Texture in Materials
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Module - 04
Texture representation
Lecture - 18
Euler Space and Orientation Matrices

Good day everyone and we will continue with the module 4, which is Texture representation and today is lecture number 18. In this lecture, we will go through and we will understand more about the Euler Space and its relationship to the Orientation matrices.

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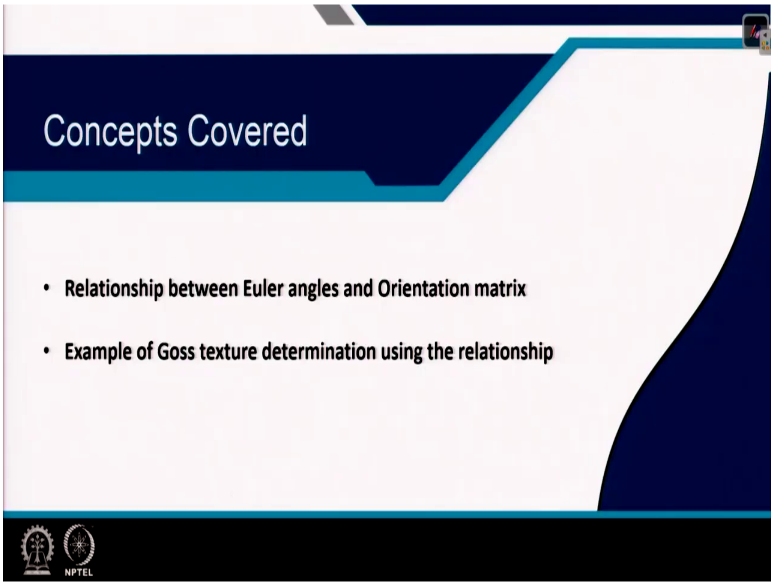

the concepts that will be covered in this lecture are relationship between Euler angles and orientation matrix and we will show an example of Goss texture determination using this relationship.

Now, you have already determined how different kind of texture components like cube component or rotated cube component, and this particular Goss texture component. which is important for transformer steels, how to determine them using by simpler rotations of phi 1, phi, phi 2 along the crystal axis means the rotation of the sample coordinate system ND, RD, TD with respect to the crystal coordinate system.

Secondly, we have find out that how we the Goss texture and the other textures could be represented using the pole figures and we have shown 100, 110 and 111 pole figures and the inverse pole figures. And, as that inverse pole figures you have to show the three different pole figures that is the ND inverse pole figure. The TD inverse pole figure and the RD inverse pole figure and we know how to find out them.

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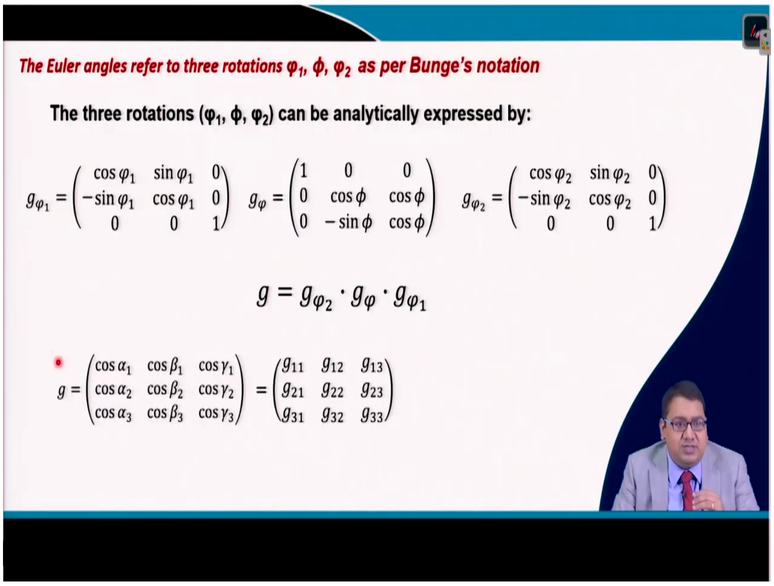

let us go ahead and see that how the orientation matrix is related to the Euler angles and therefore, thereby the Euler space, right. Euler angles are phi 1, phi, phi 2 and we use the Bunge's notation, right to determine that. if this three angles or the three rotations that is the phi 1 along ND, the phi along RD and the phi 2 along ND which we have shown when we demonstrated how to produce this Euler angles and the relationship between the sample and the crystal coordinate systems and that from the previous lectures.

