Conduction and Convection Heat Transfer

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Lecture-29
Thermal Boundary Layer- II

In the previous lecture, we were discussing about the case of Prandtl number = 1for hydrodynamic and thermal boundary layer over a flat plate. Now as we mention that, not all cases are characterized by the Prandtl number valve of 1, so we will consider the other cases of Prandtl number and or making order of magnitude analysis, we will consider some limiting values of Prandtl number.

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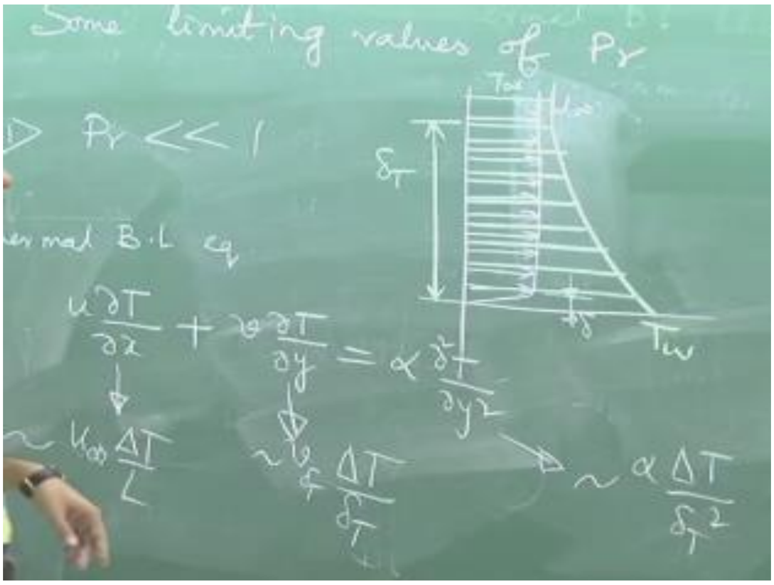

Case 1, Prandtl number much less than 1. So, if you want to draw the velocity and temperature profiles at a given location, Prandtl number much less than 1, always remember that in force convection, Prandtl number has a meaning of delta y delta t. We will see that these meaning may be disturbed, if you have natural convection. We will talk about that in details when we discuss about natural convection.

But in force convection it will be having a meaning of delta y delta T. So Prandtl number much less than 1 means, the hydrodynamic boundary layer thickness is much less than the thermal boundary layer thickness. So that thermal boundary layer thickness may be say this is, this one. We will be having temperature profiles like this. So we will try to make an order of magnitude analysis of this.

So, let us write the equation, the thermal boundary layer equation. Remember one thing, we always analyse hydrodynamic boundary layered equation within the hydrodynamic boundary layer, similarly we always analyse the thermal boundary layered equation within the thermal boundary layer. Because the domain of applicability of this equation is within the thermal boundary layer.

So this equation, if we now apply within the thermal boundary layer, what is the order of magnitude of these term? What is the order of magnitude of u? See, because delta T is much greater than delta, so for almost over the entire delta t u is u infinity. Okay, so the maximum valve of u is u infinity. This is of the order of; this v is what? v where? If you write order of magnitude of these, this value of v is v at which location?

Yes, age of thermal boundary layer, not hydrodynamic boundary layer. Because this equation has its domain in the thermal boundary layer. So this v, you write v at delta T, where v at delta T is the velocity at the age of the thermal boundary layer not hydrodynamic boundary layer * and these term as an order of magnitude of, now use the continuity equation to get v delta T in terms of u infinity.

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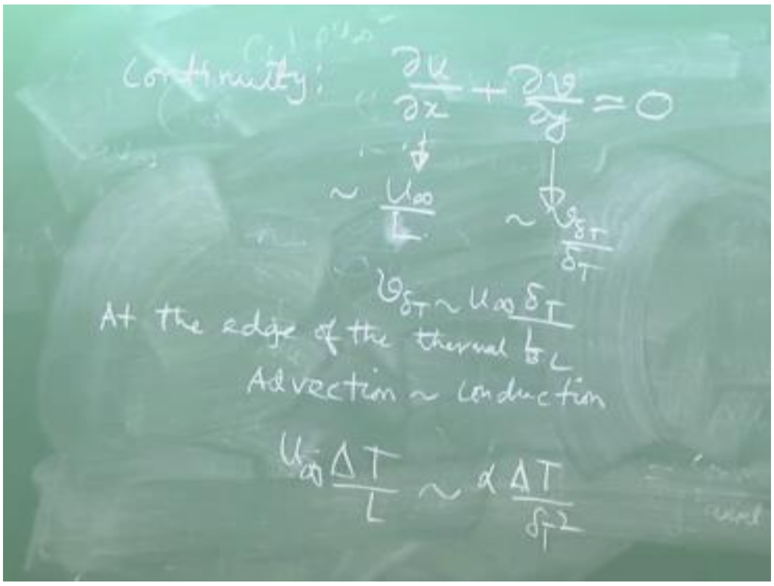

