

**Introduction to Fluid Mechanics and Fluid Engineering**  
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**Lecture - 46**  
**Pipe Flow (Contd.)**

We were discussing about fluid flows through pipes, and working out some examples. Let us continue with another example.

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The image shows handwritten notes and a diagram for a fluid mechanics problem. The diagram depicts a horizontal pipe of diameter  $\phi = 8 \text{ cm}$  and radius  $R = 40 \text{ mm}$  containing air at  $1 \text{ atm}$ . A U-tube manometer is connected to the pipe, with one end at the centerline (point A) and the other at the wall (point B). The manometer fluid is water, and the height difference between the two limbs is  $\Delta h = 40 \text{ mm}$ . The notes include the following:

- Friction factor equation:  $f = \frac{\tau_w}{\frac{1}{2} \rho U^2}$
- Given values:  $k = 0.41$ ,  $B = 5$
- Velocity profile equation:  $U = \frac{1}{k} \ln \left( \frac{y}{B} \right)$
- Relationship between centerline velocity  $U_{CL}$  and average velocity  $U$ :  $\frac{U}{U_{CL}} = \frac{1}{1 + 1.33 \sqrt{f}}$
- Final result:  $U_{CL} = 20.5 \text{ m/s}$
- Bernoulli's equation derivation for the manometer:  $\frac{P_1}{\rho_a} + \frac{V_1^2}{2} = \frac{P_2}{\rho_a}$ , leading to  $V_1^2 = 2 \Delta h g \left( \frac{\rho_w}{\rho_a} - 1 \right)$

Let us say that you have a piping system or a single pipe in this example, where a pitot tube is put, the fluid that is being handled in the system is air, and the manometric fluid is water, the difference in the height of the 2 limbs of the manometer say is  $\Delta h$  which is given, the diameter of the pipe is 8 centimeter and the pressure upstream is one atmosphere approximately 101 kilopascal.

Now what you asked, number 1 estimate the centerline velocity, estimate the volume flow rate and the wall shear stress assuming smooth wall. Given dimension of  $\Delta h$  is 40 millimeter and a relationship between the centerline velocity and the average velocity is given as  $1/1 + 1.33$  square root of  $f$  where  $f$  is the friction factor okay. So this is the description of the problem. Now let us first look into the problem from a very basic consideration of the pitot tube.

That what you really can write regarding the difference in the properties between say 1 and 2, so the point 2 is supposed to be the so-called stagnation point, and between 1 to 2 if one uses the pitot tube uses the well-known expression of the pitot tube, and that expression if you recall it does not account for any energy loss, it considers that the fluid is undergoing a reversible process without any energy loss.

And then it is possible to use the Bernoulli's equation along a streamline between points 1 to 2, so without any loss therefore, it is like  $p_1/\rho + V_1^2/2 = p_2/\rho$  this is ideal right, and this is what equation is written for the pitot tube. Now somebody who is a very casual engineer will do a mistake in what? Will do a mistake in having these points 1 and 2 at some distance apart so that head loss between these 2 becomes important.

If these 2 points are very close, then that head loss may not be important, but a very wrong approach of engineering maybe to put them at some distance apart, I mean of course they will be at some distance apart what question is how to make that affect minimum. Because you are using an idealized equation, where that affect is not present trying to predict whatever the velocity at the state 1 or at the point 1 using that, but that will itself be erroneous if head losses are significant.

Now between  $p_1$  and  $p_2$  of course you can relate that what changes by using the principle of manometric, so let us say that this height is  $h$ , so you can write the if you consider the same horizontal level you have the pressure at A is same as pressure at B that is you have  $p_1 + h \rho_{\text{air}} g + \Delta h \rho_{\text{water}} g = p_2 + h \rho_{\text{air}} g + \Delta h \rho_{\text{air}} g$ , so from here you can when you write this  $p_1/\rho$  this is  $\rho$  of what?  $\rho$  of air or  $\rho$  of water in this equation  $\rho$  of air right.

So to make use of this equation we just divided it by  $\rho$  of air, but before some terms will be cancelled out like  $h \rho_{\text{air}} g$  from both sides, so  $p_1/\rho_{\text{air}} - p_2/\rho_{\text{air}} = \Delta h g (1 - \rho_{\text{water}}/\rho_{\text{air}})$  that means you have  $V_1^2/2g = \Delta h g (\rho_{\text{water}}/\rho_{\text{air}} - 1)$  sorry that  $g$  is like this okay. So from here you can find out what is  $V_1$ , all other things are given, so you will be able to find out a value of  $V_1$ , let me just tell you that what value you are expected to get out of this one.

So you will get  $V_1$  as 25.5 meter per second. Now this is from an idealized analysis, the other important thing is that how this  $V_1$  is related to, so this  $V_1$  is what? This is velocity at a point, so velocity at a point on the centerline. So this is as good as the velocity at the centerline, one is the point on the centerline, so we need not be confused between a point and the average over a section, so this is at a point we are writing okay.

