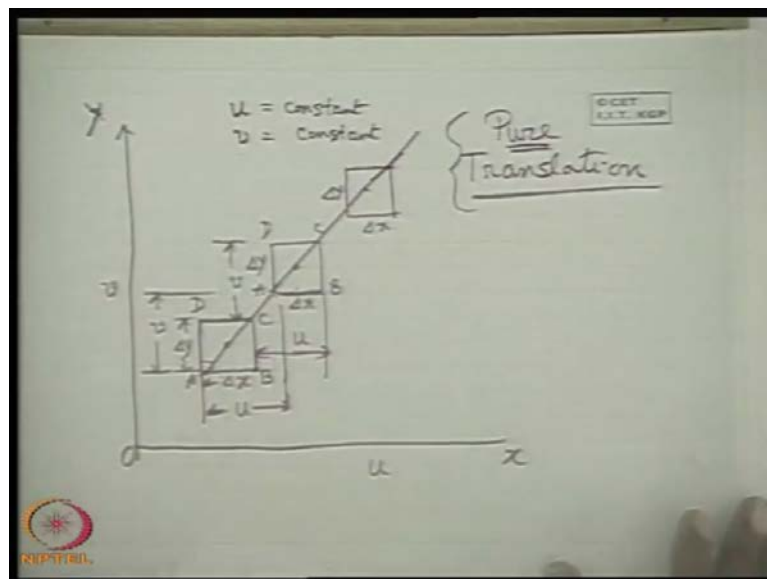


Fluid Mechanics
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Lecture - 12
Kinematics of Fluid Part - III

Good morning. I welcome you all to this session of fluid mechanics. Well, last class we just started translation, rotation and deformation. We just tried to recognize these three terms. That in case of a fluid flow the distinguishable features of its (()) that are translation, rotation, and deformation. We also recognized the difference with the motion of a solid, there were solid for solid bodies all the particles within the solid move with the same velocity for please, for liquid, for fluid. The particles, the particles constituting a fluid body move with different velocity that is the basic difference between the motion of the solid and a fluid body. So, in respect of that the three distinct features come in it course of fluid flow translation, rotation and deformation.

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Now, let us see that. In case of a motion described like this where, let us consider a two-dimensional case. Sorry, this is y, if you take two-dimensional system in x y axis. If you consider the u component of velocity that is in the x component and v is the y component velocity. So, u is constant; that means, u and v does not vary with the space coordinate. Now, in this context, I would like to tell you the translation rotation and deformation are the result of the variation of velocity components with space coordinates. So, fluid

maybe steady or unsteady if it is steady, all the pictures correspond to all times, or if it is unsteady the picture corresponds to a particular instance therefore, regarding the variation velocity component with space coordinate.

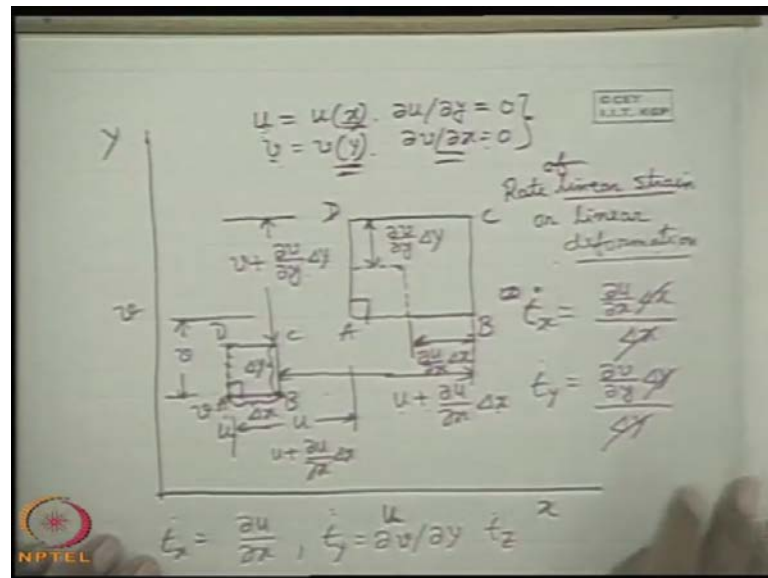
Let us consider a very simple case that is the uniform flow u v both the velocity components are constant we consider a two dimensional case. In that case, what happens if we just recognize a fluid element; we recognize fluid element whose lengths are Δx let the element is and the lengths are Δx and Δy . The element is A B C D with the lens Δx and Δy . What will happen in course of this flow field? This will be simply translated like this; this will be simply translated like this that means this will be the position respective time different.

Times this will be the position this will be simply translated; that means, the fluid will be simply translated along the resultant velocity. Now, what happens since u v are constant so each and every particle are point in this fluid element A B C D will move with equal velocities in x and y direction; that means, the lengths Δx Δy ; that means, here if we consider this is A B C D it has come here.

The length Δx and Δy ; that means, this length will remain unchanged Δx and Δy ; that means, if we consider the movement, but unit time here then if m moves by A distance u that is the x component of velocity B. This point also moves by the same amounting. Similar is the case for movement in the y direction understand that means, if this A moves; that means, this distance is v considering a unit time. Then D also moves by the same distance v ; that means, each and every point moves with the same velocity and their displacements are same. So, that the sides are not changed the in course of motion and also all the sides move paralelly that means, A B comes here paralel A D B C, they are paralel. This is simply known as translation this happens.

