MECHANICS

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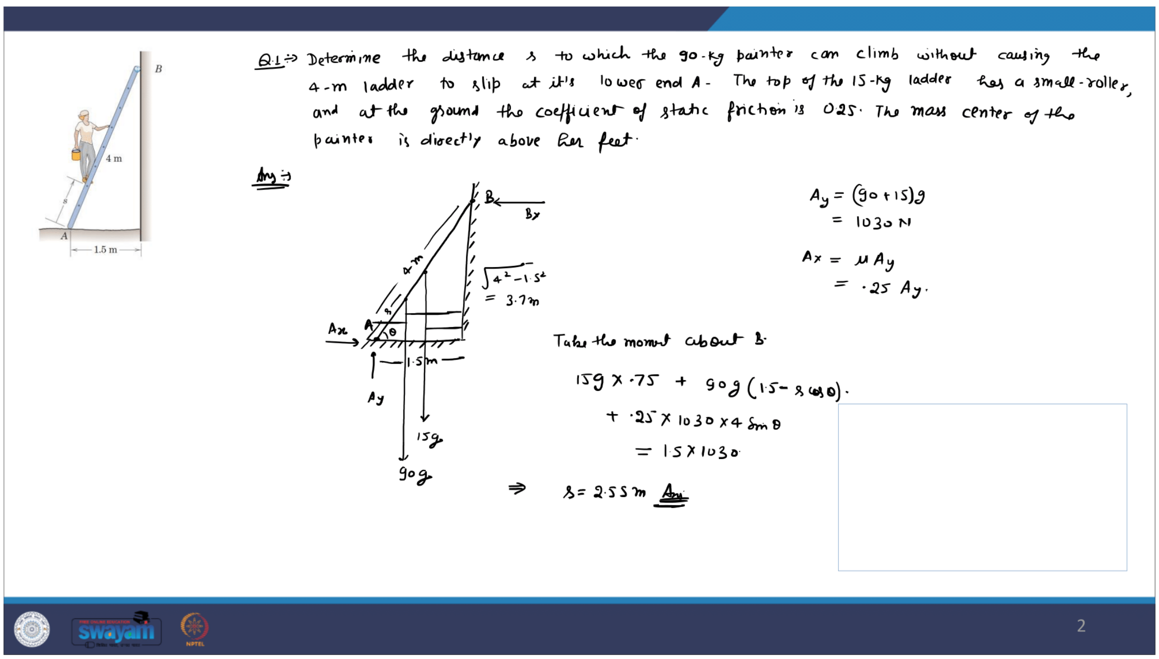
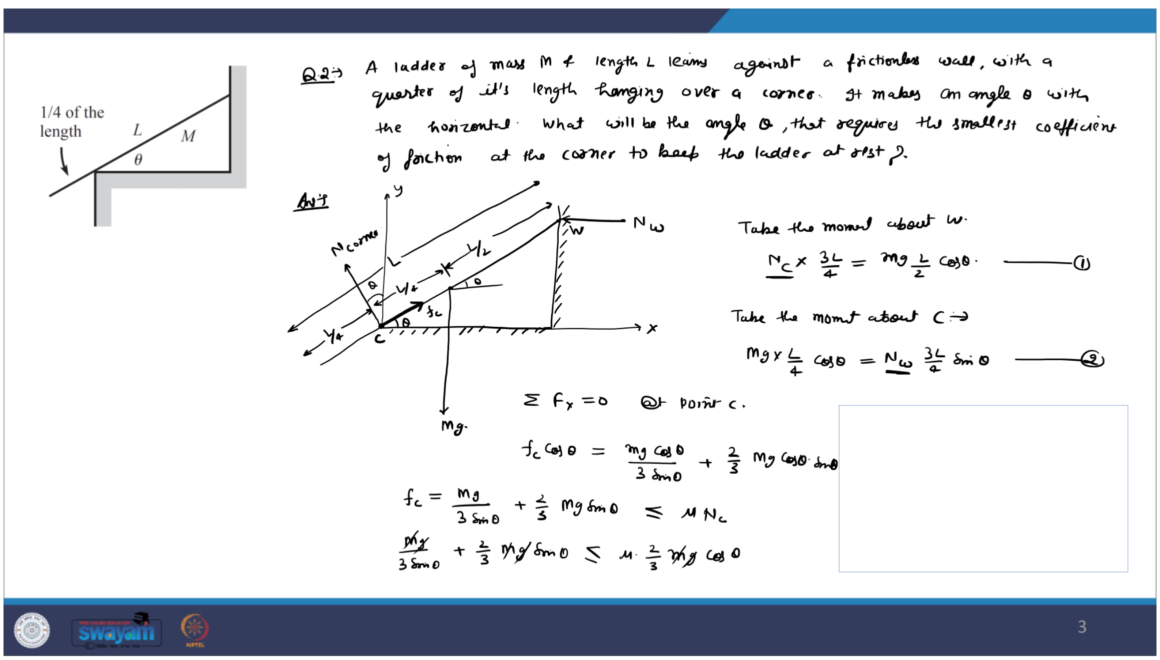
Lecture 25
Friction: examples

