Introduction to Soft Matter
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Lecture 03
Response of Elastic Solid

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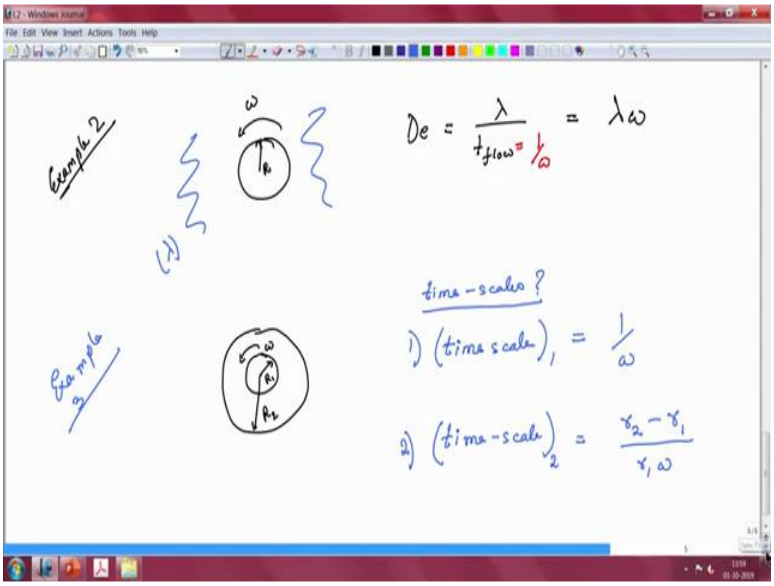

So, hello and welcome back to one more lecture on Introduction to Soft Matter. So, last time we were discussing this particular example and then I had left it for you to solve so this was example 3 and the question was what will be, how many different Deborah numbers can you identify in this particular case.

So, obviously now that we have seen that the relaxation time scale is already provided to you so the question finally boils down to figuring out what are the different time scales that you can associate with this particular problem. So, question is time scales, what are the different time scales.

So, at least one time scale you can borrow for the previous problem. So, time scale one so just a second so, time scale let say this is the one time scale, can still be given by 1 by omega but, you can also found another time scale now and that time scale this is essentially a shear rate or the inverse of the shear rate, so this mu time scale is another so, you can use this to define another Deborah number or in this case often when people use the second one, it essentially becomes a Weissenberg number.

So, with that we are done with our examples on or we are done with our introduction to time scales and Deborah numbers which are the two very important concepts, first important concept with regards to soft materials of viscoelasticity.

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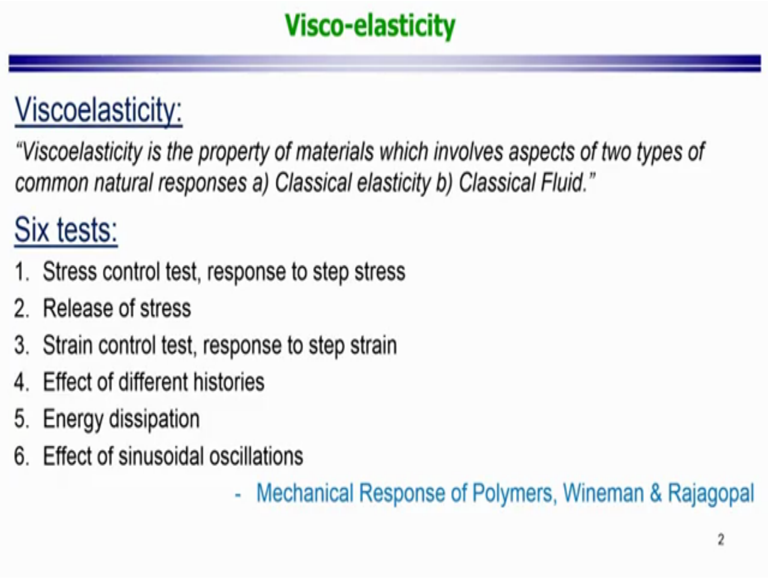

So, now what we will do is we will go back to the initially we had already discussed this in one other lecture what is viscoelasticity, and we had said that viscoelasticity is the property of materials which involves aspects of two types of common natural responses, classical elasticity and classical fluid. So, classical elastics solid or the classical viscus fluid this definition was taken from one of the text books that are part of the recommended texts for this class.

