Introduction to Fluid Mechanics and Fluid Engineering

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Lecture - 39
Boundary Layer Theory (Contd.)

In our previous lecture, we were discussing about the momentum integral equation for boundary layer in presence of 0 pressure gradient and we came up with this form of the expression and the whole objective of doing the analysis was to calculate the wall shear stress.

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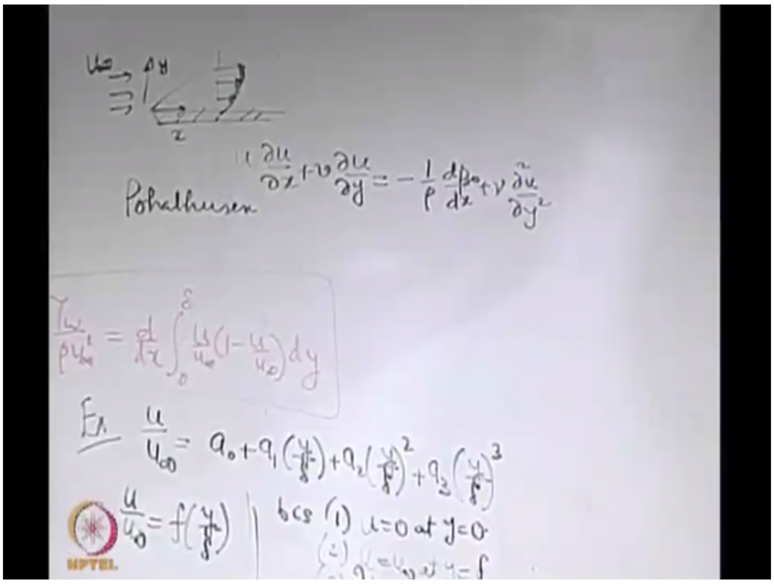

Now, if you look into this expression, you will see that, you may calculate the wall shear stress provided you know what is the velocity profile within the boundary layer. Because that is what is an unknown in the integral expression which is there. The big question is that, what should be the velocity profile. Any velocity profile, which you may choose or approximate. In general, will not be the correct one.

Because, the correct one should come out of detailed solution of the flow field and then what is exactly there within the boundary layer. But, you may always substitute this velocity profile with some approximate velocity profile, which sort of satisfies your boundary conditions. So, when it satisfies your boundary conditions and the velocity profile is substituted here, our expectation will be that the velocity profile may be inaccurate.

But in a somewhat integral form the velocity profile should give not a bad estimate of the wall shear stress and this expectation is from the fact that after all the boundary layer is very thin. So, error in the velocity profile may be important when you consider the velocity profile as such but in the integral sense, the integral of the velocity profile might not be that erroneous. That is one of the expectations.

So, whether that expectation is justified or not, the best way in which we may look into it by some example. So, let us say that we want to have a polynomial approximation of the velocity profile. So, for example, we may have different forms of this expression like one may assume a polynomial like this. Even you may consider 4th order term and that is what was done by an engineer as Pohlhausen.

So, his particular method was dedicated in the name of his honor. But, here we are not going for a 4th order polynomial. We will leave that one you as an exercise in your assignment problem. We will consider a third order polynomial. Even one may consider a first order polynomial, one may consider a sinusoidal function, so many different forms of the functions are possible.

Within the constraints of the function, you have to satisfy the most important boundary conditions. So, that this unknown constants a0, a1, a2, a3 whatever, these are determined. So, let us look into this special case as an example problem and see that how do you calculate the boundary layer thickness, the wall shear stress and the total drag force on the plate on the basis of this. So, to do that, first of all we have to understand that how do we calculate these coefficients.

So, if you have 4 coefficients, you must satisfy 4 boundary conditions to get these 4 coefficients. Let us see what are the most essential boundary conditions that you need to satisfy. What is the physical problem we are considering? You have a flat plate on which you have a free stream flow coming with a velocity u infinity and the boundary layer is growing like this and you are interested in describing a velocity profile at a section x.

So, what are the boundary conditions that you want to satisfy? What is the most essential boundary conditions that you would always like to satisfy? See, the boundary conditions should come in order of priority. Because, if you choose lower order polynomials, you may not be able to satisfy all the boundary conditions. Because, you have less number of coefficients. But, the most essential one you should always satisfy. What is the most essential one? Loosely boundary condition at the wall. So, u=0 at y=0.

Then, what happens in the outer stream? So, let us say that y=delta, y=delta in an engineering sense is equivalent to y tends to infinity in a mathematical sense. Because, outside delta whatever happens is like hot stream okay. So, literally it is the variable y/delta eta tends to infinity. But, it is as good as y=delta. So, at y=delta what are the boundary conditions? Okay u=u infinity at y=delta. Any other boundary condition at y=delta? So, u does not change further with y at that. That means, the gradient of u with respect to y=0 at y=delta.

A 4th boundary condition. Of course, you may go on adding here like in terms of boundary condition at y=delta, you may also have the second order derivative of u=0. But, more important boundary condition, which has a priority of what this is what happens at the wall. Because, at the wall, you have one boundary condition but if you really have a liberty to put more, the priority should come there because, these 2 are quite sufficient to describe what happens at the boundary layer.

But this alone is not very sufficient to describe what happens at the wall, if there are scopes of incorporating more boundary conditions at the wall, it would be better. So, look at the governing equation, let us try to satisfy the governing equation at the wall. See, these velocity profile does not understand the momentum equation right. It is just an approximate velocity profile. So, we will try to teach the velocity profile in such a way that it is in a way satisfies the constrains of the governing equation at the wall.