So because this emerged from the Bernoulli's equation between 2 points. Now this  $u$  centerline and  $u$  average they are related by this equation, how this equation comes is a bit of a background information, but it is quite simple. That is this comes from the logarithmic law application of the logarithmic law for the velocity profile, so it is assumed for this problem that it is a turbulent flow over a smooth wall of a pipe.

And in the turbulent flow so the velocity profile is taken like this in this form okay, so here  $u^+$  is what?  $u^+$  / some  $u$  reference, this one and  $u$  reference is square root of  $\tau_{wall}/\rho$ , and what is  $y$  here?  $y$  is nothing but capital  $R$ -small  $r$ ,  $y$  is the distance from the wall okay. So in this  $y$  is the distance from the wall, not any arbitrary co-ordinate, so in the pipe co-ordinate system the cylindrical co-ordinate system this is the distance from the wall.

So from this type of velocity profile, you can find out the average velocity by integrating this over the section integral of  $u$   $dA$ /the area, so and you can find out the velocity at the centerline by putting small  $r$ =capital  $R$  okay, and then sorry small  $r$ =0 that means  $y$ =capital  $R$ . Then you will get 2 expressions, remember that these  $kappa$  and  $B$  are like sort of constants,  $kappa$  is 0.41 maybe a bit more accurate, and  $B$  for a smooth pipe is close to 5,

We have mentioned this earlier when we are discussing turbulent flows. Now with that if you find out the ratio you may relate that ratio with the friction factor, how you relate the ratio with the friction factor? You know that  $\tau_{wall}=C_f$  the friction factor is what?  $\tau_{wall}/1/2 \rho u_{average}^2$ , and  $\tau_{wall}$  and  $u_{average}$  are related by or you can relate  $\tau_{wall}$  with  $u_r$  this  $u$  reference, and you may relate  $u_{reference}$  with  $u_{average}$ .

And how you relate  $C_f$  with  $f$ ? So  $C_f$  is  $f/4$  right, so if you use this expression you use this velocity profile, and for averaging also use the same thing relate that with the friction factor you will get this expression. This may be exactly derived by putting these numbers, so that is how this is there, it is not just a very magical thing just from the very basic understanding of the velocity profile and it is averaging.

Now let us say that  $u$  have this as a centerline velocity,  $u$  are having a relationship between the centerline and the average velocity, if you know that then in one way you may straightaway write the average velocity volume flow rate like that, but only hindrance is you do not know the friction factor okay. So one of the ways again maybe by the case by the trial and error, so let us say that you make a trial say  $u_{\text{average}}/u_{\text{centerline}}$ .

See trial and error is not always heat and means, it requires some intelligence and understanding of the problem. So if I give you a choice  $u_{\text{average}}/u_{\text{centerline}}$  say 3 trials one is 0.1, another is 0.5, another is 0.8, out of these 3 which one you expect to be a better trial? 0.8 is expected to be a better trial in this case why? Because remember we are talking about turbulent flow, where the velocity profile is almost uniform.

So there is not a great difference between the centerline velocity and an average velocity, so of course the average velocity will be less than the centerline velocity no doubt about it, but how much it is less it depends on the skewness in the velocity profile. So for a turbulent flow it is almost uniform and therefore, like if you have such choices maybe 0.8 or 0.85 or 0.9 these types of guesses are reasonable guesses.

And so still if you guess 0.1 or 0.5 still okay, but let us say you guess these are 2, then it is absolutely erratic because I mean that it does not matter whether its laminar flow turbulent flow or whatever, the central line velocity is always greater than the average velocity. And so these types of basic considerations should be kept in mind any time when you are having a iterated solution or a trial solution and putting a guess for that.

So when you substitute this trial you will get a value of  $f$  from this equation straight away, once you get the value of  $f$  from this is equation, then the question is that, is this  $f$  what comes out from the relationship that you get from the Moody's diagram? Here, remember that we are talking about a smooth wall, so hydraulically smooth pipe for that the friction factor should not be dependent on the wall roughness.

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Handwritten notes on a whiteboard showing calculations for friction factor  $f$ , flow rate  $Q$ , wall shear stress  $\tau_w$ , and the relationship  $f = \frac{0.316}{Re^{0.25}}$ . The Reynolds number  $Re$  is circled and has an arrow pointing to the right.

$$f = 0.0175$$

$$Q = 0.109 \text{ m}^3/\text{s}$$

$$\tau_w = 1.23 \text{ Pa}$$

$$f = \frac{0.316}{Re^{0.25}}$$

So for a smooth wall, it will depend on only the Reynolds number, and for the hydraulically smooth pipe this is like  $0.316/\text{Reynolds number to the power } 0.25$ , otherwise one may directly read from the Moody's diagram the corresponding graphically plotted values without looking into the function. So that means if you know  $f$ , you will get a Reynolds number right, and the Reynolds number is what?