What is the order of magnitude of this? This equation is analysed within what? Within thermal boundary layer, because our objective is, you can analyse the continuity equation both within thermal boundary layer as well as within hydrodynamic boundary layer. But you have to consider the objective. What is the objective here? The objective here is to get velocity at the age of the thermal boundary layer.

Therefore, we are going to apply it across the thermal boundary layer. So, u infinity/delta, sorry, L, this is of the order of v delta T / delta T. Because these 2 terms should be of the same order of magnitude, we can write v delta T is of the order of u infinity * delta t/L. So this is of the order of u infinity * delta T/L. So just like the momentum equation, we can see that in the energy equation also, these 2 terms are exactly of the same order of magnitude.

So, although here there is v and v may be much less than u, del T/ del y over compensates that as compared to del T/ del x. So you have to keep in mind that these 2 terms are of the same order of the magnitude. So these 2 terms together represent physically what? They physically represent the advection effect, that is, the transfer of heat due to fluid flow. So and the right hand side is the conduction.

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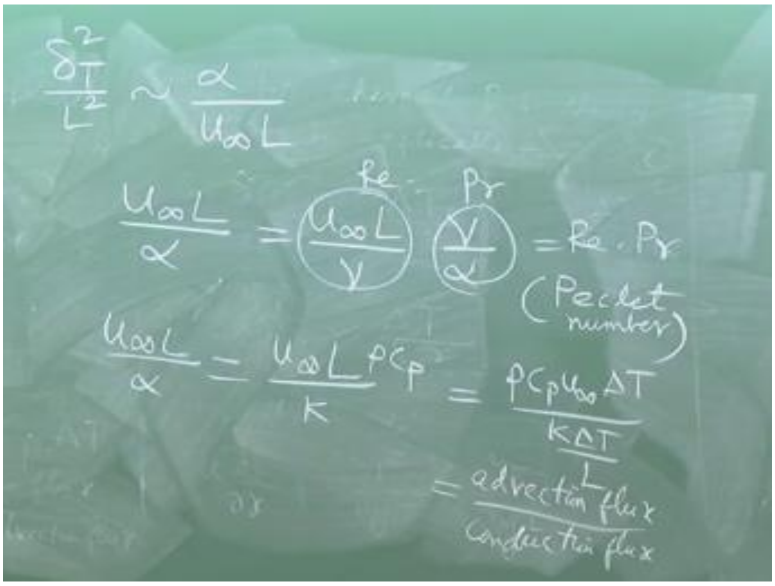

So at the age of the thermal boundary layer; that means we can write, so delta T square/L square, we can write u infinity L/ alpha. So what is u infinity L/ alpha? This is u infinity L/nu * nu/ alpha. Right, So Reynolds number * Prandtl number. This is also called as Peclet number. Now what does it physically represent? So, u infinity L/alpha, you can write it as u infinity L/k rho, cp in the numerator, right.

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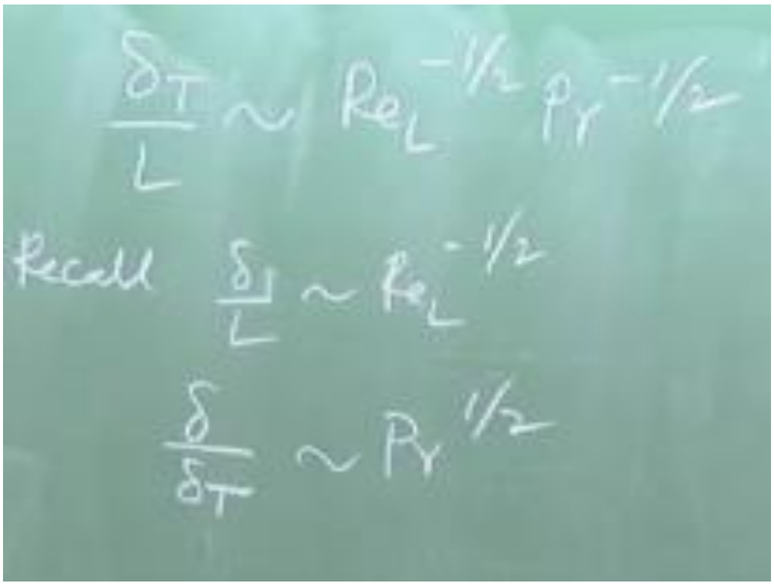