Hello everyone, welcome to the lecture again. In the last lecture, we look at the definition of friction, the kinetic friction and the static friction. We also solve couple of examples. Today, we are going to look at more examples about the friction. So, let me first write down the first problem statement.



The question statement is determine the distance s to which the 90 kg painter can climb without causing the 4 m ladder to slip at its lower end A. The top of the 15 kg ladder has a small roller and at the ground, the coefficient of static friction is 0.25. And it is also given that the mass center of the painter is directly above her feet. So, to solve this question, let us first make the free body diagram. So, we have the ground and the wall and this is the ladder. This point is A, this point is B. At A, there will be a force A_y and A_x. At B, since there is no friction and there is a roller, so therefore, the reaction force will be in the x direction. It is also given that this is 1.5 m, this is 4 m. Therefore, this length will be $\sqrt{4^2 + 1.5^2} = 3.7\ m$. Now, the weight of the ladder will act at the center and it is 15 multiplied by g. So, therefore, $15g$ and the weight of the painter is acting at a distance of s which we have to determine and it is $90g$. Let's say this angle is θ. So, we can balance the forces in the x and the y direction. So, from here we have $A_y = (15 + 90)g = 1030\ N$. Now, the value of A_x, this is because of the friction. So, either you can write down A_x or you can write down the frictional forces. Its value will be μN. So, therefore, it will be $\mu \times A_y$. So, and of course, this is the maximum value of the friction. So, this you can write down because now A_y I know and $\mu = 0.25$. It is given multiplied by A_y. $A_x = 0.25 \times A_y$. Now, to find out s, what we can do is we can take the moment about B. So, therefore, B_x will not contribute. So, let us take the moment about B. So, we have $15g \times 0.75 + 90g(1.5 - s cos\theta) + 0.25 \times 1030 \times 4sin\theta = 1.5 \times 1030$ and this gives you because $cos\theta$ is also known, g is also known. So, this gives you $s = 2.55\ m$. So, this was the example of a leaning ladder.



Now, let us look at one more question wherein the ladder is at the corner. The problem statement is following. A ladder of mass m and length L leans against a frictionless wall with a quarter of its length hanging over a corner. It makes an angle θ with the horizontal. And in the question, it has asked what will be the angle θ that require the smallest coefficient of friction at the corner to keep the ladder at rest. So, we have to find out the smallest value of μ and from there we have to find out what is that angle θ. So, let me again make the free body diagram of this. This length is $L\backslash 4$ and the reaction forces are going to act in the following way. So, from here, there will be a normal force. Let us say this is N_c,

c for corner and There will be a frictional force. So, because the impending motion of this ladder will be downward. Therefore, let us say that the frictional forces are like that. And let me call it f_c, c for corner. This point is, of course, given as c. Therefore, since the entire length is L, its weight is going to act at the middle. So, this will be mg. So, therefore, this length will also be L/4 and this will be therefore, $L/2$. Because the vertical wall is frictionless, therefore, there will be only the horizontal component. So, let me call it N_w. Okay. Now, this is the free body diagram. It is also given that this angle θ that we have to find. Therefore, this angle will also be θ. Let me fix the coordinate axis. So, let's say this is x-axis. This one is the y-axis. Therefore, this angle will also be θ from the geometry. So, first let us take the moment about w. In that case, we have $N_c \times \frac{3L}{4} = mg\frac{L}{2}\ cos\theta - - - - - (1)$.

