Convective Heat Transfer

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Lecture – 01
Introduction to convective heat transfer

So, welcome to the convection heat transfer course. So, this will be the first lecture. So, before I go on and start describing that how we will approach this entire topic. Let us have a very brief background that what is convection heat transfer per say. So, those of you who have taken courses on fluid mechanics know that fluid mechanics is basically a transport of momentum essentially.

Now heat transfer whoever have not taken convection know that heat transfer is associated with say for example, you may be familiar with problems in conduction; that means, you take a rod you heat one end of the rod and after some time you feel that the other end of the rod becomes heated, right.

So, somehow the heat has conducted through this solid medium, right and you have felt its effect. Now that is of course, quite relevant in objects which are basically not moving; that means, solid bodies; say for example, it can also happen in fluids nevertheless, but usually these are not associated with movement of the medium per say.

So, by movement of the medium, I mean that there is no transport that is happening between the 2 ends of the rod except through the process of diffusion or conduction that takes place through the rod. So, it is conduction that you most of you are actually familiar with convection may not be all that apparent, but the fact that convection is an important phenomena if you look at any application in the world, right.

Anything and that can range from micro fluidics, all the way up to large gas turbines, it can encompass like for example, biology fields in biology 2 fields in fabrication manufacturing wherever you go you will find that convection heat transfer is perhaps the most important mode of heat transfer that is available right. Now the other 2 modes of heat transfer, the conduction and the radiation they are also important, but convection is actually encompasses both conduction and advection is related to the motion, right. So, these 2 combined gives that term the convection and this is one of the most important modes of heat transfer which is commonly encountered in all spheres of life in all important applications that you are going to encounter with, right.

So, in this particular course, we are going to talk through that what are the different aspects of convection heat transfer. For example, we will first do the conservation equations that is there is a conservation of mass momentum and energy, then we will shift gears and go and look at some canonical problems like for example, flow over a flat plate, then look at the classes of solutions that how we can solve a problem for external flows.

Then we will go on to internals flow like flow through pipes and other things, then we are going to move on to natural convection natural convection, we will see what natural convection is in due course and then we are going to look at the turbulent modes of heat transfer and if time permits, we are also going to focus a little bit on the small scale nature of heat transfer as well through convection.

So, let us start. So, if I start jotting down some of the points.

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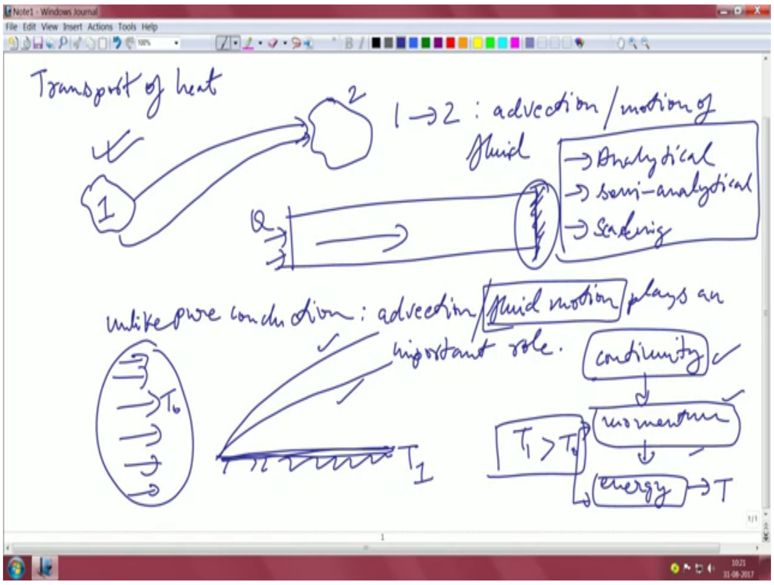
Transport of heat

1 → 2 : advection / motion of fluid

unlike pure conduction : advection / fluid motion plays an important role.