So, what is this? This is advection flux; this is conduction flux. In convection, what we are always trying to see or explore that how is advection enhancing or augmenting the heat flux with only conduction, there would have been some heat flux, with advection, how that can be augmented, that is one of the big objectives of studying convection. So you can write delta T/L is of the order of Reynolds number to the power of -1/2 x Prandtl number to the power of -1/2.

Recall that, delta/L was of the order of Reynolds number to the power of -1/2. This we derived earlier. So delta/ delta T is of the order of Prandtl number to the power of 1/2. S as we discussed that Prandtl number is the quality major of delta y delta T in force convection, you can see that, that is justified. Now our objective again is not to calculate delta or delta T.

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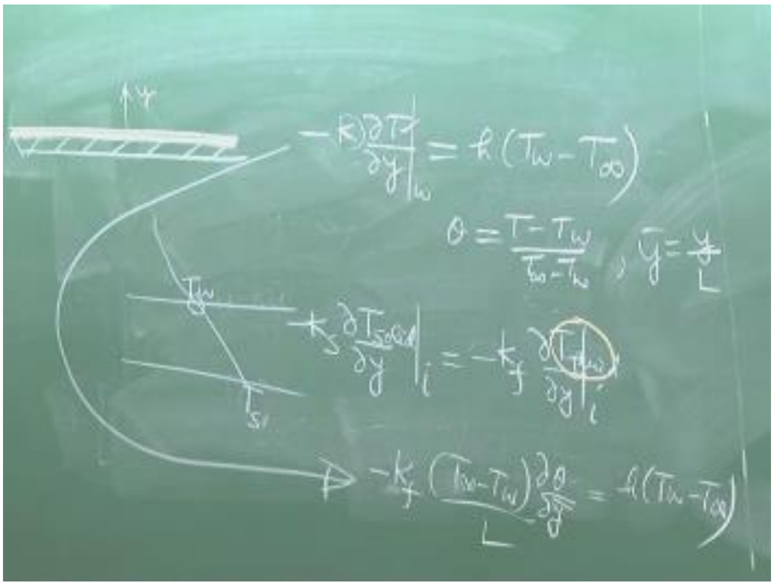

But our objective is to calculate the heat flux at the wall, which when express non dimensionally is called as Nusselt number. So, our objective is to calculate the Nusselt number. So at the interface, that is, at this location, this is the y axis, we write; so we have discussed about various issues with these equation, but let me try to iterate, because these are important issues. Let me first ask a question; is this k of the solid or k of the fluid?

Yes, so what temperature gradient we consider here? So you have a solid, so let us say this is Ts, this is T wall and then you have some temperature profile in the fluid. So, at the interface, you must have -ks, right, at the interface, we must have these. Now we have to keep in mind that in convection, our important consideration is temperature profile within the fluid not temperature profile within the solid.

So, because we are interested about temperature of the fluid here, that is what we considered here. Therefore, these case k of the fluid, but had we be interested with k with the temperature profile in the solid, these could also be k of the solid. So, whether this is k of the solid or k of the fluid, it depends on whether we are considering temperature profile in the solid or temperature profile in the fluid.

In convection, we are interested about temperature profile in the fluid, therefore this k must be k of the fluid and the other important assumption is that we are neglecting radiation. If you do not neglect radiation, there will be an additional heat flux due to radiation. Okay, what does it physically say? This is the very important physical law, what does it say? It physically says that whatever is the heat flux due to conduction at the interface, the same is heat flux due to convection.

So, that mean that, interface cannot store anything, so this is valid for both steady as well as unsteady state. It is not just steady state, but also at unsteady state, because interface cannot store any thermal energy, so this is the big difference between electrostatics and heat transfer. In electrostatics, interface can store charge, but in heat transfer interface cannot store any thermal energy; any surface cannot store thermal energy, it can only transfer thermal energy.