This phi 1, phi, phi 2 can be analytically expressed by g phi 1, which is the rotation along ND by phi 1 and therefore, it can be shown by the matrices cos phi 1, sin phi 1, 0, minus sin phi 1, cos phi 1, 0 and then 0, 0, 1. Because the rotation is taking place along g here the third column third row, right. On the other hand, the phi rotation, which is given by g phi is the rotation along RD right. the rotation is along here which is RD and then the rotation is by phi. Therefore, in this coordinate it will be cos phi and then here it will be sin phi it is a mistake in the printing and then it is minus sin phi and cos phi, right. And, g phi 2 is the rotation again along ND that is this axis and therefore, it is cos phi 2 sin phi 2 minus sin phi 2 cos phi 2, right.

g is equal to g phi 2 dot product time dot product g phi dot product g phi 1. as that g is given by this nine variables relating the angular relationship between the sample coordinate system and the crystal coordinate system and these are alpha 1, alpha 2, alpha 3, beta 1, beta 2, beta beta 3, gamma 1, gamma 2, gamma 3 and is given by g 11, g 12, g 13 using this kind of matrix. g is equal to this matrix, right.

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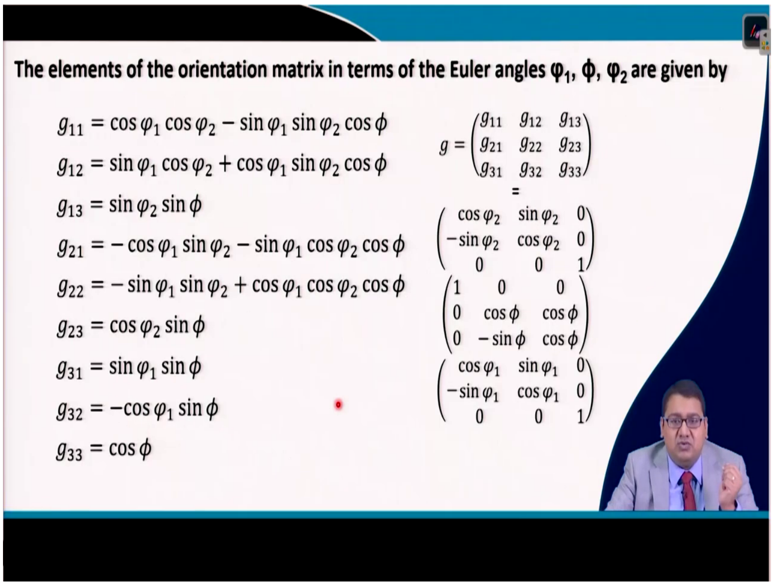

now if we can relate this means with this equation and this equation; that means, that we have the value of this matrix equal to cos this g phi 2 dot product of g phi dot product of g phi 1 and here it is sin of phi it is a mistake in printing. If we relate them then we can find out the value of g 11, g 12, g 13 and, which we have written it here up to g 33. In addition, we can find out the g 11 is cos of phi 1 cos of phi 2 minus sin of phi 1 sin of phi 2 cos of phi right like that g 11 2 is sin of phi 1 cos of phi 2 plus cos of phi 1 sin of phi 2 cos of phi. Like that, we can find out g 13, which is sin of phi 2 sin of phi right like that g 1, g 21, g 22, g 23, right. So, few important and easy to obtained solutions could be obtained from few equations like g 13 which gives the dot product between sin phi 2 and sin phi. On the other hand g 33, which is equal to cos phi could directly give the value of phi if we know the value of g 33, right. On the other hand g 33 sorry, g 32 is the relationship between minus of cos phi dot product into dot product sin phi.

