**Advanced Marine Structures**
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**Lecture - 23**
**Theories of Failure - II**

We will continue to discuss the theories of failure, which we started in the last lecture.

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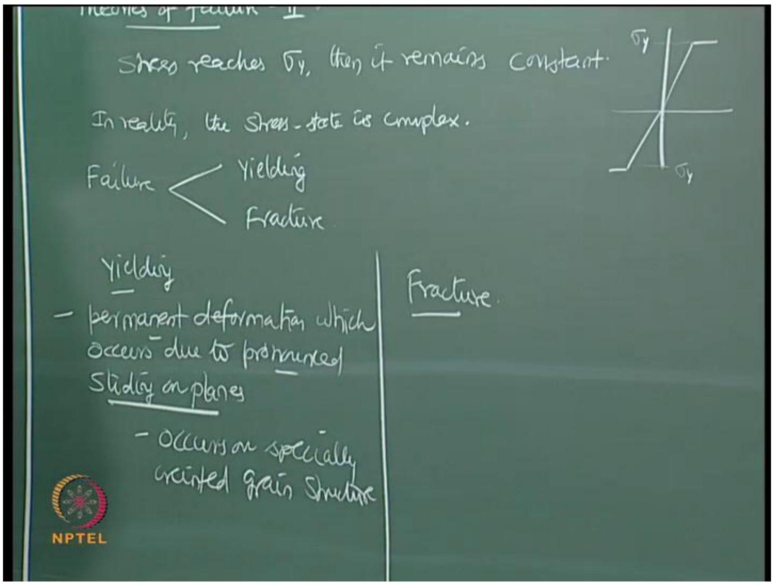


We already said in a plastic design, we assumed that once the stress reaches the yield value, then it remains constant. And ideally, the stress-strain curve looks like this. And this value is what we call as sigma y. And we also assume that, the yield value in tension and in compression is of same magnitude. This is also sigma y. This is true when the member or the material is subjected to uniaxial stress system. But in reality, in marine structures, we have got lateral loads, gravity loads arising from respectively, wind and waves as well as from the self-weight of the platform or on applied load on the top side of the platforms. Also, the wind and wave loads are not unidirectional. They can arise from any direction; it is multidirectional. Therefore, in reality, the stress state is complex. Actually, it is a multi-directional stress state – multiaxial. So, all the three sets of axis will act in a given structure. Therefore, we must now define, what is actually the failure criteria based on which the yield value should be computed.

Now, as we all understand that the yield value is a significant value, which is attained by a simple tension test. Now, there are some serious problems with simple tension test, because many values reach their ultimate state of level when a simple tension test is conducted. So, let see what those failures are and how the theories handle this. The moment we talk about failure, there are two kinds of failure, which can be initiated: one is by yielding; other is by fracture. There are two entirely different things. Let us see what is yielding and what is fracture. If you look at yielding classically, yielding is a permanent deformation. It is a permanent deformation, which occurs due to pronounced, that is, dominant sliding on planes. Actually, it is a slip; it is a sliding on planes. And this sliding occurs on specially oriented grain structure. That is the first point what we can see in the yielding. It occurs essentially as sliding on planes. And this sliding occurs on specially oriented grain structure.

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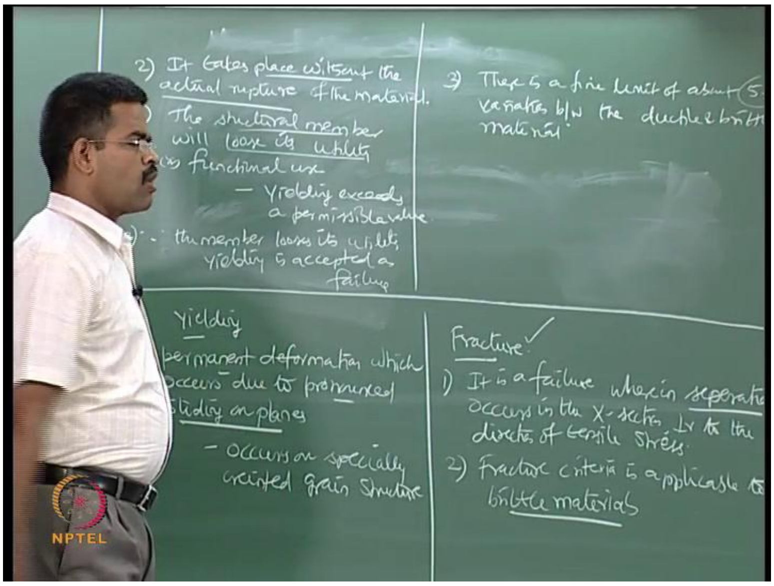