So, obviously now the next step before we try to understand viscoelasticity, should be to try and understand what a classical elastic solid is and what a classical viscus fluid is, that should be the obvious next step. Now, another thing we are going to do here right now, and I had discussed this in one the previous lectures also, that there is a certain continuum approach to understanding viscoelasticity where we do not necessarily discuss or depend upon what is the molecular constitution of the material.

We are defining, going to define macroscopic tests where information on the molecular aspects is not necessary, obviously if you have that insight it gives you more, a better understanding of the material response. But, even if that is not available to you, you can still define macroscopic tests which can help you to understand how a classical elastic solid differs from a classical viscus fluid and then we will see what a viscoelastic material is.

Which you can probably guess I mean we have already discus is lies somewhere between the two. So, in the textbook by Wineman and Rajagoapal they gave six different tests or macroscopic tests that one can do, which helps elucidate the behavior of a classical elastic solid or a classical fluid or any container, continuum essentially. So, the first test is called a stress response test or a response to step stress.

Another one is the release of stress, third one is a strain control test, or the response to a step strain and we will discuss each of these in more detail in a moment and effect of different histories. When we say effect of different histories it implies that effect of the different histories of one of the imposed variables.

Then, energy dissipation in the material and six effect of sinusoidal oscillations. Okay, and these are all provided in the, these are the six tests as per of, as per the book mechanical response of polymers by Wineman and Rajagoapal.

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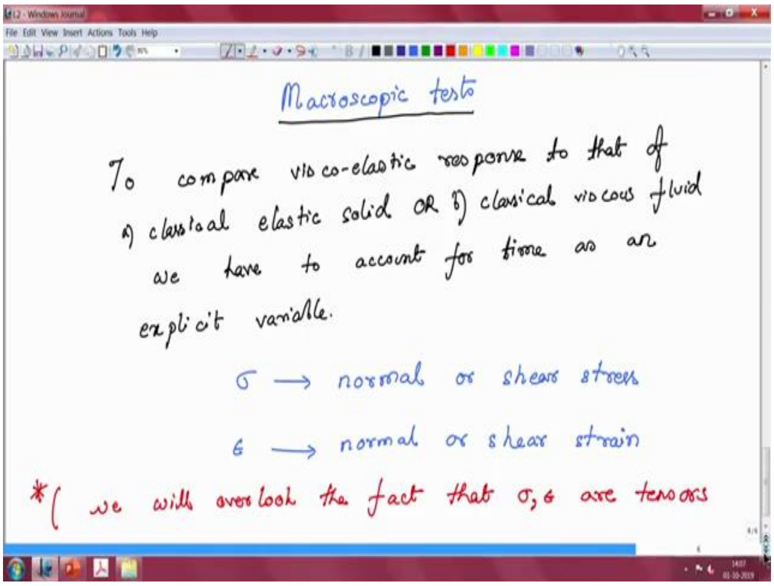