In case of a solid body if solid body is given a linear motion displacement this solid body displaces like this; that means, none of its sides are changed altered. And the sides are translated paralelly, the displacement takes place. So, that the angle included angle between 2 linear segments remains unaltered. This case is known as pure translation, or sometimes, we tell as translation. This is known as pure translation. Then u v are constant; that means, the entire flow field is invariant with the space coordinates. I think you have understood all of you.

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Now, next consider another simple case, where, just we take little this was earlier one is the simplest case. Now, we give little restriction; that means, let us now consider a situation, where u x component of velocity y component is a function of x only and v is a function of y only that means, we consider the variation of the velocity components with the coordinate axis. But take a very special case that is x component of velocity varies with x coordinate only while the y component of velocity varies with y coordinate only which mathematically means that $\frac{\partial u}{\partial y} = 0$ and $\frac{\partial v}{\partial x} = 0$. But this mathematics does not have much bearing now with the physical understanding just to write the sake of writing I am write, I write this that u is a function of x and v is a function of, if we take a special case like that then I see then I let us see what happens. In course of motion of a fluid element A B C D.

For example, A B C D, where this length was Δx and this length is Δy initially what happens. So, if we see at a later time, this be the position what we will see. Now, first of all, let us recognize one thing that when u is a function of x ; that means, each and every point. For example, in the linear element A B, which is varying with x coordinates move with different u velocities, which means B has an opportunity to have a displacement relative to a in the x direction. This is because the B point has a velocity more than u . For example, if u a point more than A, A point as a velocity u . At the point B the velocity will be u plus $\frac{\partial u}{\partial x} \Delta x$ into Δx . So, b has a higher velocity. So, by this A B linear segments get elongated continuously.

Similarly, since the v component of velocity varies with y which means that every point on the linear segment $A D$, which are varying with y coordinate had a different y component velocity, which can be told in this that D point has the opportunity to have a relative displacement over A in the y direction. So, for this type of velocity variations gives a continuous elongation of the linear segments. But another interesting fact is there because of this simplification that u is neither a function of y v is neither a function of x . In that case what happens that, if we now consider the different points in the linear segments $A B$. So, one thing is true at all points y component of velocity is not changing, this is because v is not a function of x , this is 0 . Because all these points vary only in the x coordinate. So, for this reasons the line $A B$ gets parallelly shifted. Similarly, the x component of velocity for all the points on the linear segments $A B$ does not vary. Because x component of velocity is not a function of y which means that the line $a d$ shifts parallelly.

So, as a result of which what happens there is a elongation of this element in the linear directions without, let this is the earlier one without changing. It is nearly exaggerated too much, because the elongation will be lower than the smaller than the original length. However for understanding, there will be no difficulty. Therefore, $A B C D$ without any change in the included angle; that means, the sides move parallelly, but with a linear elongation. Obviously, if we take this picture; that means, this displacement per unit time this position after an unit time. So, we can tell that this elongation is what is very simple, because if A moves by an amount u per unit time.

So, B will move by a distance. This distance this, this distance will be u plus $\frac{du}{dx} \Delta x$; that means, per unit time a will move to a distance which is the velocity here D will move to a distance what is the velocity at the. Because of the variation of u with x the velocity at B will be u plus $\frac{du}{dx} \Delta x$. Therefore, the elongation this part in x direction or displacement of B with respect to A in x direction will be $\frac{du}{dx} \Delta x$. In the similar way, if you see the A if the velocity. Here is v y component. So, A moves; that means, the displacement of a per unit time is v whereas, the displacement of D per unit time will be more than v ; that means, this displacement it is v plus $\frac{dv}{dy} \Delta y$, why? This, because if v is the velocity at A , in the velocity at D will be, because of the variation of v with respect to y will be v plus $\frac{dv}{dy} \Delta y$. This is the velocity at d . So, D will move by this amount.

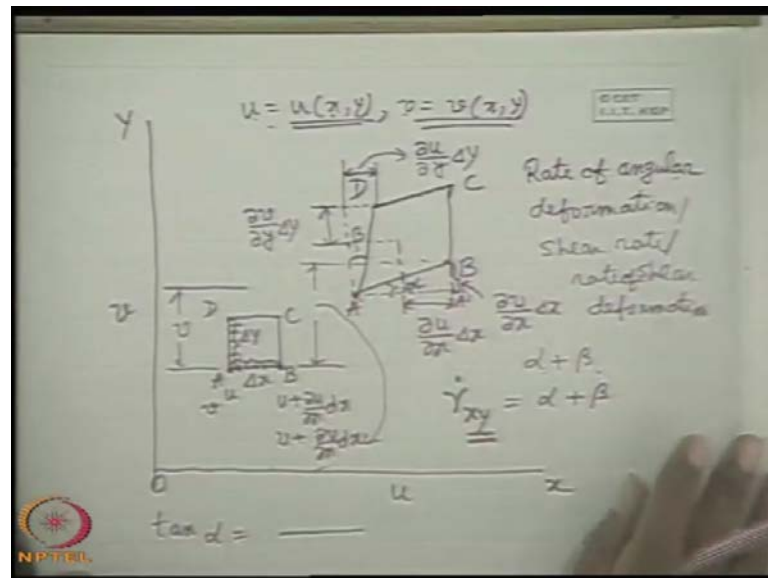
Therefore, we see that this displacement or elongation in the y direction is $\frac{\partial v}{\partial y} \Delta y$. I think this picture is clear, but these sites are displaced parallelly. There is no change in included angle. This is because u is not a function of y only x and v is not a function of x only y . Now, in this case, we define a terminology rate of linear strain or linear strain or linear deformation or linear rate of linear deformation. Why there is a linear deformation in this case. So, we see that in this case there is a linear deformation. Because this sides are linearly deformed or linearly strained there is a linear strain and linear deformation without any change in included angle shape of the body remains unchanged. And rate of linear of linear strain or linear deformation is defined as the rate of change of the length per unit length per unit time. So, per unit time this is the change in the length in the x direction.

