the kidney port and here we will find a groove this groove later I will show the shape of this groove like a pyramid,

this groove is called silencing groove if this groove is not provided then we will find when this kidney port of the barrel coming contact with this valve manifold the rate of increase of the orifice is very high and the pressure transient in the fluid will be erratic, it is unpredictable.

Whereas if you provide this silencing groove this pulses in the pressure will be controllable as well as there will be less noise. So that is why name of this groove is the silencing groove, it is actually controlling the pressure pulsation which reduces the noise, okay. So we need to calculate depending on the shape what are the areas.

(Refer Slide Time: 32:32)



Now here in details it is again shown that what might be the groove, in in this case say here the section A-A it is a flat type groove that means if you take a say this depth is remain same this depth is remain same but this is also we can say rectangular type that means you can put a cutter and then you can (33:07) and you can make this slot like this but here the groove is called ramped bottom, this is an constant width geometry this means that groove is like this but there is a slope this is a ramped, this is a flat bottom constant width, whereas here it is a ramped bottom constant width.


In this case this is a flat bottom ramped width geometry that means this is flat but this is gradually increasing, whereas here it is ramped bottom ramped width geometry that means this is also inclining and here is also if we consider the surface area that is increasing but I would say this is the most popular one which are used for this pump, this is the best one we should say and also if you look into this manufacturing of this groove will be more expensive

than this because we have to make this depth which is stepper you can just imagine of machining, this is easy this also may not be difficult we can incline the cutter but this is very difficult but perhaps this might be the better one for the silencing point of view, whereas widely used one is this one, we will analyse this one in the next slides.

(Refer Slide Time: 34:58)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of Actual Pump Flow (Contd....) :


The total discharge flow of the pump is equal to the net flow generated from each piston chamber instantaneously positioned over the discharge port. Dividing this sum total by the nominal flow (equation 5.19-21), we get the normalized flow as:


$$Q = \frac{\pi}{ZA_p \frac{d_p}{2} \tan(\alpha) \omega} \sum_{i=1}^{Z'} [\text{sign}(p_i - p_d)] C_{di} A_{oi} \sqrt{\frac{2}{\rho} |p_i - p_d|} \quad \dots (5.19-23)$$


The pressure in each piston at an instant is estimated from the consideration of bulk modulus as follows:

$$\frac{dp_i}{dt} = \frac{-\beta}{V_i} \left(Q_i + Q_{leak_i} + \frac{dV_i}{dt} \right) \quad \dots (5.19-24)$$

Q_{leak_i} is the leakage that occurs due to the clearances between the piston and bore and/or any other leak paths that exist in the design of the piston chamber.



17 

The total discharge flow of the pump is equal to the net flow generated from each piston chamber instantaneously position to over the discharge port dividing this sum total by the nominal flow we get the normalized flow as in this case we have consider the elaborate equations and this is now coming in this form, earlier I have shown the simplified one where here we have consider the area, coefficient of discharge, sign of the pressure, etc but it is more or less similar we have just replaced the flow equations with this detail equation.

Now the pressure in each piston at an instant is estimated from the consideration of bulk modulus as follows. Now we are considering inside that total volume inside and then we are considering also bulk modulus, why we have consider such things because with the increase and decrease in the pressure this there will be effect on the fluid compressibility which will ultimately control the pressure which will control the pressure.

So to find out the pressure fluctuation we need to consider the compressibility of the fluid inside the pistons. Now Q_{leak_i} is the leakage that occurs due to the clearance between the piston and bore and or any other leak paths that exists in the design of the piston chambers. Normally while we are calculating this pressure development we need to consider

the leakage past the pistons and the bore that is most important. So that leakage we have to consider within this equation sorry.

(Refer Slide Time: 37:18)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of Actual Pump Flow (Contd....) :
 Considering the leakage flow is laminar, it is expressed as:

$$Q_{leak_i} = K_{i\theta} p_i = K_\theta p_i \quad \dots (5.19-25)$$

$K_{i\theta}$ is the leakage coefficient which may vary depending on the capillary leakage passage length which is dependent on θ and the clearances in i^{th} Chamber. (See Figure- 2). It is to be found experimentally.

However, considering more or less same clearances in all piston-cylinder sets we may assign K_θ instead of $K_{i\theta}$.

If the stroke of piston is short then K_θ may be taken constant for a pump (hydrostatic unit).

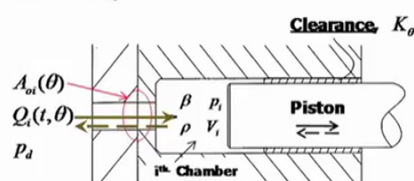


Figure 2 : A typical Piston inside Barrel cylinder.