So, the transport of heat, if you look at the board or the transport of heat here; for example, there is a blob of fluid. Let us call that fluid blob one it moves to another location which is called say location 2. So, basically, what it does is that fluid blob one transports whatever energy that it carries to the location 2, right. So, in this case the energy is in the form of heat. So, this transfer from 1 to 2 has happened due to advection right advection or motion of fluid, correct.

Now, if this was conduction this would have been a rod say you apply heat to one end of the rod after some anyway you put your palm on this side of the rod or some kind of a sensor on this side you will feel the heat after sometime, right. So, that would be conduction through the medium without this kind of a motion that you see over there right.

So, conviction essentially as you can understand it is a much more what we call much more enhanced process because there is a bulk motion that is involved. So, it is not a slow process it is actually supposed to be a little fast right compared to pure conduction. So, unlike pure conduction; right; so, unlike advection slash fluid motion plays an important role.

Now, if you look at it from the scheme of things whenever there is fluid motion; that means, there will be a lot of fluid dynamics that will be involved because first you have to know the fluid dynamics of the flow field right, let us take an example, let us take this example of a flat plate, right, this flat plate and then, there is a flow that is approaching the flat plate. Now this flat plate what might sound to be a very why am I placing a fact flat plate in a flow field this flat plate can be say for example, a gas turbine blade a heated gas turbine blade which you need to cool otherwise this blade will actually crack.

This is the most common gas turbine as you know is the most common form of power generation which derives the power in a in a aircraft or powers your house and all these things. So, it actually produces 25 to 30 percent of the world's power, right. So, in a gas turbine that turbine blades need to be cooled. So, that they do not crack and they do not fail, right. So, you can imagine that this media heated plate at a temperature T 1 and this may be a flow of cold fluid which is at a temperature T naught such that T 1 is greater than T naught right. So, what you are essentially trying to do is that you are trying to extract the heat out of that plate; right.

Now, if there was no flow if there was no flow in the absence of natural convection this plate would have lost some amount of heat to the surroundings correct that would happen right that is also happens; say for example, if you place a heated you know heated rod in the in an ambient environment with no flow say for example, you will find that this will cooled definitely it will cool down because heat will flow from high temperature to low temperature no matter what, but it will take a very long time for it to cool, right. So, in a gas turbine environment that time scale is something that you cannot afford, right.

So, in order to achieve the rapid cooling of a blade what do you do the most common thing is to pass a flow over it this is the same reason that if you have a say you are drinking tea, right. So, what happens is that you sometimes actually you know blow air over the tea to cool it down correct that is what we normally do. So, you just blow a little air then you sip the tea, right, why do you do that?

You are basically creating a flow over the surface of the tea why that is because you are trying to create a convective mode of heat transfer so that you can extract heat at a very rapid rate. So, essentially that is the same thing if you look at the PPT once again or in the journal you will find that this is a plate and there is a flow that goes over the plate, right.

With the sole purpose of cooling it, right. Now in order to describe this particular phenomena, say I as an engineer, I want to know that how this particular plate is going to get cold, right, what will be the temperature distribution in the near vicinity, right, how effective I can actually cool the whole thing for knowing those answers, right, we have to first analyze the problem and what is the analysis first you have to solve, what is the flow field right; that means, what will be the flow field this flow which comes out nice and uniform as it encounters the plate, right, what will be the flow field looks like once we know the flow field, then you can use the flow field basically to solve the equation. So, the hierarchy is very simple first you need to solve continuity which is basically mass conservation.

Next you need to solve the momentum which is basically your fluid dynamics equation right and ultimately you need to solve your energy equation which if you solve it properly will give you the temperature field right now normally all these things may be actually coupled with each other specially these 2 what do I mean by coupling? Coupling I mean that say for example, take this example that when you actually have temperature in a flow field the density changes right because density is a property which is a function of temperature right and the momentum equation, we will see shortly has got density as one of the most important terms, right. So, there could be that with energy.