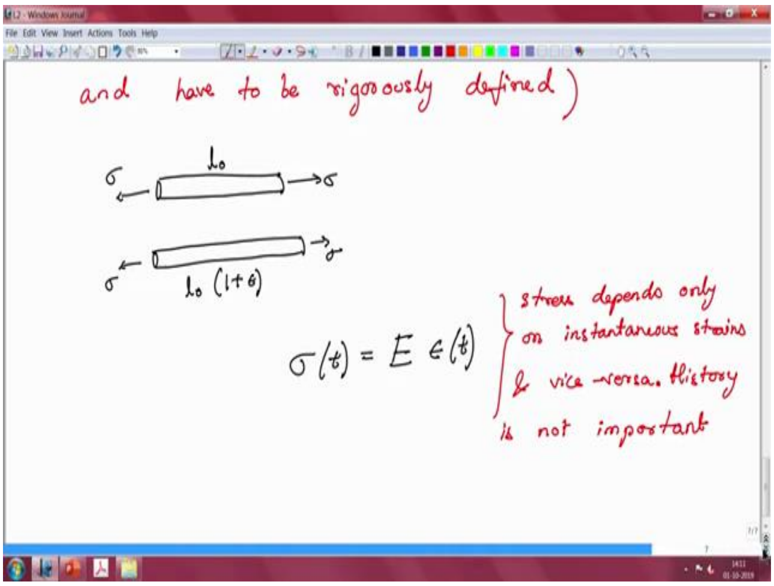

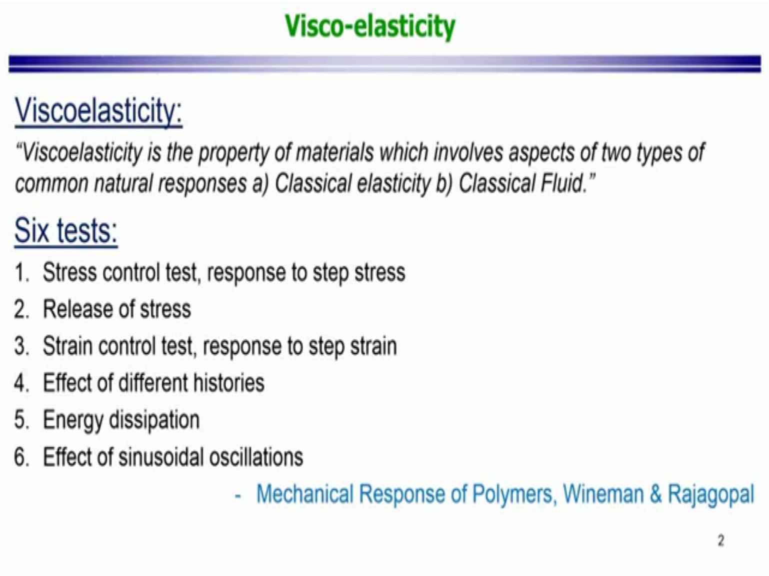

So, we will start a new page here so, now we want to discuss macroscopic tests okay, but before we get into it we have understood that to compare the viscoelastic response, so to compare viscoelastic response to that of a) classical elastic solid or b) the classical viscus fluid. We have to account for time as an explicit variable.

So, what do we mean by that? We mean that the relationship between stress and strain when it comes to a classical elastic solid or a classical viscus fluid, typically the constitutive relationship does not account for time explicitly, it is already there and we will, so to get there we let us just quickly introduce a couple of variables that may be using quite a bit.

So, let us introduce sigma which we will be using for normal or shear stress, and epsilon which is again going to be a normal or shear strain. So, we are going to assume that you have some understanding of what stress and strain are but I am also going to make another note which is, so in a bracket I will make a note that we know if you are familiar with stress and strain we know that stress and strain are tensors and that they have to be very rigorously defined in many cases.

So, we are not going to do that, at the moment we are going to take a very simple, undergraduate as you might have seen in the undergraduate classes of very simple understanding of stress and strain we will assume at the very moment okay. So, we will neglect so, we know that stress and strain are, so, we will overlook the fact that sigma, comma epsilon, are tensors.

And have to be rigorously defined and we are going to that because, we are currently do not require the tensorial notation so much and we are just trying to get some initial intuitive understanding so we just going to treat it almost it like as if it is a scaler quantity. We will come back in one of the later lectures we will introduce stress and strain as tensors when we discuss constitutive, more formal discussion of constitutive equations we will come to that.

Also, what do we mean by strain, so, here we are still going to go by some of the or very early intuitive ideas that we had built up about what stress is, for example, if you have a rod and then the rod is of l naught size then when you apply at normal stress then the rod increases the length and becomes l naught 1 plus epsilon as a response to this.

So, we will stick to this definition, this definition only works for very small strains and if you have stress that you have a very large values then there are other ways of defining strain but for the time being, for the most of this course we are not going to require that and so, we will confine our attention to this.

But, I am just letting you know that this is not a complete description of or rather what we are going to take is a very simplistic understanding of these quantities. Now, I just said, in the previous, just in a few moments ago that we have to account for time explicitly, so what do we mean by that.

So, when it comes to a solid material you are already familiar with the very famous rule which is given as, so E here is a modulus quantity so here what we do is, the strain or the stress or the vice versa so the we can say that stress depends only on instantaneous strains and vice versa. So, when we write this, we already we are assuming that history affects are not important because the stress at any given point of time depends exactly on the strain at that time and not a value of strain in the past and vice versa is also true.

So, now that we have discussed this, let us, come. So, we are going to break this idea and we are going start introducing a time more explicitly in our conversation and when we do that we are going to go back. So, I am just going to quickly refer back to the six tests that we had discussed. So, the six tests were, okay so the, let us just keep to the first test.

Sorry, so the step control test or the response to step stress and the release of stress. So what do these things mean?