So, at the wall, if you look at the governing equation, so, if you have, let us write the momentum equation, the boundary layer equation basically. This is the boundary layer equation. When you consider flow over a flat plate, essentially it is a 0 pressure gradient. So, this term is not there. Let us try to satisfy this at the wall. At the wall, u=0 right by loosely boundary condition. At the wall, v=0, by no penetration boundary condition.

That means, you must have the second derivative of u, with respect to y, 0 at the wall right which follows from the governing equation. So, this is not a boundary condition that you directly find out intuitively but, if you want to make sure that the momentum conservation is satisfied, then that is what you get. So, the remaining what is easy that you substitute these 4 conditions so you will get 4 algebraic equations involving a0, a1, a2 and a3.

So, from that, you find out a0, a1, a2 and a3. So I am not going into the details of the algebra because it is too trivial.

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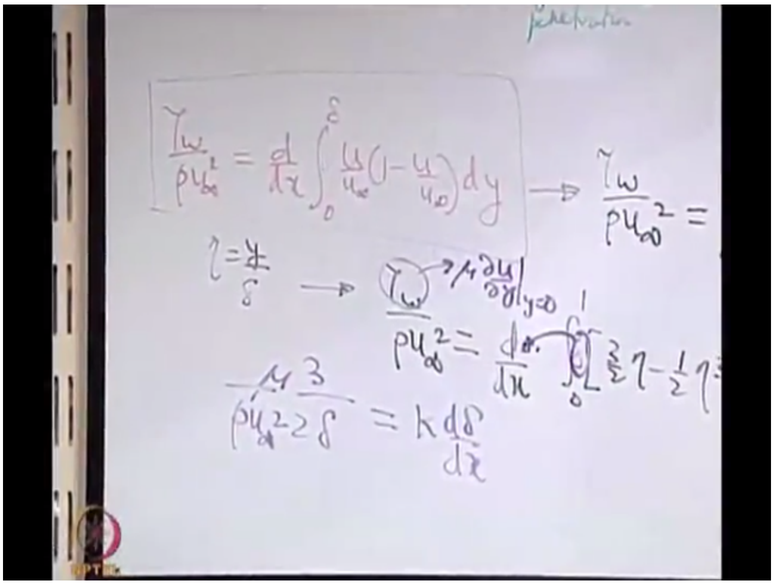

So, then what you will get out of this is u/u infinity =3/2 y/delta-half y/delta whole cube. So, the remaining terms will go away okay. Now, at least once you get out of these whether your algebra is correct or not at least you should check that it satisfies most of the essential boundary conditions that at y=0, u=0 and at y=delta, u is u infinity. Now, what we will do with this velocity profile? We will use this velocity profile for evaluating the integral, which is there in the expression.

So, to do that we will write tau all/rho u infinity square=ddx of integral 0 to delta. Now, you will write 3/2 y/delta-half y/delta whole cube*1-3/2 y/delta + half y/delta whole cube dy. With an obvious substitution of variable that is eta=y/delta, you may write this as tau all/rho u infinity square=ddx of integral 0 to 1, then 3/2 eta-half eta cube*1-3/2 eta + half eta cube d eta and then multiplied with a parameter delta.

Because dy=delta*d eta okay so change of variable is there. So, important thing is, first of all, this delta is dependent on x, but this integral evaluation is not dependent on delta. So, you can take this delta out of the integral. So, it walls down to evaluating some sort of polynomial expression and then coming to the integral. Again let us not waste any time in like doing this very simple algebra and simple integration.

Let us say that, out of the integration what you will get? You will get a number. Say, you get a number k, which comes out of the integral of this entire expression. It is a definite integral, so, it will come like a number. Now, what is the tau all? Tau all is mu*this one at y=0. So, that also you may calculate from assumed velocity profile. So, what is your assumed velocity profile? So, that is this one and you may calculate the first order derivate at y=0.

So that will be 3/2 delta because remaining term will have y, which will be 0 at y=0. So, you will have mu*3/2delta rho u infinity square is also there in the denominator=k*d delta dx okay. So, then you can integrate this and once you integrate this, you will get delta as a function of x. So, let us write the integration.

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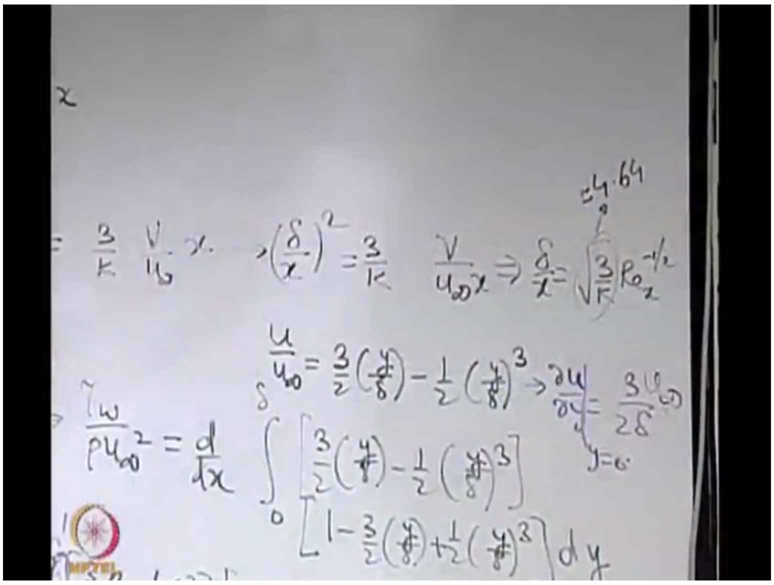

So, you have delta d delta=mu/rho u infinity square 3/2k dx right. So, if you integrate it, delta square/2=3/2k mu/rho u infinity square x + constant of integration. As we know that, in the limit as x tends to 0, you have delta tends to 0. That is the edge of the plate, where from the boundary layer grows. And therefore, you must have c=0. So, you see the growth of the boundary layer is like an initial value problem, where you start with x=0 and whatever condition is at x=0, all the subsequent conditions for delta may be evaluated.

