Introduction to Atmospheric and Space Sciences
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Lecture - 34
Stable, Unstable and Neutral Atmosphere

Hello dear students. So, we will continue our discussion on atmospheric stability. So, in the last class we have seen that, if you write the virtual temperature of the environment the rate at which it changes and compare it with respect to the virtual temperature of the air parcel will get three conditions which are stable, unstable and neutral right. So, before we go ahead and write it in various other forms let us recall the basic definition of virtual temperature and why we need something called as the virtual temperature.

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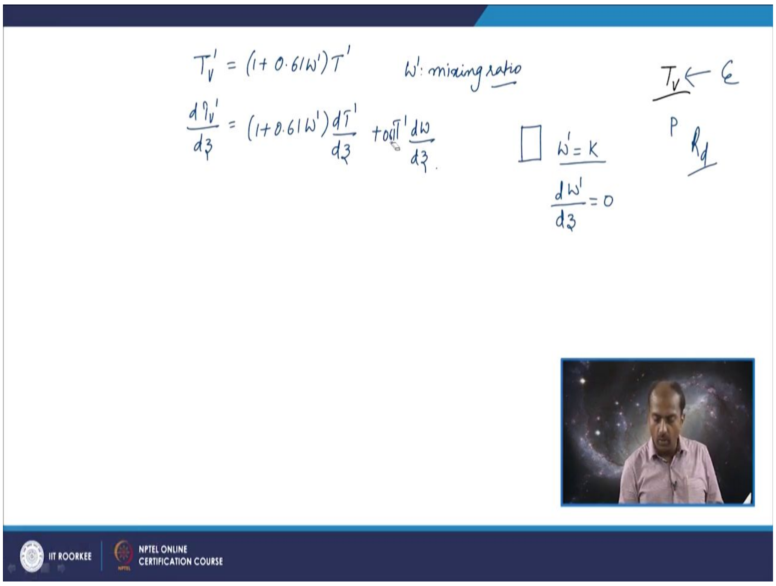

$T_v' = (1 + 0.61w')T'$

w': mixing ratio

$\frac{dT_v'}{dz} = (1+0.61w')\frac{dT'}{dz} + 0.61T'\frac{dw}{dz}.$

$w' = K$

$\frac{dw'}{dz} = 0$

$T_v \leftarrow \epsilon$

P

R_d

So, virtual temperature is that temperature that you should heat the air. So, that density becomes equal to the density of the moist air right here. So, the basic idea is if you want to calculate pressure knowing the volume and the temperature you will have to use a gas constant and this gas constant should be specific to the type of gas that is taken into account. And if you replace the air molecules with water molecules, every time you repeat every time you change the concentration you will have to calculate the gas constant for that particular chemical composition.

So, calculating with gas constant for every single change does not make sense. So, what we did was we modified the temperature and within this temperature we have defined a ratio of the molecular weight of water with respect to molecular weight of air with respect to molecular weight of dry air right. So, here so this is the basic definition of virtual temperature. So, here let us write. So, the virtual temperature of the air parcel from the earlier discussions I will directly pull this equation.

So, which is 0.61 w prime T prime. So, the virtual temperature of the air parcel the primed variables are representing the air parcel and unprimed variables represent the environment. So, virtual temperature of the air parcel is 1 plus 0.61 times the w prime and T prime. So, T is the temperature as it is w prime is the mixing ratio right.

So, let us take a derivative with respect to height d T v prime divided by dz is simply 1 plus 0.6 1 w prime d T prime by dz right. It should also be written as T 0.61 T prime dw by dz. But, generally we do not write that; because we take the air parcel to be an adiabatic entity. So, it is not exchanging energy and matter with respect to the surroundings or with the surroundings or with the environment.

So, that means the air and the level of moisture inside the air parcel will remain as it is they would not change as it moves around right. So, that means that the mixing ratio w prime is a constant. So, the mixing ratio inside the air parcel will not change with respect to the height. So, that means we can exclude this particular term. So, this term is not relevant for us when we write it right.