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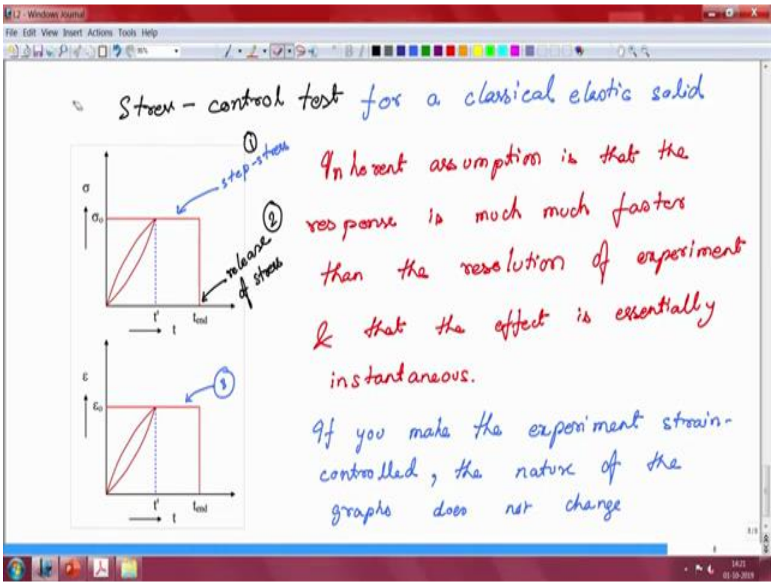

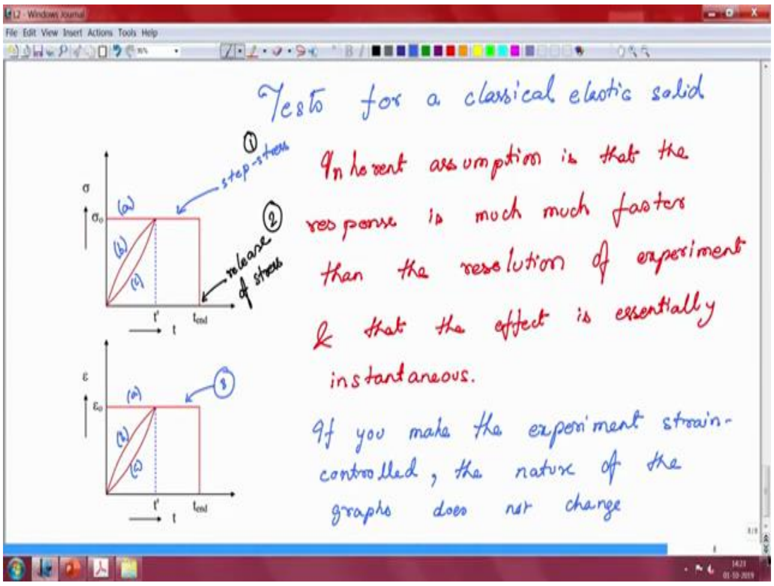

Visco-elasticity

Viscoelasticity:

"Viscoelasticity is the property of materials which involves aspects of two types of common natural responses a) Classical elasticity b) Classical Fluid."

Six tests:

1. Stress control test, response to step stress
2. Release of stress
3. Strain control test, response to step strain
4. Effect of different histories

2

So, let us discuss the first one, which is the stress control test. So, I am just going to insert, just a second, so we are going to use this image here, so in this test what we are asking is, that we have applied so stress control test should automatically tell you that the variable that is experimental variable that is under your control is the stress.

So, you are applying a known amount of stress that say sigma naught in this case and then you are measuring strain as a response of the material. So, the first diagram is the input in a sense and the second diagram to the bottom is the output.

So, this is, this curve the first one, this curve is your step control test, belongs to your step control test, so this is step stress so you are provided a known value of stress to the system and then you are going to measure what the output is, what we see for a classical elastic solid so, just second a here, so what we are going to discuss is, so this is let me, so as I said first we want to understand a classical elastic solid so, for the next few moments our discussion will be for a classical elastic solid.

So, this diagram tells you what happens when you apply a step stress to your system, what you get for a classical elastic solid is that the moment you apply stress or stress of sigma naught your system instantaneously registers a strain of epsilon naught and this strain holds for as long as you keep your stress to that value. So, here the inherent assumption is, so inherent assumption is that the response is so, is much, much faster than the resolution of experiment and that the effect is essentially instantaneous.