Note that the contribution of N_w will not be there because we have taken the moment about w. Similarly, let us take the moment about c. So, we have

$$mg \times \frac{L}{4} cos\theta = N_w \frac{3L}{4} sin\theta - - - - - (2)$$

And again, note that since we have taken the moment about c, therefore, the contribution of N_c will not be there. Now, to find out the relation among them, let us take or let us balance the forces along the x-axis. So, let me say that $\sum F_x = 0$ and let us say we are balancing this at point c. So, we have

$$f_c cos\theta = \frac{mgcos\theta}{3sin\theta} + \frac{2}{3} mgcos\theta sin\theta$$

So, I have balanced the forces along the x-axis and the value of N_c and the value of N_w, I have taken from equation number 1 and 2 and then take the component along the x-axis.

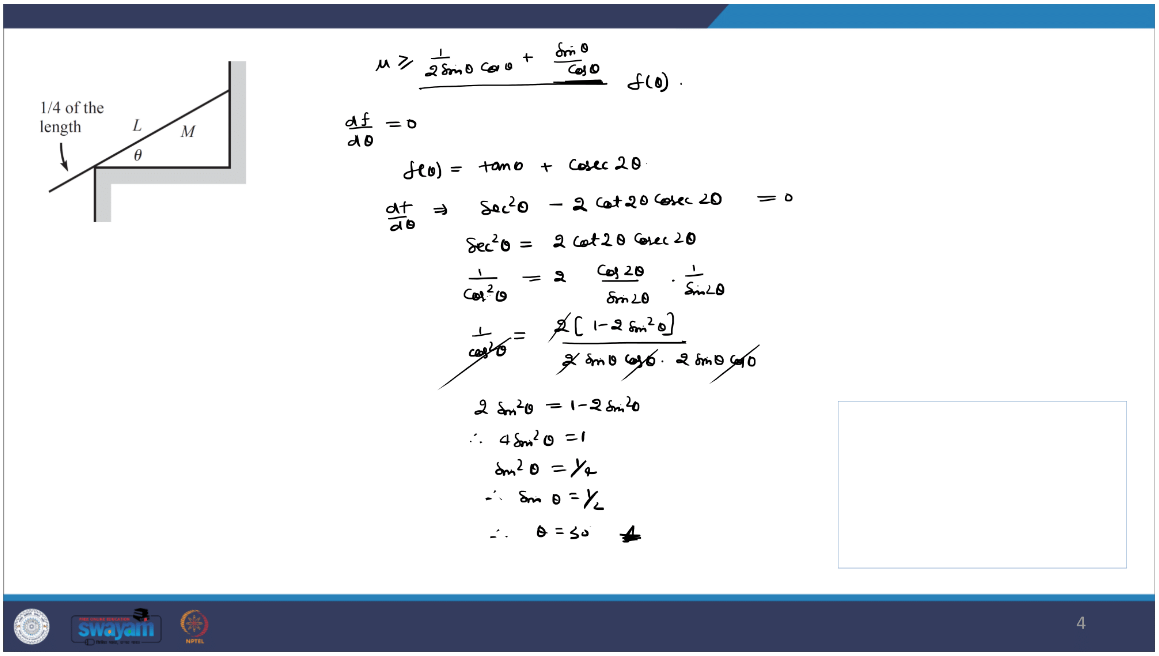
$$f_c = \frac{mg}{3sin\theta} + \frac{2}{3} mgsin\theta \leq \mu N_c$$

