Heat Exchangers: Fundamentals and Design Analysis
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Lecture – 48
Micro scale Heat Transfer

Welcome to everyone. If you recall we have started a new topic in our last lecture. We want to know regarding micro heat exchangers. And for knowing regarding Micro Heat Exchanger, it is important to know that we should have some it is important to know micro scale heat transfer. We should have some idea regarding the convective heat transfer which takes place when the channels are narrow. So, this is called micro scale heat transfer and we have started this. I will go back to one slide which we have seen in our last lecture, so, here.

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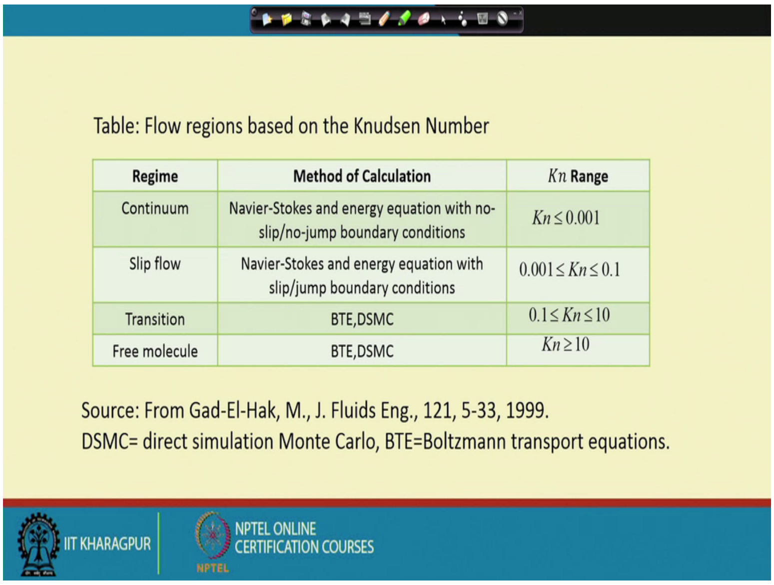

So, this slide we have seen in our last lecture. So, here we have defined a number called Knudsen number and this Knudsen number is given by lambda given by the expression lambda by l, where lambda is the mean free path of molecular collision and l is the characteristic length of the engineering system which we are considering. So, depending on Knudsen number we can have continuum flow when Knudsen number is less than 0.001, we do not have any difficulty this is our conventional flow situation. Though we can have a very small channel, but Knudsen number is in this range. So, we can analyse it as we have analysed our earlier problems of heat transfer fluid flow.

Then we can have a different value of Knudsen number or a range of Knudsen number Knudsen number is greater than 0.001, but less than 0.1. Then we will have slip, at the wall we will have slip, at the wall we will have a temperature difference between the wall and the fluid which is called temperature jump. And, slip flow basically we will use the Navier-Stokes equation we will use the energy equation, but boundary conditions will be different. So, here you can see that if we have to have heat exchangers operating in this range so, we have to be careful.

Then transition, where the Knudsen number is greater than point 1 but less than point 10. So, we will have we cannot have our conventional question for fluid flow and heat transfer that is Navier-Stokes equation etcetera we cannot have and we have to have either Monte Carlo simulation or the Boltzmann transport equation. Free molecular flow Knudsen number greater than 10. So, here again we will have the Boltzmann transport equation or Monte Carlo simulation. Let me explain this thing little bit.

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So, I think that would be good to let us go to the white board and in the white board let me explain the. So, you see, so, let us say this is a solid surface. In conventionally if we see that the, fluid that is made up of number of particles. So, what will happen all these particles, they will have random motion from the physics we know that they will have random motion. When they are having random motion then what will happen, so, the particle will collide with each other let us say two particles are there. So, they will collide or this particle will collide with another particle and they will also collide with the wall and when they are colliding with another particle or colliding with the wall, so, their direction of motion and their velocity both may change.

Now, when there are very large number of particles. So, at one instant of time millions of particles are colliding with the wall and how to deal them either from the statistical physics or from the continuum we have learnt and basically when this kind of pictures are there, we are not bothered what is happening at the particle level we are we can deal them as continuum level and we basically have different kind of laws etcetera like momentum equation on Navier-Stokes equation energy equation we can apply them.

But, what happens if the number of particles are small, then it may happens like this that these particles will have some sort of a mean free path for collision to take place. When the number or numbers are small the particle numbers are small then what will happen? The mean free path will increase or when the channel dimension that is small, but there are enough number of particles, so, in that case also the linked of the mean free path becomes comparable with the dimension of the channel. So, this is the second situation we are considering and now we will have different kind of different kind of physical phenomena which is not present in case of the conventional flow so far what we have considered.