NPTTEL

18

But the problem is that say here I have shown particularly this leakage is important also there might have some leakage through this point also but we need to consider these leakage characteristics which is two factors we should consider one is K_θ and another is the clearance. Now the clearance is from the geometric dimensions measuring those dimensions considering the tolerances we can easily estimate what will be the clearances there but K_θ is the coefficient which we leakage coefficient we need to estimate. You can see this is there that Q leakage is defined by the instantaneous pressure in i th chamber multiplied by this constant.

Now here the detail given $K_{i\theta}$ is the leakage coefficient which may vary depending on the capillary leakage passage length which is dependent on θ , θ is the angle that means this length is varying this capillary passage length is varying so that is important depending on that this coefficient will also vary, so depending on θ this will vary and the clearance in i th chamber. However, this $K_{i\theta}$ is found mostly experimentally.

Now this clearance normally for a pump we may consider all chamber is having chamber and piston is having equal clearance but actually it is also varying so depending on that I would say for each and every chamber the $K_{i\theta}$ will be different for different θ angles I mean for the same θ angles when it is coming at the same say suppose if we know this

length is equal for pistons p_i is also equal for pistons but depending on this clearance this will vary however this we may take almost constant.

However, considering more or less same clearances in all piston cylinder we can assign K_{θ} is equal to K_i θ is equal to K_{θ} θ so which I have used simply will K_{θ} . Now this K_{θ} again will be will vary on the stroke length total stroke length but if the stroke length is not very high say small stroke length then you will find that there is a little variation in K_{θ} that means for a particular type of pump we may consider K_{θ} is constant and there will be ofcourse depending on the length this K_{θ} will vary but we may consider for a particular pump the K_{θ} is a constant.

Again such pump is one pump is tested and K_{θ} is found out experimentally we may consider all other pumps we can scale up even if we can make a chart from which say 7 pistons pump for a 1000 RPM, one is giving 5 litre per minute and another is giving 25 litre per minute probably there K_{θ} will be same.



(Refer Slide Time: 40:52)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of Actual Pump Flow (Contd....) :

The instantaneous rate of change of pressure in the i^{th} . piston-chamber is given by:

$$\frac{dp_i}{d\theta_i} = -\beta_b \frac{[\text{sign}(p_i - p_d)] C_{di} A_{oi} \sqrt{\frac{2}{\rho} (|p_i - p_d| + K_o p_i) - \left[A_p \frac{d_p}{2} \tan(\alpha) \sin(\theta_n) \right] \omega}}{\left[V_o + A_p \frac{d_p}{2} \tan(\alpha) \cos(\theta_n) \right] \omega}$$

... (5.19-26)

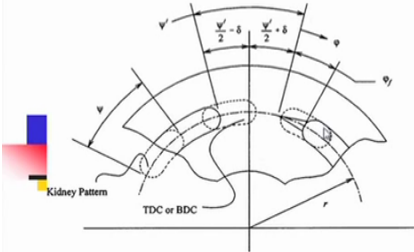



The instantaneous rate of change of pressure in the i^{th} piston chamber is given by this is simply that I am not showing the derivation of the equations but if you derive dp_i by $d\theta$ if you do it and consider all the equations appropriate equations will easily arrive into this expression. Now this is difficult to remember suppose if I ask a question of that definitely I will give you this formula do not worry about that but you should understand and here what we have done inside that as it is a under root we have to take care so this should not become minus so we have taken the mod of that, okay and β is the bulk modulus of the oil we are

using this again vary with temperature and pressure ofcourse but for a pump performance within a range may be temperature range may be ambient temperature 45 degree centigrade to 70 degree centigrade and pressure may be varying from 0 to say 15, 20 Mega Pascals but we will consider only one bulk modulus for such oil.

(Refer Slide Time: 42:15)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Analysis of Actual Pump Flow (Contd....) :
Determination of Port area A_{oi} :



Referring to the Fig. 4, the port area remains at a maximum constant when it is completely over the discharge or suction manifolds. In the transition region, the port area gradually increases to a maximum from zero when the piston is entering into manifold zone from the dead zone (i.e., from TDC). In case of pump the oil is being exposed to high (working) pressure & in case of motor the oil is being exposed to low (discharge) pressure. Similarly the port area gradually decreases to zero from its maximum opening when the piston is leaving the manifold zone to the dead zone (from BDC).