The second point what we can discuss in yielding is that, it takes place without the actual rupture of the material. The actual rupture does not occur in yielding. Then the question comes, why yielding is called as a failure? Because actual rupture is not occurring. The structural member will lose its utility or functional use when this yielding exceeds a permissible value. There is a permissible value, beyond which the structural member will lose its utility. Since the member loses its utility value, yielding is accepted as failure; otherwise, yielding is not a failure, it is only a slipping between the planes actually. Yielding is nothing but, sliding between the planes, which are critically oriented depending upon the grain structure of the material. So, it does not cause actually a rupture in the material at all. But we say that, yielding is a failure, because of a simple reason, when that yielding exceeds certain value, permissible value, then the structure loses its functional utility value. That is the difficulty.

Therefore, we say, yielding is an accepted mode of failure. Alternatively, if we look at fracture; fracture is a failure wherein separation occurs. There is a distinct separation occurs in the cross section perpendicular to the direction of tensile stress. Fracture criteria is applicable to brittle materials – is very important. If you have steel, which is predominantly elastic material, a ductile material, (( )) supposed to use the theory of fracture for an elastic or ductile material like steel, because it is essentially applicable to brittle materials. I will talk about this in theories of failure now – which is a brittle failure; which is a ductile failure; and, why.

The third point which is very important for fracture as a failure mode – actually, there is a fine limit of about 5 percent variation between the ductile and brittle material in terms of its fracture. It is a very fine mode of failure, which has got hardly any distinct difference between a ductile, brittle material. So, actually, we are not able to make out from the mode of failure very clearly and distinctly or explicitly that, the failure has caused because of fracture or it is because of yielding. That is why both of them are considered to be a failure mode as far as theories of failures are concerned.

Now, you will agree that, in both the cases, we can talk about failure, especially in plastic analysis and design; we say that, the stress in the material reaches the yield value as sigma y p and then remains constant at the value. And that sigma y p value has been taken from or for a material, which is subjected to axial tensile test. But there are very serious limitations or there are very serious problems, which are depicted in an axial tensile test, which causes or creates confusion in this whole analogy. Let us see what are they.

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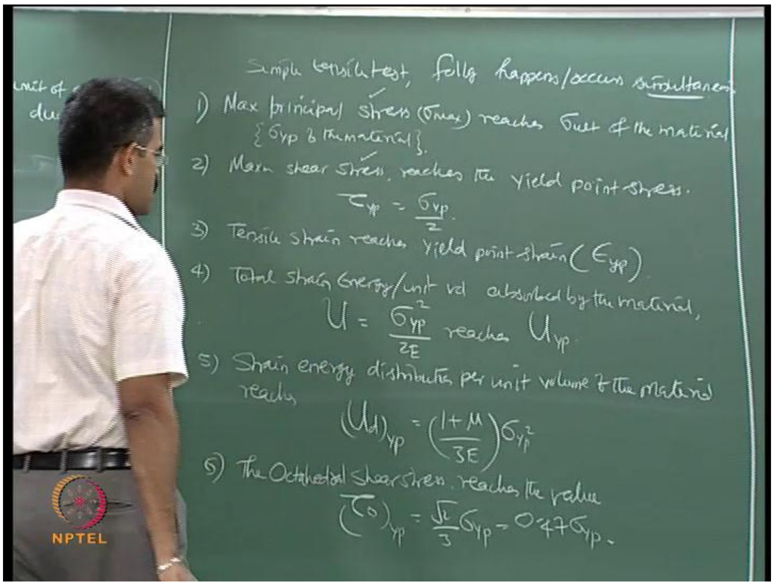


When you talk about simple tensile test, which is a bench mark test to estimate the yield value for any given material, following things or following happens or I should say occurs simultaneously; that is the problem. Remember, they occur simultaneously; you do not notice this. You may be thinking that, when you try to pull the material axially along its length, it only elongates and fails till sigma y p and then sigma u is reached. But that is not true. They are about seven things, which happens simultaneously; which post a very serious problem of understanding the failure by which the material actually failed.

