Convective Heat Transfer

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Lecture – 06
Scaling Analysis – Energy l

In the last class, we did the x momentum equation reduced it in the boundary layer by assuming that the delta the thickness over which the over the bound; that is, the thickness of the boundary layer and the velocity varies over that small thickness is much less compared to the variation of velocity in the in the x direction essentially, right.

So, that was the main conjecture, where we said that this variation is a lot sharper lot higher compared to the variation in the along the length of the plate. That was the main understand right.

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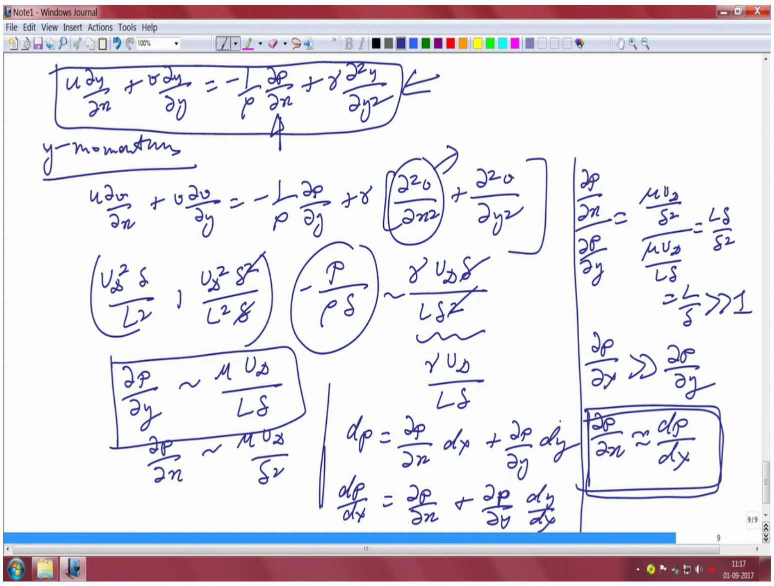

So, the y momentum equation if you write it is d v by d y u y square. So, that was a y momentum equation. Now let us once again substitute the individual terms, right. U we already established what is the length what is the order or the scale of v from the continuity.

So, it is u square delta by L square, that is the first term, then u square delta square by L square delta, which is once again gives you the same thing. So, once again the magnitude of the forward terms are the same, all right. This gives you now pressure divided by rho delta that is; the second term the third term basically over here will be the gamma u infinity, u infinity delta by L delta square.

So, that will be the that will be the term, that we will have this is the final term this term we; obviously, know is going to be small, right. Once again because of the same argument that we made in the last class. So, this goes down. So, ultimately what we have is this particular term boils down to u infinity L delta, right.

So now we come to a crucial juncture over here. So, your d p by d y as you can see is proportional to something like something like a mu u infinity divided by L delta, right. That is what we are getting because if the pressure term has to be proportional to this this is what if all the terms has to be equally important to the equation right; that means, their orders have to be comparable, right. That is the basic logic if one term is that is how we got read of this remember. That this term is way smaller than the other terms right. So, it is almost like you have a lot of things one few things are. So, small that you can basically discard it.

Similarly, d p by d x on the other hand if you look at it d p by d x, is basically given by u infinity by delta square this comes from the previous definition, right. Now what we can do is that if you write d p, how is it given by d p by d x into d x plus d p by d y into d y, correct? You can write it like that. So, or in other words d p by d x is equal to d p by d x like this plus d p by d y into d y by d x, correct? You can write it like that also. Now if you take I am trying to do it in one sheet.

So, that you can get a better feel. So, d p by d x divided by d p by d y if we do this, what will happen? You will get mu u infinity delta square divided by mu u infinity L delta right. So, this will give you L delta by delta square, right, which is basically L by delta which is much much greater than one., Right. Or in other words what we are saying is that d p by d x is much much greater than d p by d y, right. Or in other words of other words your d p by d x can be written like the ordinary differential of pressure, got it? Fine.