Fig. 4 : Typical valve Plate (Kidney Pattern Port).

NPTTEL

20

Now determination of the port area we need to find out the port area as I have told so we must consider in details what will be the port area. Referring to this figure the port area remains at a maximum constant when it is completely over the discharge or suction manifold that means when this kidney port is on this main manifold, okay then it is the maximum and maximum area is nothing but this area kidney port area so that is the A_{oi} maximum and this is usually you may consider that this is a rectangular port.

So what is done actually this if we would like to find out this area simply we consider this is a straight line and this is another straight line and this is half circle, this is another half circle that means if this is included by this ϕ angle then we actually take the centre from here to here and then ϕ into sorry this angle into this radius will give this distance that distance into thickness will give the area of the rectangular and then one circular area which is just the diameter we may consider equal to the thickness of this port, okay.

So that is the maximum area of a oil, what might be the minimum area that when this is coming over this this small angular area that means this is again something like a pyramid so if we consider the truncated tip of the pyramid area through that point will be our the area we

will show that area calculations later not in this lecture but later we will show this calculations.

In the transition region, the port area gradually increases to a maximum from zero when the piston is entering into the manifold but keep in mind this width I mean this length of this kidney port is more than this silencing groove so here this when it is gradually coming over that this port first it will cover the silencing groove then it will start covering the main area of the manifold, so until this end is coming on the manifold full manifold that means this until this is reaching here area is gradually varying initially on the groove then on the main manifold that we have to carefully calculate.

(Refer Slide Time: 45:36)

ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE PUMP :
Flow and Flow ripple – (Numerical Example):

Table- 1: Data of a pump.

	Description	Value
1.	Bulk modulus of the fluid, β	$10^9 Pa$
2.	Discharge coefficient of the orifice, C_d	0.62
3.	Angular extent of the barrel orifice (one chamber), ψ	30°
4.	Angular extent of a silencing groove, ϕ_f	11°
5.	Maximum opening area of the silencing groove, A_g	$2.25 \times 10^{-5} m^2$
6.	Maximum area of each barrel port, A_k	$3.75 \times 10^{-4} m^2$
7.	Leakage coefficient, K	$10^{-12} m^3 / Pa$
8.	Swash plate tilt angle, α	$0.314 rad (18^\circ)$
9.	Nominal volume of a single piston chamber, V_0	$22.85 \times 10^{-6} m^3$
10.	Pitch radius of Piston spacing on barrel, $r = d_p / 2$	$5.501 \times 10^{-2} m$
11.	Rotational speed of the barrel/block, ω	$235 rad / sec$
12.	Density of the fluid, ρ	$850 kg / m^3$

Note: $\phi_f = \psi'$ and $\delta = 0$ [Ref. Fig.- 2].

NPTEL 21

Now what this area calculation as I have told that I will see you later show you later that how this area is being calculated but here I have shown you only the flow ripple considering this is a real pump. Now this data let us study this this bulk modulus beta is normally 10 to the power 9 Mega Pascals, sometimes it is taken it is sometimes 9 into 10 to the power 8 something like that it is taken.

Coefficient of discharge is 0.62 that means this I would like to mention that usually these grooves are made in such a way that orifice so that we can use this coefficient 0.62 we have studied the Von Mises criteria when to determine this C d usually we design the orifice to have our coefficient of discharge is equal to 0.62. However, as I told for depending on the pump pump manufacturers they also can define this, what should be the coefficient of discharge we should consider or we can find experimentally in the laboratory.

Now angular extent of the barrel orifice that means that kidney port on the barrel that is usually one chamber is 30 degree, angular extent of silencing groove 11 degree that is the pyramid type we have taken 11 degree, maximum opening area of the silencing groove A_g is 2.25×10^{-5} meter square. Now here what is the maximum opening of this groove that I will show later that what area actually we are considering from the orifice it is not on the plane on which the barrel is moving rather this is barrel plane and below this this is the ramped groove so we consider this area for the groove maximum of that is A_g not the surface area we will show later.

Maximum area of each barrel port A_k that means this is 30 degree opening for that it is taken instead of mentioning the groove width we have taken that this is the opening of the barrel port that means this is the maximum area of the barrel port this means that initially when it has started opening the area will vary from 0 to 2.5×10^{-5} until this groove is completed after that from 2.25×10^{-5} to 3.75×10^{-5} this area will increase and then when this kidney port of the barrel will come on the kidney port of the valve plate then fully then the maximum opening will be 3.75×10^{-4} and this will continue upto the end, ofcourse at the ending there is no groove but that will also gradually decrease the kidney port will gradually move from the main manifold groove.