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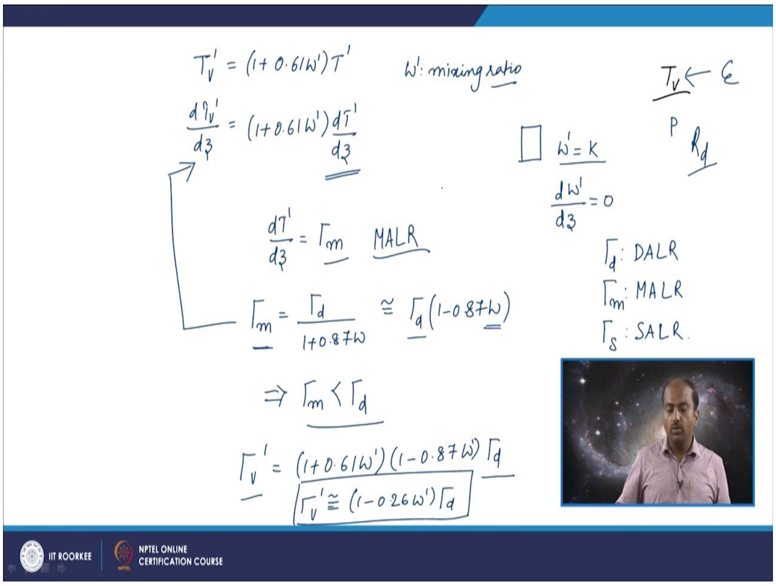

$T_v' = (1 + 0.61w')T'$ w': mixing ratio $T_v \leftarrow \epsilon$

$\frac{dT_v'}{dz} = (1+0.61w')\frac{dT'}{dz}$

☐ $w' = K$ P R_d

$\frac{dw'}{dz} = 0$

$\frac{dT'}{dz} = \Gamma_m$ MALR

Γ_d : DALR

Γ_m : MALR

Γ_s : SALR

$\Gamma_m = \frac{\Gamma_d}{1+0.87w} \cong \Gamma_d(1-0.87w)$

$\Rightarrow \Gamma_m < \Gamma_d$

$\Gamma_v' = (1+0.61w')(1-0.87w)\Gamma_d$

$\boxed{\Gamma_v' \cong (1-0.26w')\Gamma_d}$

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So, here what we have the other way is that. So, the rate at which the temperature changes inside the parcel is 0.61 times w prime d T by dz right.

So, this is the temperature change. So, if there is moisture inside the air parcel. So, we know that d T prime by dz the rate at which temperature changes inside the air parcel is gamma m. What is gamma m? Gamma m is moist adiabatic lapse rate right. So, we know that from our earlier discussions, that the gamma m can be written in terms of gamma d as 1 plus 0.87 w right.

So, what is gamma d? Gamma d is dry adiabatic lapse rate, gamma m is moist adiabatic lapse rate, gamma s is saturated adiabatic lapse rate. These 3 are different any way right. So, what does it mean? So, here we have already discussed gamma m will be less than gamma d. Why is it? So, because the possibility of moisture will bring along the possibility of saturation and saturation will have some amount of latent heat release.

So, this latent heat release into the air parcel will decrease the rate at which temperature decreases with respect to height. Hence gamma m is less than gamma d. So, we can say that we can rewrite this to an approximation as gamma d times 1 minus 0.87 w right. So, the moist adiabatic lapse rate, in terms of dry adiabatic lapse rate and the mixing ratio right.

So, if the mixing ratio is larger what you get. You get gamma m even a slightly more deviated away from the dry adiabatic lapse rate right. So, let us say we substitute this into this equation. So, we can write gamma v prime the virtual lapse rate 0.61 w prime into 1 minus 0.87 w times gamma d.

So, far this is virtual temperature of the air parcel and temperature of the air parcel. So, the starting expression is just for the air parcel. Now by including gamma m the moist adiabatic lapse rate we have brought a relation between the virtual lapse rate of the air parcel and the dry adiabatic lapse rate right. So, we can write gamma v prime isapproximately equal to neglecting the small terms. We can write 1 minus 0.26 w prime gamma d.