So therefore, rate of linear deformation in x direction. It is symbolized as ϵ_x ; that means, in x direction with a dot usually is recommended. Because it is the rate is the change of displacement in the x direction per unit time per unit length. So, per unit time this is the elongation in the x direction for a linear, linear segment in x direction divided by its original; that means, simply $\frac{\partial u}{\partial x}$. Similarly, ϵ_y is $\frac{\partial v}{\partial y} \Delta y$; that means, this is the elongation or strain or deformation per unit time in the y direction divided by its original length. So, $\frac{\partial v}{\partial y}$. So, this can be written for in general for a three-dimensional flow $\frac{\partial u}{\partial x}$ ϵ_y dot is $\frac{\partial v}{\partial y}$ and ϵ_z dot is $\frac{\partial w}{\partial z}$.

Therefore, we see that when in a general three-dimensional flow, if the component of velocity is a function of that space coordinate; that means, x component is a function of x coordinates; that means, $\frac{\partial u}{\partial x}$ is non zero. Similarly, v is a function of y . So that $\frac{\partial v}{\partial y}$ is nonzero they exist. Similarly, w the z component of velocity is the function of z only. So, that $\frac{\partial w}{\partial z}$ is non-zero, then the a fluid body gets simply elongated in the linear direction.

That means all the sides in z direction also sites will be elongated this is the rate of linear deformation that is elongation per-unit original length per unit time. But the shape of the body will remain unchanged since the u component of velocity is neither a function of y and z v component of velocity is neither a function of x and z and w component of velocity neither a function of x and y . So, this is another simple case. Now, you come to the most general case; most general case; most general case.

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You can now appreciate very much, what is that general case that where u and v in a two-dimensional frame of reference both are function of x y . Because there is no need of considering u is a function of x and v is a function of y . In most general case of two-dimensional flow; two-dimensional flow means where the velocity components are functions of only 2 space coordinates and we consider only 2 velocity components. This is the definition of a two-dimensional flow. So, where did in a two-dimensional flow the most general case is that 2 velocity components u and v should be function of both x and y . In this case what happens? In this case, it is very simple to recognize that, if there is a element $A B C D$ initially in the shape with the Δx that is $A B$ length $A B$.

And if we define this length $A D$ as Δy that means a rectangular fluid element of $\Delta x \Delta y$. Now, what happens that? Since u is a function of y along with the function of x . What happens, if we consider different point on the linear segment $A B$. It is not only having a velocity different u component of velocity, it is having different y . It is having different y component of velocity obviously, because v is a function of x . Now, for which now we can say that each and every particle on the linear segment has got a relative displacement over any particle here. For example, a both in x and y direction that means, now you can take that, if this is the velocity u at A . The velocity u at B will be u plus $\frac{\partial u}{\partial x} \Delta x$. Similarly, the y component of velocity v at A , so y component the velocity at B will be v plus $\frac{\partial v}{\partial x} \Delta x$, which means, now v has an opportunity to

have a relative displacement over a in both x direction and y direction. So, because of this relative displacement in y direction line A B will be tilted like that.

So, it will not move parallelly. Similarly, we consider the line segment AD. You see clearly, that because of a change in v with y . So, it will be linearly elongated as we have discussed earlier. But since u is a function of y also; that means, this direction velocity for all elements particles on ad will be different. Because u is now a function of y for which the pointed d has got an opportunity to have a relative displacement over a not only in the y direction, but also in the x direction, which means, then the line AD will not be parallelly shifted. So, AD will get a tilt clear therefore, the situation is like this. Let us consider, this is the, the old one. Now, now this is the situation. So therefore, A B C D. Now, you see there is a distortion in the shift. This clearly understood now: obviously, from our discussion, we can tell that this is, what this is $\frac{\partial u}{\partial x}$, the linear elongation in the x direction.

So, this point B has got a relative displacement over A in x direction is this one. So, what is this one, please tell me what is this one, $\frac{\partial v}{\partial x} \Delta x$ very good. This is because this relative displacement of B with respect to A per-unit time; that means, if you consider this position after unit time is nothing. But a extra velocity v which the B point has over A, that is v plus $\frac{\partial v}{\partial x} \Delta x$. That means if A moves with a displacement per unit time v , then this B point will move; that means, if I take a projection here this will move by this value. That means, v plus $\frac{\partial v}{\partial x} \Delta x$, because this has a higher velocity in the y direction with respect to A by this amount. So, this will be the relative displacement, very good.