So, here when you apply the stress, the step stress immediately the step stress registers in epsilon naught and this immediate is really immediate in a sense that the experimental resolution is much, much bigger than the response time. So, any material you can understand would take some amount of time but in case of classical elastic solids they are so small, that the typical experiment would not register them and what you would end up seeing is essentially an instantaneous response.

Now, for the moment, for the time that we are going to discuss the continuum mechanics problem. We are going to treat this response as really instantaneous, when it comes back to the problem of, when we start looking at the molecular nature we will see why this response is very fast, and will also get some idea of how fast this response is really because it cannot be infinitely 0, it has to be a finite number.

We will come back to that problem, but at for the time being we will just consider this as instantaneous. Now, so this is the first test and what it says is it just continues for as long as you keep it so, as long as you hold the strain, at that value sorry, stress at that value the stress, sorry as long as you hold the stress at that value the strain will also be held at the particular value.

Now, our second test was the release of stress so we had seen that our, the second test was the release of stress so, what happens when you release the stress. That also I have already draw, this diagram has already drawn so, it already shows you so this indicates here release of stress.

So, basically you should imagine that we are doing the experiment and we are measuring the output and here at t end we have release the stress and the stress now goes back to a value of 0, what ends up happening, is that your strain, also goes back instantaneously to 0. So, this is, this was your second test. So, this was in a sense.

Now, the third test just going back to this the third test was the strain control test so, in this test we change the input variable. So, we changed the input variable and now, this bottom diagram becomes your input and the top one so you, the strain is now the input is the experimental condition and then you are measuring stress.

So, what happens for a classical elastic solid when you flip the experimental control, what do you think happens? It does not changed it is the same that remains, right, so, here so if you make the experiment strain controlled as opposed to stress control, the nature of the graphs does not change.

So, this in a sense becomes your third experiment. Let me take this of and I will just make it more general we will just say, these are we are discussing tests for a classical solid. So, we have discussed three important points. Now, comes the fourth point which is the effect of different histories, when we say effect of different histories it implies how did you reach that value of stress or how did you reach that value of strain, and if we change it what happens, so in this diagram we have already indicated a few things.

So, this is one path on the top so let us call this path A, this is another path, path B by which you are reaching sigma naught. And this is another path, path C by which again you are reaching sigma naught but its time histories are different in the three cases. So, what happens here the corresponding diagram is already drawn for you and what you see is that if epsilon naught is reached by different time histories, sorry if sigma naught is reached by different time histories, epsilon naught will also be reached by different time histories.

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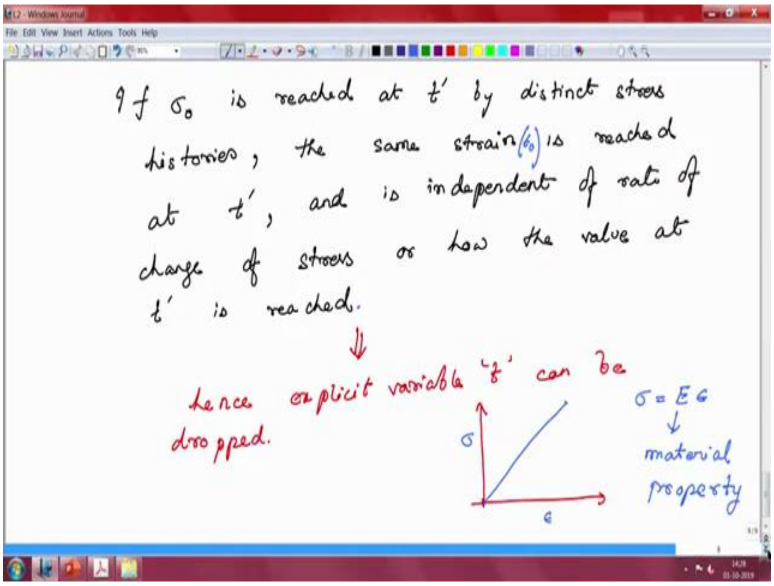