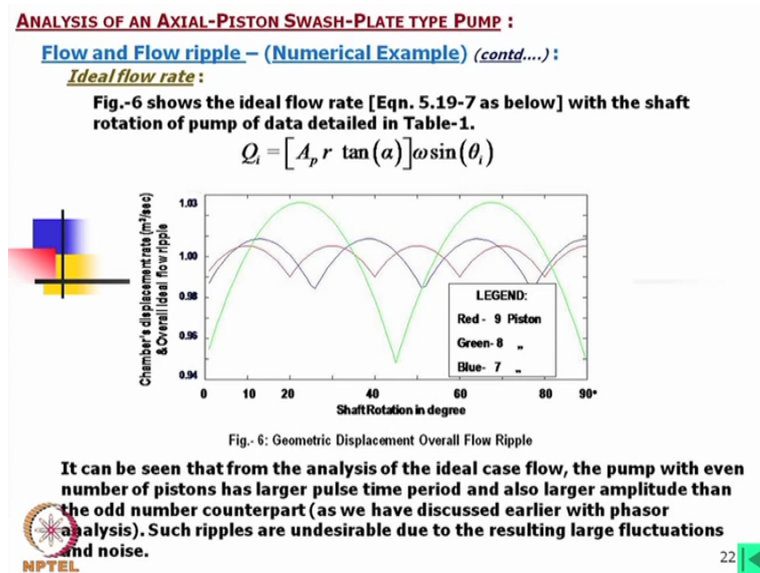
That is why you will normally find that in pump even in motor as the silencing groove are put in one direction only for the best performance we will find the direction of rotation is mentioned there. So we should we must maintain that direction.

Now leakage coefficient K here we have even we have not used the K_{θ} because K is constant for a pump, irrespective of the stroke length we have taken just single K . Swash plate tilt angle α is 18 degree, nominal volume of the single piston V_0 is 22.85×10^{-6} this is the nominal volume and the pitch radius we have taken 5.501×10^{-2} meter you can this means that how much 0.05 meter this means 5 centimetre 5.5 centimetre is the diameter.

Now here I have not mentioned what is the number of pistons this just I would like to mention the number of pistons for the same parameter we have taken the number of pistons 7, 8 and 9 for the analysis but this is basically a data of a 7 piston machines. Rotational speed just for one calculation we have taken 238 rad per second how much it will be it will be perhaps 1500 RPM like that and this density is 850 Kg per meter cube, density of the oil I would like to mention that if it is given not given you can take it from 830 to 850 anything in

between the effect of this on such calculations for that much variation will not be much, okay here I have given this two angle and delta is equal to 0.

(Refer Slide Time: 51:56)





And then for that we have plotted these equations, look at this for 9 pistons the ripple is coming like this please note that we have considered the overall volume displacement is same displacement we have not mentioned the diameter of the pistons. So therefore what we have considered from the average flow is same considering that for 9 pistons you can see what are the ripple and for 7 pistons the this one is the 7 pistons for blue one but as you see for the 8 pistons it is increasing.

Now this we have already shown that in case of even pistons this will be higher by the normal phasor equations, here actually we have put the we have used the actual equation to plot the flow and then we have shown this, what will be the ripple. It can be seen that from the analysis of the ideal case flow ideal case flow the pump with even number of pistons has large pulse time period and also larger amplitude then the odd number counterpart as we have discussed earlier in phasor analysis such ripples are undesirable due to the result in large fluctuation and noise that means we should not go for even number of pistons.

(Refer Slide Time: 53:35)

References:

1. Manring, N. D. 2000. [The discharge flow ripple of an axial-piston swash-plate type hydrostatic pump.](#) *ASME Journal of Dynamic Systems, Measurement, and Control*. 122:263-268.
2. Manring, N. D. 2003. Valve-plate design for an axial piston pump operating at low displacements. *ASME Journal of Mechanical Design*. Technical Brief. 125:200-05.
3. Mandal, N. P. , Saha R. and Sanyal D. 2008. Theoretical simulation of ripples for different leading- side groove volumes on manifolds in fixed-displacement axial-piston pump. *IMechE (UK) J. Systems and Control Engineering*. (Part-I) 222:557-569.



23

So these are the references I suggest that you should read all these three papers to understand this phenomena this how this flow ripples flow are calculated in details but this we have consider only a simplest pump this pump may be more complicated depending on the pivoting point of the swash plate and others but that is only you have to add you have to take care of the geometric and trigonometric relations there, so thank you for listening.