Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 9

Lecture – Lec53

53. Reynold's Stress Modelling (RSM): governing equations - II

So, let us take the diffusion rate D_{ij}. This time this this particular term requires modeling because this has lot of unknowns. And we have three terms here the pressure diffusion rate, viscous diffusion rate and turbulent diffusion rate. So, pressure diffusion rate we have similar argument like in the when we model the pressure diffusion rate of turbulence kinetic energy. What did we do there? Sorry, the pressure diffusion rate of the turbulence kinetic energy was this modeled? No, we did not model.

So, what is the argument that we gave? So, we said this is very small compared to the other terms in simple canonical turbulent flows and it is also not possible to measure this in experiments. So, given this argument it was ignored we did not model this, but I did show you the data where if you deviate away from a smooth surface where let us say you introduce some roughness or some obstacles flow separation the pressure drag is very important there. So, the pressure diffusion also becomes very large. So, but we were going to use the similar assumption here.

The pressure diffusion rate is also omitted in this equation here. So, the pressure diffusion rate, the first one is pressure diffusion rate. So, this particular term is $-\frac{\partial}{\partial xk}$ of you have $\frac{p'u_j'}{\rho}\delta_{jk} + \frac{p'u_i'}{\rho}\delta_{jk}$.this term is neglected again for the same arguments, this term is neglected again with the same arguments that is difficult to measure this right. not possible is able to measure in experiments because it is a correlation of pressure fluctuation and the velocity fluctuation.

So, that is not possible. And the other argument is that in from the DNS data we see that DNS data shows that it is small compared to the other diffusion rates in simple or I can just say canonical shear flows. So, therefore, this is neglected not modeled at all ok, not modeled. We are omitting the pressure diffusion rate term. So, we have turbulent diffusion rate and the viscous diffusion rate.

So, the second one is let us say the viscous diffusion rate. which is $\frac{\partial}{\partial x_k}\left(\nu \frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right)$. Do we need to model this viscous diffusion rate? No, because Reynolds stress are available here just like the production rate term right modeling is not required here the term is readily calculated ok. So, no need to model since ui prime uj prime is available to us. are available to us.

So, we can just like the same argument as the viscous diffusion rate of turbulence kinetic energy that was $\frac{\partial k}{\partial x_j}$ there ok j was the repeated index. So, so far it is straight forward then that means we have another term which is the turbulent diffusion rate the 27 turbulent diffusion rate. this cannot be ignored because in a turbulent flow the turbulent diffusion dominates turbulence also transports in addition to it dissipates I told you right it also transports. So, turbulent diffusion rate is very important. And if you recall how we modeled a turbulent diffusion rate term for the k equation we use what is called a gradient diffusion hypothesis analogous to your Fourier's law of conduction right.

So, we are going to use that, but we will see that there are there is slightly a different option here. We can fall back to the original option or we can slightly make it little better ok. So, recall the recall the gradient diffusion hypothesis, hypothesis used while modeling k equation right. in the k model what did we do? The k model equation for modeling the turbulent diffusion rate term which was $-\frac{1}{2}\left(\overline{u_i' u_i' u_j'}\right)$ right, $\frac{\partial}{\partial x_j}$ of this term in the k equation. I am talking about going back to the k equation now first.

So, $\frac{\partial}{\partial x_j}$ of this particular term was modeled as modeling modeled as $\frac{\nu_t}{\sigma_k}\left(\frac{\partial k}{\partial x_j}\right)$ $\frac{\partial}{\partial x_j}$ of that. So, this particular correlation term was replaced by a gradient diffusion hypothesis that is where we use this $\frac{\nu_t}{\sigma_k}$ was an isotropic diffusion coefficient. So, where $\frac{\nu_t}{\sigma_k}$ was isotropic diffusion coefficient. So, it is dimensionally consistent, we continued in the k equation model like this. But there was an issue here especially when we come to Reynolds stress modeling.