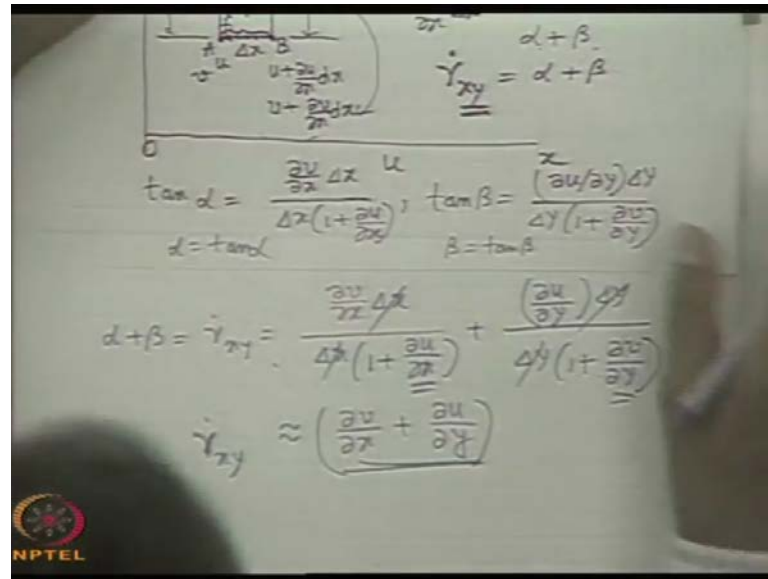
Similarly, in this direction obviously, if we take a projection this displacement as we know this is the linear deformation that is $\frac{\partial v}{\partial y} \Delta y$. Similarly, what is this value; this value will be this is nothing but a displacement of D with respect to A in x direction and this is responsible for $\frac{\partial u}{\partial y} \Delta y$, because it is a change in u velocity over a change of length Δy . Now, if I denote this angle as α and this angle as β , well α and β . Now, here many terminologies have to be defined. First we define the rate of these are the terminologies you have to know rate of angular deformation; rate of angular deformation or sometimes we can tell shear rates many names are there or rate of shear deformation; shear deformation.

So, these are the terminologies, the rate of angular deformation rate, shear rate, a rate of shear rate or rate of shear deformation of a fluid particle at any point we define. How do you define? Definition is very important, rather than the different terminologies right of angular deformation or shear deformation at any point is defined as the total rate of change in the total angle between 2 linear segments of a fluid element, which were initially perpendicular; that means, if this be the definition that means it is the rate of change of angle, between to linear segments of a fluid element which were initially perpendicular. That means if we consider AB and AD 2 linear segments of a fluid element which were initially perpendicular. So, the per-unit time the change in its angle is $\alpha + \beta$. Therefore, the rate of angular deformation or shear deformation is $\alpha + \beta$. And this is symbolized as $\dot{\gamma}_{xy}$. Gamma is the terminology or nomenclature for this dot is for the rate and xy is given, because this deformation is taking place, because of a tilt of the linear segment.

So, this happens only in the xy plane above the z -axis. In case of a three-dimensional flow there will be a deformation of the surfaces in the other plain yz or xz plane. So therefore, this plane specification is given $\dot{\gamma}_{xy}$. In this case only, one plane because this is a two dimensional case it has got only one plane. So, in this case, it will be $\dot{\gamma}$ or $\dot{\gamma}_{xy}$ you can write $\alpha + \beta$. Now, from a simple geometry, you see that in this triangle AB letters give some name A dash A, A dash B is the school level. So, $\tan \alpha$, we can write, you see $\tan \alpha$ is what this divided by this length A B A dash divided by A A dash $\tan \alpha$. I think $\tan \alpha$, you understand A dash B divided by; that means, $\tan \alpha$ becomes equal to what, $\frac{\partial v}{\partial x} \Delta x$ divided by Δx into $1 + \frac{\partial u}{\partial x}$.

Similarly, we can write $\tan \beta$ is equal to $\frac{\partial u}{\partial y}$. Sorry, $\frac{\partial y}{\partial y} \Delta y$ divided by Δy into $1 + \frac{\partial v}{\partial y}$. Now for small angles α β this deformation small, considering the small, we can write α is equal to $\tan \alpha$, as you know for small angles, the angle tangent of the angle equal to the angle itself tangent and sign of the angles become equal to the angle for the series you can appreciate that.

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So therefore, we can write alpha plus beta is equal to gamma dot x y is equal to what this plus this. That means, if I write this plus these, let me write is equal to del v del x delta x divided by delta x 1 plus del u del x plus simply this quantity del u del y delta y divided by delta y. Now, considering this values to be small; that means. the angles are small rate strain rates are small. We can neglect the higher-order term product of these 2. So, that it will make a simplification, we will simply get del v del x plus del u del y del v del x plus del u del y. So therefore, we get the shear rate or the rate of angular deformation in x y plane is simply del v del x plus del u del y. Now, we appreciate, del v del x plus del u del y this you can see at home also this the product we neglect. So, ultimately this will be del v del x plus del u del y yes. So, everything will be this you take this value you cannot find out you take this 1 plus del u del x to the power minus 1, 1 plus del v del y to the power minus 1 expanding and neglect the higher-order and find this, this is very simple mathematics you are to follow it.