What are those things? One – the maximum principle stress – sigma max reaches sigma ultimate of the material. So, for plastic analysis, we can even say that, it reaches sigma y p of the material. That is one thing, which we all know. Second – maximum shear stress reaches the yield point stress. Let us say the symbol; we can say tau y p reaches sigma y p by 2. So, this also occurs simultaneously. Shear is also reaching half of yield value, which is significantly high. The third thing – tensile strain also reaches the yield point strain, which I can say epsilon y p – total strain energy per unit volume absorbed by the material, which is given by capital U – sigma y p square by 2 E also reaches U at yield point. That is occurring simultaneously. The fifth point – strain energy distribution per unit volume of the material reaches strain energy distribution at yield point – 1 plus mu by 3 E of sigma y p square. So, it reaches the strain energy distribution at yield point. This occurs simultaneously. Now, the sixth issue – the octahedral shear stress; the octahedral shear stress reaches the value, which is equal to tau o – octahedral, which is root 2 by 3 of sigma y p, which is 0.47 y p. Now, the question comes, if maximum principle stress, maximum shear stress, maximum tensile strain, total strain energy, strain energy distribution and octahedral – all of them reach the respective yield point value simultaneously; what is the confusion? Where is the confusion? The confusion starts here.

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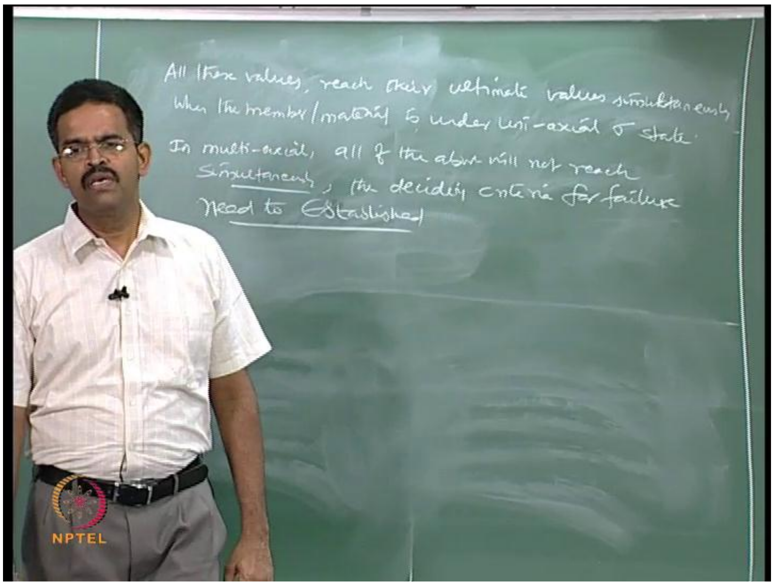


Now, all these values reach their ultimate values simultaneously when the member or I should say the material is under uniaxial stress state. In multiaxial, all of the above will not reach simultaneously. Therefore, you will not know what should be the selection criteria to decide that the failure has reached. So, there will be a confusion here, whether should we consider that the failure has occurred when the maximum principle stress has reached sigma y p; or. should we consider that the maximum shear stress when it reaches half of sigma y p, the failure has occurred. What will be the deciding criteria to conclude that the failure has occurred? It is a big problem. Since the rule reach simultaneously, the deciding criteria for failure need to be clearly established. You should know whether it has failed because the maximum principle stress is reached; sigma 1 or sigma 2 has reached sigma y p; or, the given stress state tau y p has reached sigma y p by 2.

What is the criteria, which is fulfilled in a given multiaxial stress state system, so that you declare that failure has happened? Now, the entire question is, if you do not know which is the condition, which is dominating towards failure; we will not know whether sigma y p has to be considered or sigma y p by 2 has been considered, etcetera. There is a big problem here, because so far, in plastic analysis and design. We always think that, when sigma y p is reached, when the plastic hinge is formed, redistribution starts occurring if the structure statically indeterminate. And we can keep on finding out the true collapse load using either lower bound or upper bound theorems. Now, here the difficult is what is that value which is reached to declare that the structure has failed? So, we do not know what are the failure criteria. So, the failure criteria need to be established very clearly.

Therefore, there are about five theories of failure, which has been suggested in the literature; based on which, you can always closely examine the failure mode and say that, it is failed because of what value of this has reached the ultimate. So, it is essential that, we need to establish what is the criteria we are considering for failure – so-called failure. And that failure can occur in two ways: one is yielding, which is predominant in ductile material; other is fracture, which is predominant in brittle material.