So, this is the definition of the frictional force. Therefore, we can write down

$$\frac{mg}{3sin\theta} + \frac{2}{3} mgsin\theta \leq \mu . \frac{2}{3} mgcos\theta$$

This gives me a relation between μ and θ. So, mg will get cancelled and we can rewrite this as $\mu \geq \frac{1}{2sin\theta cos\theta} + \frac{sin\theta}{cos\theta}$

Okay. Let me call this function as $f(\theta)$ and to find out what is the minimum value of this μ, we have to minimize this function and that will give us θ. So, to minimize this function, $\frac{df}{d\theta} = 0$. So, let us calculate the differentiation of this.



So, we have $f(\theta) = tan\theta + cosec2\theta$

So, let us minimize this $\frac{df}{d\theta} \rightarrow \sec^2\theta - 2cot2\theta cosec2\theta = 0$

$\rightarrow \sec^2\theta = 2cot2\theta cosec2\theta$

$$\rightarrow \frac{1}{\cos^2\theta} = 2\frac{cos2\theta}{sin2\theta}\frac{1}{sin2\theta}$$

$$\rightarrow \frac{1}{\cos^2\theta} = 2\frac{(1-2\sin^2\theta)}{2sin\theta cos\theta}\frac{1}{2sin\theta cos\theta}$$

$\rightarrow 2\sin^2\theta = 1 - 2\sin^2\theta$

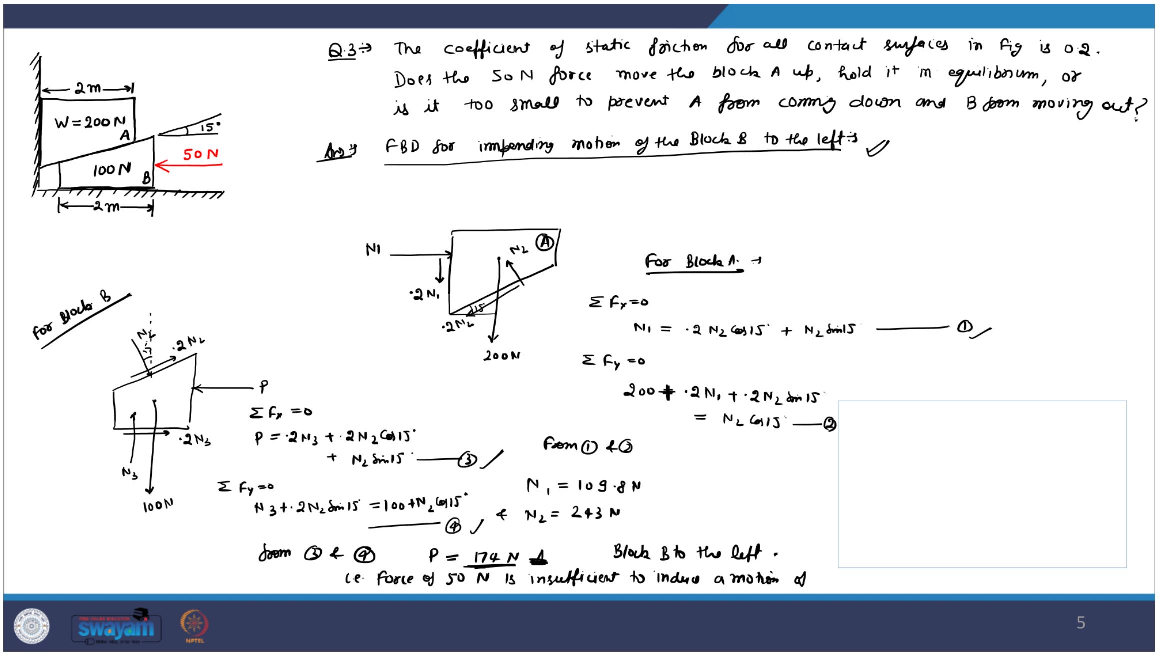
$\therefore 4\sin^2\theta = 1$

$$\therefore \sin^2\theta = \frac{1}{4}$$

$$\therefore sin\theta = \frac{1}{2}$$

$\therefore \theta = 30°$

That means that for this $\theta = 30°$, our μ will be minima and this is the problem statement to find out θ for which the coefficient of friction is smallest.