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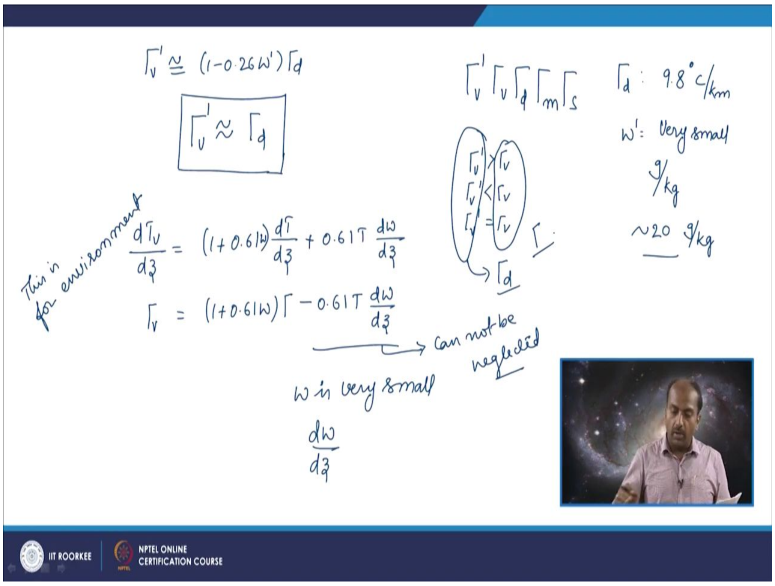

$\Gamma_v' \cong (1-0.26w')\Gamma_d$

$$\boxed{\Gamma_v' \approx \Gamma_d}$$

This is for environment

$$\frac{dT_v}{dz} = (1+0.61w)\frac{dT}{dz} + 0.61T\frac{dw}{dz}$$

$$\Gamma_v = (1+0.61w)\Gamma - 0.61T\frac{dw}{dz}$$

can not be neglected

w is very small

$\frac{dw}{dz}$

$\Gamma_v' \; \Gamma_v \; \Gamma_d \; \Gamma_m \; \Gamma_s$

$\Gamma_v' > \Gamma_v$

$\Gamma_v' < \Gamma_v$

$\Gamma_v' = \Gamma_v$

Γ_d

Γ

Γ_d : 9.8°C/km

w': very small

g/kg

~20 g/kg

so what did we get gamma v prime is approximately equal to 1 minus 0.26 w prime into gamma d. So, gamma d the dry adiabatic lapse rate is 9.8 degree Celsius per every kilometer right. So, we have seen that the mixing ratio inside the air parcel is generally very small. It is very small and we always mention in terms of number of grams of water vapor per kg of the air right. So, it is typically the highest value that we have taken. So, far was 20 gram per kg.

So, it is a very small number. So, that means that I can equate or I can neglect this term and write gamma v prime is approximately equal to gamma d. So, what does it mean? So, the rate at which the virtual temperature changes inside the air parcel is almost equal to the dry adiabatic lapse rate right. So, let us write what and all lapse rates that we have taken. So, far gamma v prime is the lapse rate of T parcel, gamma v is the virtual temperature lapse rate of the environment right, gamma d is the dry adiabatic lapse rate, gamma m is the moist adiabatic lapse rate, gamma s is the saturated adiabatic lapse rate.

So, if it is the case now what we can rewrite the conditions of stability just in terms of the dry adiabatic lapse rate. But, before that let us so let us remember them gamma v prime greater than gamma v gamma, v prime less than gamma v and gamma v prime is equal to gamma v. So, my point is so these 3 terms can simply be replaced in terms of gamma d right.

so it is also important that to note that the mixing ratio remains constant inside the air parcel, but for the environment the mixing ratio is not constant. And for the air parcel due to the adiabatic nature we have taken the mixing ratio to be a constant otherwise it cannot be a constant. So, for the environment let us say d T v by dz is again 1 plus 0.61 times w times d T by dz.

Now, we do not have any primed coordinates. So, this is for environment plus 0.61 T dw by dz right. So, this is gamma v the virtual lapse rate of the environment is equals to 1 plus 0.61 w times gamma is just a gamma the rate at which temperature changes with respect to height minus 0.61 T d w by dz right.

So, we can say that we cannot simply ignore this term and we can again write gamma v as gamma because it is not possible. See w is very small ofcourse, but dw by dz need not be small right. So, the w as a number is small, but the rate at which this is changing with respect to height need not be so small right. Now, what does it mean. So, this term on the right hand side cannot be neglected cannot be neglected right.