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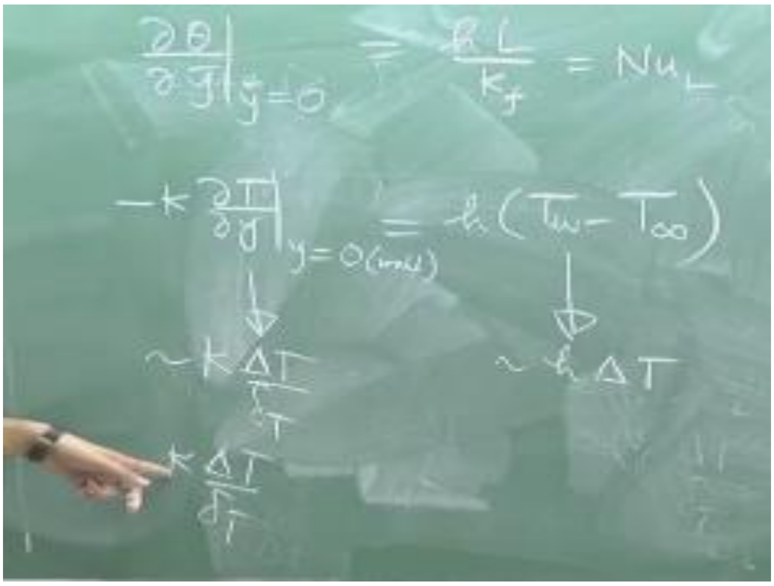

So now if you define, let us say you define theta is T-T wall/T infinity- T wall and let us say non dimensional y is y/L. So you can see that non dimensional temperature gradient at the wall is this non dimensional parameter, which is nothing but the Nusselt number. So Nusselt number in a way represents the temperature gradient at the wall in a non-dimensional form, that is what it does.

Now in this particular problem, let us write this –k del T /del y at y=0 or wall. So what is the order of magnitude of this? k delta T/; what is the order of magnitude of this? So order of magnitude y, these 2 must be the same. That means, k delta T/delta T, remember in convection, when we are writing k, its k of the fluid. So in this same definition, if it is k of solid, that becomes the Biot number, which we used in conduction.

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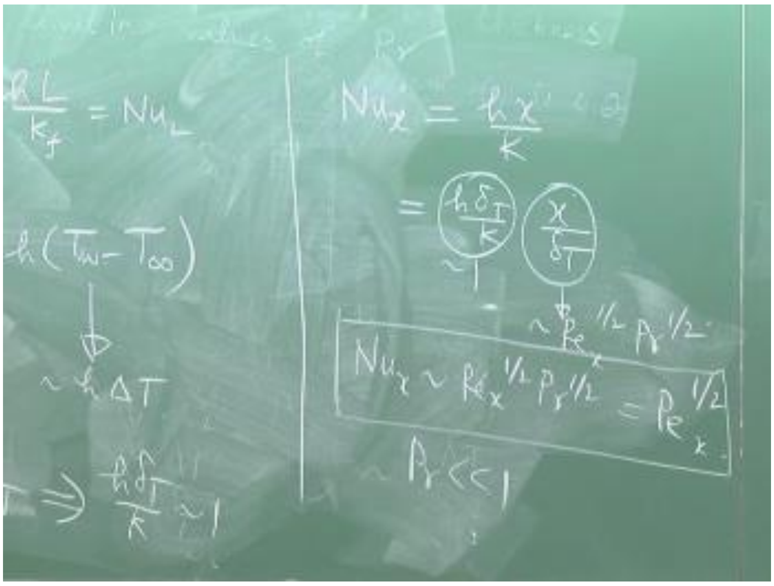

So Nusselt number at a given x, the maximum value of x is x=L, but at any local x, that is = hx/k. This is nothing but, h delta T/k * x/delta T. h delta T/k is of the order of 1 and x/ delta T; you can see here, just think of L replaced by x, Reynolds number L will be replaced by x. It will become local Reynolds number. So x/delta T will be Reynolds number to the power of 1/2 * Prandtl number to the power of 1/2.

So this is of the order of Reynolds number to the power of 1/2 * Prandtl number to the power of /2. So, Nusselt number is of the order of Reynolds number to the power of 1/2 * Prandtl number to the power of /2 or Peclet number to the power of 1/2. This is for Prandtl number much less than 1. What kind of practical fluid will have Prandtl number much less than 1?