Now, you have got reinforced concrete, which has got steel as well as concrete; whereas, concrete is brittle; and, reinforced steel is ductile. So, what is the failure you will consider as a designer? So, you are estimate of w c or true collapse load may go wrong if you do not know exactly, what criteria you must consider for the failure (( )). So, let us say that, it is therefore, important that I must study the theories of failure. I am not getting into the detail of fracture mechanics. There are other courses in NPTEL given by other professors in IIT Madras as well as other institutes, which can speak very well in detail about fracture mechanics. So, I would advise you; if you really want to know more in detail about the failure modes of multiaxial stress state members and materials, you must look into those courses for your interest.

But, for completion sake, benefit of the viewers, what we will do is, we will take up a very brief introduction of all the theories of failure at one go; quickly compare them and debate on that and make our problem; as a designer, try to solve the problem and understand which failure theory should I consider and which I should not consider and why. We will discuss that. So, that ends the discussion appreciating the importance of theories of failure related to plastic design. Remember, I am not talking about fracture mechanics at all here. But, the necessity of understanding failure is, I would not establish by which mode I am promoting or accelerating the failure in literature. I would like to know this.

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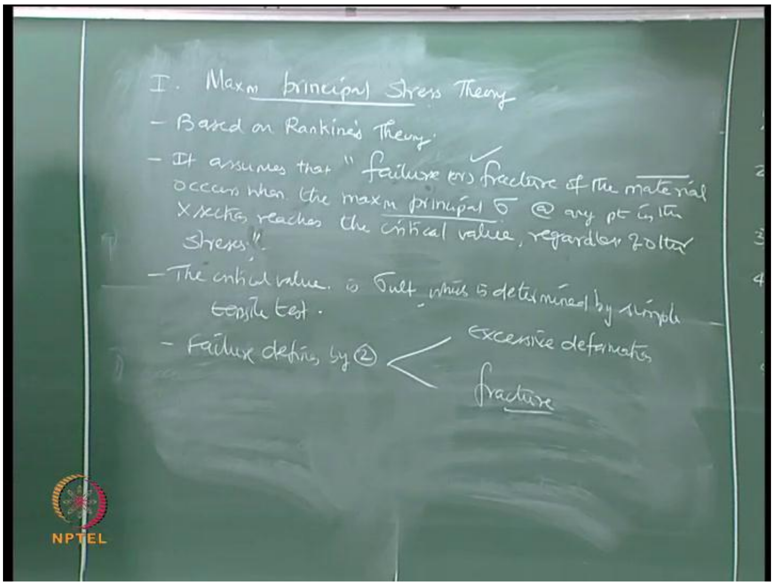


The first theory which has been established by people in theories of failure is maximum principle stress theory. This is based on what we call as Rankine's theory. This theory states that or I should say it assumes that, there is no statement of these theories (( )) it is not a theorem. It is a theory. It assumes a very important fact; what is that fact? Failure or fracture of the material occurs… Here, we are not talking about the structure; we are talking about only the material. Failure or fracture of the material occurs when the maximum principle stress at any point in the cross section reaches the critical value regardless of other stresses. What do you mean by this statement? The statement says, if the material is subjected to multiaxial stress state – biaxial or triaxial; if the maximum principle stress in the given system reaches its critical value; whether others have reached the value simultaneously or not, let us not brother about that. Failure occurs. That is already here. So, failure occurs when the maximum principle stress reaches the critical value.

Do not bother about what is happened the other stresses. Regardless of other stresses, this statement is true. That is one assumption or that is the statement made by this theory, which follows the failure mode or recommends the failure. Now, what is the critical value? The critical value is sigma ultimate; it is the ultimate stress, which is determined by simple tensile test. We can find out the sigma ultimate of the material. Now, in such cases, this theory defines failure by two ways: one – the failure can be due to excessive deformation; two – can be due to fracture; interesting. Maximum principle stress does not talk about yielding. It is a clear cut failure. It is a distinct explicit failure. There is no slipping or there is no sharing happening between the critically oriented granule structures, talks about clear explicit fracture; can be also due to excessive deformation, which can occur essentially from yielding.