if we know the value of phi from g 33 then we can find out the value of phi 1; on the other hand g 31 is equal to sin phi one times sin phi. If the value of phi and g 31, then we can get the value of phi 1. Similarly, for g 23 if the value of phi we can get the value of phi 2 and g 13 also the same thing. depending upon which ever will be suitable we would we will be able to use it to obtain the value of phi 1, phi, phi 2 from the orientation matrix using this relationship.

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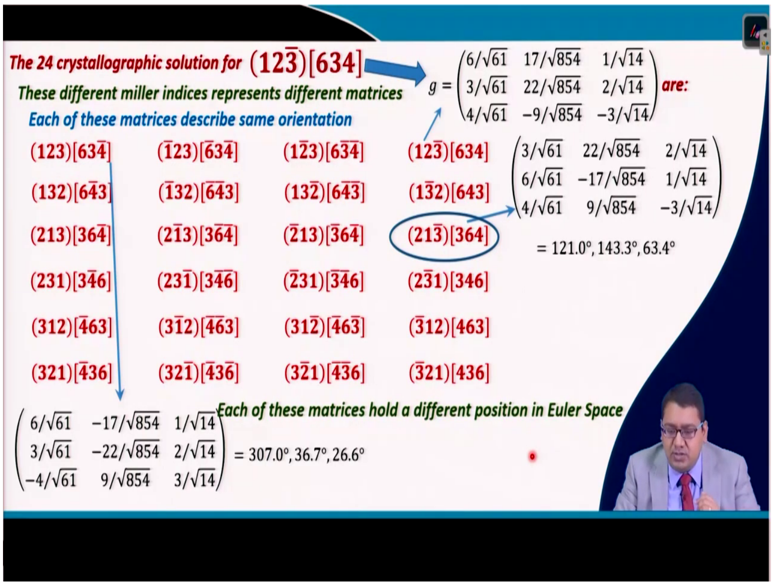

Now, let us take the same example and that we have taken the same example of 123 bar 634 because this 123 bar 634 lies it does not lie in any corner of the triangles 24 triangles present in the stereographic projection. These this texture is present somewhere inside the 100, 110, 111 triangle 24 triangles like this present in the stereographic projection. there are definitely 24 different crystallographic solutions present for this particular type of orientation.

In addition, this texture component or orientation example is given in the book of Olaf Engler and Valerie Randle where they have given in much detail that how various crystallographic related solutions lead to various Miller indices. Then how it relates to form various orientation matrices, different orientation matrices like we have shown in the previous lecture. Then how these different matrices will produce different phi 1, phi, phi 2 values and will produce different angle axis pair and the different robotics vector, but all of them though show different numerical values in terms of degrees and they represent the same orientation. If we have this four different orientations which can also lead to obtain the 24 different orientations like this. and these are in terms of Miller indices; then in terms of each we can get a different value of g and we are showing here that this for this particular g we have got the value of phi 1, phi, phi 2 to be 307 degrees phi 1 phi 36.7 degree 26.6 degree for phi 2.

On the other hand, if we look into another one and we can see for a different orientation matrix for this different Miller indices we have a different Euler angles phi that is phi 1, phi, phi 2 which is 121.0 degree, 143.3 degree, 63.4 degrees for this phi 1, phi, phi 2.

we for if we calculate for each one of them we will get different values of phi 1, phi, phi 2; that means, it is component will show intensity at different positions in the Euler space or in the ODFs. But, we should understand that these all means different Miller indices representing different matrices and each one showing different positions in the Euler space will actually is describing the same orientation or the same texture components, right.