The Reynolds number is based on the average velocity right  $\rho u_{\text{average}} * D / \mu$ , so once you get this  $f$  you will get a Reynolds number, and that means you will get the average velocity, put that average velocity back here, and see that you get a new  $f$ . So in this way you iterate till you come to a convergence okay. So important consideration is that to have a distinction between the centerline velocity and the average velocity.

The whole understanding is when you are applying the sort of Bernoulli's equation between the 2 points and neglecting losses, you are talking about only velocities at the points. Whereas

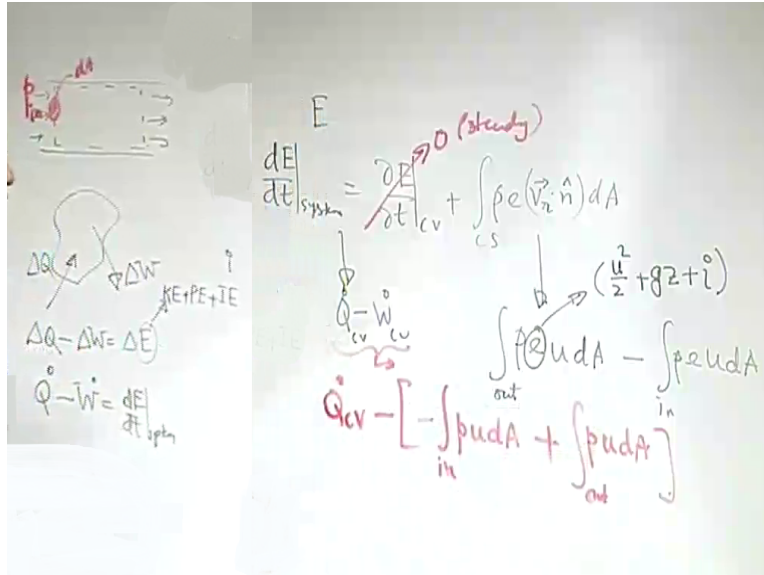
whenever the head losses are calculated, those are based on the Reynolds number, which takes reference velocity as the average velocity over the section not velocity at a point that is the key concept that is used for solving this problem.

So let me give you the answer, I mean once it is converged then the remaining calculations are very straight forward, and I need not repeat, but let me just give you the answer. So the  $f$  = the converged value of  $f$  is 0.0175, then the  $Q$  is 0.109-meter cube per second and  $\tau_{wall}$  is 1.23 Pascal okay. So these kinds of practical examples are important because in practice you have energy losses or head losses.

Next what we will see is that we have till now discussed about the head loss, but how is the head loss related to the energy of the fluid that may be interesting to us, because in the very beginning of our course when we are talking about in inviscid flows we were discussing about the Bernoulli's equation, and we found out later that the Bernoulli's equation sort of gives the mechanical energy balance for a system for flowing fluid.

Now therefore, here we are seeing that even we might be tempted in using the Bernoulli's equation, but because of certain losses that may not directly be applicable there might be certain errors, so these losses must have some relationship with the energy consideration in the pipe flow. So let us look into a bit more details of the energy consideration in pipe flow.

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The objective will be to figure out that how the head losses are related to the total energy balance, let us say that you have a pipe of whatever sections, say a circular pipe as an example and we are looking for the fluid in the pipe, and trying to write an expression for the energy balance. So for any conservation equation we may start with the Reynolds transport theorem, so let us start with the Reynolds transport theorem, where  $E$  is the total energy of the system.

So we can write  $dE/dt$  for the system, where capital  $E$  is the total energy and small  $e$  is the energy per unit mass okay. So now let us make certain assumptions for simplification that let us assume that it is a steady situation, so the unsteady terms go away. So when you have this steady situation, the next thing what you do? The next thing what you do is you try to write this expression for  $dE/dt$  of the system.

So that it is what it gives the total rate of change of energy of the system, and so if you have a system like this of whatever arbitrary configuration, the total energy change of a system is something of fixed mass and identity, and for that the total energy change is given by a particular form of the first law of thermodynamics. So you are having basically some interaction of heat and work.

So you have let us say there is a heat transfer to the system say  $\Delta Q$ , there is a work done by the system say  $\Delta W$ , these are positive sign conventions that we will follow. So any heat

transfers to the system we consider as positive, any work done by the system that is energy flowing out of the system because of work we consider as positive. So let us say that some heat is transferred to the system.

And as an example some part of that is used to do work, the remaining will change the energy of the system. So you have  $\Delta Q - \Delta W = \Delta E$  = the change in energy of the system, obviously I am not writing these terms in a very formal way, whenever in the next semester when you will be studying thermodynamics you will be studying in details of how to formally write all these terms, but we are just trying to make use of this, and I am just trying to be at your level.