So, the x coordinate here acts like a time coordinate. As if we have started with time=0, instead of time=0, it is x=0. And as you march along x, in the positive direction. You see that the boundary layer grows. So, this is called as a marching problem. So, instead of marching with respect to time, you are really marching with respect to space or position. So, now you can find out what is delta/x. So, delta square will be 3/k. We will always write mu/rho as nu, because kinematic viscosity is what that governs the physical situation.

So, nu by u infinity square x. So, you can see clearly that delta, that is with square root of x, which we have also seen from the previous analysis like the order of magnitude analysis and the Blasius solution. Now, if you want to find out how delta/x is scaled. So, basically what you have to do? You have to divide by x square. So, if you divide it by x square, then one important thing is we have missed one u infinity here. Because this u should be I mean it is u*u infinity is this one.

So, somewhere u infinity has to be there. So, one infinity we have missed. So, this u infinity and this infinity will cancel. So, eventually we will have only one u infinity here right. So, for evaluation of the tau all, there was one u infinity. This is u infinity that has to be there. Now, if you divide it by x, so delta/ x square is 3/k*nu/u infinity x. that means delta/x is square root of 3/k* Reynolds number to the power -half okay.

So, if this is calculated, this comes out to be, this particular number, roughly if I remember correctly, roughly 4.64. So, see this is somewhat erroneous because if you remember the Blasius solution, this was high. So, instead of 5, it is coming out to be 4.6 or something like that. But the important thing is that the scaling behavior is similar. So, the order of magnitude gave delta/x of the order of Reynolds number to the power -half. The exact solution is like 5* Reynolds number to the power -half.

Some approximation it is deviated from the exact solution. But, we are not really that much bothered about this one. This is somewhat disturbing because the approximate solution is somewhat deviated from the exact one. But, this is what is not our focus. Our focus is the wall shear stress calculation. We are never expecting that it will give the correct boundary layer growth but our expectation is that at least integral of the velocity profile will nullify the error in the velocity profile to some extent and get a better estimation of the wall shear stress.

So, let us calculate the wall shear stress.

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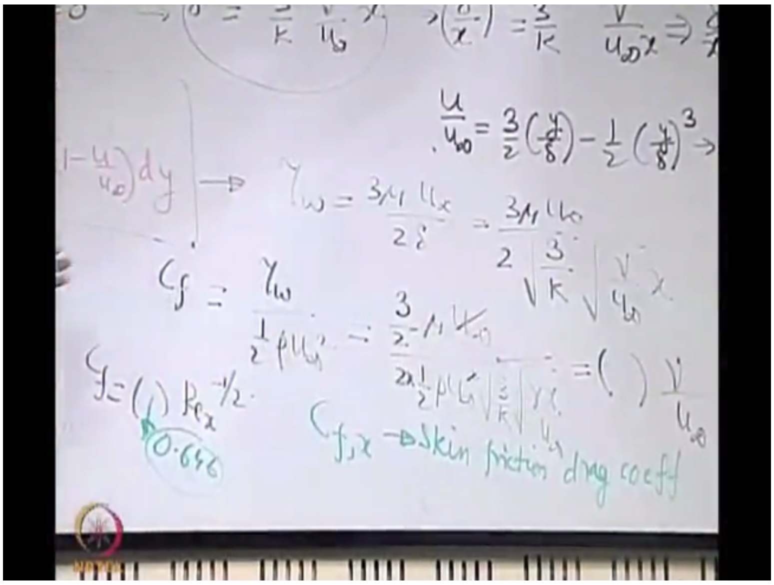

So, if you write the wall shear stress tau all that is 3 mu u infinity /2 delta that is what and delta you may substitute as a function of x. so, delta is nothing but 3 mu u infinity/in place of delta, we will write square root of 3/k square root of nu/u infinity x right. So, what we will get essentially? Say tau all, what is our matter of interest is the non dimensional form of the tau all. So, the non dimensional form of the tau all is given by the cf, which is the friction coefficient, which is tau all/half rho u infinity square.

So, you divide this by half rho u infinity square. So, one of the u infinity's get cancelled, then you are left with there is one mu/rho, that means it is basically something * **"Professor - student conversation starts"** (()) (19:26) which one? Sir, x. This x is (()) (19:34). No, just I am using this expression. So, delta is square root of this one, whatever * square root of x okay. That is what is substituted here. **"Professor - student conversation ends"** So, this one something*nu/u infinity*root of nu x/u infinity.

So, that means cf, is coming out to be soothing * Reynolds number to the power -half. See, this becomes square root of u infinity, this numerator becomes square root of nu. So, square root of nu/u infinity x right. So, this constant if it is evaluated, this comes out to be 0.646 okay. And the exact solution, the Blasius solution gives this one as 0.664. and this is really quite close. So, the error between these 2 may not be that significant for a calculation for an engineering calculation.

And therefore you see that remarkably although the velocity profile was erroneous, although the boundary layer thickness was quite erroneous, this is also erroneous but the error has somewhat gone down. And that is because of using this in the integral form. So, the whole expectation is that the functions may be inaccurate but, area under the function has area under the curve that may be approximately the same, despite the error in the functions themselves.

So, this is, this sometimes is called as skin friction coefficient. So, sometimes this is given with a subscript x. So, cf with subscript x for cf has a function of x and the name of these is known as skin friction drag coefficient. Why it is so? Because we will later on see that, there may be other mechanisms of having a drag force on a body. So, here this drag force is originated out of the frictional action or the viscous action. So, this is given a name of skin friction drag coefficient.