So, this is quite clear now that using this y momentum equation, we have shown that basically d p by d y is a very small quantity compared to d p by d x, right? And therefore, you can write d p by d x as an ordinary differential, right? Cool up to this particular point. Now returning to the x momentum equation to x momentum equation, what we have is that u infinity square by L is therefore, equal to 1 over rho d p by d x.

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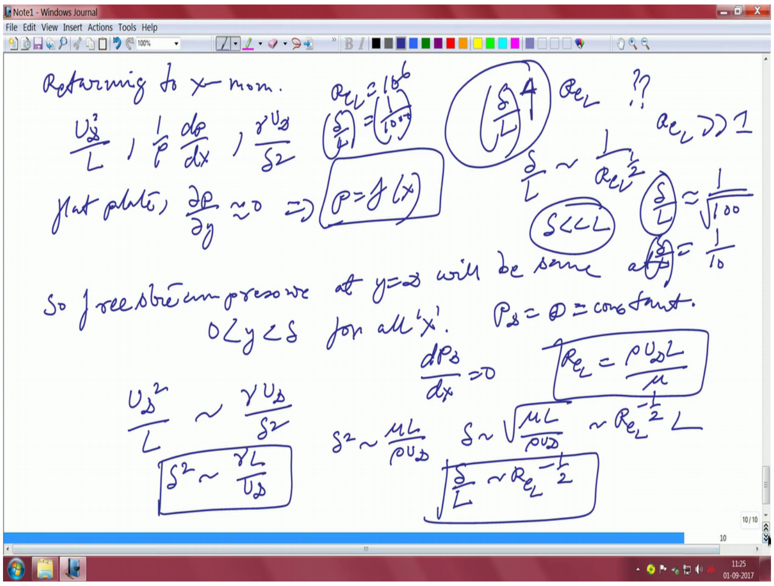

And these are the other terms and delta square, right. These are the 3 terms that you have as of now. Now recall at this particular point that this is a flat plate, correct? And d p by d y is practically equal to 0, that is what we proved right. So, this leads to that p is a function of f x only that is, how we could write it as d p by d x that is the ordinary differential. So, let us consider if this is the case; that means, pressure anywhere in the flow field will be only a function of x, it would not be a function of y, right..

Based on this for the so free stream pressure free stream pressure at y equal to infinity, right will be the same will be same at y equal to y greater than 0 greater than delta, for all x you got it? So, across anywhere in the boundary layer at any distance from the leading edge, right. Your pressure within the boundary layer will be the same as whatever is a pressure that is outside the boundary layer.

That means as if the outside boundary layer pressure is imposed within the boundary layer, got it? That is what it means because, there is no variation with respect to y there is no variation with respect to y correct? So, the whatever is the free stream pressure; however, the free stream pressure varies. That is what is imposed within the boundary layer correct. But the free stream pressure unfortunately in the case of a flat plate with a uniform flow is basically equal to constant, right..

So, that means, the pressure within the boundary layer will be also constant right. So, in other words it says that d p infinity by d x is equal to 0, right. So, that means, your d p by d x is also equal to 0. So, that means, inside the boundary layer in a flat plate there is no pressure gradient, right. There is no pressure gradient in a flat plate remember I am emphasizing on the word flat plate.

Do not try to apply this result to an incline plate and things like that, we will see what that is. But 2 important concepts that you are taking out from here one is pressure variation in the y direction is negligible. Second thing is that the free stream pressure is imposed within the boundary layer. So, if the free stream pressure varies with x, the pressure within the boundary layer will also vary with x. In this case the free stream pressure is constant, it does not vary with x. So, the pressure within the boundary layer also does not vary with x, right.

So, based on these 2 therefore, we are left with only 2 terms. One is a convective term which is u square by L. Remember, the 2 terms in the convective derivative a of the convective acceleration at the same order write this u square by L. And that must be the same as u infinity by delta square; which is the other term which is the viscous term on the, right. Hand side of the equation right.