So, two situations one can think of, either the fluid is rarefied; that means, there are very less number of particles. So, before collision one particle traverses a large link which is comparable to the physical dimension of the system or the physical dimension of the system itself is very small, so that it becomes comparable with the free mean free path of the particles for collision to take place. Now, that is one thing we can understand that why the physics are different and then it is very important though we are not concerned all the time that energy transfer momentum transfer for these things it is very important to know that how the particles are interacting with the wall.

If the particles are interacting with the wall at a very large number at one instant of time there are millions of impacts of the particles with the wall then we get one behaviour and we deal them with one type of law in physics, type of laws in physics and when there are small number of interactions then we deal them with another law of physics or another types of law another sets of law of physics. So, in case of micro scale heat transfer we are concerned with the second problem; that means, here as the number of particles has reduced. So, we will have very less number of interaction with the wall and then we have to see how we can deal with that.

Again, we will not go this is not the place to discuss micro scale heat transfer in detail or particle physics in detail for rarefied kind of situation. But, the scientists have developed certain rules certain laws when this second condition which is unique condition that exist. So, some of those laws we will discuss and some of the physical insight I would like to give you as our discussion process, ok. Let us now go to the next slide.

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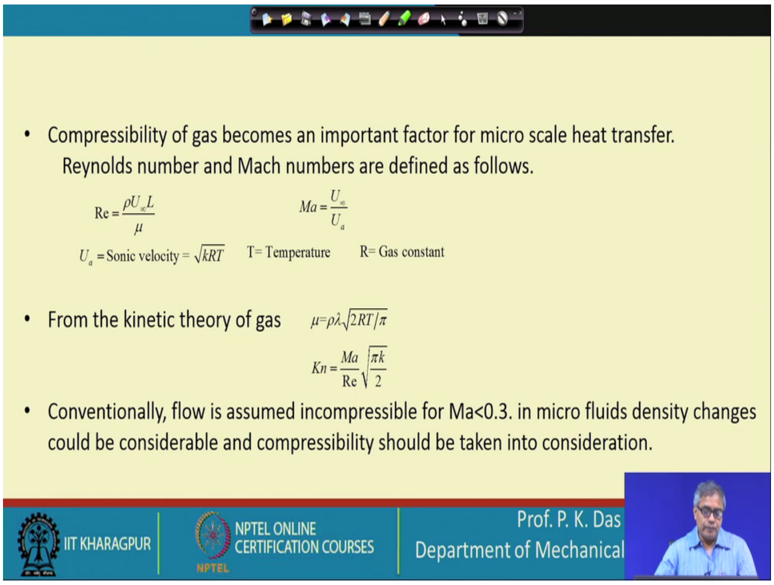

So, we have defined we have defined Knudsen number and based on Knudsen number we have got different kind of different kind of flow phenomena we could classify different kind of flow phenomena, that is continuum flow, slip flow, transitional flow, free molecular flow like this. So, Knudsen number is important and another thing we will see that Mach number also becomes important because you see there is a change of density. We are considering rarefaction or during the flow particularly micro channel flow there could be a change in density.

So, compressibility of the gas please look into this compressibility of the gas becomes an important factor for micro scale heat transfer. Let us define these two we can define

Reynolds number like this and we can define Mach number and then in the Mach number, Reynolds number you see density has come the velocity has come characteristic velocity, characteristic length and viscosity and in Mach number we have got the characteristic velocity and the local sonic velocity. Local sonic velocity that will be given by root over kRT; R is the gas constant, T is the temperature and k is the ratio of the specific heats, ok.

So, from kinetic theory of gas again mu can be obtained by this particular formula and from there we can define Knudsen number again we are coming back to Knudsen number, but here you see that two important one dimensional number we have included that is, one is your Mach number and another is your Reynolds number. So, you see this is this is very interesting and important to see.

Initially I have defined Knudsen number in terms of the length free molecular mean free path of the molecular collision which may be difficult to determine for a particular flow situation, but knowing the basic flow parameters, obviously, one can define Mach number and Reynolds number. So, from there one can get the idea of Knudsen number and again Knudsen number or ranges of Knudsen number are associated with the different types of flow. So, one can get the knowing the Knudsen number one can tell what kind of flow one expect to get and then we can decide regarding our method of analysis.