The subscript x to indicate that it is local friction coefficient. That means as you change x, this will vary. Now, one is interested at the end to calculate the total drag force on the plate. So, how do you calculate the total force on the plate.

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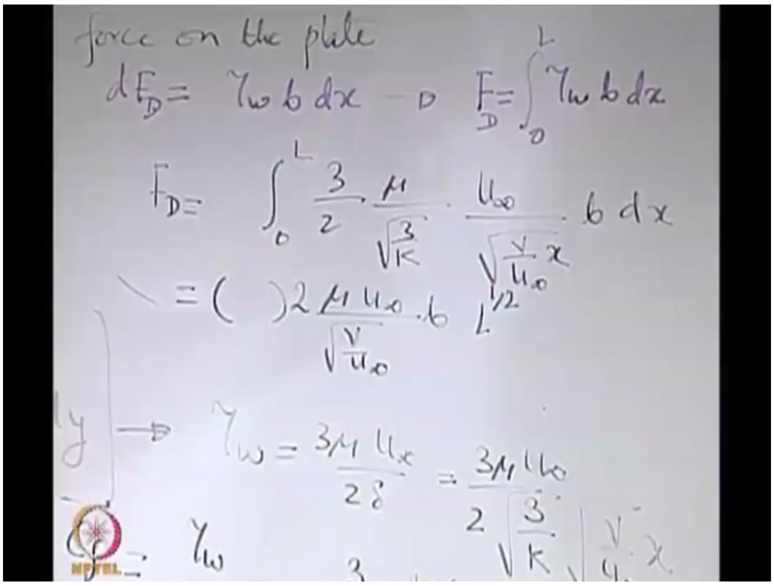

To calculate the total drag force on the plate. Basically you have to keep in mind that the wall shear stress varies locally with x. and it is important to take care of that local variation. So, to take care of that local variation, what we may consider is, see let us say at a distance x, we take a small strip on the plate of thickness dx. So, the plate is having some width so, it is basically there is a strip, which has some width. So, let us say that, the width of the plate is b.

So, the element that we have considered is something like this. And the shaded portion, which is there, this is where it is exposed to the fluid. And that is where there is a shear stress between the fluid and the solid. So, how do you calculate that, what is the total shear force on this element, whatever is the shaded element. So, let us say that dF is the shear force on the element. So, what is dF? So, tau all is the wall shear stress that *b*dx okay. So, if you want to find out the total one.

Of course you have to integrate it so, the total F, let us give it subscript d to indicate that it is a drag force. So, FD will be integral of tau all b dx from x=0 to x=L, where you know that how tau all varies with x. So that is already known to us. So, Fd let us write integral 0 to L in place of this one, we will write 3/2 mu/square root of 3/k u infinity/nu/u infinity x*b*dx.

So, again if you consider the form like there will be some numbers based on what is the value of k, then mu u infinity divided by square root of nu/u infinity b, then just integral of x to the power -half dx. So, it will become this one. Now, just like the local skin friction coefficient is a non dimensional representation of the wall shear stress locally. There is also a non-dimensional representation of the drag force. And that is given by coefficient of drag. So, let us write what is the coefficient of drag.

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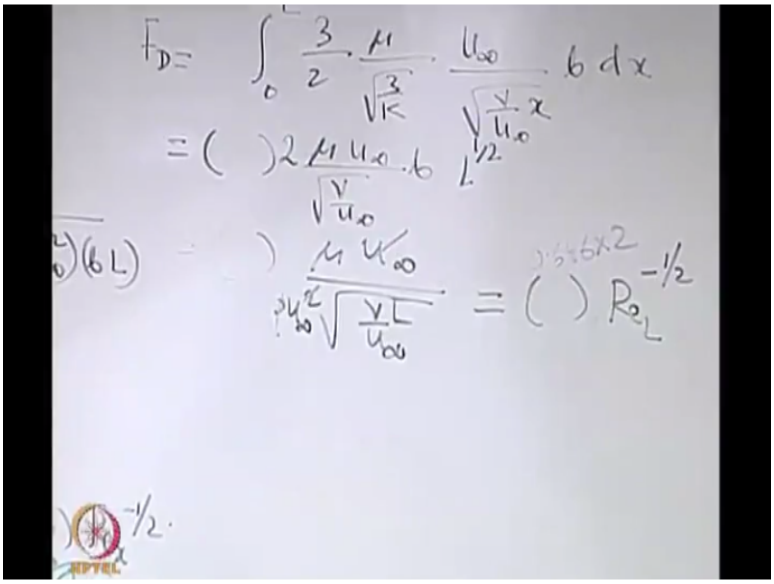

So, the coefficient of drag is given by FD divided by something reference. See, this is a force, so this should be represented by some equivalent half rho u infinity square that is there times some reference area. So, what area you will choose here? So, if L is the length of the plate, b*L. We will see later on the depending on the shape of the body, the area are engineering conventions of how what reference area should be taken. So, these fundamentally is like some reference area.

For flow over a flat plate, it is like this b*L, but for different shaped bodies, we will see if the flow passed a circular cylinder. May be flow passed a aerofoil section. How these reference areas will change from one to the other? It is nothing very fundamental. It is an engineering convention that what reference area you choose for designating the coefficient of drag. Now, here you can write it in like you will get some number times mu u infinity/square root of nu/u infinity then you get rho u infinity square*you will get a square root of L.

Because numerator square root of L, denominator is L. So, again you will get one of the u infinity gets cancelled out. So, this is something * Reynolds number to the power -half. But Reynolds number is based on L. so, this Reynolds number is not based on any local coordinate but, the total length of the plate. So, this gives the total effect and this coefficient is just double of this 0.646. So, it is whatever like 0.646*2.