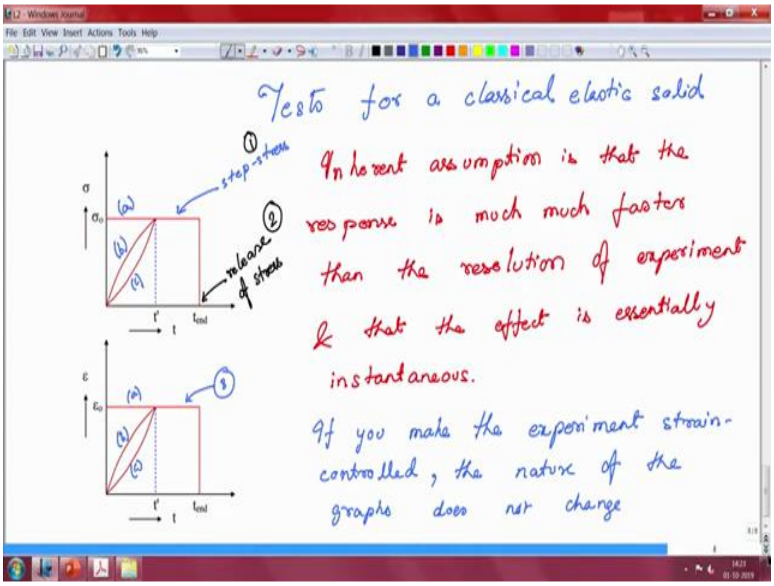

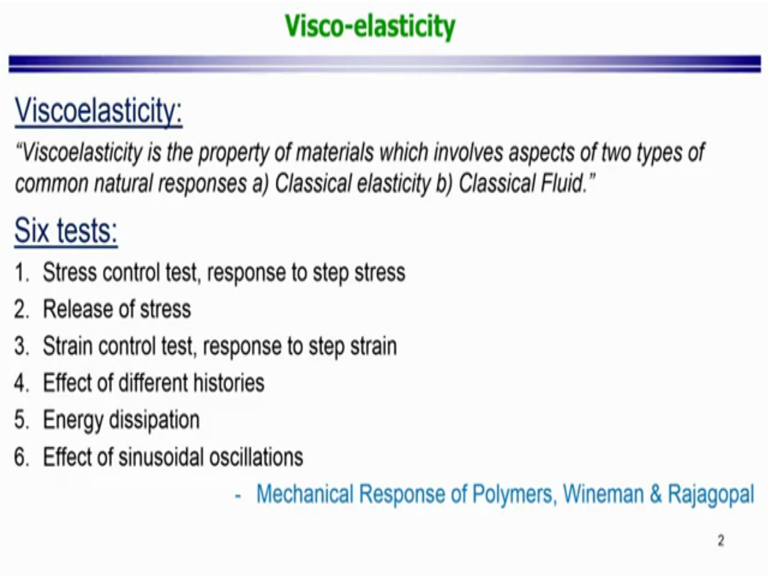

So, what you see is, if epsilon naught is reached at, did we have time here, let me quickly check, so t dash so, let us say this time by which it is reaching sigma naught is this time called t dash. If epsilon naught is reached at, sorry let me just change this, this was sigma naught, if sigma naught is reached at time t dash, by distinct stress histories the same strain is reached at t dash, and is independent of the rate of change of stress or how the value at t dash is reached.

So, if you reach sigma naught but distinct stress histories the same strain epsilon naught is going to be reached at t dash and it is independent of the how fast you are changing your stress to reach that and it is also independent of how this particular path was taken. So, this is a very very important issue with classical elastic solids and this is basically what helps you get, so when you are doing an experiment you are always doing a time history type of experiment where you are taking one value and you are changing it with time or you are reaching in a arte of certain time, and you are recording the time response of the material.

But since, sigma naught and its corresponding epsilon naught is independent of how you reach that value hence, this explicit variable t can be dropped so, hence explicit variable t can be dropped. And thus, irrespective of who has done the experiments with what history, you will, always end of reaching us the same conclusion if many people were doing the different experiments or different histories, all of the different peoples is still is the plot sigma versus epsilon they get the familiar straight line curve.

Where, the slope is a function of the material itself so the slope gives you a material response. So, that is why we write this as sigma equal to E epsilon where this is a modulus you already know this where this modulus is a material property. Sorry, okay, so just going back so these different strain histories these are basically your fourth type of test so that leaves us with two tests still that is energy dissipation and effect of sinusoidal oscillations.