So that we can proceed further. Now when you write this energy keep in mind that this energy is the sum total of kinetic energy, potential energy and anything else other than kinetic and potential energy which is a function of the internal configuration, which we call as internal energy. So let us just symbolically write it kinetic energy + potential energy + internal energy, in books of thermodynamics internal energy is given symbol of  $u$ .

As you have noticed maybe earlier, but here because we already use  $u$  for velocity, we use just  $i$  as a symbol for internal energy to avoid the confusion in the terminologies. Now this equation you can also write as a rate equation, so you can write  $\dot{Q} - \dot{W} = dE/dt$  of the system, so this we can write  $\dot{Q} - \dot{W}$ ,  $\dot{Q} - \dot{W}$  of what?  $\dot{Q} - \dot{W}$  for a system, but in the limit as  $\Delta t$  tends to 0 when it is derived, this is same as this for the control volume as well.

So this is as good as  $\dot{Q}_{\text{control Volume}} - \dot{W}_{\text{control volume}}$ , what is the control volume? So you have chosen some control volume which is across which some fluid enters in the pipe and it leaves the pipe, and the boundary of the control volume is shown by this dotted line. Now let us concentrate on the right hand side, first of all this control volume is stationary, so that the relative velocity and the absolute velocity they are the same.

So this will be  $= \int \rho e$ , so  $\int V \cdot n \, dA$  for the outflow it will be positive and the inflow it will be negative, so we can say that  $\int \rho u \, dA$  for outflow boundary -  $\int \rho e u \, dA$  for the inflow boundary right. Because we have now lost the vector sense,



so we have put the proper algebraic sign to represent a vector sense. Now next is to split different terms based on like what are the important effects.

So heat transfers, so heat transfer of course there may be some heat transfer to the system or away from the system say you are heating the wall of the pipe is heated, so there may be a heat transfer from the surrounding to the system, if it is not heated then also there may be a heat transfer because of the temperature difference between the ambient outside and the fluid that is there in the pipe, and how such temperature difference may be created we will try to see.

Now if you consider concentrate on the work done, so what is the work done here? By the fluid in the control volume. First of all, you have the fluid let us consider the inflow, the fluid is can entering with pressure  $p$ , so it is putting some energy to the control volume as it displaces some fluid and enters it, so what is the corresponding work done, see we have related this with the flow energy or flow work, so that is same what we are referring here.

So if you have let us say a small element of area here say  $dA$ , so the elemental work done is  $p \text{ in} * dA * u$  the displacement, here we are writing the rate so the rate of the displacement there is the velocity, so  $p \text{ in} dA * u$  integral of that over the entire area sign+ or-, you see this consistency of the first law of thermodynamics, see any energy in the form of work it if it is transferred from the inside of the system to the outside it is positive.

Here, the energy is being put into the system, so that is negative work in terms of the work. So that means you have  $Q \text{ dot-}$  so you have -integral of  $p u dA$  for the in, and for the outflow it will be+ okay, so that is the left hand side expression that we are having. Now the next is we are assuming that  $\rho$  is a constant for this problem or for this discussion, so when  $\rho$  is a constant we can take  $\rho$  out of the integration, in place of  $e$  what we can write? This is energy per unit mass.

So first say if you write first kinetic energy  $u^2/2$ , potential energy  $gz$  and internal energy per unit mass  $i$  okay, so if you collect all the terms what you get at the end? So you get  $Q \text{ dot} C_v$ , let us say that you take these terms of integral  $p u dA$  to the right hand side, so if you take these

terms to the right hand side you will see that it will club up with this one's  $u^2/2 + gz + i$  that you will have one  $p/\rho$ , because  $\rho$  is there as a multiplier, this is just  $p$  alone, to adjust with that you will have one  $\rho$  multiplier outside, so in the bracket what will enter is  $p/\rho$ .

So you will have this = say if you take  $\rho$  outside then integral of  $p/\rho + u^2/2 + gz + i$   $dA$  same thing for the inflow. Now the next important thing is the integration of whatever integral appears in the 2 inflow and the outflow boundary terms. So when you write this integration, you have to keep one thing in mind that you have to be careful whether the properties are varying over the cross-section or not, because these are integrals over the cross section.

So let us assume that the pressure is not substantially varying over the cross-section, and that is in a way through that we have seen that the major pressure gradient is along the  $x$  direction. Then  $u$  will definitely vary with the cross section, because you have a velocity profile, it is not a uniform flow, the potential energy effect that also you may consider that the pipe diameter is not so large that there will be a great difference in potential energy effect.