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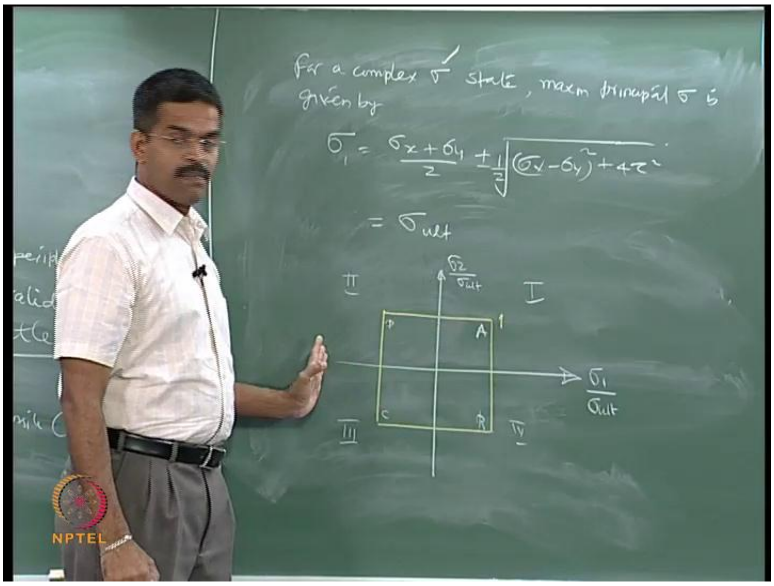


So, for a complex stress state… I am actually using this symbol for stress just to make it short – sigma. For a complex stress state, the maximum principle stress is given by sigma 1 is equal to sigma x plus sigma y by 2 plus or minus the standard equation – root of sigma x minus sigma y square plus 4 tau square; where, tau is the shear stress in the given system. And this theory states, this should be equal to sigma ultimate. When the maximum principle stress reaches the critical value, it says failure has occurred regardless of whether sigma x is reached sigma y, I mean, sigma yield; sigma y has reached sigma yield; tau has reached sigma y – yield by 2; regardless of all of them, is the maximum principle stress reach the critical value; it says the failure has occurred. And that failure can be either way – excessive deformation, which can result from post-elastic deformation of yielding; can be due to fracture also.

Now, this can be graphically represented by a plot. Let us say this is sigma 1 by sigma ultimate; and, this axis is sigma 2 by sigma ultimate. And may plot simply a square, which is 1. These values are unique.

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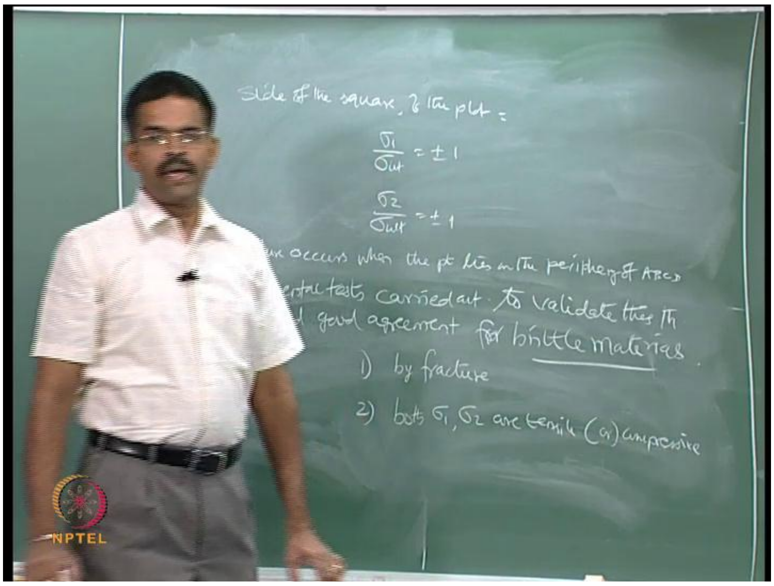


So, I should say, the side of the square of the plot is nothing but sigma 1 by sigma ultimate is equal to plus or minus 1; sigma 2 by sigma ultimate is equal to plus or minus 1. If I name this as ABCD, we can now say that, failure occurs when the point lies on the periphery of ABCD. If the point lies within ABCD, failure has not occurred, because the value is not equal to 1. If it falls exactly at the periphery of this square, we can say the failure has occurred.