So, based on this you can readily get that your delta square will now become u L by in u infinity, got it? If you just compare the order. So, therefore, delta square can be also written as mu L by rho u infinity. So, delta can be written as mu L by rho u infinity; which is also nothing but Reynolds number with respect to L by this. So, delta by L is varies as Reynolds number to the power of minus half. As you know the definition of Reynolds number is what rho u rho u infinity in this particular case, rho u infinity into L by mu right.

So, that is the definition of Reynolds number right. So, you can see that your delta by L varies as Reynolds number to the power of minus half. So, that means, what is delta by L delta by L is basically the ratio. You can also call a something like a slenderness ratio of the boundary layer right; that means, delta by L should go up, right. As Reynolds number if the Reynolds number goes up what happens? 1 over the Reynolds number it becomes. So, you can see that once again this is given as Reynolds number to the power of minus half right.

This is 1 over the Reynolds number right. So, if the Reynolds number becomes a very big quantity. As you increase the Reynolds number what will happen? As you decrease the Reynolds number what will happen? So, depending on that the slenderness ratio will change. So, is live left as an exercise is a very small exercise, right.

That you can guess and you can plot write, that as the Reynolds number increases, or the Reynolds number decreases the slenderness ratio; that means, the ratio of the ratio of the delta and L, what does this ratio vary? How does this ratio actually vary does it go up or down? But as you can see that Reynolds number is actually a large number usually. We are dealing with flows which is much greater than Reynolds number is much much greater than 1, right. It is not a stucation flow, right. It is laminar, but still the Reynolds number is high right.

So, based on that you can readily see this by delta. Say, the Reynolds number is say thousand right. So, this will be like 1 over thousand root. So, as you can see this will become a small number. So, our initial assumption that delta is much much smaller than L is very justified. So, you can readily see and if the Reynolds number becomes a 10 to the power of 6. This particular ratio will become if it is 10 to the power of 6. So, if this is say this is say 100 that may be easier for you to understand.

So, this will be 1 over 10, right. If the Reynolds number becomes equal to 10 to the power of 6, what happens? What is 10 to the power of 6 the root of 10 to the power of 6. So, once it becomes 10 to the root of 10 to the power of 6. So, you can see this delta by L will become something say it is 10,000 it becomes right. So, as you can see this number is even smaller, right.

So, that the difference between delta and L is substantial. So, the slenderness essentially increases, right. As you go on increasing the Reynolds number. So, that is the takeaway point that is coming out of this particular exercise, got it? Now for any if we just replace L by any measurable any reasonable x, right..

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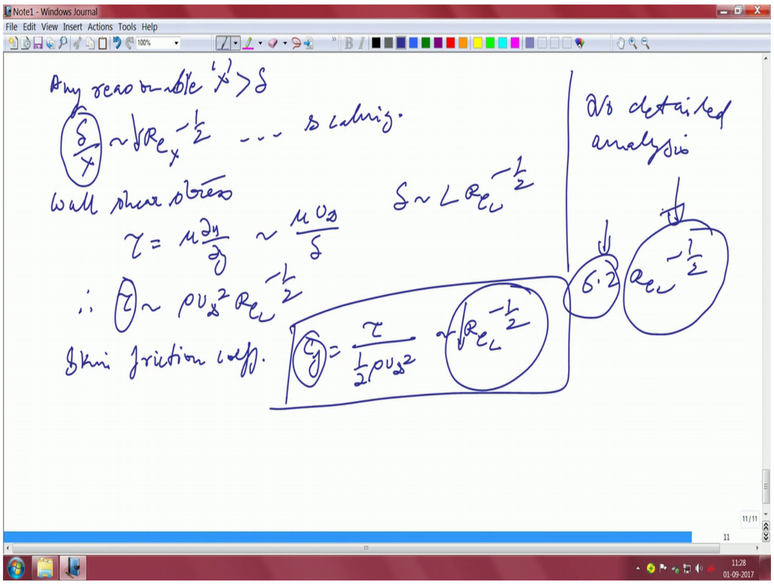

Which is greater than delta; that means, at some point you do not consider the full length. You can also show that this will be the same. This will be now given as Reynolds number based on that x location, not with respect to L anymore. This comes simply from the scaling right. So, if we now know this can we calculate the wall shear stress? Because as we know as engineer our main aim was to find out the drag.