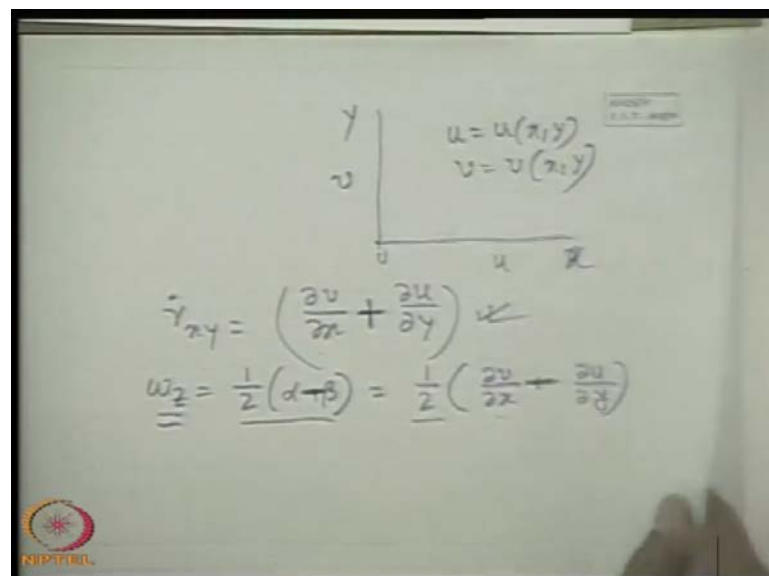
So, this becomes del v del x del u del y. Now, again this is the final picture, the most general picture, we will be finding so many things from here. Now, again with respect to this picture apart from rate of angular deformation or shear rate another concept comes that is rotation. What is mean by rotation of a fluid element? So, when a fluid body is deformed like this. In course of its motion, because of a very general case in the velocity components are function of all space coordinates then the fluid body deforms like this.

We define 2 quantities, one is the rate of angular deformation, another thing which is defined is the rotation. Now, we have to know, what is this?

So, rotation at any point is defined as the average angular velocity of the 2 linear segments, which were initially perpendicular. Just like this, it is the rate of change of angle which is the deformation. It is another way, it is the average or arithmetic mean of the angular velocities of 2 linear segments. Now, when this A B gets tilted like this; that means, it has got a rotation, a rotation. Since, it is coming; that means, it has got an angular velocity about A at A about the z-axis. Similarly, as A D is shifted like this not parallelly; that means, it gives a concept of angular velocity about z-axis at A. That means, while the line A B has an angular velocity in this direction line A D has got an angular velocity in this direction.

So, in that case, if alpha and beta are the angles made per-unit time, as we have found out otherwise, we cannot tell the displacements like $\frac{\partial u}{\partial x}$ $\frac{\partial v}{\partial y}$ there is; that means, that I have considered per-unit time then you can tell the angular velocity is alpha for this A B and for this AD angular velocity is beta. That means, the angle made per-unit time. But velocity is the vector; that means, this should have a sign or sin.

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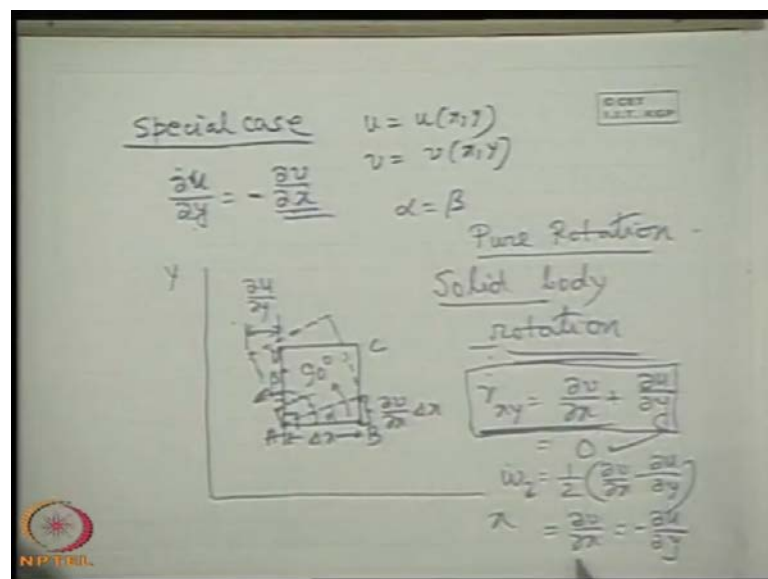


If we take a conventional nomenclature as which one we will take as anticlockwise, if which is usually taken as the positive. That means alpha is positive beta is negative. So therefore, from there is we can write, that the rotation, rotation is defined with omega.

About the z-axis for the point A is half of the alpha plus beta; alpha plus beta; alpha plus beta. So, what is alpha plus beta? So, if you write this half of alpha beta; that means, alpha is half of this plus this, which becomes equal to as I did half del v del x plus. Sorry minus, because beta is minus. So, I write half alpha minus beta with the minus sign del v del x minus, del v del x minus del u del y. So therefore, we see the deformation rate in the x y plane is del v del x in two dimensional case del u del y in two dimensional coordinate system; that means, x and y where u and v component of velocity varies with both the space coordinates. The most general case that is x and y.

In that case is fluid body has got a rate of angular deformation in the x y plane which is given by del v del x minus del u del y. Sorry, del v del x plus del u del y very good and fluid body has got a rotation about the z-axis. At any point, which is given by half del v del x minus del u del y. So, this is the concept. Now, in a three-dimensional case, if we consider now, before that of course, I like to tell you some special cases, let us consider some special cases.