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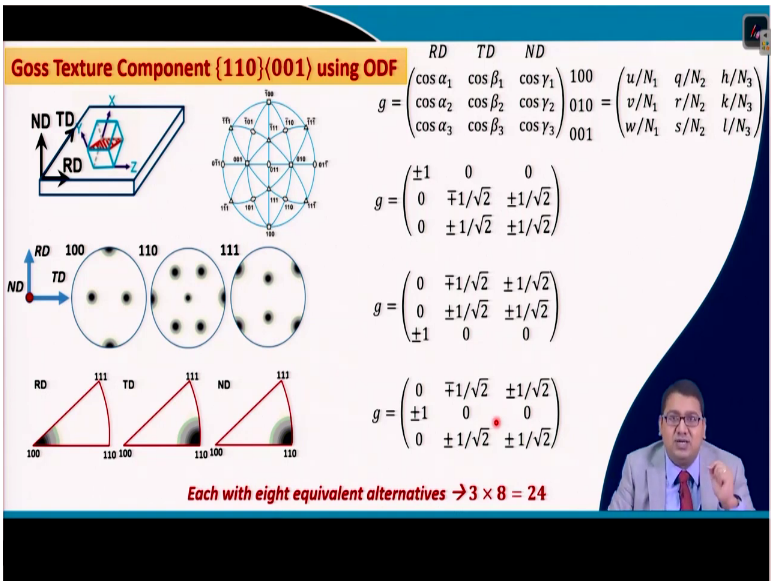

that if we go to the example as I said when we started this lecture that we will show the example of the Goss texture component and which is 110, 001 using the ODF. And, before starting it you should know that Goss texture is a very important texture for electrical steel and it are and this texture is used for transformer.

Because the a rolled steel plate or wire used in a transformer if it is having the Goss texture it if it is hot rolled and dynamically recovered and recrystallized and have proper Goss texture, it has a most lower magnetic permeability and magnetic loss. And, therefore, the supply has a much much higher efficiency than those products which does not have the Goss texture component.

It is very important and as I have been using this equations showing g with respect to cos alpha 1 cos alpha 2 beta 1 beta 2 gamma 1 relating it to RD, TD and ND and we are showing this because to show that how this is related to the Miller indices. And therefore, we can find out u, v, w, q, r, s, h, k, l as the Miller indices for the RD, TD and ND directions or planes. We also know that the Goss texture looks like this from our previous lectures and if we are given a crystal structure with this kind of crystal structure with X, Y and Z and that this is parallel to the plane which contains the RD and the TD this red plane of the crystal and it is related to RD, TD and ND like this and we know that this texture is called the Goss texture, right.

If we have given this kind of figure or this kind of Miller indices or even if anyone of this pole figure has been given. For example, say if we have been given this 100 pole figure with the intensity points at here we can see from this pole figure and tell that ok, it shows a two-fold symmetry because you that is either symmetric like this or it is symmetric like this.

It shows a twofold symmetry and this twofold symmetry may lead to form because at the center it has the 100 axis because 100 axis has a twofold symmetry, right. If we can if we can take the 10 sorry not the 100 axis 110 axis, which are the 2-fold symmetry. Now, if we can take the 110 stereographic projection and as we have shown earlier and we put it over this and we superimpose it with the 100 pole figure then we can see the other poles, right.

And, if these poles if these poles if these poles matches with the 100 poles of the stereographic projection then our intuition is correct. Now, see though I understood that this could be intuited because these are very simple examples to make you understand that how the texture works.

any one of this pole figure say it is if you if we are given a 110 pole figure, then also it can super impose to show the 110 points and if we are giving the 111 pole figure then also we can superimpose to form the 111 points.

Now, from this pole figures we can determine that ok this texture is given and therefore, it is 110, 001 type texture which is the Goss texture. Now, the similar thing could be easily obtained from the inverse pole figure where the RD is given at 100, the TD is at 110 and the ND is also at another 110 perpendicular to the both RD and ND.