So, where does it leave you it leaves you that the idea was the mixing ratio keeps changing. We need to use a separate gas constants every single time right. So, so in this equation by considering the mixing ratio to be a constant with respect to height I can of course, replace gamma v primes with gamma d. But, I cannot do a simple substitution as that to replace gamma v in terms of gamma or anything else right. What I can say that the virtual temperature lapse rate of the air parcel is equal to the dry adiabatic lapse rate right.

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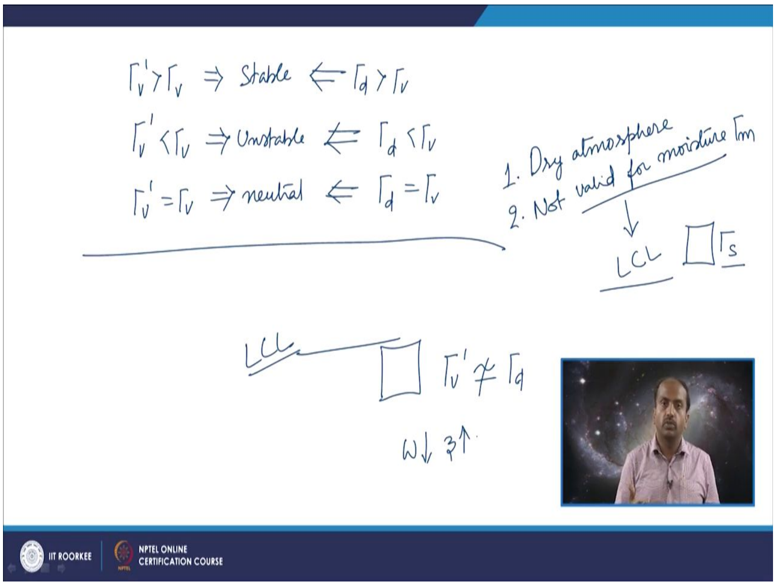

$\Gamma_v' > \Gamma_v \Rightarrow$ Stable $\Leftarrow \Gamma_d > \Gamma_v$

$\Gamma_v' < \Gamma_v \Rightarrow$ Unstable $\Leftarrow \Gamma_d < \Gamma_v$

$\Gamma_v' = \Gamma_v \Rightarrow$ neutral $\Leftarrow \Gamma_d = \Gamma_v$

1. Dry atmosphere
2. Not valid for moisture Γ_m

$\downarrow$

LCL $\square$ Γ_s

LCL $\square$ $\Gamma_v' \neq \Gamma_d$

$W\downarrow$ $z\uparrow$

So, I can rewrite the conditions just. So, let us say let us have them parallelly gamma v prime greater than gamma v which is stable. We can rewrite this in terms of gamma d is greater than gamma v; both of them indicating a stable situation or gamma v prime less than gamma v which is unstable. Writing in terms of dry adiabatic lapse rate gamma d less than gamma v and gamma v is equals to gamma v neutral which is also equivalent to writing gamma d is equals to gamma v right.

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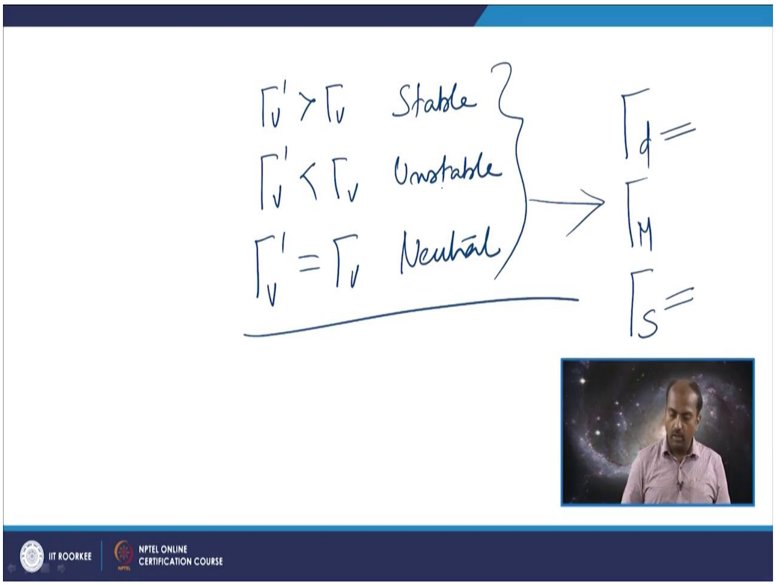