So, momentum equation is influencing the energy that is because you have advection now in this energy equation the density changes because of the temperature field this density change once again feeds back into your momentum field right and that creates what we call a 2 way coupling between the 2; that means, you cannot solve the 2 equations independent of each other.

But in most of the cases you will find that the coupling is only one way; that means, the momentum influences the energy which it would because it is a flow I mean we will see that the energy equation will have the flow terms, right. So, you need to solve the flow field before you come to the energy equation right standard, but we or will not deal with situations where the reverse happens; that means, there will be energy which will change the density in such a way that the momentum will be altered.

We are going to do natural convection, but there will be some approximations that we will make where we need we do not need to deal with this kind of an acute problem right. So, that is what it is right. So, the order of things still remains the same you have to solve continuity momentum and energy.

So, you cannot directly go and jump and solve the energy equation in conduction, it is possible because there is no flow understand the thing in conduction, it is possible to solve the energy equation as is right, but when you are dealing with convection you cannot solve the energy equation as is right you first have to come through the momentum right. So, once unless you solve these 2, there is no way you can actually describe this canonical problem where there is a flow over a flat plate heated plate.

So, you have to first know the flow field in this case the flow field will look like something like that right once you know the flow field then you can use that to solve the energy or the temperature field right which will look something like that and once you have those 2 informations you can find out that how this plate can be effectively cooled or effectively heated whatever may be the purpose, right.

So, as a first step in the first lecture what we are going to do we are going to look at the continuity equation first because our order of business will be first, we will start looking at the conservation equations because those are the building blocks. Once we have a thorough idea of the conservation equations then we will move to all these canonical problems right because once we do the canonical problems properly, you will get a feel that what convection heat transfer is right and any canonical problem we are going to solve it in try to solve it in actually 3 ways ok.

One will be the complete analytical solution; that means, you can just use pen and paper to solve the whole thing as you know that fluid dynamics equations are non-linear. So, solving them analytically poses a very big challenge, but in certain cases you can especially where the non-linear terms are small or 0. We can make certain approximations certain special cases.

The second class will be some kind of approximate solutions. So, we will talk about integral formulations and things like that and the third and the most important thing from an engineering point of view we are going to spend a lot of time on doing this scaling analysis because why scaling analysis is an important because this will enable you to take a pen and paper and essentially work out the physics of the problem and get the correct relationships without going into too much math or experiments, right.

So, that gives you from an engineering perspective a great feel of the problem. So, you know that how my heat transfer is happening, right without spending a lot of time in doing very complicated simulations or experiments. So, that is vital because that gives you a lot of insights. So, essentially this will be the way that we will work out if you look at. So, there will be some analytical approaches there will be some semi analytical which we will see what those are right and there will be of course, this scaling. So, these 3 approaches are going to give us a complete idea of each and every canonical problem that we can get hold off right, but remember most of the problems in real life will be much more complicated and then this.

So, that will require perhaps extensive experimentation or it might actually require complicated CFD simulations, but these will offer you the great insights on the nature of the problem. So, that later on when you go to more complicated situations you have a feel you have the fundamentals worked out. So, that is the first thing. So, let us look at the first set of equations that we will do. So, that will be mass conservation or continuity; this is important.