So Prandtl number much less than 1 means, the thermal diffusivity is much more as compared to the kinematic viscosity and that is true for liquid metals. So, in steel industry or any industry involving liquid metals, this kind of analysis is very important where if you have a flow of liquid metals, you can safely assume that the Prandtl number is much less than 1.

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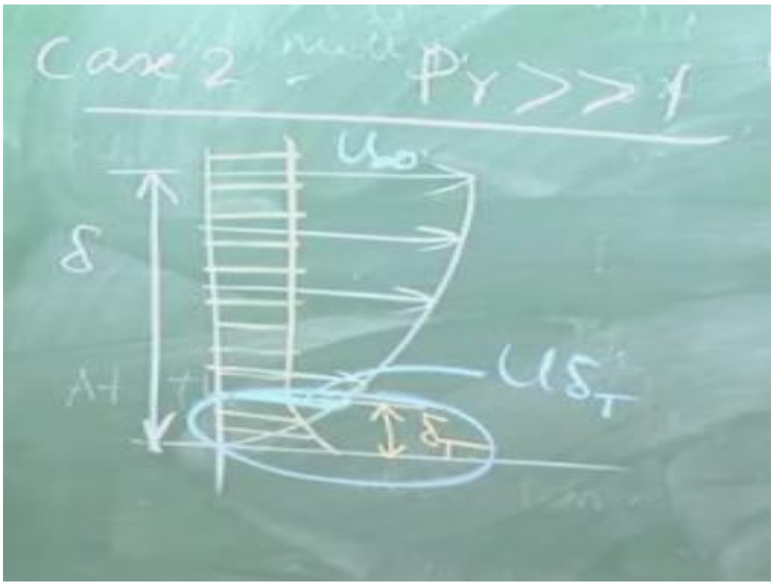

We will consider the other limiting case before we stop today and that is Prandtl number much greater than 1. So Prandtl number much greater than 1 means, delta much greater than delta T. So if you draw the velocity and the temperature profiles, let us say this is delta and; let us say this is delta T. So, remember that our domain of analysis, is this one for the thermal boundary layer equation.

Now, what is the key difference between the previous case and this case? In the previous case, at the age of the thermal boundary layer, we could consider u =u infinity, but now here u is not= u infinity; u is this one, which is not u infinity. So if you make an order of magnitude analysis, let us call these as u at delta T. u at delta T is not u infinity; u infinity is this one.

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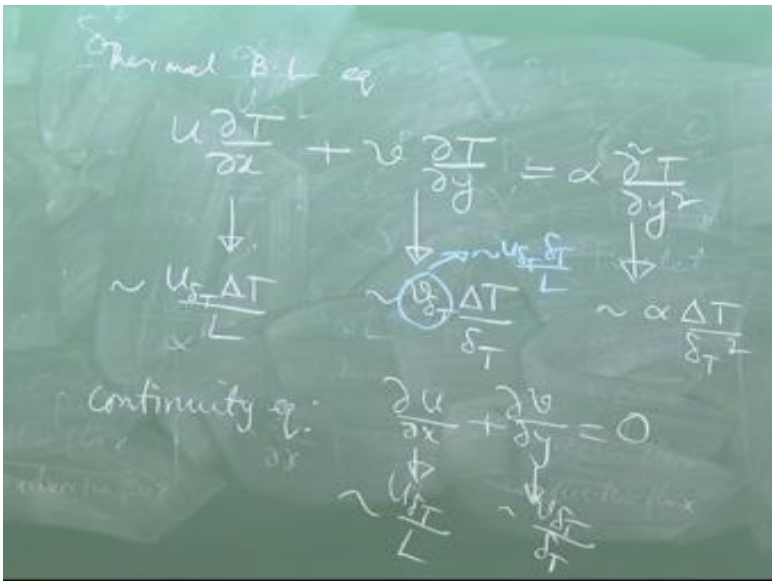

So thermal boundary layer equation; what is this? this is v at delta T. Now we will use the continuity equation. So this is of the order of; remember we are interested within the thermal boundary layer. So u delta T/L, this is of the order of v delta T/delta T, that means v delta T is of the order of u delta*delta T/L. Again, the same conclusion that these 2 terms are of the same order.