The g matrix corresponding to this Goss component texture 110 001 could be either 100, or one could be plus minus and then if it is plus then it is the this is RD 100 and the ND will be 0, 1 by root 2, 1 by root 2 because root 2 comes from the root over of square root of 1 square plus 1 square plus 0 square which is 2. Either if this is plus this both of them are plus, if this is minus both of them are minus; that means, both this are same orientation right both of them are of same orientation and we can find out TD by cross product of ND with RD that is ND cross RD and therefore, we got this value for TD.

Now, that there could be another g matrix because there could be number of solutions 24 different solutions as we have said, right. The g matrix could be 001. the position of 1 changes and now it is here and with respect to that the position of 1 by root 2, 1 by root 2, 0 has changes from this to this so that the RD direction lies in the ND plane.

If the ND planes is changing then the RD direction will change and so that if we cross product ND with respect to RD we must get a TD which is perpendicular. All the axes should be perpendicular to each other and therefore, this solution exist. and for the another solution that is 0, 1, 0, 1 by root 2, 0, 1 by root 2 and the TD with minus 1 by root 2 plus 1 by root 2 exist or in other way we can have the minus 1, but it is showing the same texture. There are three different g values and each g value can have 8 equivalent alternatives, right. if we calculate there are 8 inter means equivalent alternatives for e g values and this can be even observed from the stereographic projection of the and that is why I have given the 110 stereographic projection here.

And, that if we consider 010 or any 110 poles of the family of 100 planes and you can see that it is actually shared between 1, 2, 3, 4, 5, 6, 7, 8 triangles and therefore, instead of having 24 alternatives we only have 3 alternatives. In case of the Goss texture because the RD, RD and ND lies in the corners of this stereographic triangles and it does not lie in the center or somewhere in between inside these triangles.

And, therefore, the poles are been shared by 8 triangles in case of 010 and 4 triangles in case of 110 and therefore, there are only three alternatives of orientation matrices in case of the Goss texture component and this is also because of the symmetry. Now, if we take the this example and we can take 1 by 1 this g matrices and solve with respect to the relationship the equations that we obtained from the orientation matrices and the phi 1, phi, phi 2. let us do that in the next slide.

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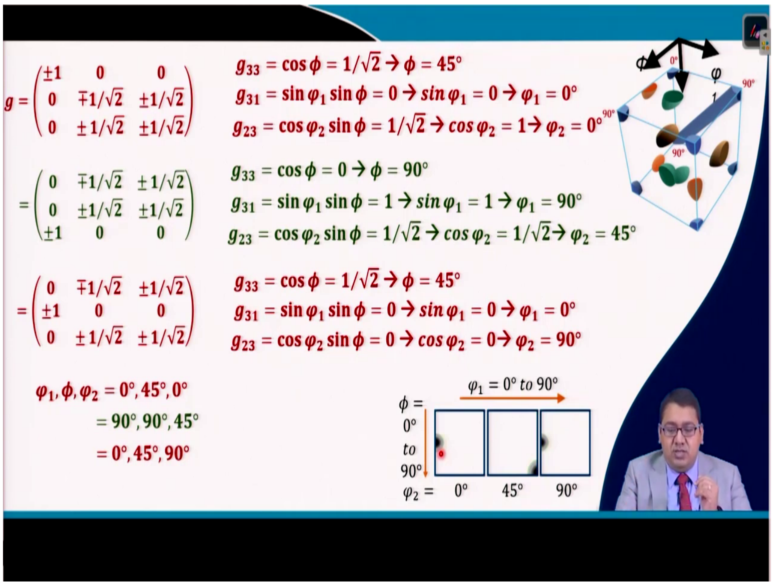

let us take out this g matrix 1, 0, 0, 0, 1 by root 2, 1 by root 2 my sorry, 0, minus 1 by root 2, plus 1 by root 2, 0, 1 by root 2, 1 by root 2. this is RD, TD and ND and we have already have solved that how by rotation of phi 1, phi, phi 2 along ND, RD and ND again to obtain the positions of the Goss components cube components rotated cube components.