So, we will see that this equation now essentially assumes that if diffusion along the x_j direction is 0 for example, then there is no then the total diffusion can go into 0. So, I mean it essentially looks into the gradient $\frac{\partial k}{\partial x_j}$. So, if $\frac{\partial k}{\partial x_1}$ let us say is 0, it assumes that there is no diffusion. are essentially that is the whole modeling point of it that we are looking into a gradient of that one. Gradient becomes 0, diffusion becomes 0.

To save that we have another option here. So, so here let us say call this equation 1. Note that equation 1 assumes that if gradient along x_j direction is 0, then the diffusion rate is 0. So, to save this we would like to introduce an anisotropic diffusion coefficient ok. So, we would like to have what is called an anisotropic diffusion coefficient here or we can consider a general gradient diffusion hypothesis.

is given by this is the reference Dalian Harlow 1970. So, according to this general gradient diffusion hypothesis we will make use of an anisotropic diffusion coefficient instead of this eddy viscosity nu t. So, what we do is we model this $\frac{\partial}{\partial x_j}$ of $-\frac{1}{2}\left(\overline{u_i' u_i' u_j'}\right)$ this is modeled as we essentially take $\frac{\partial}{\partial x_j}$ of I take another term here anisotropic diffusion coefficient which is C_k a model constant $\frac{k}{\varepsilon}$ at time scale introduced to make dimensionally consistent form and the anisotropic diffusion coefficient instead of eddy viscosity we use Reynolds stresses because Reynolds stresses is being calculated which represents the anisotropy in the flow. So, the Reynolds stresses comes in here which is $\left(\overline{u_j' u_k'}\right)$ ok and then your ∂x_j. So, here you had $\frac{\nu_t}{\sigma_k}$ that is the isotropic diffusion coefficient that is being now replaced with $\overline{u_j' u_k'}$ which represents the.

So, this represents the or contributes to accounts for anisotropic behavior, nature of turbulence and $\frac{k}{\varepsilon}$ is introduced for a dimensional consistency, a time scale a time scale introduced for dimensional So, this particular term is what we have here, this is your anisotropic diffusion coefficient. So, the dimensions has to be the same on both the sides here. If you look into it, it has to be consistent. So, whatever has appeared here instead of mu t by sigma k, if you go and look into it, you would get the same dimensions as the $\frac{\nu_t}{\sigma_k}$. So, this is the idea of this introducing an anisotropic diffusion coefficient in the k model equation.

one can do this. Now we will use the same idea to model the turbulent diffusion rate of the Reynolds stresses ok. So, similarly we can say now before that we have to justify right we just said that you know the diffusion goes to 0 if gradient along x_j is 0 right. If gradient along x_j direction is 0 diffusion is going to 0. Now you see how does this So, we can say here, for example, in the j direction, wait, wait, wait, there is a change here, there is some, this has to be, this is, there is a tensor rules are being, tensor rules are not correct because j is repeated thrice. So, this is $\frac{\partial}{\partial x_j}$, this is ok, this has to be different, ∂x_k here.

J was repeated thrice. Yes. So, this particular part is included. So, that this is a sum of now three terms. So, $\frac{\partial}{\partial x_j}$ if I expand it now you will see. For example, if I take this as the direction x1.

So, here if I take x1 direction, I would get it as $c_k \frac{k}{\varepsilon}$ if j if the k is the repeated index we will say that this is $u_j^{'} u_1^{'} \frac{\partial k}{\partial x_1}$ + I have $c_k \frac{k}{\varepsilon}$ $u_j^{'} u_2^{'} \frac{\partial k}{\partial x_2}$ + $c_k \frac{k}{\varepsilon}$ $u_j^{'} u_3^{'} \frac{\partial k}{\partial x_3}$ right. So, now even if $\frac{\partial k}{\partial x_1}$ is 0 in one of the directions the other two directions will save this one here. So, it is a sum of three different terms there. So, the gradients are even if it is 0 in one direction the other two directions can save you right. So, that is the advantage here.

the term that is inside I am not looking into the $\frac{\partial}{\partial x_1}$ of this the internal term itself as sum of the three terms here. Here the whatever is in the round bracket that was only $\frac{\partial k}{\partial} (\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial x_j})$. So, in that particular gradient is 0 the entire thing will be 0 that particular term. Now that term is now sum of three terms. So, even if gradient is 0 and 1 it will be saved here.