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Handwritten derivation of the energy equation for a control volume in a pipe. The derivation shows the balance of energy fluxes and work done by pressure forces. The final result is the definition of the kinetic energy correction factor  $\alpha$ .

$$\frac{p_1}{\rho} + \alpha_1 \frac{\bar{u}_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{u}_2^2}{2} + gz_2 + \left[ \frac{I_2 - I_1}{\rho} - \frac{\dot{Q}_{cv}}{m} \right]$$

$$\dot{Q}_{cv} = \frac{p_2}{\rho} \dot{m} + \frac{\rho}{2} \int u^3 dA + mgz_2 + \dot{m}i_2 - \left[ \frac{p_1}{\rho} \dot{m} + \frac{\rho}{2} \int u^3 dA + mgz_1 + \dot{m}i_1 \right]$$

$$\alpha_2 \frac{1}{2} \dot{m} \bar{u}^2 \quad \alpha = \text{K.E correction factor}$$

$$\alpha \frac{1}{2} \rho \bar{u}^3 A = \frac{\rho}{2} \int u^3 dA$$

$$\alpha = \frac{\int u^3 dA}{\bar{u}^3 A} = \frac{\int \left(\frac{u}{\bar{u}}\right)^3 2\pi r dr}{\int \left(\frac{u}{\bar{u}}\right)^3 2\pi r dr}$$

So certain terms of these you may vary confidently take out of the integral assuming that those are constants, so like you can for example write say you divide  $\dot{Q}_{cv}/\rho$ , so one you have  $p/\rho$  or let us do one thing we will divide by  $\rho$  in the next stage. We will now see that if you

take  $p/\rho$  out of the integral, then what you are left with in the integral is integral of  $u \, dA$  that is the volume flow rate that multiplied by the density is the mass flow rate, so we call it  $\dot{m}$  okay.

Next is you have  $\rho/2$  integral of  $u^3 \, dA$ , so this is remember we are writing for the outflow, so let us give some names to this areas, 1 for the inflow area, and 2 for the outflow area, so this is  $p_1/\rho$ , this is integral over the section 1. And then similarly, this is  $\dot{m} g z$ , if you assume that the temperature is also not varying over the section, then the internal energy  $u$  may assume to be a constant over the section, so+ say  $\dot{m} i$ , this is 2 right yes okay,  $\dot{m} g z$  this is  $2 i^2$ .

Then -similar terms for 1, so  $-p_1/\rho \dot{m} - \rho/2$  integral of  $u^3 \, dA$  for the 1,  $-\dot{m} g z_1 - \dot{m} i_1$  okay, now let us say that we neglect this effect, we do not neglect this effect of velocity variation out right, but we somehow make up for our negligence. See if we do not consider this effect altogether and say that the consider that the same velocity, velocity is same as the average velocity is there, then one approximation to this term could be  $1/2 \dot{m} u_{\text{average}}^2$ .

Because this is what? This is like kinetic energy, because  $\rho u \, dA$  is like  $\dot{m}$  and that  $u^2$  square is like this one, but this is erroneous, why this is erroneous? Because this is not exactly same as this this one, because  $\dot{m}$  is what?  $\rho u_{\text{average}} A$ , so  $u_{\text{average}} u_{\text{average}}^2$  is  $u_{\text{average}}^3$  that is not same as integral of  $u^3 \, dA$ , so this is an error, and that error has to be adjusted with the multiplying factor say  $\alpha$ , which we call as kinetic energy correction factor.

So this  $\alpha$  at the section 2 this may be different at different sections, because velocity profiles may be different in general over different sections. So this  $\alpha$  is known as kinetic energy correction factor, so what is this correction factor all about? This is the correction factor to correct the kinetic energy from a hypothetical consideration that it is based on the average velocity to the real kinetic energy that is there integrated over the cross-section.

So the kinetic energy correction factor will be what? So you have  $\alpha \dot{m} u_{\text{average}}^2 = \rho u_{\text{average}} A u_{\text{average}}^2 = \rho/2$  integral of  $u^3 \, dA$  okay. So you can now write an expression for  $\alpha$  as  $\alpha = \frac{\rho/2 \int u^3 \, dA}{\dot{m} u_{\text{average}}^2}$  this one okay, so if it is for a

circular pipe, so this is as good as  $u/u$  average whole cube  $dA$  is  $2\pi r dr/\pi R^2$  from 0 to  $R$ . So you can calculate the kinetic energy correction factor given the velocity profile.

Now can you tell, whether it will be more for laminar flow or turbulent flow? Laminar flow, why it should be more for laminar flow? So it depends on the  $u/u$  average right, so  $u/u$  average it is  $u$  deviates from  $u$  average significantly more for laminar flow, so you will have a more significant value of this one deviated from one, so if  $u$  was  $=u$  average throughout then kinetic energy correction factor would be 1, if it is slightly deviates from  $u$  average then it will be very close to 1.

But if it is largely deviating from  $u$  average say consider the fully developed laminar flow through a circular pipe, so  $u$  is  $u$  centerline is  $2u$  average, so you can see there is a large difference, and that 2 factor will be there if you consider this kinetic energy correction factor. So it will be a large value, so I will leave it you leave it on  $u$  as an exercise that you calculate the kinetic energy correction factor for fully developed laminar flow through a circular pipe.