The last point that is also very important please note it carefully conventionally flow is assumed incompressible for Mach number less than 0.3 in micro fluids density changes or in micro scale fluid flow density changes could be considerable and compressibility should be taken into consideration. So, you see Mach number is important in this case and density change as there is density change the compressibility effect of fluid flow of course, it is associated with gas flow mainly, so, that should be taken into consideration.

So, these are the changes what we can see that if really we have got a heat exchanger in the micro scale the scales how we have to determine that I have given some hints. So, then the physics changes and specific kind of or appropriate type of fluid flow equation and heat transfer equation we have to consider. So, now, late us proceed to see what kind of equation we have to take care, I mean we have to consider and we will try to learn this things with some simple example simple problems.

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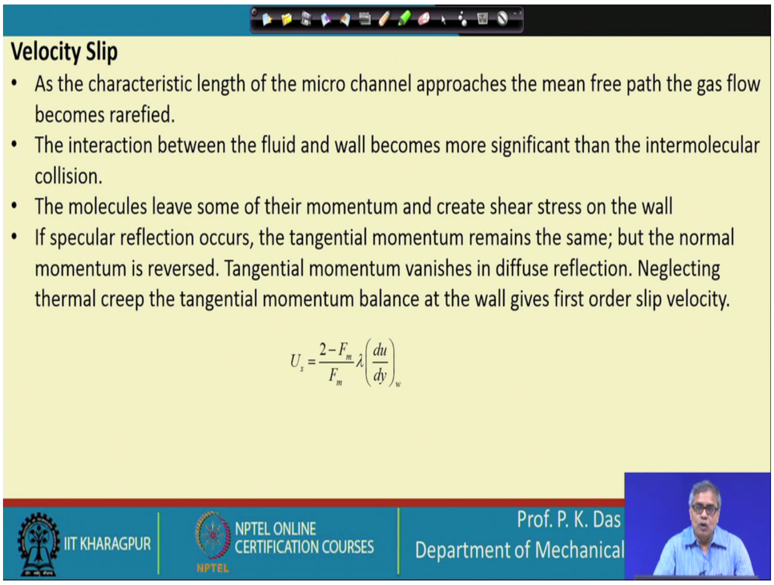

So, so far two things I have told regarding your micro scale heat transfer one is velocity slip and another is temperature jump. So, let us see what is velocity slip. As the characteristic length of the micro channel approaches the mean free path of gas flow mean free path the gas flow becomes rarefied. This we have I have explained with some sort of a sketch of the particles etcetera. So, then the interaction between the fluid and wall becomes more significant than the intermolecular collision.

So, this is also one thing as I have told that now, the interaction is not like before. When there are large number of molecules that the gas is not rarefied or the channel is not of smaller dimension. So, large number of collisions used to take place; now, number of collisions are reduced. So, then what will happen, the molecules leave some of their momentum and create shear stress on the wall the molecules will leave some of their momentum and there will be some sort of a change in the shear stress.

If specular reflection occurs the tangential momentum remains the same, but the normal, but the normal momentum is reversed and let me read it and then I will explain. The tangential momentum vanishes in diffuse reflection and neglecting the creep thermal creep basically the tangential momentum balance at the wall gives the first order slip velocity. So, U s is this slip velocity or the velocity of fluid at the wall. At the wall, if the wall is stationary we should have got 0 velocity, but here we can see that there is a velocity and that velocity depends on number of parameters we will come back to this parameters later on, but it depends on du dy at wall; that means, the velocity gradient at the wall this is important. It depends also on the lambda; lambda is the mean free path of the particles for collision.

Now, what has been told let me explain that thing little bit let me.

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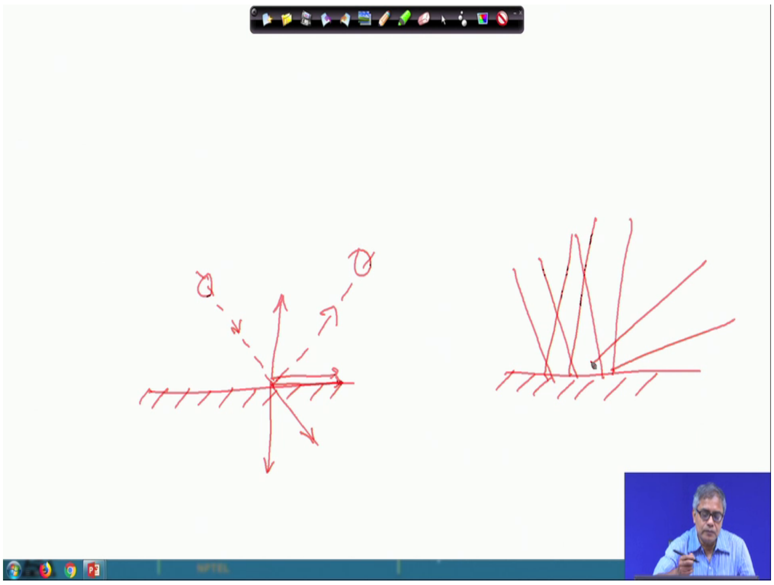