In addition, that there are three positions where Goss components are visible if this is the Euler space with 0 to 90 degree phi 1, 0 to 90 degree phi and 0 to 90 degree phi 2. This position, this position and this position are the position of the component, but in this case let us forget about that geometrical rotations and visualization stuff and let us do it mathematically.

if let us consider that g 33 and this is equal to cos of phi, right. if we say that what is g 33? g 33 is 1 by root 2 and let us take the positive one. 1 by root 2 of then this means that cos of 45 is 1 by root 2. Phi is 45 degree, right. We know the value of phi. we can put that in g 31 which is equal to sin phi 1 times sin phi equal to 0 because if you look at g 31 here this is 0, right. Sin of phi is sin of 45 is again 1 by root 2. 1 by root 2 sin of phi is 0 this means sin of phi 1 is equal to 0. This indicates that phi 1 is equal to 0 degree, right. Let us take again any other variable of g matrix that is g 23 in this case. g 23 is this 1 right g 11, g 12, g 13, g 21, g 22, g 23. g 23 is plus 1 by root 2 right which is equal to cos phi 2 sin phi, right? Sin phi is again 1 by root 2, right. This shows that 1 by root 2 cos phi 2 is equal to 1 by root 2 this makes cos phi 2 equal to 1 and this shows that phi 2 is equal to 0 degree, right.

What we get? We get the values of phi 1, phi, and phi 2 for the Goss texture to be 0 degree, 45 degree, 0 degree for the orientation matrix, which is shown here, right. Now, if we go and take another orientation matrix [FL], now that if we look into the ODF and because it is the phi 2 equal to 0 degree section.

Let us show the phi 2 equal to 0 degree section, where phi is shown vertically that is from 0 to 90 degree and phi 1 is shown horizontally that is 0 to 90 degrees. In addition, the position is 0, 45, 0 that is 0 along phi 1, 45 along phi and 0 along phi 2 and this is the position of the Goss texture where it will be obtained, right. Let us take the orientation matrix which is 001, right for the RD, 1 by root 2, 1 by root 2 for 0 for the ND and minus 1 by root 2 plus 1 by root 2 0 for the TD, right. Now, in this case let us take the same g 33. g 33 is equal to cos of phi and if you take cos of phi g 33 is 0 here. phi is cos of 0 cos of phi is equal to 0 when phi is equal to 90 degree, right.

Let us take again g 31 the same one, where g 31 is 1 here in this case. sin phi 1 and sin 90 sin phi which is sin 90, right is equal to 1 [FL]. sin 90 is equal to 1, right. sin phi 1 also becomes equal to 1. phi 1 is equal to 90 degree then, right. We have solved and obtained that phi 1 equal to 90 degree and phi equal to 90 degree again. Let us take g 23 again and this time it is cos phi 2 sin phi. sin phi is sin 90 degree, right. Sin 90 degree is 1 and g 23 is this one. g 23 is sorry, this one 1 by root 2. that cos phi 2 then becomes equal to 1 by root 2 right, then phi 2 is equal to 45 because cos 45 is 1 by root 2. in this way we can find out that in this case the phi 1, phi, phi 2 is 90 degree, 90 degree, 45 degrees right 90 degree, 90 degree and 45 degrees.

In addition, let us show that in terms of ODF and we can show that in the 45-degree section because we show the phi 2 equal phi 2 sections and in this case, it is 45. We are showing the 45 degree section and phi 1 is equal to 90 and then phi equal to 90 to get the intensity point here, right. Now, let us go to the third orientation matrix for the Goss texture and which is 010 for the RD, 1 by root 2 0 1 by root 2 for the ND and minus 1 by root 2 1 by root 2 for the TD.