So, the wall shear stress in this particular case will be tau is equal to mu d u d y, which basically is mu u infinity by delta, right. Delta we already know, that is L into Reynolds number to the power of minus half. So, therefore, this gives tau as rho u infinity square into Reynolds number to the power of minus half all, right. Skin friction coefficient if we talk about, because some engineers will talk about skin friction coefficient, ç f that will be equal to tau divided by half rho u infinity square, that will become Reynolds number L to the power of minus half, got it?

So, all these things happened, without any detailed analysis. No detailed analysis. We did not do any detailed analysis at all, right? Correct there is no detailed analysis there is no real math that we solved. Yet, we found out what is a functional variation of c f, right. Or what is the functional variation of tau. We know that tau now depends on the Reynolds number in a certain way. As you increase or decrease the Reynolds number, tau actually increases and decreases accordingly. So, these are all without any detailed analysis.

So, that is the powerful nature of the scaling that without doing any detailed math we are able to identify the key functional groups over which the c f tau and all these things will depend. We also know how the boundary layer thickness will depend compared to the distance from the leading edge, right. But why didn't we need to do a comprehensive analysis simple reason is that we do not know what sits here these are all scales; that means, within one order we are, correct? So, but there will be some coefficient some number that will sit here right. So, in this some number can be say 5.2 into Reynolds number of minus half, that may be the actual answer. But this 5.2 you can never get through scaling arguments for this you need to solve it in details, right.

So, that is there lies the caveat, but you still know that what is the significance what is the functional numbers or the non-dimensional parameters over which skin friction coefficient should depend on. So, this is a good back up the envelope calculations that one can do to easily get an answer to this particular question, got it? Some let us look at the thermal boundary layer now.

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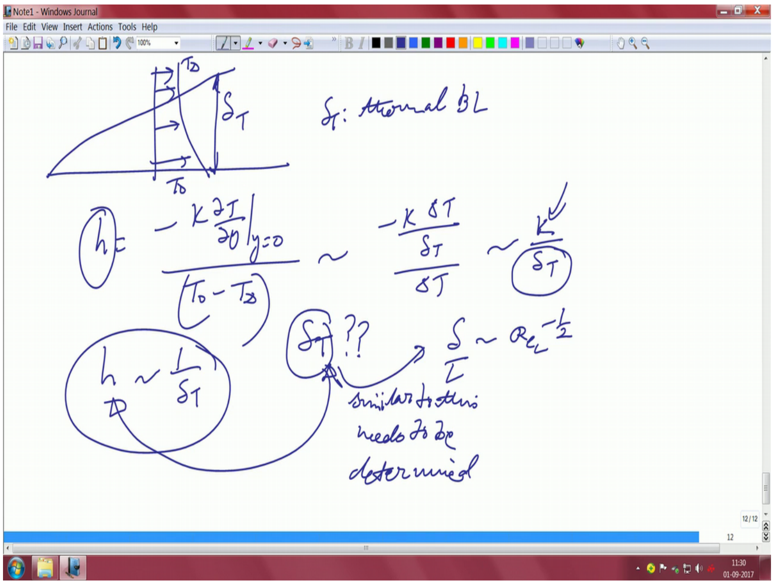

So, thermal boundary layer as we know it is like this. Once again looks something like that, this is T naught this is T infinity this is your delta T as I said that delta T and delta we still do not know the association.

But we know that delta T is basically the thermal boundary layer. And we know h is nothing but minus k d T d y at y equal to 0 divided by T naught minus T infinity, right.

Or in other words this will become k some kind of a delta T by the thermal boundary layer thickness divided by delta T. So, this will become k over delta T.

So, as we know that your heat transfer coefficient is proportional to the inverse of the thermal boundary layer. Because k is a constant normally for all the applications. So, the heat transfer coefficient is inversely proportional to the thermal boundary layer thickness. So, as the thermal boundary layer thickness becomes very large, h becomes very small, if the thermal boundary layer thickness is very small h becomes very large, right. So, this is a h is therefore, equal to 1 over delta T.