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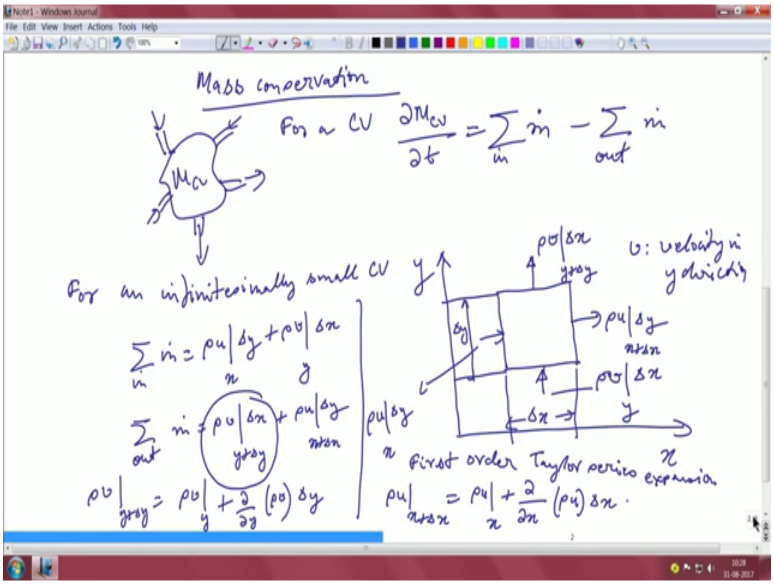

Mass conservation

For a CV $\frac{\partial M_{cv}}{\partial t} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$

For an infinitesimally small CV

$$\sum_{in} \dot{m} = \rho u|_{x}\,\delta y + \rho v|_{y}\,\delta x$$

$$\sum_{out} \dot{m} = \rho v|_{y+\delta y}\,\delta x + \rho u|_{x+\delta x}\,\delta y$$

$$\rho v|_{y+\delta y} = \rho v|_{y} + \frac{\partial}{\partial y}(\rho v)\,\delta y$$

First order Taylor series expansion

$$\rho u|_{x+\delta x} = \rho u|_{x} + \frac{\partial}{\partial x}(\rho u)\,\delta x\,.$$

So, if you observe, let us take any arbitrary control volume. So, this is a control volume which can have multiple inlet and outlets correct. So, there can be some inlets some outlets; that means, mass is entering into this control volume through multiple ways and the mass inside the control volume can be allowed to change.

So, that is m CV. So, for a control volume; so, control volume in short as CV m CV by d t is basically summation of all the inlet mass minus on the outlet mass correct. So, whatever mass is coming in minus whatever mass is going out that would dictate what is the change of mass within the control volume right, this is a very simple thing that you have done in thermodynamics and other cases as well, right.

So, there is a lot of inflow outflow. Now if I make this control volume infinitesimally small control volume, let us take this particular picture this is the control volume though I have drawn it big it is actually small and let us do it in 2 D, but potentially this can be done in 3 D as well.

So, there is no problem, but 2 D is easier for people to understand this length is delta x delta x being a very small number this length is basically delta y delta y is once again a small number right. So, what is happening? So, this is a very small control volume with delta x delta y that kind of a thing. So, what happens is that mass enters into this control volume through this phase leaves this control volume through the other phase right similarly mass enters this control volume through this phase and leaves the control volume through this phase right phase. So, this particular phase the mass can be written as rho v sorry rho v evaluate it at y into some delta x where v is basically the velocity in the y direction right v velocity in y direction correct.

So, you can understand this is the mass rho into v should give you and then multiplied by delta x and this is a unit volume. So, you can basically the other phase the 3 D part of the face is basically one. So, and what is leaving out this will be still rho v, but it is evaluated as y plus delta y. So, these 2 Vs are not the same one v is evaluated here at y and one v is evaluated at y plus delta y this is also multiplied by delta x correct.

So, similarly on this face what will happen write it here is rho u, right, evaluated at x into delta y right and what is leaving on the other side is rho u evaluated at x plus delta x into delta y correct. So, this is the situation, all right. So, once again u is evaluated at x u is evaluated at x plus delta x these 2 us are actually different. So, coming back now if we write all the inlet mass now within this control volume right what it will be it will be rho u in evaluated at x into delta y correct plus rho v evaluated at y into delta x correct.