So, the anisotropic diffusion will help. So, now I am going to use the same concept here to model the RSM. So, similarly I can say, similarly modeling the turbulent diffusion rate in the RSM equation. So, we can say that the d_{ij} term here not the dk, d_{ij} let us say T or the turbulent component, d_{ij} turbulent component this is essentially $\frac{\partial}{\partial x_k}$ of - $u_i^{'} u_j^{'} u_k^{'}$ the 27 unknowns. This is now being modeled as modeled as analogous to what we did here the general gradient diffusion hypothesis.

I will have $\frac{\partial}{\partial x_k}$ ok and then I would take c_k the model constant. $\frac{k}{\varepsilon}$ at time scale and then I will introduce the Reynolds stresses which is $u_k^{'} u_m^{'}$ here followed by the Reynolds stresses $\overline{u_i^{'} u_j^{'}}$ by ∂x_m. There k was introduced in the k equation, the repeated index k so that there is no tensor rules not being followed. Here m follows here. So, it is a sum of three extra terms basically.

k is the divergence term but m is introduced so that this is sum of three terms here internally inside. So, k is also repeated, but for let us say $\frac{\partial}{\partial x_1}$ when k equal to 1 this is sum of 3 terms plus when k equal to 2 it is sum of 3 terms. So, it will be like sum of 9 terms. So, again the same argument here this is introduced for anisotropic diffusion coefficient. So, this particular part here is the anisotropic diffusion coefficient.

coefficient ok. But we have to check that whether it is dimensionally it is correct or not. So, here I get meter square by second cube, that is what I get here. Do I get this here? It is important. this last part is Reynolds stresses meter square by second square by meter. So, I get meter per second square meter square per second square $\frac{k}{\varepsilon}$ is second meter here.

So, I get meter square per second cube. So, dimensionally consistent and this particular $\frac{k}{\varepsilon}$ is actually introduced for that to make it dimensionally consistent. But there is a problem this particular if you implement like this now the question comes why did we do this? This is great anisotropic diffusion, but we did not do this in the k modeling equation that is because this introduces numerical stability issues. So, when you implement like this it leads to it is already strongly coupled this leads to some coupling issues, but one can try it out for a given problem it may work ok. So, one issue here is that this is numerically unstable not be in all flows, but one can try it out to see in general it is numerically unstable to use this form of modeling using an anisotropic diffusion coefficient.

So, what we do this is of course, option 1 to model. The option 2 is to fall back to the isotropic diffusion coefficient like before ok. So, the option 2 is to go back to option 2 is to again use isotropic diffusion coefficient. That means the model is the $\frac{\partial}{\partial x_k}$ of your - $u'_i u'_j u'_k$ this is now modeled as $\frac{\partial}{\partial x_k} \frac{\nu_t}{\sigma_k}\left(\frac{\partial u'_i u'_j}{\partial x_k}\right)$. Numerically stable, but we have introduced isotropic diffusion coefficient or I can easily say it is here it is already here ν_t.

But where do I get nu t here? This we are solving for Reynolds stress model. We can use the same formulation right $\frac{k2}{\varepsilon}$, but I do not have to model k here. what is k sum of 3 normal stresses divided by 2 that is available to you. So, you are computing the turbulence kinetic energy. So, ν_t is lot better here you do not have to model it, but the formulation you can use for ν_t right.

So, mu t you can still use it as say ν_t is $\frac{k^2}{\varepsilon}$ term. introduce Cν if you like or not that is depends on the model, but k and ε are available to you or I have not talked about ε , but at least where k is sum of your stresses. So, this is available to you being computed. So, it is available. So this is numerically stable, numerically a stable option while computing, but other can also be used anisotropic diffusion coefficient many flows it may work.