And it is therefore also quiet close to the result that one gets from the exact solution or the Blasius solution. So, what we get out of these exercise is, we have seen that how by using an approximate velocity profile, we may estimate how the boundary layer thickness grows with x, how we calculate the wall shear stress and how we calculate the total drag force and their non dimensional versions. Again, because this is originating out of a skin friction force.

So sometimes this is given with a subscript CDF, F for skin friction because as I have mentioned, we will encounter such examples where there will be also other mechanisms of the drag force not just the wall shear stress okay. Next, we will try to go through some of the important concepts or important terminologies related to the boundary layer and these terminologies are displacement thickness and momentum thickness.

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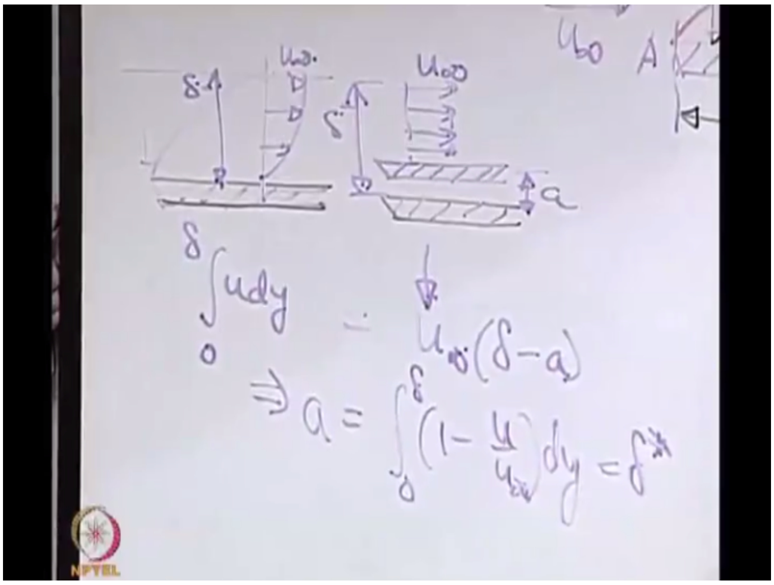

So, first terminology that we will try to understand is displacement thickness delta star. What it is? Let us say that, you have a flat plate but there is no growth of boundary layer. So, then what happens, if you have uniform flow, uniform flow will remain uniform and that means the plate is having no effect. So, if there was a stream line like this, this stream line would move parallel to the original direction. So, when you have uniform flow, the stream lines are actually parallel to the flow direction.

Let us say this is one of such undisturbed stream lines. Now, this is what that does not happen in reality. In reality what happens? In reality you have the growth of the boundary layer. Let us say that the boundary layer is growing and the boundary layer is growing like this. So, because of the growth of the boundary layer what happens, what will happen to the stream line? This stream lines will originally be parallel to each other but when they come close to the plate, will the stream line.

So, we have decided or we have come to a conclusion that the stream line cannot remain just like this. So, it has to get shifted or deflected. Question is, it will be deflected upwards or downwards. Upwards. So, let us try to see that why it should get deflected upwards. So, let us say that because of the effect of the boundary layer the stream line has got deflected upwards. So, this same stream line which was undisturbed now it has got deflected upwards. The reasoning is very straight forward.

If you consider 2 sections let us say that we consider one section here say section A and another section here say B. if the flow is steady, whatever is the flow rate through section A should be the same flow rate through section B. Because, see this black line, this is a stream line so there cannot be any flow across it okay. Now, if the same flow rate has to be maintained. See this is the region where the flow is slowed down.

To compensate for that, the stream line should be upwards so that this extra flow + this flow becomes same as this flow right. So, the stream line has to move upward not downward. Question is how much does the stream line move upward? Let us say that, we have chosen such a reference that this is the delta at the given x. Let us say that this shift of the stream line is delta star. We want to find out what is this delta star.

Nature is not important for us. This is just a rough sketch, what is important is that whether it is shifted upwards or downwards. Because like we have not having any attention on what happens across the stream line. Because by definition of the stream line, there is no flow across the stream line okay. So, let us try to find out this delta star. What would be the basis by which we may find it out?

That the mass flow rate at the sections A and B must be the same. So, you must have m dot at the section A same as m dot at the section B. So, if the uniform velocity was u infinity at the section A it is rho*u infinity*delta*the width. Let us say the width is what? B. the width will get cancelled in both the sides so let us not just write the width. For B, you see there is one portion within the boundary layer.

So, that is integral 0 to delta udy, where u is the velocity as a function of y here. Outside the boundary layer, you have u=u infinity. So +rho integral of delta to delta + delta star*u infinity dy. Basically that *u infinity*delta star. So, from here if you simplify one more step, what we get? You will get u infinity*delta=integral 0 to delta udy+u infinity*delta star. This term, you can also write as integral 0 to delta u infinity dy all the same.

The whole idea is that we want to club these 2 terms together. So, from here you can write delta star=0 to delta 1-u/u infinity dy by dividing all the terms by u infinity okay. So, this is known as displacement thickness. So, physically what it is indicating? Physically it is indicating may be a displacement or a shift in the stream line because of the existence of the boundary layer. It may also be looked from a different angle.

Let us say that we are trying to consider a case when this is a flat plate with boundary layer and retardation of the flow close to the wall and we consider an equivalent case where we do not consider the effect of the wall directly but what we do is, we make a shift of the wall. So, the new location of the wall becomes say like this. What should be this shift so that this problem with a boundary layer is equivalent to a problem with a uniform flow.

So, this is a problem with a boundary layer, where we are trying to have the same flow rate but no boundary layer. That means there will be a uniform flow. So, let us say that the uniform flow is like this with a velocity u infinity. So, here there is a growth of the boundary layer so we have our delta as a function of x. Here you do not consider the growth of the boundary layer, but to avoid that analysis what you do?