These are the this is the mass that is entering inside the control volume as you can see from the arrows right then there are mass which is going out this is m dot right. So, that will be rho v evaluated at y plus delta y into delta x correct plus rho u into x plus delta x into delta y. So, put a marker here. So, that you do not get confused. So, this is the inlet and the outlet mass, right. Now how do we evaluate terms like this that is a question right because we have to somehow cast it, right, in order to get an expression cast it in some familiar form. So, what is the commonly used thing that we use is first order Taylor series expansion Taylor series expansion.

So, what does the expansion look like let us say for example, you take rho u at x plus delta x right, this term, you take u basically expand it around rho u at x plus this is a first order only, right. So, you know what will be the truncation term rho u into delta x correct. So, that is the first expansion. So, similarly you can work out for rho v y plus delta y right you will first have rho v evaluated at y plus d by d y right into rho v into delta y right. So, that is the expansion of each of the term right got it. So, this is the first order Taylor series expansion first order only. So, we are retaining the first order term only. So, when you evaluate the exit velocity you can expand it around the inlet velocity plus whatever is the slope essentially this is the gradient the second term is essentially the gradient right now the whole thing, good.

So, now that we have established this now the job becomes very easy.

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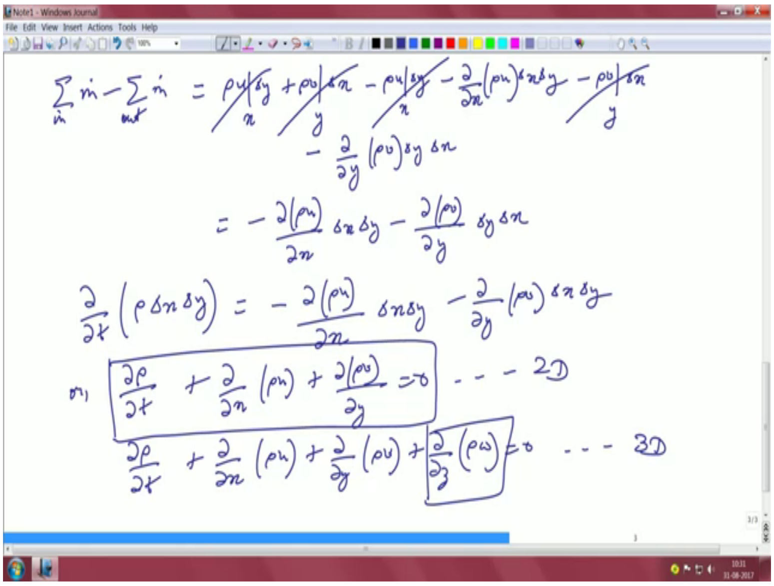

$$\sum_{in}\dot{m}-\sum_{out}\dot{m} = \cancel{\rho u|_{x}\delta y} + \cancel{\rho v|_{y}\delta x} - \cancel{\rho u|_{x}\delta y} - \frac{\partial}{\partial x}(\rho u)\delta x\delta y - \cancel{\rho v|_{y}\delta x} - \frac{\partial}{\partial y}(\rho v)\delta y\,\delta x$$

$$= -\frac{\partial(\rho u)}{\partial x}\delta x\delta y - \frac{\partial(\rho v)}{\partial y}\delta y\,\delta x$$

$$\frac{\partial}{\partial t}(\rho\,\delta x\,\delta y) = -\frac{\partial(\rho u)}{\partial x}\delta x\delta y - \frac{\partial}{\partial y}(\rho v)\delta x\,\delta y$$

or, $\boxed{\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial(\rho v)}{\partial y} = 0}$ - - - 2D

$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \boxed{\frac{\partial}{\partial z}(\rho w)} = 0$ - - - 3D

So, to say so; that means, when you actually have in m dot minus m dot out, right, you have to evaluate this right. So, what we do let us assemble all the terms together that is this is x plus rho v y delta x minus rho u evaluated at x into delta y minus d by d x rho u delta x delta y minus rho v into y delta x minus rho v by rho y rho v to delta y into delta x got it. So, as you can see most of these terms will actually drop out because you are expanding it, this terms will drop out. So, you will be left with an expression which will be rho u delta x into delta x into delta y minus rho v into delta y delta y into delta x, right. So, that will be the expansion.