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Special cases; what is that special case? Let us consider. Now, we can make many special cases. Let us consider this special case, when u v variation is such that del del u del y, let del u del y is minus del v del x. Let us take that u is a function of x y. And v is a function of x y in such a way that del u del y is minus del v del x. So if the take this special case then what happens? Just you think that this is the x y plane, then what

happens? Let us consider this as the body here just at the same place we consider. We are not going somewhere else, then what happens, $\frac{du}{dy}$ is $\frac{dv}{dx}$. You will see this thing will happen. The included angle will remain same, why this is $\frac{du}{dy}$ $\frac{dv}{dx}$ Δx . Let this is Δx A B C D, A, and this is the dotted one is the final position. Now, I want to show that, if $\frac{dv}{dx}$ and $\frac{du}{dy}$ are same in opposite sign in. Now, or earlier what we did be considered please see $\frac{du}{dy}$.

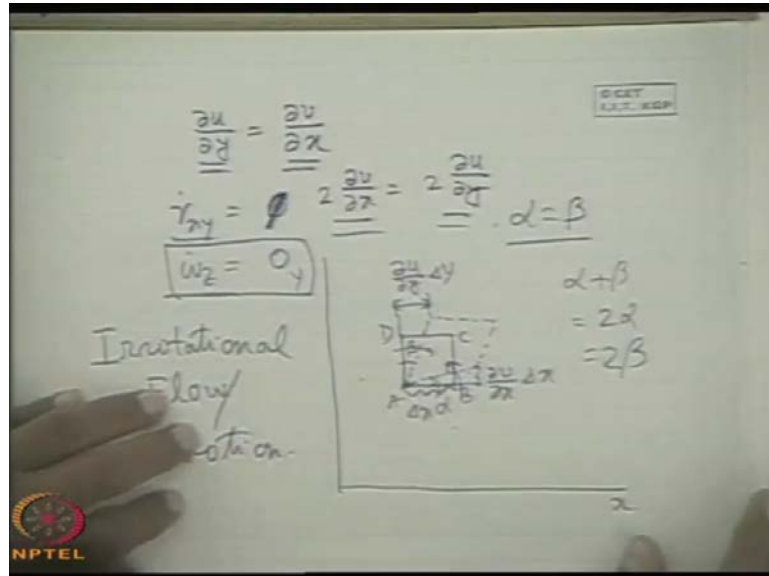
Now, $\frac{dv}{dx}$ and $\frac{du}{dy}$ are in the same direction, same sign; that means, here what happened? The $\frac{dv}{dx}$ is positive; that means, v increases with increased in x , that is why B has got a displacement over A in the positive x direction. Similarly, here we have considered the most general case of $\frac{du}{dy}$ positive; that means, the u velocity increases with increasing y . That means, D has got a displacement in x direction over a in its positive direction; that means, this one. So, that these sides have been displaced like that. But if we consider now for example, $\frac{dv}{dx}$ is positive; that means, the v component of velocity increases with the increase in Δx .

So, B has got a relative displacement over A in y direction in this direction that is positive y direction. Now, if I consider the $\frac{du}{dy}$ is minus; that means, if it is positive $\frac{du}{dy}$ is negative. That means, D will have a displacement in the x direction less displacement over A. That means, in other way you can tell that it has got a y , x direction displacement in this direction. So therefore, we can tell that the particle will move in this way. That means, in the same direction the linear element will be rotated. And since, they are equal in magnitude; that means, α will be equal to β , α will be equal to β . Because if these 2 are same, if you recollect α is $\frac{dv}{dx} \Delta x$ divided by Δx . We can write that, if we neglect this next term the Δx ; that means, it is $\frac{dv}{dx}$ and β will be $\frac{du}{dy}$.

So that means, that this $\frac{du}{dy}$ and $\frac{dv}{dx}$ terms if they are in opposite sign. But equal in magnitude then what happens both the linear elements will move in the same direction and (()) they are equal, what happens included angle remains same. That means, this is the earlier included angle and this is the final included angle both of them remain 90 degree. This type of rotation is known as, this type of motion is known as pure rotation are more easily understood, or conceptually solid body rotation. Solid body rotation means the rotates as a solid body, this is solid body rotation. So, this is solid

body rotation; that means, a solid body rotation like that by making the included angles same.

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So, this is known as solid body rotation, if we consider another special case where $\frac{\partial u}{\partial y}$ is equal to $\frac{\partial v}{\partial x}$; that means, they are equal in sign. In that case from the definition γ_{xy} becomes 0, γ_{xy} is not 0. Here the γ_{xy} becomes 0. I am sorry, I am sorry please. Come to this here, what happens this is a solid body rotation and pure rotation. Now, what are the values you see γ_{xy} use $\frac{\partial v}{\partial x}$, plus $\frac{\partial u}{\partial y}$ here γ_{xy} is 0. And here, one fine find out the ω_z . It is half of $\frac{\partial v}{\partial x}$ minus that is $\frac{\partial v}{\partial x}$ is minus $\frac{\partial u}{\partial y}$ or $\frac{\partial u}{\partial y}$ is minus $\frac{\partial v}{\partial x}$. That means, it is either $\frac{\partial v}{\partial x}$ or equal to minus $\frac{\partial u}{\partial y}$. Obviously, this is just like the solid body rotation, where the rate of shear deformation is 0, because there is no change in the included angle. So therefore, for a solid body rotation or pure rotation, there is no angular deformation.