You ship the plate a bit upwards because the effect of the boundary layer is, it has the reduction in the flow rate. So, if you still want to use u infinity, you make a shift of the plate somewhat so that your effective area of the flow gets reduced. So, that multiplied with u infinity should give the same as this one. So, let us say that this shift of the plate is A. What is our constraint? Our constraint is that the flow rate should be the same.

This is the hypothetical uniform flow; this is the real boundary layer flow. So, the velocity profile is like this. This is u infinity. So, what is the flow rate here? The flow rate here is integral of 0 to delta udy. What is the flow rate here? U infinity*delta-a. of course I am just considering the volume flow rate per unit width of the plate. So, rho*b that term, I am not considering. So, if you equate this 2, what you will get from here is, what is a? A is nothing but integral 0 to delta 1-u/u infinity dy.

So, this is nothing but delta star. So, what does it mean? It means that the displacement thickness also may be looked at as a hypothetical displacement of the solid boundary so that the remaining flow within the length delta may be perceived as a hypothetical uniform flow. Still, predicting the same correct flow rate okay. So, the advantage of this is that, sometimes you do not want to analyze the details of the boundary layer but you just want to have a gross estimate of the flow rate.

If you know what is delta star, then what you may say is that, within this length, this-delta star whatever is there, the velocity is uniform. So, it is not that it is actually uniform, it is a pseudo situation, with which you are matching with the exact situation. What you are not compromising with is the flow rate prediction. So, the pseudo situation and the correct situation are giving the same flow rate, that is the basis of this.

Now, although the mass flow rates over the section a and b are the same, but the momentum flow rates are not the same. And we will see that what is the difference in the momentum flow rates over the sections a and b.

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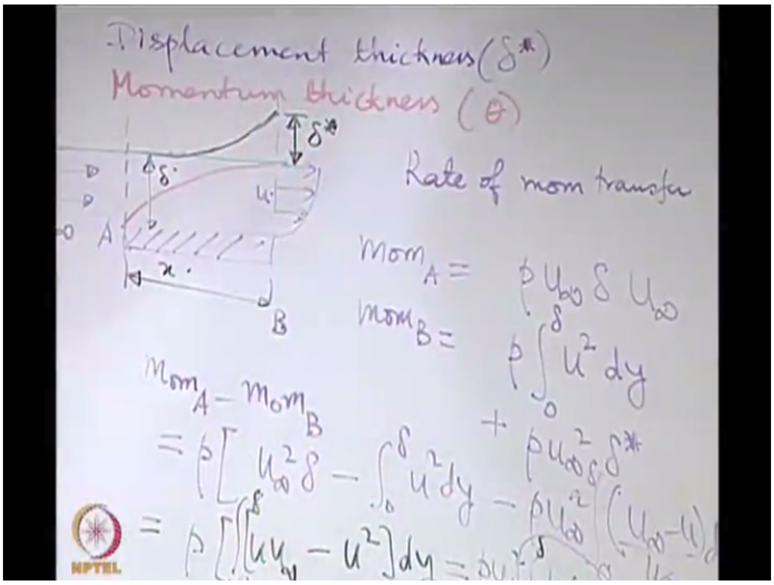

So, let us write what is the momentum flux or may be rate of momentum transfer. So, the rate of momentum transfers at A, what is this? So, what is the rate of mass transfer at A? rho u infinity*delta, that multiplied by u infinity is the rate of momentum transfer at A. what is the rate of momentum transfer at B? again one part within the boundary layer, another part outside the boundary layer.

So, whatever is within the boundary layer, you have rho integral of u*u, that is u square dy from 0 to delta + it is basically rho u infinity square*delta star. That is the top portion. Top portion outside the boundary layer, velocity is u infinity. So, if you want to find out what is the difference between these 2, momentum flux, rate of momentum transfers at A and B, the difference of these 2.

So, that you have rho*u infinity square delta-integral 0 to delta u square dy-rho u infinity square*delta star is what? Delta star is u infinity-u/u infinity integral of that dy from 0 to delta. So, one of the u infinities get cancelled out okay. So, when one of the u infinities gets cancelled out, the next step we can have a simplification, let us go for that simplification. So, this becomes, first of all this becomes -rho u infinity square delta. So, there is one u infinity square delta, then there is -1 u infinity square delta.

So, this term gets cancelled out with the first term. So, what remains is rho u infinity-u square dy from 0 to delta. Uu infinity from the last term and -u square from the second term okay. So, now what you see that, this is not 0 right, therefore there is a difference. Now, is this positive or negative? This is positive, because u infinity is >u. So, u*u infinity is > u square. So, that means this is actually a representative of the very important fact that, there is a momentum deficit at the section B.

So, whatever was the rate of momentum transport at the section A, the rate of momentum transport at the section B is somewhat < that. And this is indicator of how less it is. So, this you can write as rho*u infinity square, just taking u infinity square as reference. SO, u/u infinity*1-u/u infinity dy. Just a normalized or non dimensional way of writing is the term, which is there in the integral. And see, this term in the integral is the term that we have encountered in the momentum integral equation.

Tau all/rho u infinity square was ddx of this quantity. And this we call as theta, which is also given as symbol of momentum thickness. So, momentum thickness, that is given by theta. And what is important is that, this momentum thickness, what it physically indicates? It physically indicates the deficit in the rate of momentum transport across 2 sections because of the development of the boundary layer.

That is the physical implication of this. But, mathematically this is expressed just by this integral.