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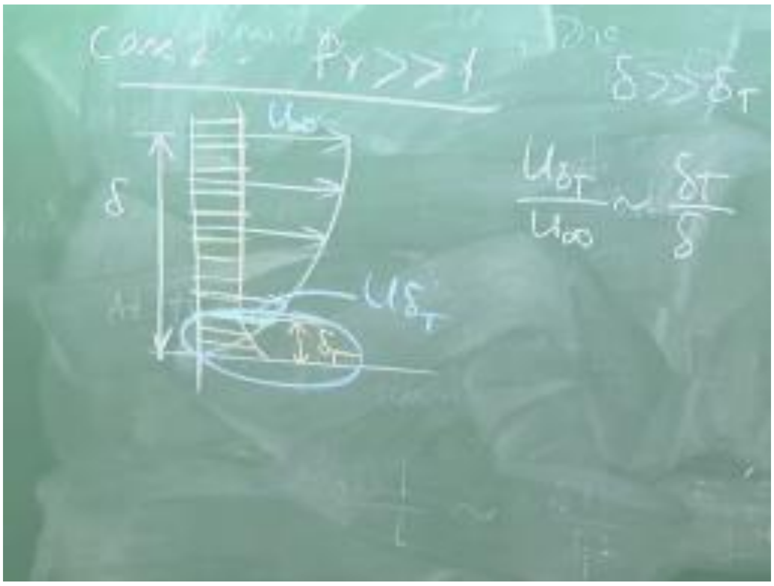

Now one important question is because we already do not what is u delta T? what is the way in which we can estimate the order of magnitude of u delta T? See order of magnitude why is? We can use the proportionality that u delta T/ u infinity is of the order of delta T / delta, right. This much divided by this much= this length divided by total length, order of magnitude wise, okay.

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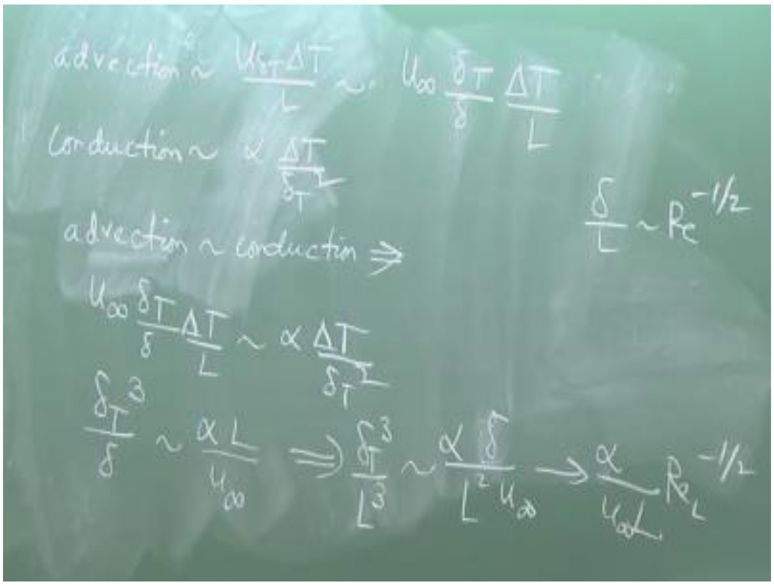

So we can write that advection is of the order of u delta T* delta T/L, that is of the order of u infinity and conduction. So, we can write delta T cube /L cube, that means we are dividing both sides by L square, so it is alpha delta/L square u infinity. Delta/L, what is delta/L, is of the order of the Reynolds number to the power of -1/2. Delta y/l does not depend on Prandtl number, so this is of the order of alpha/ u infinity L* Reynolds number to the power of -1/2.

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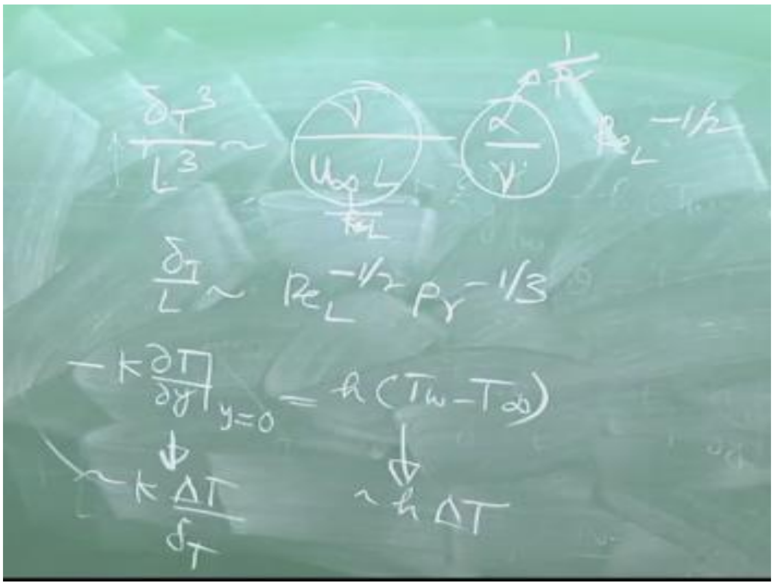