So, two types of two types of collisions have been mentioned or rather the particles let us say this is our surface and a particle let us say it is coming like this making an angle and this can go back like this. So, these kinds of collisions are called specular collision and here more or less it obeys the laws of laws of reflection, ok. So, we can get and some sort of momentum exchange also we can try to apply over here depending on how their direction of motion is changing.

So, direction of motion it is changing as we can see that one particle came here and going back like this. So, when it came like this so, there will be two component of momentum; one is in this direction and another is in this direction. When it is going back like this, it will also have two component one is like this and another is like this.

So, now, you if you read that regarding specular reflection whatever I have told that that is or whatever I have given in the slide. So, that can be related from the small figure which I have shown here that one particle is coming and then it is getting re-bounce back by the wall particle is coming at an angle and then it is getting bounce back by the wall.

Now, this is your specular reflection, then there could be another kind of interaction. The particles are coming let me draw number of this thing and they are going at random angles. So, they are going back like this. So, this is your diffuse reflection, ok. So, both can happen when the first will happen, if the wall is very smooth let us a ideal is smooth then we will have specular reflection and if they are the wall is totally rough then we will have totally diffuse reflection and if it is kind of a practical surface which is not totally smooth not totally rough, then we can have both type of reflection, ok.

So, all the engineering system the surfaces are practical surfaces they are not totally rough they are not totally smooth and so, we can get some sort of a reflection which is mixture of some particles are reflecting by your laws of specular reflection or following specular reflection and some particles are having the diffuse reflection. So, this is what we will have. So, let us go back to the, sorry. So, now if we see the equation, this is the slip velocity 2 minus F m by F m into lambda du dy at the wall. So, this is the wall velocity gradient it is not difficult to understand lambda is the Knudsen number sorry lambda is the mean free path for molecular collision.

So, this is also we have introduced, F m is some sort of an accommodation coefficient, ok. So, let us go to the next slide and then F m can be understood.

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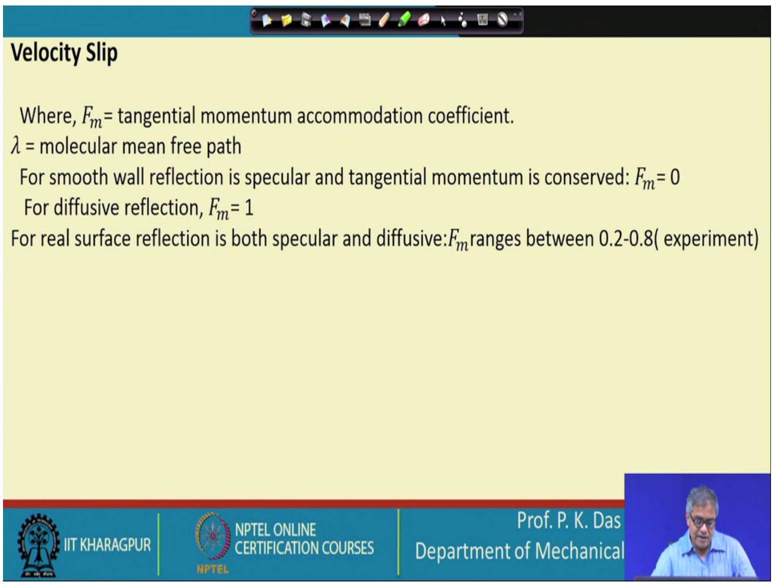

Where, F m is the tangential momentum accommodation coefficient, lambda is the molecular mean free path. For smooth wall reflection is specular and tangential momentum is conserved that we have seen and then F m is equal to 0 for diffuse reflection F m is equal to 1 for real surfaces reflection is both specular and diffusive and then F m ranges between 0.2 to 0.8, ok. So, it ranges between 0.2 to 0.8 and this is obtained experimentally as there will be some sort of a randomness in real surface. So, these values again this value has got a range. So, this is not obtained from the theory this has been obtained experimentally.