Just substitute the velocity profile  $u/u$  average  $=2(1 - r^2/R^2)$ , and then just do the integration. Now you see that this kinetic energy correction factor if you put, let us see that what equation you will get at the end, so now that has divide all the terms by  $m \dot{Q}$  okay, so if you divide all the terms by  $m \dot{Q}$  you have  $Q \dot{Q}/m \dot{Q} = p_2/\rho + \alpha u^2/2$ , because you have already divided by  $m \dot{Q}$  which is  $\rho u$  then  $+ g z_2$  internal energy term we just write separately  $-p_1/\rho + \alpha u_1^2/2 + g z_1$  internal energy 2 - internal energy 1 right.

So we may just rearrange it little bit to write that you have  $p_1/\rho + \alpha u_1^2/2 + g z_1 = p_2/\rho + \alpha u_2^2/2 + g z_2$  internal energy 2 - internal energy 1 - this one right okay. So many times when you say  $\alpha$  maybe it is better to write  $\alpha_1$  and  $\alpha_2$ , now if you consider these  $\alpha$ s as one this will look like a modified Bernoulli's equation, that here you have the total mechanical energy at 1, here you have the mechanical energy at 2.

And you have a term here the correction term, this correction term if it is 0, then it is just like the Bernoulli's equation that you have studied earlier. So sometimes this is known as modified Bernoulli's equation, again that is a very wrong concept, this has nothing to do with the Bernoulli's equation except the form, because Bernoulli's equation you are writing between 2 points, here you are writing the equation between 2 sections 1 and 2, so be very, very careful.

This is a very important misconception that people have, many times you say that in some of the industrial applications even the kinetic energy correction factor is omitted, and then still it works. It works beautifully because many of the engineering flows are so turbulent that kinetic energy correction factor is very close to 1, that means considering that or not considering that does not matter, but it is a matter of negligence or understanding.

So if you understand that it has to be there, but for a highly turbulent flow you neglect it that is one thing. But a very bad thing is you do not know or do not understand that this has to be there, so that misconception has to be avoided. So do not take it as a modified Bernoulli's equation, better we just call it as an energy equation which looks like a modified form of Bernoulli's equation, the terms in the Bernoulli's equation adjusted with something.

So what is this now? We will now concentrate on the physical meaning of this, so internal energy 2-internal energy 1 what is this? So you have basically let us say that you have so at the section 2, say at the section 1 the fluid has entered. At the section 2 let us say that heat transfer is 0, let us say that you have insulated the wall of the pipe, so that there is no heat transfer across the control volume. So then what do you expect that term to be positive or negative?

This is pure physical understanding do not try to go for any mathematics to describe it, think about this you have because of this viscous effects the relative motion between various fluid layers, it is as if like you are having one of your forms with the other. So you have the frictional resistance because of relative motion between various fluid layers, and because of that frictional to overcome that frictional resistance what will happen?

There will be some work that is necessary, but that work is not a useful work, so the entire work is dissipated and where it is dissipated it is dissipated in the form of intermolecular form of energy. So the fluid gets overheated, so it increases the temperature of the fluid, so because of the viscous action whatever work is necessary to overcome that, that is eventually manifested in the form of at increased temperature this is known as viscous dissipation.

So whenever you will be studying maybe heat transfer later on, you will go through the details of what is viscous dissipation, but it is very important to have a qualitative understanding of it that you have velocity gradients between the fluid layers, and there is a relative motion between the fluid layers to overcome that resistance some energy has to be spent or some what needs to be done, but that work is not manifested in the form of useful work.

So that what it only does is it increase the temperature of the system through the internal energy rise. So we can conclude that this term is always positive, now you may say that I will have a heat transfer which is more than this one, but see spontaneously that effect is not going to be there, in a limiting case what may happen, see when the system is overheated you have a higher temperature than the surrounding. So you will have a heat transfer.

So this spontaneous heat transfer is what? This spontaneous transfer itself is negative, so here you have to remember that this was put with a sign convention that heat transfer to the system is positive, here the system is getting overheated, so there will be heat transfer from that to the surroundings, so that itself will become negative.

So the sum total of that will be positive, that means what you can say that this represents the total mechanical energy at section 1, this represents the total mechanical energy at section 2, this is a positive term. That means the total mechanical energy at section 2 is  $<$  total mechanical energy at section 1, so there is some loss of energy and that is manifested in the form of the head loss that we have seen.