So delta T cube/L cube is of the order of; alpha/ u infinity L means u infinity L / nu, we put the nu in the numerator * alpha/ nu, right* Reynolds number to the power of -1/2. So this is what? This is Reynolds number. Yes, 1/Re and this is 1/ Prandtl number. So delta T/L is of the order of Reynolds number to the power of -1/2 * Prandtl number to the power of -1/3, right.

Because this becomes Reynolds number to the power of -3/2, 1 cube root of that is, Reynolds number to the power of -1/2, right.

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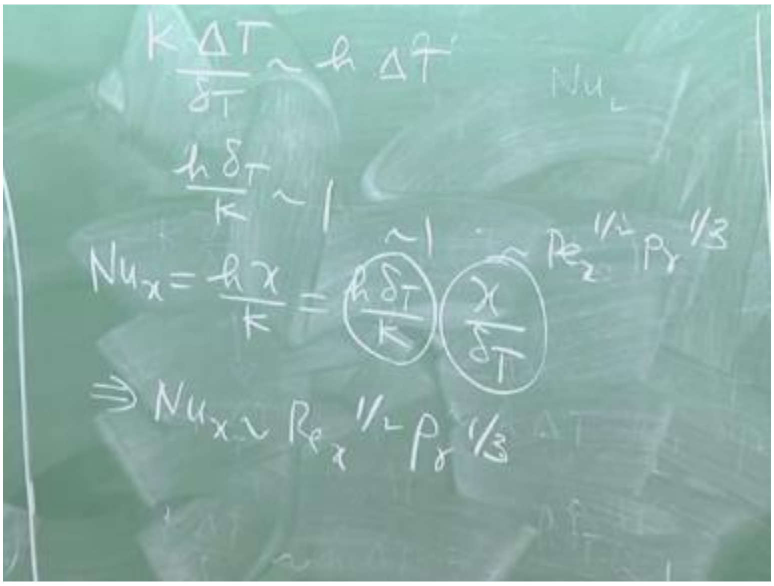

So finally –k del T/ del y at y=0 = h*T wall- T infinity, so this is of the order of k*delta T/ delta T, this is of the order of h*delta T. So we can write k delta T/ delta T is of the order of h delta T, that means h delta T/ k is of the order of 1. This is same as the previous case. So the Nusselt number, that is hx/k= h delta T /k*x/delta T, which is of the order of 1. At x/delta T is of the order of Reynolds number to the power of 1/2* Prandtl number to the power of 1/3.

So Nusselt number in this case is of the order of Reynolds number to the power of 1/2* Prandtl number to the power of 1/3. So, you can clearly see that the difference between the limiting case of the Prandtl number much less than 1 and much greater than 1 is here the Prandtl number dependence is -1/3 or 1/3 depending on which expression you are considering and there it was 1/2.

But if you find out what is delta T/delta? Or delta/delta T, that will be related to the Prandtl number. For Prandtl number much less than 1, delta T/ delta is sorry, delta/ delta T is of the order of Prandtl number to the power of 1/2. For Prandtl number much greater than 1, delta y/ delta T is of the order of Prandtl number to the power or 1/3. So only the power dependence gets changed, interestingly the Reynolds number dependence does not get change.

Because Reynolds number dependence followed from delta/x is of the order of Reynolds number to the power of -1/2. That does not understand what is Prandtl number, so Reynolds number dependence is the same but Prandtl number dependence in one case it is to the power of 1/2, for Prandtl number much less than 1 and in another case to the power of 1/3 for Prandtl number much greater than 1.

Typically, what are the fluids for which Prandtl number is much greater than 1, typically oils. Because oils have very kinematic viscosity so, on one side Prandtl number much less than 1 is for molten metals on another side Prandtl number much greater than 1 is for oils. The case closed to 1 will be valid for fluids like air, water, these type of fluids. We stop here today. we will again continue in the next lecture. Thank You very much.