This theory was verified by experiments. The experimental tests carried out. So, the experimental tests were carried out to validate this theory showed good agreement for brittle materials. There are two reasons for this; how people let us say concluded this. There are two reasons for this. First reason is the failure observed was by fracture; and, obviously, we know, fracture will occur generally in brittle materials. The second reason was, they said that, the theory was well-validated when both sigma 1 and sigma 2 are tensile or compressive; I mean this theory is very good in a biaxial stress state when both the stresses are of the same nature. What does it mean?

Let us go back to this figure here. Now, I want to make you a small correction in the statement here. The principle stress, which I gave you earlier, needs a correction. Please add half here; I have left it there. If you look at this figure, which plots the failure surface of the material based on principle stress – maximum principle stress; if both the natures are positive, that is, tensile; or, both the natures are negative, that is, compressive; then it means in quadrant 1 and quadrant 3, this theory is having no problem; it is perfectly predicting the behavior. This theorem has difficulties in quadrant 2 and quadrant 4. In these two quadrants, it means when the stresses are unlike in nature – this can be tensile, this can be compressive; in this case, this can be tensile, this can be compressive. In that situation, this theorem is not able to predict the failure correctly.

Now, in reality, if you see the structures – marine structures or the members in the marine structures, there is no guarantee that, in a biaxial stress state or in triaxial stress state, all nature of stresses will be same; the nature can reverse also. Therefore, this theorem, which says that, once the maximum principle stresses reach the ultimate value or yield value, failure has occurred, is good for brittle material, because the failure was noted in experiments by fracture. So, there is a question here that, can I use the theorem or this theory to understand the failure of ductile material.

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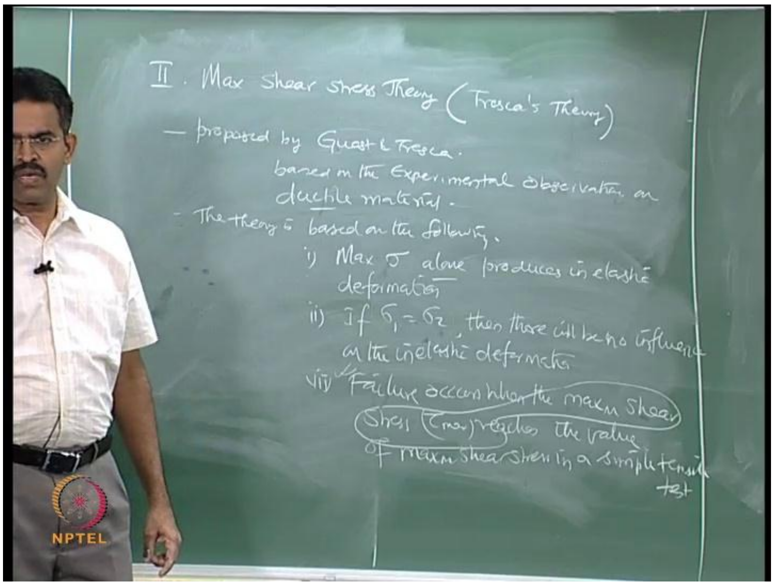


Let us talk about the second theorem. The second failure theory is maximum shear stress theory. It is also called as Tresca's theory. This was proposed by Guest and Tresca. That is why it is named after him called Tresca's theory. This theory was proposed based on the experimental observations on ductile material. This theory is based on the following.

It says maximum stress alone produces inelastic deformation if the stress 1 is equal to stress 2; then there will be no influence on the inelastic deformation. On the other hand, in a biaxial stress state, a word of condition that, where two stresses are of the same magnitude; then there will be no influence on the inelastic deformation at all, because they are well-balanced.

We will see what is the problem here exactly when we plot the theorem. It says the failure occurs when the maximum shear stress reaches the value of maximum shear stress in a simple tensile test. In a given stress state, find out the maximum shear stress. If that value reaches the corresponding maximum shear stress value, which you know from a simple tensile test, then it declares that the failure has occurred. And this failure is predominantly happening in ductile material. It means this theory says, do not look at the principle stress value reaching ultimate or yield; look at the maximum shear stress value. Now, there are two divisive opinions about the maximum principle stress theory and maximum shear stress theory given by Tresca.