So this expressed in the form of head that is if you divide all the terms by  $g$ , then it is expressed in the form of unit of length or head that is nothing but the head loss that we have calculated for

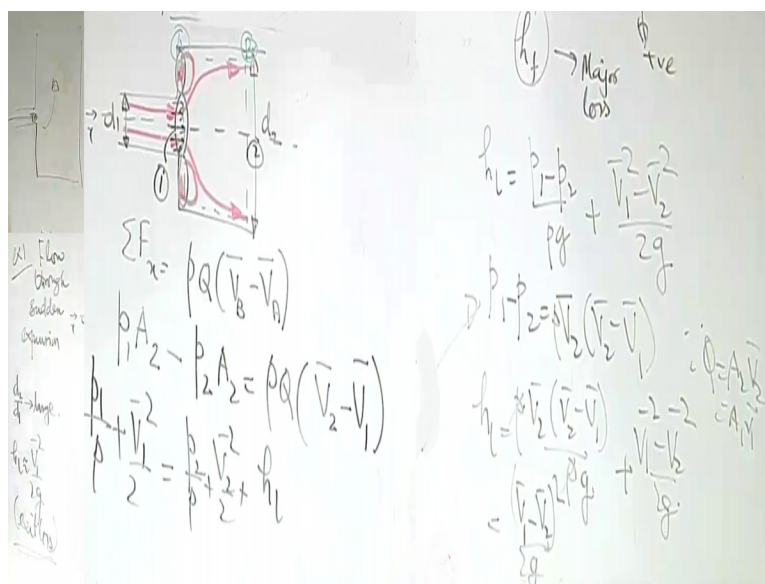
the pipe flow problems. Now consider one important thing, if we ask you that what is the work done by the fluid to overcome the wall shear stress, the shear stress at the wall, what will be that? Yes, what should be the work done to overcome that?

Keep in mind one thing, how do you calculate the work? Some force or here the rate of what? So some force multiplied with some velocity, so what where are concentrating at the wall? At the wall there is some shear force. What is the velocity of the fluid relative to the wall? 0, that means there is no work done to overcome wall shear stress at the wall. Again this is a very, very important thing.

Because these are loose wrong concepts that because there is wall shear stress, there is some work done to overcome wall shear stress and that is why fluid is losing energy and all those things people given as explanations. But you have to be very, very particular in this, where is the work if the velocity is 0, so there is no work done to overcome the shear stress at the wall that work is 0, only whatever is the work internally that is manifested in the form of this internal energy change.

But not there are sort of a useful work by the displacement at the wall, obviously because it is a no slip boundary condition.

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So from this analysis what we understand is that we can cast the energy consideration between 2 section 1 and 2 as  $p_1/\rho + \alpha_1 u_1^2/2g + z_1 = p_2/\rho + \alpha_2 u_2^2/2g + z_2 + h_L$  remember all these average velocities +  $z_2$  + head loss, which is the positive term. This is sometimes known as modified Bernoulli's equation or better to say this is an energy equation, modified mechanical energy equation, which still represent conservation but considering a loss.

Now what could be this losses? What could be the sources of these losses? One of the sources the losses is because of the viscous effects that we have already discussed that is the head loss  $h_f$ , we could characterize it that how it is different for laminar flow turbulent flows and so on. But there also could be other types of losses, and other types of losses are possible because of other changes present in a piping system.

For example, you have a small pipe, now that small pipe is getting changed into a larger diameter, now this is something where there is a loss, why there is a loss? See of course the diameter change is there, but why diameter change induces a loss, so if you consider the streamlines like this, the streamlines because of the sudden change in cross-section will be having their curvature in this way. So locally what will happen? There will be eddies formed in this way.

These eddies do not participate in contributing to the energy of the main flow, so whatever energy is there associated with the rotation of these eddies, there is a loss so far as the main flow transmitter is concerned. So this is also a loss, these loss was not taken into account for calculating the  $h_f$ . Similar, things may be there for like if you have valves in a piping system, because those are creating some resistance.

If you have a valve, if the valve is totally closed the fluid cannot flow, if the valve is partially closed and partially open the fluid may flow. So obviously there may be other forms of resistances, and those losses because of the other forms of resistances are known as minor losses in a piping system. So we will now look into minor losses in a piping system, again this minor loss is a misnomer.



Because sometimes so when there is something minor, there is something major, so what is this major? Major is this head loss due to friction this we called as major loss. But in many practical considerations minor loss becomes  $\gg$  the major loss, so it is the name major and minor should not be confused in a literal sense, so just because originally the loss considerations where there from the pipe friction considerations.

And in one way it is major, because in a pipeline if there is nothing else at least the physical resistance because of fluid friction is there over a length, you may not have a valve, you may not have a sudden change in cross-section, but the length of the pipe itself is present. So the reason of naming the major losses this loss will be there, other losses minor losses or losses they may be there or may not be there depending on what fittings are there in a piping system.

But if they are there sometimes they are much much more important than the major loss, so the relative importance need not be misunderstood. So we will look into some examples for which give rise to the minor losses, so the first example is flow through sudden expansion. So here what is important is we are able to see that there is some loss, and let us say that the diameters of the smaller and the larger pipes are  $d_1$  and  $d_2$ .

We are interested to find out the loss, because of these flow through sudden expansion. And when we find the loss due to sudden expansion, we isolate the effect of the loss due to the length of the pipe, so we only consider the loss due to this sudden expansion effect not the length effect. So what we do? We just take a control volume and try to apply the Reynolds transport theorem momentum conservation.