So, that means, if we can evaluate what this delta T is going to be you can easily say what will be the dependence of h. So, like we determine that delta is proportional to L to the power of minus half that we did, right. Similarly, if we can established a similar thing from for delta T; that means, the thermal boundary layer thickness we are done right. So, if we can establish something which is similar to this, similar to this needs to be determined, right. Something similar to this needs to be determined, am I right? It is clear from this particular point. So, let us write to the boundary layer form then once again for the thermal boundary layer.

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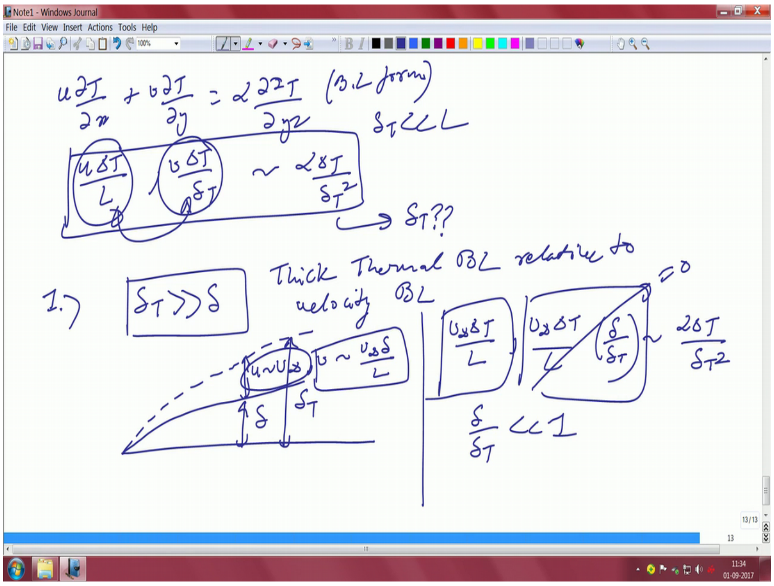

So, it is what u d T by d x plus v d T by d y is equal to alpha delta square T by delta y square. Once again, I have not included the axial gradient, simply because for the same reasons as previously mentioned that this is the boundary layer form. So, delta T is much

much smaller than L, right. That was original form that we had same thing is valid for delta T, we do not know that whether delta T is greater than delta or not, but at least we know that it is much much smaller than L. So, we can neglect that particular gradient.

So, the convection terms therefore, can be written easily v delta T by delta that should be proportional to alpha delta T by delta T square right. So now, we need to develop something on this delta T. What is this delta T? What it can be write? What will be the expression for that, this is the order form, this is the scaling form of the energy equation, right. As we once again see these 2 terms are of the same order. If you substitute for v you will get the same order. Now there are 2 situations that are possible. Situation one, when delta T is much much greater than delta. That is the first situation right; that means, we have a thick thermal boundary layer, right relative to the velocity boundary layer correct?

So, the situation is like this. This is your delta, I am drawing the other 1 by a dotted line this is your delta T. So, that is what we have right. So now, you can see that in this particular configuration, the velocity boundary layer is much much thinner compared to the temperature boundary layer. So, outside this in this particular zone, you must be equal to u infinity, right. Because it is already in that free stream region, right. And v should be the same as u infinity into delta by L, whatever is the v the relationship of v is, right. Now in this particular configuration if we write it now in this expression.

This particular expression what you will get is, u infinity into delta T by L, that is the first term, right. The second term will be u infinity into delta T by L into delta by delta T, it is proportional to alpha delta T by delta T square. Now in this particular expression what you see is that this particular term is; obviously, much much smaller than this, because your delta by delta T what do you see, as delta by delta T expression. Delta T is much much greater than delta that is what our initial assumption was, right?