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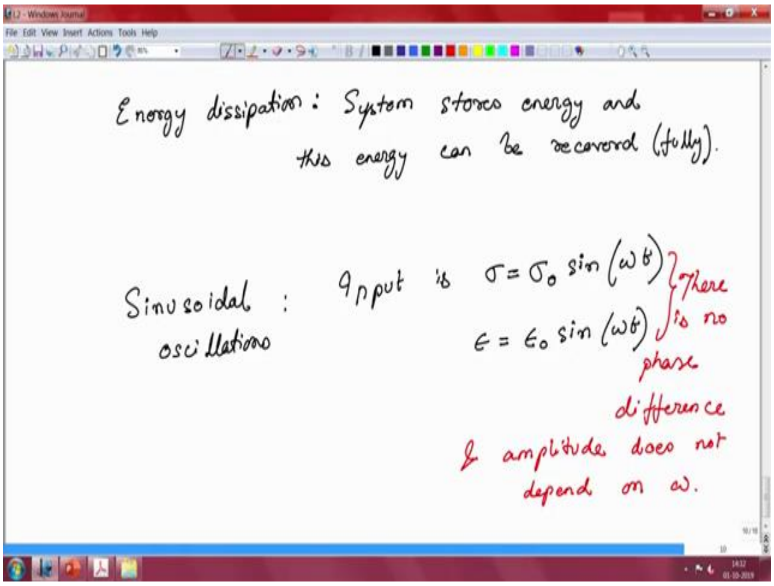

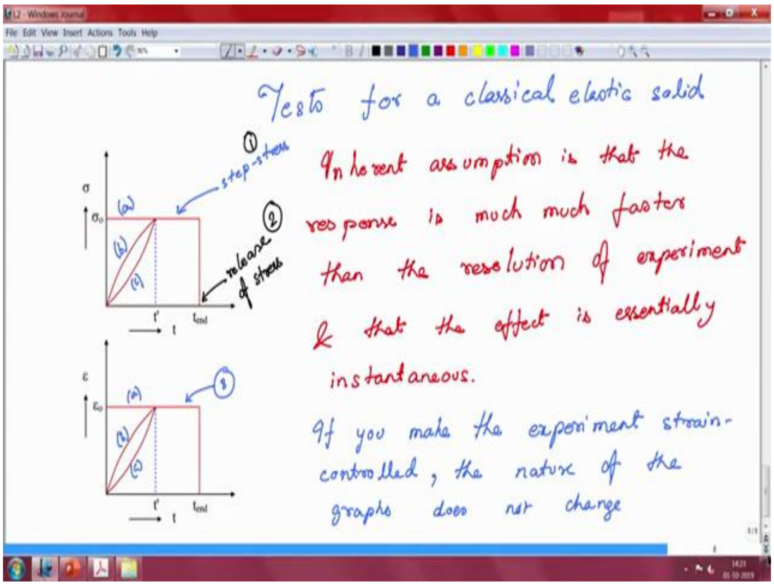

So, with regards to energy dissipation. We have seen that just go back that when you apply a known amount of stress the system goes to a certain amount, registers a certain strain and this strain holds for as long as you want infinite. That is just our simplification obviously that cannot be true but, for the theoretical or a for a theoretical prospective, the idea of a classical elastic solid is that, it can hold that amount of strain for an infinite amount of time.

And obviously as you release the stress this strain also comes back so, whatever energy that you put in or in the work done is recovered when you release the stress. So, the system stores energy for the time that you want and then the moment you leave it, the energy is recoverable so, when it comes to energy dissipation all energy, so the system, just a second, so the system which in this case is the classical elastic solid stores the energy, the applied energy and this energy can be recovered fully so, that we have one so this was the fifth test.

And we have the final one which is effect of sinusoidal oscillations. So, in sinusoidal oscillations let say your input is sigma equal to sigma naught sin of omega t, so you are deliberately applying a sinusoidal oscillation in this case, and then you are measuring the output so, what is your output going to be, we have already seen that stress and strain are proportional so this is going to be some other epsilon naught again sin of omega t.

So, here please note that there is no phase difference between the two so between the input and the output they are always in phase. And amplitude does not depend on omega as you have already determined that sigma equal to (())(34:05) modulus times and epsilon. Okay, so, today what we did in our today's lecture is that we started looking at the continuum mechanics approach, we started taking a continuum mechanics approach to understanding viscoelasticity and we have taken the first step in that direction by trying to get, trying to figure out what other requisite tests for us to understand and we found that there are six different tests.

And these six different tests that are suggested in the textbook we have applied it to a classical elastic solid and we have seen what the classical elastic solids response will, be and in the next class we will look at a classical viscus fluid.