So let us say that we take this section as section A and take this section as section B, so for this control volume, if we want to write the Reynolds transport theorem. So the resultant force along the x direction if it is x, I am just writing the final simplified form because we have discussed about such problems many times, so we assume it is a steady flow, and  $V_B - V_A$  right. Now what are the forces acting on the control volume? So you have a pressure force on the 2 ends right.

So when you have the pressure force on the 2 ends, see also there is shear force here at the wall, but we are neglecting that effect, because that effect is already considered in the major loss okay, so that does not mean that that effect is not there, we are isolating that effect from the minor loss effect. So the pressure force, so what is the pressure distribution at 1? Let us at A, so we have a section 1 which is say for here upstream and the section 2 which is at downstream.

Let us consider that the velocity profiles are approximately uniform at 1 and 2, when it is possible? It is possible when it is almost a very highly turbulent flow. So let us assume that the flow is highly turbulent, so that the velocity profiles are almost uniform that is the kinetic energy correction factor is not important. Now when you have say you want to write the force due to pressure here, see it is important to note that the pressure is acting here in this way.

And this is let us assume that this pressure and this pressure is not greatly different, and that will not be greatly different because these kinetic energies are not greatly different, and this length we are not considering so large that there will be a huge loss of head because of the friction. So you here you have the pressure at this one roughly same as  $p_1$ , if  $p$  is not located far away from the section A, if it is quite close then that is possible.

So what we are considering is that in addition to the pressure acting over this part, the same pressure also acts over the upper and lower parts, this is an assumption. So and that assumption is well justified, because the change in pressure from this one is not failed so easily by this one, because this is just a small recirculating region, so the change in pressure is failed only when the proper bounding streamlines are making it feel.

So here it is just a local recirculation this does not understand so easily that what would be the change in pressure from this to the subsequent section, so you have  $p_1$  so the entire pressure here is like  $p_1$  and the area is like  $A_2$ , so that you have to understand, it is not the area of 1 but the total area over which as if this  $p_1$  is acting, so it is basically  $p_1 \cdot \text{area of the section A}$ , area of the section A and B are the same.

So this  $\rho V_1^2 A_1 = \rho Q$  now  $V_B$  and  $V_A$ , so  $V_B$  is like  $V_2$  that is fine, what about  $V_A$  see the velocity is here are not contributing to the energy, so  $V_A$  is  $V_A$  average is like roughly it is like taken as  $V_1$  average, again it is it is an approximation. So what it considers is that the average velocity is like this is the part of the section where the velocity effects are important, and this part of the section just has a velocity which is the same as the average velocity as this one, and this is totally distributed uniformly.

So that is what is so there are lots of approximation involved, but many of these approximations are not so bad, if the flow is highly turbulent, now that is number 1. Number 2 if you can write the difference, so practically you are considering this as like section 1, and this as like section 2, section 1 is this part this small part, so if you are now write the energy equation like you can write  $p_1/\rho + V_1^2/2 = p_2/\rho$ .

So kinetic energy correction factor we are not considering, because we are resuming it close to  $1 + V_2^2/2 + \text{head loss}$ , also the points are so located we are neglecting the change in potential energy. So the head loss we can calculate, so divided/g to call it unit of head, head loss is  $(p_1 - p_2)/\rho g + V_1^2/2 - V_2^2/2$ , now  $p_1 - p_2$  you can write in place of  $Q$  you can write  $A_2 V_2$  right, so that  $A_2$  gets cancelled from the 2 sides.

So you get this as  $V_2^2 - V_1^2$ , so this is since  $Q = A_2 V_2$  which is same as  $A_1 V_1$  that is from this equation. So then you can substitute that  $h_L$  in place of  $p_1 - p_2$  you will have  $V_2^2 - V_1^2 = \rho g h_L$ , so there is a  $\rho$  here that there is a  $\rho$  here right  $\rho$  \* this one, so  $\rho$  gets cancelled out, then  $V_1^2 - V_2^2 = 2g h_L$ . So if you simplify this it will be  $(V_1 - V_2)^2 = 2g h_L$ , you can clearly see that just one step.

So what we can get from this one? This is a very interesting thing that I mean this shows that this has to be always a positive thing, so the head loss is positive, and it is a function of the difference in the average velocities over the 2 sections. One special case is that let us say this is a small pipe entering into a very large reservoir, so you have a small pipe like this it is entering into a large reservoir, so what is happening is fluid is exiting from the pipe to a reservoir.

And then that is a special case of this one with  $d_2/d_1$  very large, and then what does it become? This becomes approximately  $V^2/2g$ , this is known as exit loss. So exit loss is very important in engineering, because it signifies the exit of the fluid from a pipeline to a reservoir. So it is a special case, where the ratios of these 2 sizes are grossly different, otherwise this is the formula straight away you can use.

So this is an example of a minor loss, in the next class we look into some other examples of a minor loss. Let us stop here, thank you.