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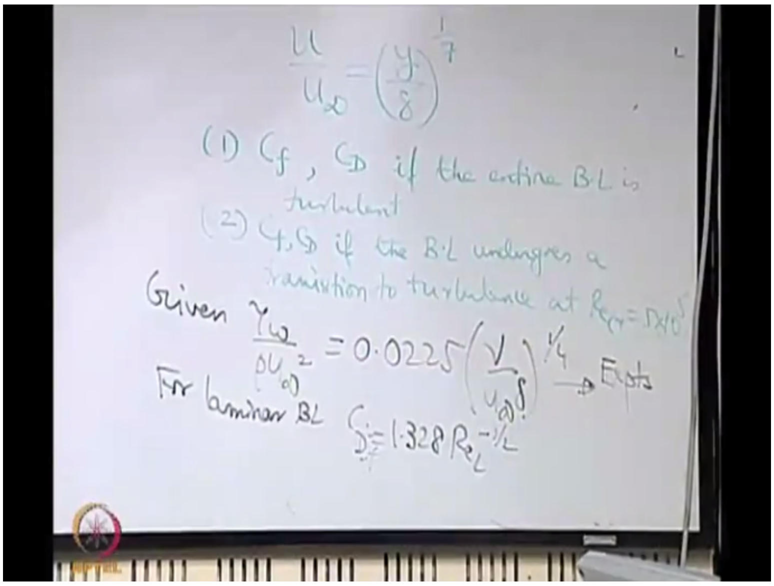

Sometimes, one also uses a third parameter which is known as the shape factor H, which is given by delta star/theta. The ratio of these 2. Why it is given a name of a shape factor is somewhat obvious. The obvious is that, this ratio depends solely on the shape of the velocity profile. Because, if you see the integrals, this integrals of course they themselves depend on the shape of the velocity profile but sort of even their ratio also therefore depends only on the shape of the velocity profile.

Shape of the velocity profile means, u/u infinity as a function of yy delta. That is the shape of the velocity profile. So, once that shape is fixed, this is also automatically fixed. So, that is why we call it a shape factor and given a velocity profile, it is possible to determine the shape factor. Now, the important thing is that we have till now, consider the boundary layer have got a flat plate and that is one of the very simple cases, but even in this very simple case, we have not considered one thing that is the effect of turbulence.

We have considered that means implicitly that, the boundary layer develops as a laminar boundary layer. But, in reality, the boundary layer may have a transition towards turbulence. And what is the Reynolds number, that is important here? The Reynolds number here is dependent on the parameter the axial location x. So, you see that as you move along the plate, this Reynolds number increases.

Beyond a critical Reynolds number, you have the inertia forces dominating so much that a slight disturbance may trigger the onset of turbulence and then the laminar boundary layer changes its characteristic to a turbulence boundary layer. May not be abruptly but at least over a given distance. And the critical x or the critical Reynolds number at which this transition occurs for flow over a flat plate is roughly 5*10 to the power 5.

So, you see that the critical Reynolds number depends on different geometries. See, for flow through a circular pipe, the critical Reynolds number was roughly of the order of 2000, there the reference length was the diameter of the pipe. Here, the reference length is the axial coordinate. There the reference velocity was the average velocity. Here the reference velocity is the free stream velocity u infinity.

So, the critical Reynolds number is therefore not a magical number which is true for all cases like depending on the situation, your reference length changes, you reference velocity changes and the convention changes altogether. Now, what happens to the growth of the boundary layer if you have such a transition? To understand that, let us work out a problem by which we will best illustrate it.

So, just note down this problem that consider the turbulent boundary layer over a flat plate for which u/u infinity for which u/u infinity is given by y/delta to the power of 1/7 okay. So, this is the velocity profile for the turbulent boundary layer. So, when you say u, remember this we are not writing it explicitly but this is the average velocity. This is not the instantaneous velocity with fluctuations.

So, u/u infinity is y/delta to the power 1/7. This is given for the turbulent boundary layer. What you have to determine, number one, the CF and CD, that is the local skin friction drag coefficient. And the total drag coefficient on the plate for the flow over the plate, if the entire boundary layer is turbulent and number 2, the same things CF and CD, if the boundary layer undergoes a transition to turbulence at Re critical=5*10 to the power 5 okay.

Then, what is given, given is that at the wall, this is what is given at the wall. We will see that why this is given at the wall. This is from experiments. This is actually a real data from experiments conducted by Blasius. So, this is from experiments and for laminar flow, laminar boundary layer, it is given that CDF=1.328*Reynolds number to the power -half okay. So, with this problem statement, let us try to work out this problem and this will give us some idea.

First of all, this is the velocity profile which is again for turbulent flow, it is not possible to determine the velocity profile exactly. So, this is just a conflict of the experimental velocity profiles and therefore, it is not a very accurate one. The loss of accuracy is important at some place and let us see what is the place.

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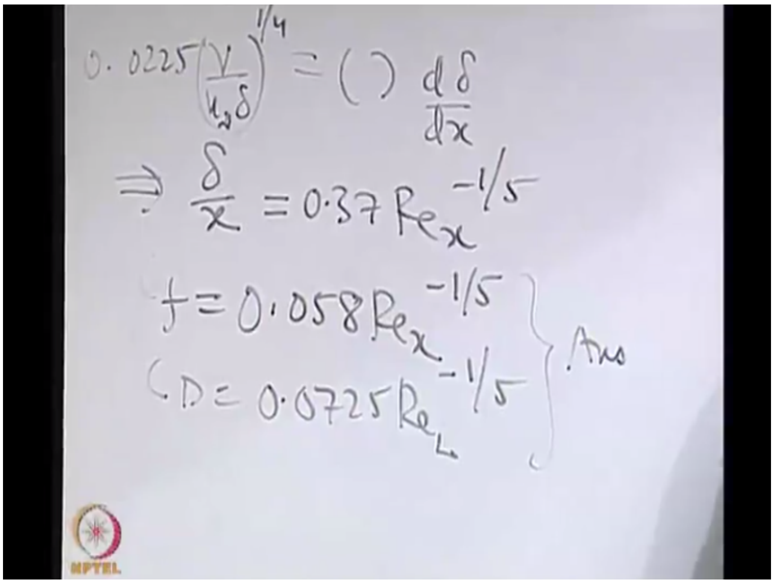

So, let us first write tau all/rho u infinity square=ddx of integral 0 to delta u/u infinity*1-u/u infinity dy. So, this part you may substitute u/u infinity as a function of y/delta and then you will get this as something*d delta dx that is fine. Substitute u/u infinity as y/delta to all these. Now, what about tau all? See, if you want to calculate mu du dy at the wall, you see that, that you cannot calculate using this velocity profile.