Well, so, with this let us do one thing we will go to some other slide. But, so far whatever I have told let us try to recapitulate. It will be go to recapitulate little bit what we have done so far. Now, we have we want to do or we want to know regarding micro heat exchangers, which are small heat exchangers which are different from the conventional heat exchangers and the difference comes that the channels, which these heat exchangers used they are basically they are very small.

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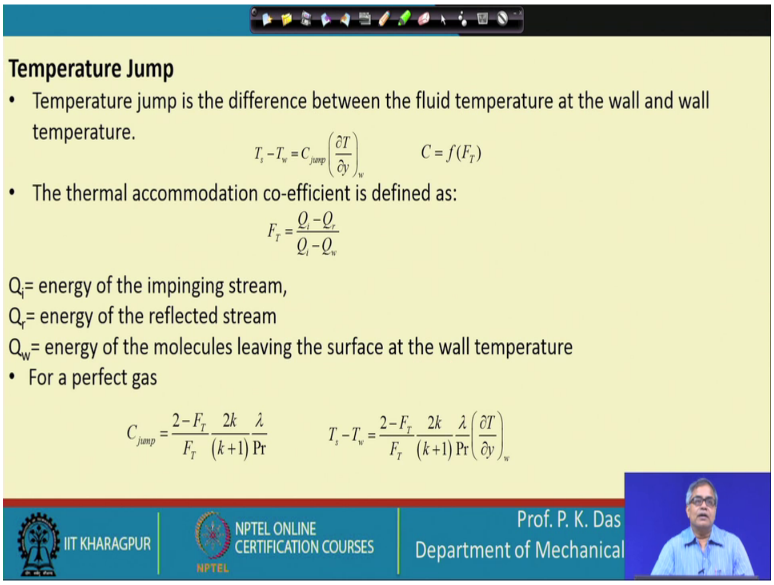

They are they are very small and due to these small size of the channels we will have different kind of fluid flow behaviour, rather we may have different kind of fluid flow behaviour. In small channel which are known as micro channel we can have different behaviour depending on one characteristic number or non dimensional number which is known as Knudsen number. Knudsen number gives a comparison between the mean free molecular path and the characteristic length of the system which we are using or which we want to analyze.

Now, if it is small this if the Knudsen number if the channels are small then what happened, the Knudsen number becomes large and when the Knudsen number becomes large then we go to the to the domain of micro channel or micro fluidics. So, micro channel flow different kind of unique phenomenon occurs to start with what one can see that there will be slip and there will be velocity jump. To slip we have described sorry, there will be slip and there will be temperature jump to slip we have described now we come to temperature jump. So, temperature jump is the difference between the fluid temperature at the wall and wall temperature.

Now, this particular formula you see we have given temperature jump this is T s minus T w; T s is the fluid temperature at the wall T w is of course, the wall temperature; that is C jump, jump coefficient and dT dy at the wall again temperature gradient that the wall C jump obviously, you can understand that this is some sort of a coefficient and this is a function of F t, F t is the thermal accommodation coefficient and is defined as F t is equal to Q i minus Q r divided by Q i minus Q w. Q i is the energy of the impinging stream I have shown it that particles are impinging on the wall. So, let us say that particle which is impinging on the wall it is coming with some sort of an energy which is Q i. So, this is energy of the impinging stream Q r that is the energy of the reflected stream. So, after impingement the particle is getting reflected. So, the energy with which the particle goes back it is that energy.

Then, Q w of the energy of the molecules leaving the surface at the wall temperature suppose, the molecules that left the surface with the wall temperature then it should have some energy. So, that is equal to your Q w for a perfect gas C jump is given by this particular formula where F t is the accommodation coefficient thermal accommodation coefficient then ultimately we get T s minus T w is equal to that is the C expression of C has been given and then from the formula which have been described earlier from there we can get the temperature jump.

So, you see we got the temperature jump and we got the slip velocity these two and both of them they are dependent on the mean free path of the molecules lambda, certain accommodation coefficient some properties and obviously, on the gradient of velocity at the wall and gradient of temperature at the wall. So, this is how we get the velocity slip and temperature jump.

So, let us stop here today with this we will continue in our next class and we will see that considering velocity jump and sorry velocity slip and temperature jump how one can get the expression for pressure drop or other friction factor and the heat transfer coefficient. So, that will be giving us some idea that how to deal with convective heat transfer at micro scale.

Thank you.