Now, let us look at another problem statement and it is following. The coefficient of static friction for all contact surfaces in figure is 0.2, does the 50 N force move the block A up, hold it in equilibrium or is it too small to prevent A from coming down and B from moving out. To find out whether the 50 N force will move the block A up or downwards, let us find out the force that is required to push the block inside or to pull the block outside. So, for that, let us make the free body diagram for impending motion of the block B to the left. Let me make the free body diagram of block A. So, this is for block A. From the wall, there will be a reaction force. Let me call it N_1. And since block B, we are assuming or we are pretending that it is moving inward. Therefore, the block A is going to lift up. Therefore, the frictional force will be downwards and it will be $0.2N_1$ because $\mu = 0.2$. Its weight is given. It is 200 N. And when the block B is going to move in, the reaction force on it will be upward. Therefore, on block A, it will be downwards. So, this is how the friction force on block A is going to act. Let us say its normal force is N_2. Therefore, this will be $0.2\ N_2$. Now, to find out what is the value of N_1 and N_2, let us balance the forces along the x and y direction.

So, $\sum F_x = 0$. This gives me

$N_1 = 0.2N_2cos15° + N_2sin15° ----(1)$

Now, let us balance the forces along the y direction.

$200 + 0.2N_1 + 0.2N_2sin15° = N_2cos15° ----(2)$

So, from equation number 1 and equation number 2, we have two unknown and rest of the parameters are given. So, therefore, from 1 and 2, we can find out N_1 and N_2 and $N_1 = 109.8\ N$ and $N_2 = 243\ N$. Now, let us look at block B. So, this is the free body diagram of block B. So, external force P is acting on it. N_3 will act upward. Its weight is given, it is 100 N. Now, as I said, we are pretending that the block is moving to the left. Therefore, its frictional force will be like that and it will be $0.2N_3$ and here you will have $0.2N_2$ because we have normal force N_2 and of course, it is acting at 15°. This completes the free body diagram of that. Now, again, let us balance the forces in the x and y direction.

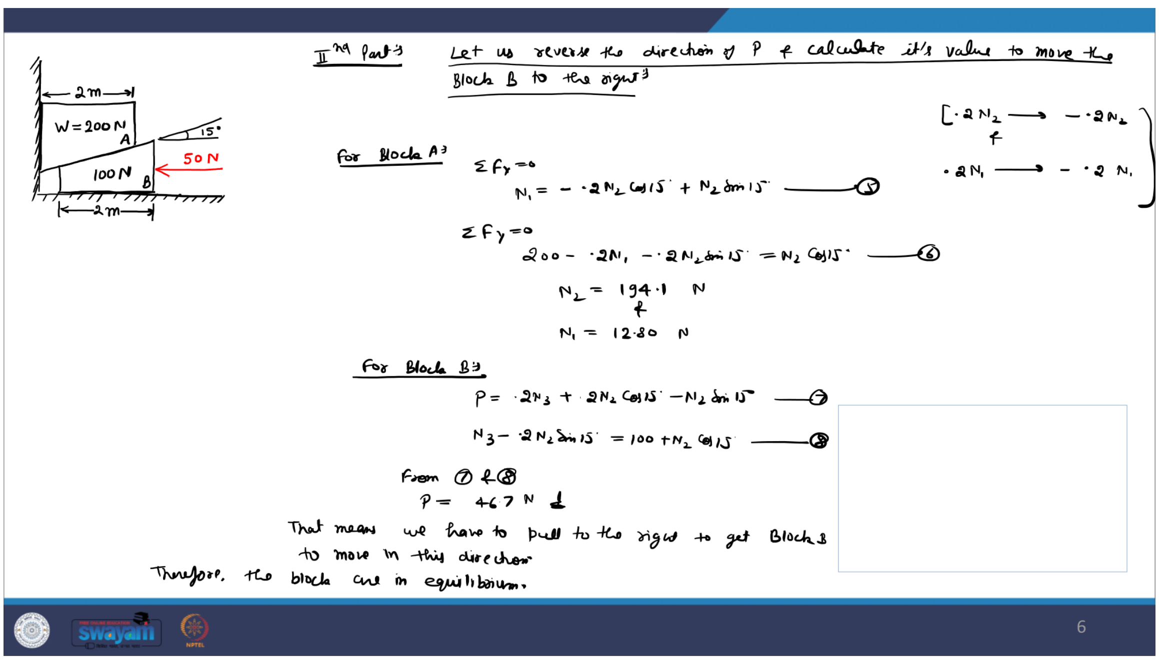
$\sum F_x = 0$. This is for block B.