So, this velocity profile is not exactly valid at the wall for calculating the wall shear stress. And that is why, the wall shear stress has to be separately determined from experimental conditions. And that is what it has been experimentally determined. See, fluid mechanics is such a subject where you cannot independently go or grow with theory or experiment. You have to somehow make a good combination of these for understanding the physical principles.

So, tau all/rho u infinity square, you will have 0.0225*nu/u infinity*delta to the power 1/4. So, then it is straight forward, you integrate this and get delta as a function of x. so, if you integrate this, you will get let me just tell the answer quickly, you will get delta/x=0.37*Reynolds number to the power -1/5 okay. So, once you get delta/x, it is easy to calculate the CF and C just as we did for the previous cases. So, let me give you the answers at least so that you can verify later on.

So, Cf will be 0.058*Reynolds number to the power -1/5 and CD is 0.0725*Reynolds number to the power -1/5 okay. So, these are the answers for the first part of the problem. But, this is not a very realistic representation why? Because we know that the boundary layer does not become turbulent from the very beginning. The Reynolds number is based on the local x. Local x is initially very small so it is always initially laminar and then it becomes turbulent. So, the real picture may be something like this.

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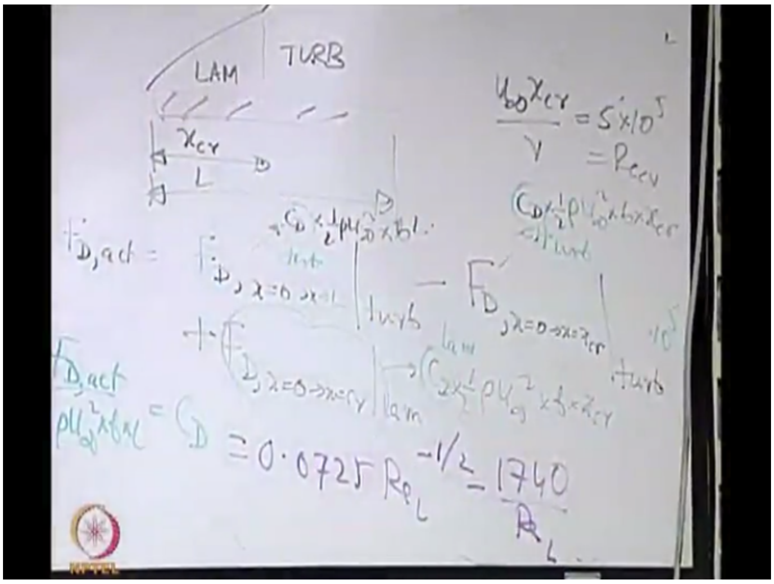

So, you may have the flat plate in this way. You have first a laminar boundary layer, then it undergoes a transition to turbulent boundary layer. Let us say that this is the length x critical over which it is laminar and then beyond that, it is turbulent. So, the question will be that, what is the length of your plate? If the length is > x critical, how do you find out x critical? X critical is given by u infinity x critical/mu is 5*10 to the power 5, that is Re critical.

So, given u infinity, you can always find out, what is x critical. If your length of the plate is > x critical, then in reality, there is a great chance that the remaining flow is turbulent. So, your actual drag force is what if you say calculate the total drag force has artifact or outcome of that, the entire boundary layer is turbulent, there is an error because of the presence of the laminar part. So, that error we have to nullify.

So, what we can do? We may consider that it is F drag force, that is the drag force width from x=0 to x=L considering it as fully turbulent. Then subtract the drag force from x=0 to x=x critical for turbulent and add the drag force for that part for the laminar okay. So, as if the full thing was turbulent, then you subtract the turbulent part from the initial and at the laminar part okay.

And you have to keep in mind that, when you are writing the corresponding the CDs, so, this will be CD*half rho u infinity square *b*L. Here, it will be CD*half rho u infinity square b*x critical. And here, also it will be the same. CD*half rho u infinity square b*x critical. The difference is that, in one case the Cd is the laminar Cd, in another case, this is the turbulent CD. So, this is turbulent CD, this is the turbulent CD and this is the laminar CD.

In the problem, already it is given what is the expression for the laminar CD, you have to be careful in place of L, you have to now use the x critical as the reference length. And then, if you make a simplification of this, the net drag force is the F drag force actual/half rho u infinity square*b*L. It is the net CD and the answer of this will come out to be 0.0725 Reynolds number to the power -half-1740/Reynolds number. This is the answer to this.

So, if you just substitute and make a simplification, just check that whether you get this final expression. So, what this final expression says that, there is a correction because of a part of the boundary layer being laminar and not the entire boundary layer being turbulent. And this the expression of the composite CD that takes care of the fact that a part is laminar and then it is turbulent.

So, with this we stop our discussion today and in the next class, we will start with the discussions on boundary layer, where you have the effect of the pressure gradient. Till now, we have considered the boundary layer without the effect of pressure gradient. And boundary layer with the effect of the pressure gradient, we will start discussing from the next class. Thank you.