So, therefore, delta by delta T must be much much less than one right. So, in if that is the situation therefore, this term will be become will become negligible. Now how is that that the velocity term, which we said that they are normally of equal magnitude becomes negligible here. That is because your velocity boundary layer is very thin over here right. So, outside the velocity boundary layer in most of the space, right. You are outside the velocity boundary layer your scale of velocity u is u infinity right. So, therefore, there the v velocity scale will be negligible. Because you are outside the velocity boundary layer. And there is no free stream it is only a free stream horizontal component of the velocity right. So, outside the boundary layer there your v velocity will be negligible. Because you are outside the velocity boundary layer. Because the velocity boundary layer is very thin.

So, there are 2 or 3 arguments over here. Your thermal boundary layer is way thicker than your velocity boundary layer that is the first assumption, right. Because of that in most part of the thermal boundary layer, right. You are outside the velocity boundary layer, because you are analyzing a flow condition. Where the velocity boundary layer is very thin compared to the thermal boundary layer right. So, in most part of the flow field, inside the thermal boundary layer, remember. The analysis here is done for the thermal boundary layer the velocity scale is almost like u infinity right.

And not only that because you are outside the velocity boundary layer, right. The v component of the velocity will be negligible. So, there is no doubt that this is the correct expression that we have got all, right. Now let us look go to the next page. Based on this now what we have is alpha into delta T by delta T square is proportional to delta u delta T by L..

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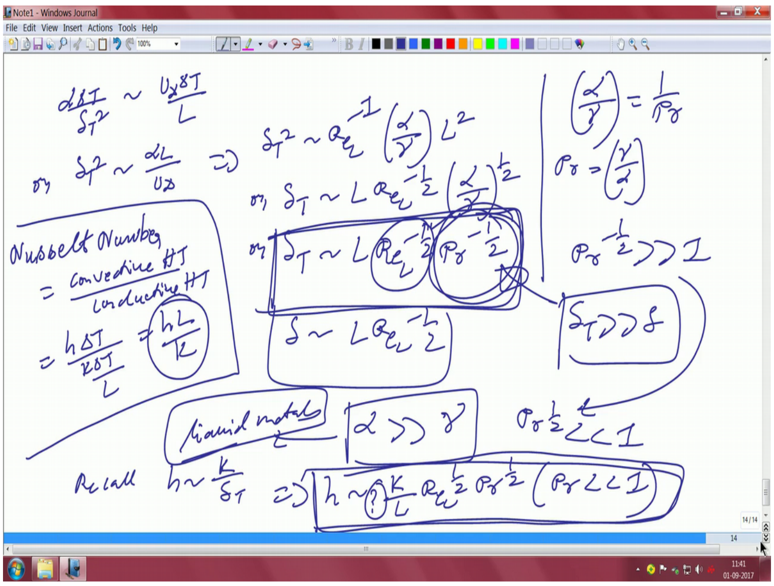

Or delta T square is proportional to alpha L by u infinity all right?

So, that is what we get over here. So, therefore, now we can do a little bit of jugglery this leads to delta T square is proportional to Reynolds number to the power of minus half alpha by gamma into L square, right. Or in other words, delta T is proportional to sorry this is not Reynolds number to the power minus 1. So, delta T will be equal to L Reynolds number to the power of minus half into alpha by gamma raised to the power of minus half.

Now, the key quantity over here is alpha by gamma, what is alpha by gamma? Alpha by gamma is nothing but the inverse of the Prandtl number. Prandtl number is nothing but gamma by alpha right. So, is basically a ratio of the momentum diffusivity divided by the thermal diffusivity is a relative ratio that so, a Prandtl number greater than one means the momentum diffuses much faster than the than the thermal diffusivity. Whereas, Prandtl number less than 1, essentially means the opposite. So, in this particular case, what you have is your delta T is your delta T is therefore, proportional to L Reynolds number to the power of minus half Prandtl number to the power of minus half, clear?

So, as we can see this is very similar to your delta, delta was this no this is very similar to your delta, except now this Prandtl number comes into the picture. If Prandtl number is equal to one both the deltas are the same; that means, your thermal and momentum boundary layers are exactly the same the overlap on the top of each other alright. So, knowing one is equivalent to knowing the other, right.