$P = 0.2N_3 + 0.2N_2 cos15° + N_2 sin15° - - - - (3)$

and then we have $\sum F_y = 0$ that gives you

$N_3 + 0.2N_2 sin15° = 100 + N_2 cos15° - - - - (4)$

Again now from equation number 3 and 4 you can find out what is the value of P. And it comes out to be 174 N. Now, in the question statement, it has given that P was 50 N. Therefore, this 50 N force is not sufficient to push this block inside because just now we have calculated that the value for P should be 174 N. So, this implies that force of 50 N is insufficient to induce a motion of block B to the left.



Now, let us reverse the direction of the force and find out what is the force that is required to pull this block out. So now, let us reverse the direction of P and calculate its value to move the block B to the right. So, note that the only changes that are going to happen is the direction of N_2 and N_1 are going to reverse. So, in the previous part, wherever you have $0.2N_2$, this will become $-0.2N_2$ and $0.2N_1$ will become $-0.2N_1$ because the direction of the frictions are going to change. So, let me just write down for block A. Now, $\sum F_x = 0$. You can look at this equation. And as I said, the direction of the frictional force are going to change. And of course, P also we have reversed. So, therefore

$N_1 = -0.2N_2 cos15° + N_2 sin15° - - - -(5)$

and then we have $\sum F_y = 0$. So, you have

$200 - 0.2N_1 - 0.2N_2 sin15° = N_2 cos15° - - - -(6)$

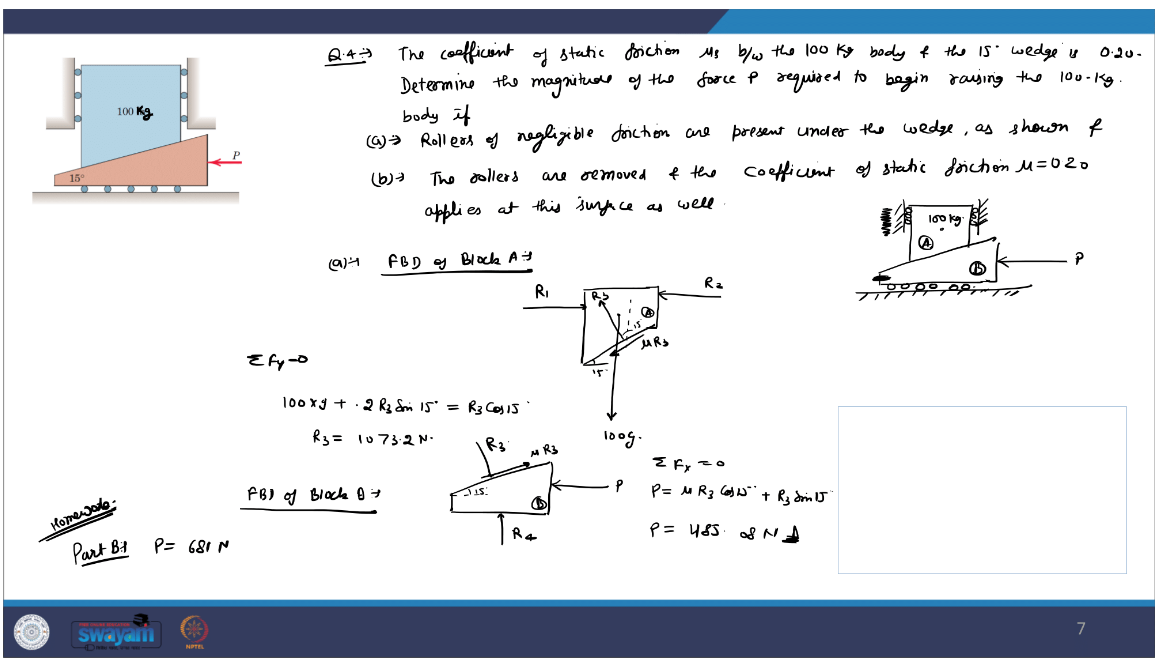
and from 5 and 6, you can find out what is the value of N_2 and N_1. So, you get $N_2 = 194.1\ N$ and $N_1 = 12.80\ N$. So, for block B, again we can use equation number 3 and 4 and we can change the direction of the frictional forces. So, this will be