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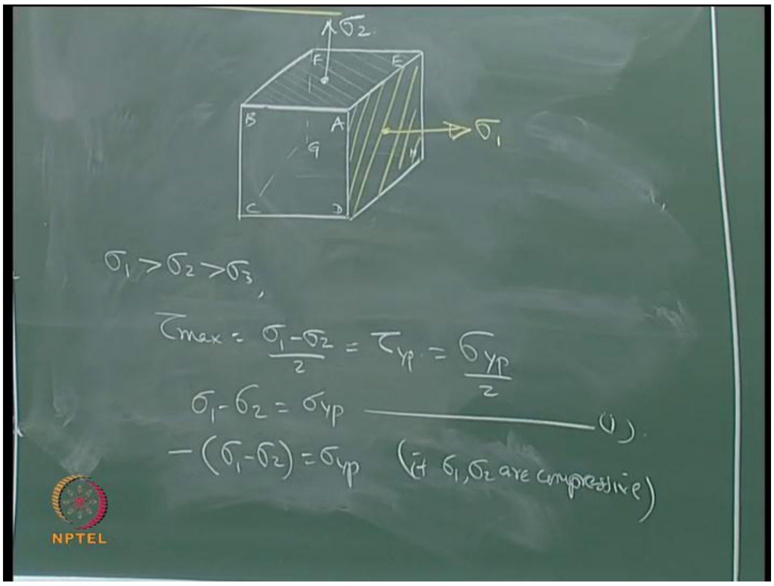


Now, the question comes here is, when will an account for the maximum shear stress in a simple tensile test when it reaches the yielding point. Let us say for a biaxial stress state. Let us take two surfaces: this is one surface; this is the other surface. I call normal to the surface as sigma 1 and I call normal to this surface as sigma 2. And I name these corners as A B C D E F G H. In a given biaxial stress state, we already know that, if sigma 1 is equal to sigma 2, then it need not influence the failure. So, I should say that, sigma 1 greater than sigma 2 greater than sigma 3. So, in that case, tau max, that is, the maximum shear stress should reach the corresponding maximum shear stress value in a simple tensile test at yield point. It means tau max should be equal to sigma 1 minus sigma 2 by 2; which is otherwise equal to tau y p; y p stands for yield point; which is also equal to – we just now saw in a simple tensile test – we see that, many things happens simultaneously; we also saw the shear stress. This is equal to the yield value by 2. Therefore, I can straight away say sigma 1 minus sigma 2 is sigma y p. This equation is true when both sigma 1 and sigma 2 are same nature, let us say, tensile. If they are compressive, then I should say minus of sigma 1 minus sigma 2 is equal to sigma y p.

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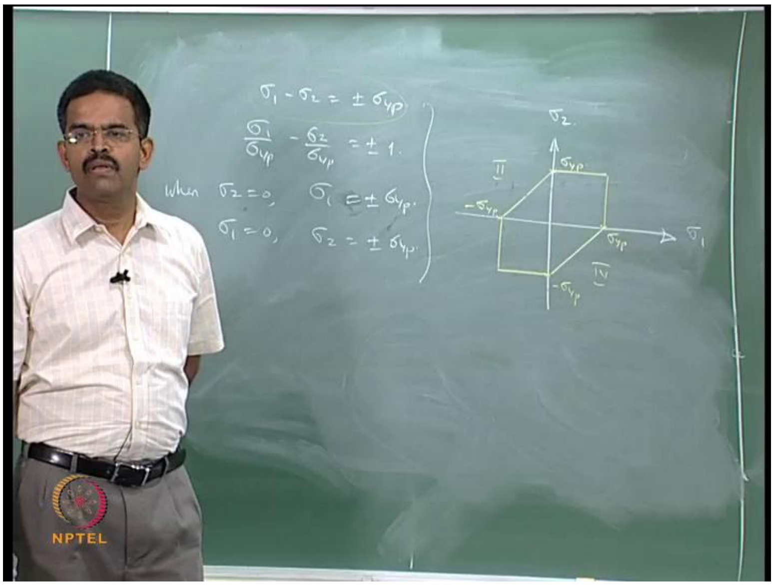


Therefore, I can say sigma 1 minus sigma 2 is equal to plus or minus sigma y p; or, sigma 1 by sigma y p minus sigma 2 by sigma y p is equal to plus or minus 1. Now, when sigma 2 is 0, sigma 1 will be simply plus or minus sigma y p; when sigma 1 is 0, sigma 2 will be plus or minus sigma y p. Now, I want to plot this as a failure envelope for a given material in a biaxial stress state. I can plot it like this. So, this value is sigma y p. This value is also sigma y p. This value is minus sigma y p. And this value is minus sigma y p. That is what this equation says. For quadrants 2 and 4, either of them should be 0. For example, let us say in this quadrant, if sigma 2 is 0, that is, sigma 2 is 0; only sigma 1 value is there; it will be plus or minus sigma y p. So, this line represents the argument of either one of them being 0.