Let us take g 33 once again which is cos phi and g 33 is equal to 1 by root 2. phi is again equal to 45 degree for this case right and let us take g 31 and if we take g 31 g 31 is 0, right. And, if g 31 is 0 then sin phi 1 times sin phi is equal to 0; that means, sin phi 1 times sin 45 is equal to 0 and as sin 45 is 1 by root 2 this means that sin phi 1 equal to 0 and this makes phi 1 is equal to 0 degree, right. Now, let us take another, which is g 23, right. Now, g 23 is this one. g 23 is equal to 0 is equal to cos phi 2 sin phi sin phi is sin of 45 degree which is equal to 1 by root 2 and therefore, cos phi 2 becomes equal to 0 and cos phi 2 if is equal to 0 then phi 2 becomes equal to 90 degree right. we can find out that phi 1 phi sorry, phi and phi 2 becomes 0, 45 and 90 degrees. In this case as because it is phi 2 equal to 90 degree section we are showing that and in this case the position is 0 along phi 1 and 45 degree along phi. we can find out the positions of the Goss texture that we earlier found out by rotations of the specimen coordinate system to get to the crystal coordinate system using the Bunge's notation, right, in the in earlier lectures and therefore, we could find out that mathematically also we can solve out this relating it to the orientation matrix.

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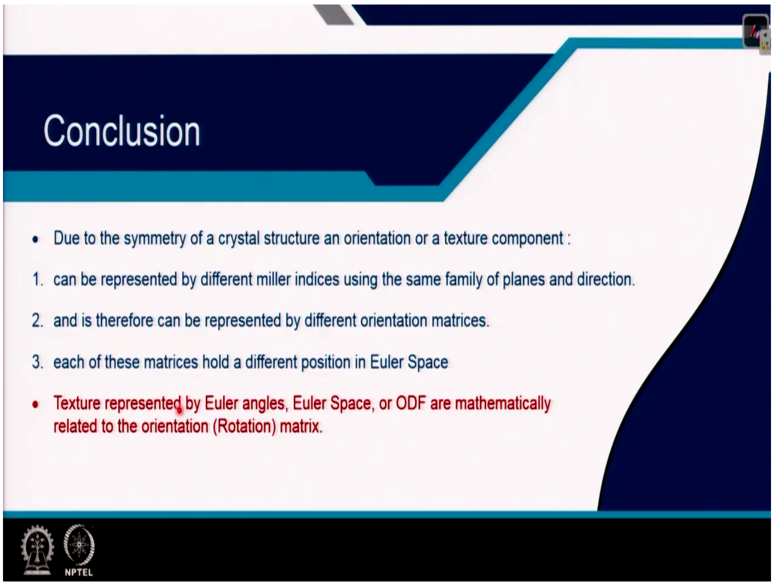

The conclusions that we can get from this lecture are that due to the symmetry of the crystal structure, right and particular orientation a particular orientation or a particular texture component can be represented by different Miller indices using the same family of planes and directions. It means it can be shown by poles of same family of planes and directions and it will be many, right.

Therefore, what happens that it can be represented by different Miller indices sorry yes of course, it can be represented by different Miller indices and therefore, by different orientation matrices. In case of cubic symmetry, it is 24 different solutions, in case of cubic symmetric SEP it is 12 different solutions, in case of orthotropic system it is sorry, orthogonal system it is 4 solutions. And then for each solution or each Miller indices or different orientation matrices there is a different position that is a different phi 1, phi, phi 2 in the Euler space.

Therefore, it the same component could be will be observed repeatedly in the Euler space in different positions. As we have shown for the Goss texture, we can observe the same orientation at three different positions and the number of positions is lowered because the miller indices of the Goss components are present at the corner edges of the triangles 24 different triangles in the stereographic position, right in the stereographic projection, right.

The final conclusion is that texture represented by Euler angles that is Euler space or in ODFs are mathematically related to the orientation or the rotation matrix and therefore, these are the ways how different textures are being measured and these are related to symmetry. We will continue our course and we will talk more about symmetries affecting the representation of texture in further detail in the next lectures.

Thank you.