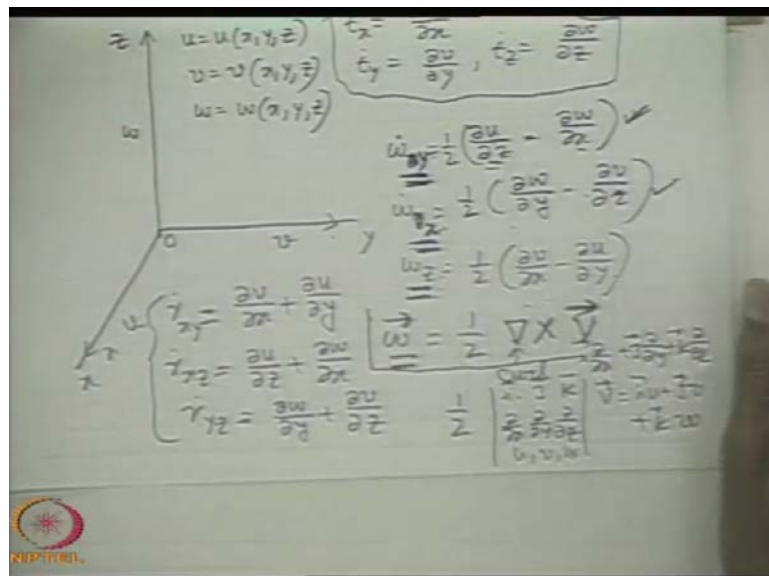
So, these value is 0 and angular velocities arithmetic mean of the 2 linear segments is nothing but the angular velocity of the body as a whole which is $\frac{\partial v}{\partial x}$ either $\frac{\partial v}{\partial x}$ or $\frac{\partial u}{\partial y}$ very simple. So, we can understand, what is mean by a solid body rotation. And the solid body or pure rotation, where the rate of angular deformation is 0 and angular velocity exist and it is given by this value. But now take a special case, where $\frac{\partial u}{\partial y}$ is $\frac{\partial v}{\partial x}$. In this case, γ_{xy} is not 0. It is either twice del

$v \frac{\partial}{\partial x}$ or twice $\frac{\partial u}{\partial y}$. But in this case by definition $\omega \cdot z$ is 0, because this is equal to this with the same sign. So, $\omega \cdot z$; that means, rotation is 0. What is this situation, this situation is very simple to concede x and y direction.

Let us consider the initial particle like that element like that A B C D, final one will be like this where this α is equal to β , they are of the same sign that means, if $\frac{\partial v}{\partial x}$ is positive, this is $\frac{\partial v}{\partial x} \frac{\partial x}{\partial x}$, if this is $\frac{\partial x}{\partial x}$. So, $\frac{\partial u}{\partial y}$ is positive also; that means, in the positive direction there is a displacement. So, it is $\frac{\partial u}{\partial y}$ is also positive. So therefore, both of them are in the same sign. So, the 2 linear elements will move in the opposite direction. See 2 are negative, than this was moving in this direction, this was moving in that direction that means both of them will be moving in the opposite direction, which we first considered in general case.

In that case, what happens? The α is equal to β ; that means, the angular velocities of the 2 linear segments initially perpendicular will be same. But will be in the opposite direction that is why by definition of the rotation the arithmetic means becomes 0. So, rotation becomes 0, but it has got definitely a shear deformation or angular deformation, because rate of change of angle is $\alpha + \beta$ or 2α or 2β .

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So, this either by this or this, this type of motion is known as irrotational motion, irrotational flow or motion. So now, we see that, we have recognized 3 distinguishing features, that is translation deformation and rotation. Now, let us write in case of a, after

this physical understanding that in case of a three-dimensional system $x y z$. We can write ϵ_{xx} . Now terminologies are known that is the rate of linear strain is $\frac{du}{dx}$. And you consider u as a function of $x y z$, what is u_x component velocity? What is v_y component? What is w_z component? So, v as a function of $x y z$ and w as a function of $x y z$. Now, ϵ_{yy} is $\frac{dv}{dy}$ and ϵ_{zz} is $\frac{dw}{dz}$. This defines the linear strain rates in three direction; that means, 3 sides elongates in this way.

Now, ω_x ; that means, about x -axis ω_y and ω_z . We have already recognized $\frac{1}{2}(\frac{dv}{dx} - \frac{du}{dy})$ in defining. So, what we have considered, we have considered the anti clockwise direction rotation or angular velocity is positive. Similarly, with respect to x direction, you see which one will be positive anticlockwise rotation. That means, in this direction rotation and this direction rotation will be responsible by the z component of x component of velocity. In the z direction, this direction rotation is the positive, one here you have see the $\frac{dv}{dx}$, there you have seen the $\frac{dv}{dx}$. Here, we will see the $\frac{du}{dz}$. So therefore, ω_x is $\frac{1}{2}(\frac{du}{dz} - \frac{dw}{dx})$. So, this is the positive, this is the negative. So, $\frac{du}{dz}$; that means, the linear element here will have a positive angular velocities, that is responsible for a change in the x component velocity with respect to z similarly with y direction what will be this linear; that means, about y . I have written, ω_x will be in $y z$ plane.