So, this becomes the failure envelope. So, if a point of a stress combination in the material lies along the periphery of this hexagon – 1 2 3 4 5 6, is an irregular hexagon; if it lies along the periphery, failure is occurring for the ductile material. If it is within this hexagon anywhere, failure is not occurring. Now, you see, compare quickly this quadrant and this quadrant with that of the maximum principle stress theory; you will see that they are equal. But if you compare the second and fourth quadrant, the values given by, suggested by the maximum shear stress theory and that of the maximum principle stress theory are different. So, there is a question mark here.

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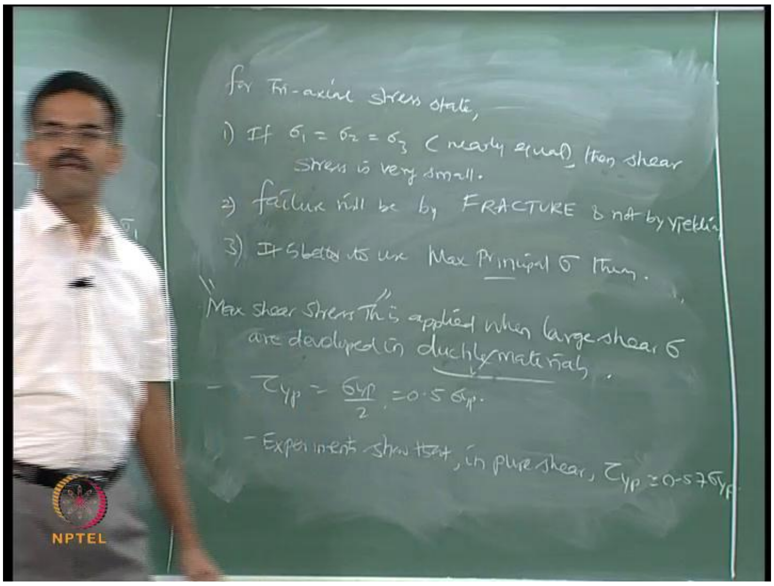


Let us look at the next theory. The third theory talking about the failure is… Before looking at the third theory, let us look at the triaxial stress state also. For… Because this is biaxial. For a triaxial stress state, if sigma 1, sigma 2 and sigma 3 are nearly equal; for example, you have a stress state, where all the three stresses are nearly equal; then the shear stress effect is very small. It is very small. Therefore, in that case, failure will be by fracture and not by yielding. In that case, it is better to use maximum principle stress theory, because it is by fracture. Then, one may ask me a question, what is a critical application where I must use maximum shear stress theory? Maximum shear stress theory can be used when large shear stresses are developed in ductile material. So, this theory is very good for ductile material provided that large shear stresses will develop.

Now, you may ask me a question; sir, I want a ductile material. We agree that the ductile material develop large shear stresses; but still this theory cannot be used when the magnitude of them in all principle axes are almost equal, because then the shear of it is negligible. So, there are two constraints here. You must also see what is the likely magnitude of these stresses coming across the different sections or different axes; at the same time, is the material is showing a ductile behavior or not. Then, this theory can be used; otherwise, it cannot be applied. We have already said that, in maximum shear stress theory, the value of tau y p is taken as sigma y p by 2, which is 0.5 sigma y p. Experiments have shown that, in pure shear, this value is approximately equal to 0.57 sigma y p. Therefore, this theory gives me conservative results. It gives conservative results; it gives me safe results.

The third theory, which talks about failure, is what we will discuss in the next class, because we have got 3, 4 and 5. Then we will talk about the problem applicable to all the five theories. Then we will pick up and compare them and use as a designer, which theory should I use. This we will discuss in the next lecture. Do you have any questions in the failure theories of 1 and 2, that is, maximum principle stress theory and Tresca's theory? Do you have any questions? Have you understood the complexities advised with these theories in unlike stress states of quadrant 2 and quadrant 4? You can see the clear difference. Any questions? So, we will continue in the next lecture talking about the other remaining three theories of failure; and we will do some problems on this.