$P = 0.2N_3 + 0.2N_2 cos15° - N_2 sin15° - - - -(7)$

and we have

$N_3 - 0.2N_2 sin15° = 100 + N_2 cos15° - - - -(8),$

let us say this is equation number 8 and again from 7 and 8 as we have done in the last part, you can find out what is P and you can see that $P = 46.7\ N$. So, this implies that we have to pull to the right with this 46.7 Newton force to get block B to move in this direction okay. Therefore, the block are in equilibrium okay because To pull the block, we need 46.7 N force and to push the block, we need 174.7 N force and we are applying only 50 N force. Therefore, the block will be in equilibrium.



Now, let us look at one more question on the same concept and the problem statement is following. The coefficient of static friction μ_s between the 100 kg body and the 15° wedge is 0.20. Determine the magnitude of the force P required to begin raising the

$100\ kg$ body if (a) Rollers of negligible friction are present under the wedge as shown and (b) The rollers are removed and coefficient of static friction $\mu = 0.20$ applies at this surface as well. The concept of this question is same as the previous question. So, here we have these rollers and we have a block which is being raised by applying a force P. The weight of this block is $100\ kg$. Let us call it block A and let us call it block B. So, in the question statement, we have asked to find out what is the value of P so that the $100\ kg$ block is getting lifted. So, let us look at the free body diagram of block A. So, we have the following situation. Note that at these ends, we have a roller support. Therefore, there is no friction. So, we have only R_1 let's say and R_2 force. Since this block B is having an impending motion towards the left, therefore the friction on block A will be like this and its value will be μR_3 where R_3 is the normal reaction. The weight of the block is of course, going to act downwards and it is given that it is $100\ kg$. So, therefore, weight will be $100g$. This angle is of course, $15°$. So, let us balance the forces along the y direction. $\sum F_y = 0$. So, we have $100 \times g + 0.2R_2 sin15° = R_3 cos15°$ and we know the value of $sin15$ and $cos15$. So, this gives you R_3. and $R_3 = 1073.2\ N$. Now, let us look at block B. So, this is the free body diagram of block B. The force P is acting on it. Therefore, the impending motion on block B will be left. Therefore, the frictional force are going to act upward μR_3, where R_3 is the normal force. And from the bottom, there is going to act a reaction force R_4. There is no frictional force because it is a roller support and it is frictionless. So, let me balance the force along the x direction. $\sum F_x = 0$. So, we have $P = \mu R_3 cos15° + R_3 sin15°$ and Again, the value of R_3 is known, the value of μ we know and $cos\ 15$ and $sin\ 15$ also we know. Therefore, $P = 485.08\ N$. That means we require this much force to move this block towards the left. Part (b), wherein the rollers are removed and replaced by a frictional surface, you can take it as a homework problem and the final answer for part (b) is $P = 681\ N$.

With this, let me stop here. See you in the next class. Thank you.