So, this actually tells you what is the mass going in mass going out what is the rate of change of mass within the control volume that will be given by this rho delta x into delta y right because the other side of the control volume always said it was unity right. So, this should be now equal to equal to this particular term. So, now, or so, if you now transfer the terms of the right hand side to the terms on the left hand side and if you did I if you take off the delta x and delta y out you will find this expression takes this nature right if you want to write it in 3 D. So, this is actually strictly in 2 D right if you want to put it in 3 D. So, that will give you where this is the additional term due to the 3 D effect.

So, at z is the other direction z or z is the other direction and w is basically the velocity component in the z direction. So, based on this is what your continuity equation looks like clear. So, this is what the continuity equation looks like right. So, you can write it in a little bit more compact way.

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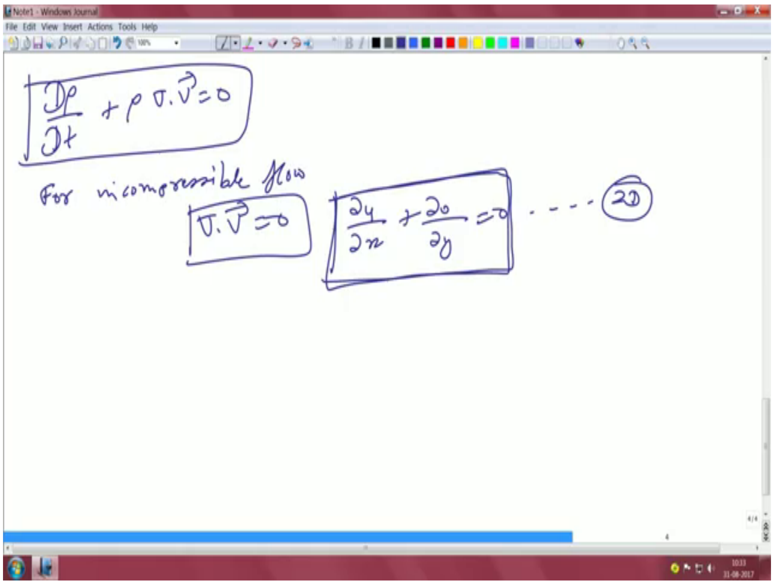

$$\boxed{\frac{D\rho}{Dt} + \rho\, \nabla\cdot\vec{V} = 0}$$

For incompressible flow

$$\boxed{\nabla\cdot\vec{V} = 0} \qquad \boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \; \text{----} \; (2D)$$

So you can also write it by the using the material derivative this is the most compact way of writing the continuity equation got it. So, if you expand this you will basically get what I wrote for the 3 D or the 2 D depending on what your flow configuration is. So, this is the compact notation now for incompressible flow things become a little bit more simple, what is incompressible flow; that means, incompressible flow means that there is no density change, right.

So, when there is no density change for an incompressible flow all we get this reduces to is equal to 0 right in other words if we cast it in a proper form this means in 2 D, you can write it like this will be the most common form that we will use right. So, this is in other words it says that the volumetric dilatation is actually equal to 0.

So, this is the most common form of the equation that you will get d u d x plus d v d y is equal to 0 if there is a third dimension that will be plus d w d th z will be also equal to 0 right. So, this is incompressible right when density is not changing with the flow field. So, this basically completes that from this continuity equation all that in most of the flows which are kind of incompressible we are going to deal with mostly incompressible flow you will find that this is the most common form of equation that you will use; that means, there is no volumetric dilatation of the flow field there is no volumetric dilatation of the flow field that is what this actually essentially means.

So, in the next lecture, we are going to look at the situation in which we will look at the conservation of momentum.