But on the other hand, in this case we have found out an expression; which relates the thermal boundary layer with the Reynolds number and with the Prandtl number of the flow field Prandtl number is a property dependent parameter remember that. So, these are the 2 important things and this is valid. For Prandtl number Prandtl number much much greater than 1, right. Because your delta T was much much greater than delta, right. Because your delta T was much much greater than delta, correct?

So, the Prandtl number is obviously much much greater than 1 or the rather I am sorry, the Prandtl number sorry extremely sorry over here delta T is much much greater than delta. So, Prandtl number to the power of minus half is much much greater than 1. So, therefore, this leads to Prandtl number to the power of half is much much less than 1. So, this is the situation; that means, where the thermal diffusivity is greater than the momentum diffusivity, right. So, that means, your alpha is much much more than gamma, right. Because that is the inverse right.

So, alpha is much much greater than gamma. So, this is the situation in the case of liquid metals, they show this behavior liquid, metals actually show whether thermal diffusivity is much higher than the momentum diffusivity right. So, recall that h is proportional to k over delta T, right. That was what we said. So, therefore, h therefore, becomes k by L Reynolds number to the power of half Prandtl number to the power of half for Prandtl number much much less than 1.

So, you can see the heat transfer coefficient can be easily worked out through this scaling argument as a function of Reynolds number which is the flow. Remember, I said that the heat transfer coefficient is a function of the flow and it is a function of the property. So, Prandtl number is a function of the property Reynolds number is a function of the flow k is also a function of the property of whatever fluid that you are dealing with. So, for Prandtl number much much less than 1; that means, for liquid metal and similar such families you have your h given in this particular form right.

Once again, we do not know what sits here, that is still a question that we need to answer, that can be only answered when you actually solve the equation in a proper way right. So, similarly before we go to the other case. Let us define something called Nusselt number right. So, Nusselt number as you may be familiar with is nothing but the convective heat transfer divided by the conductive heat transfer. Do not confuse it with the butte number. So, it is basically h into delta T divided by k into delta T by L which is nothing but h L by k.

So, as you can see therefore, go to the next slide. As you can see over here now that we have found out what h is Nusselt number therefore, becomes let us not put the equality over here.

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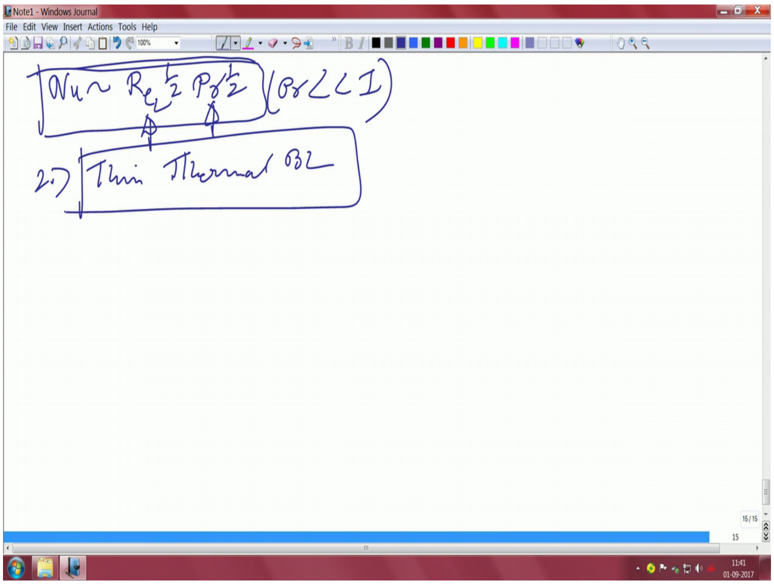

Reynolds number to the power of half Prandtl number to the power of half for Prandtl number much less than 1, you can see Nusselt number is one quantity that you are very familiar with. So, heat transfer coefficient and Nusselt number are basically one in the same. So, you can see that they are all functions of flow function of properties right. So, we have looked into the case of a thick boundary layer. Next class what we are going to do? We are going to look at the case of a thin thermal boundary layer. So, that will be the topic for next class.

Thank you.