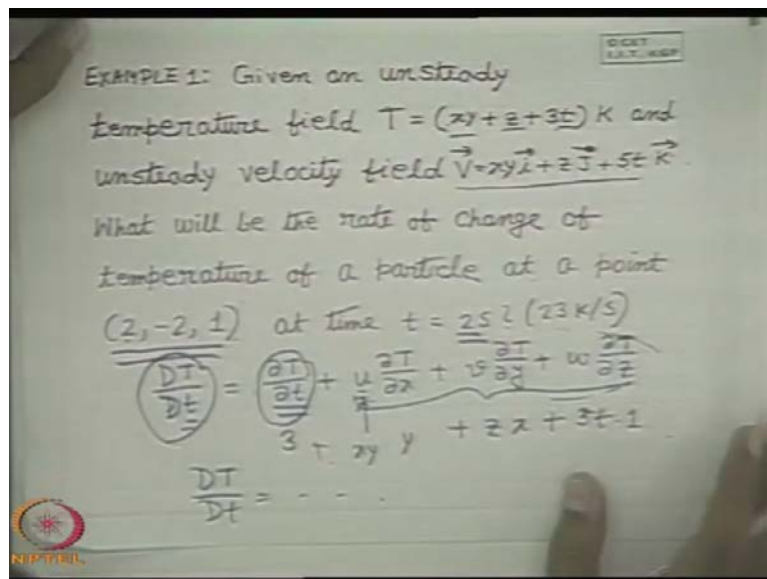
So, this will be ω_y . So this will be ω_y . I am sorry, because this is the y direction. So, I have taken $x y z$ like that. So similarly, ω_x ; that means, this will be in the $y z$ plane. Obviously, so this will be the $y z$ plane. So, this will be simply the $\frac{dw}{dy}$; that means, rotation in this direction will be positive, which is responsible for change in the y direction y component of velocity change in this, is the change in the w component of velocity with $\frac{dw}{dy} - \frac{dv}{dz}$. So, this you can make it, I am sorry, because it is just that $x y z$. So, ω_x , this is ω_y , this is ω_z . So, this is the angular velocity, the rotation $x y z$; that means, this $x y z$ means the rotation about z axis rotation about x -axis rotation about y -axis.

So therefore, we see the rotation is a vector which is got 3 distinct component $x y z$. So, this vector can be written as half of curl of the velocity vector, you know this this operator is known as Curl that means, del operator in this and vector. So, if we write this,

we will get the different components like this will next definitely you know these things that if, you write this, what will be the term half i j k curl operator is like that del del x del del y del del z this operator is i del del x i j k are the unit vectors j del del y plus k del del z. So, if you make a cross product u v w then from this determinant you can find out x y z component like this. That means, these 3 components appear in this form. We can write in a vector form for the vector angular velocity, which is got 3 distinct component, this is your preliminary vector knowledge you know.

Now similarly, in this case there will be angular deformation in 3 planes, one will be in x y plane. Here, there is no problem, because all are additive del v del x del u del y. So, in x y plane, the deformation is responsible for u v velocities and their dependents with xy. Similarly, in x z plane x z plane it will be, because of w and u and it will be crossed differential del u del z plus del w del x x z plane. Similarly, for y z plane the deformation in the y z plane will be responsible for y component and z component of velocity; that means, del w del y plus del v del z.

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So, these are the 3 distinct components of deformation rates and 3 distinct from one ends of angular velocities. In which case, the angular velocity being a vector can be represented in a vector form. Like that see v bar is the velocity vector v is the velocity vector and having a component i u plus j v plus k w i j k are the unit vector in x y z. We

can write this curl; that means, the cross product of del with velocity vector v which gives distinct components like that ω_x , ω_y , ω_z clear no confusion.

Now after this please, we come to an example. Let us come to an example please write given an unsteady temperature field, T given an unsteady temperature. Field T is equal to $x^2 + y^2 + z^2 + 3t$ Kelvin and unsteady velocity field v like this. There is an unsteady velocity field like this. So, the, what will be the rate of change of temperature of a particle at a point, this at time t is equal to 2 seconds. So, this is simply your application of the substantial derivative. Now, in a flowing field the rate of change of T $\frac{D T}{D t}$ $\frac{D T}{D t}$ big t is the temperature in Kelvin. This is the function, the t varies with x , y and z and time and this is the velocity field can be written as $\frac{D T}{D t}$ plus, you know that u that this is the temporal change.

So, this is the convective change $\frac{D T}{D t}$ into $\frac{D T}{D t}$ which is u the way we derived. If you recall that change of any property in a flow, which is property, which is convected in a flow field. So, it should not be velocity only it may be a temperature. So, it is total change with time will be its temporal change with time that time temporal derivative and convective derivative. So, a straightforward application so what is $\frac{D T}{D t}$, $\frac{D T}{D t}$ here, the first-term is 3, what will be u ? Here x , y now, simply school level thing. What is $\frac{D T}{D x}$? What is v_z ? What is $\frac{D T}{D y}$? What is w ? w is $3t$, it is $3t$, $5t$. I am sorry, I have seen there. I am sorry; I am sorry, $5t$ and what is $\frac{D T}{D z}$ 1. Now, this plus; this plus; this plus this is a function of x , y , z , t . So, at this point and that this value of t and this value of x , y , z now, it is school level thing you can find out what is the value of $\frac{D T}{D t}$. This is simple straightforward application of using the concept of temporal and convective derivative to find out the total derivative. In case of time, it is the total change in time is this local changing time plus the change with convection.

Thank you for today.