$\Gamma_v' > \Gamma_v$ Stable

$\Gamma_v' < \Gamma_v$ Unstable

$\Gamma_v' = \Gamma_v$ Neutral

$\rightarrow$

$\Gamma_d =$

Γ_M

$\Gamma_S =$

So, I mean what have we achieved. We started from this where the conditions of stability are written in terms of virtual lapse rates. By assuming that the mixing ratio remains constant inside the air parcel we have been able to replace virtual lapse rate of the parcel with dry adiabatic lapse rate a more realistic number that we know for sure right and we rewrote the condition for stability as this right.

So, how long is this going to be true? So, this is true when you take a completely dry atmosphere that means, devoid of any moisture right then if you include the moisture then these parameters will not be able to tell you whether that mostly stable unstable whatever it is. Then one more most important thing is of course, not it is valid only for the dry atmosphere and not valid for moisture right.

Now, when you add moisture another important parameter that you should consider or take in to account is the lifting condensation level. So, we see that as it rises the saturation level keeps on increasing and at the lifting condensation level the parcel temperature or the lapse rate is to be expressed with gamma s, but not gamma d . If it is moisture then it has to be gamma m and if it is saturated you have to express it in terms of gamma s.

So, these conditions as it is are not valid always. So, whether the air parcel has some amount of moisture or if the air parcel is saturated things like that. So, this analysis or this criteria are valid only below the lifting condensation level right. So, we can say that the first saturated parcels when you have moisture what will happen if it reaches LCL. If you have moisture then gamma v prime cannot be equal to gamma d.

What will happen? If you have moisture when it reaches the lifting condensation level immediately it will lead towards the condensation and condensation will leave out products such as water or water droplets and the release an amount of heat. This heat is called as the latent heat latent heat of evaporation or condensation. So, if it is the case then at the lifting condensation level what will happen to the mixing ratio.

Mixing ratio will remain a constant inside the air parts and that is what we have assumed. But if it reaches the lifting condensation level mixing ratio will decrease as you increase the height. Why does it happen? Because of the formation of water droplet us the moisture content available in the air parcel will decrease because it has led to the formation of droplets. Now in that key situation let us see how it works out.

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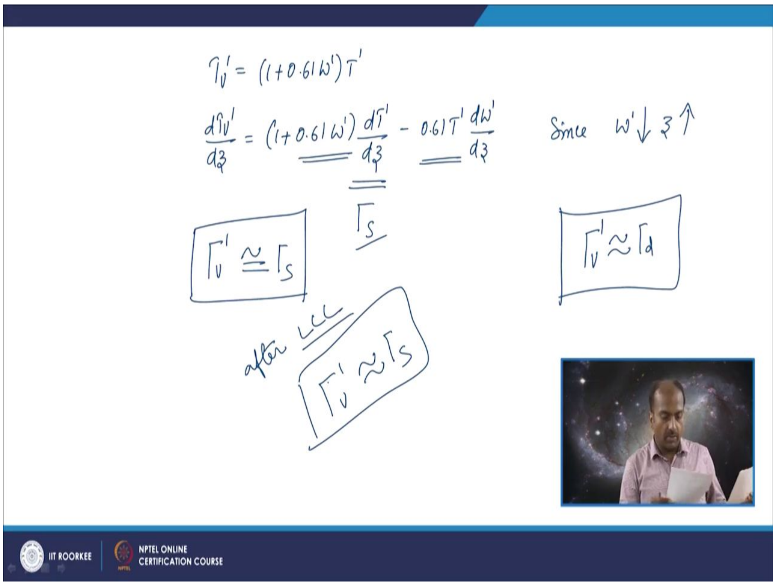

$$T_v' = (1+0.61w')T'$$

$$\frac{dT_v'}{dz} = \underline{(1+0.61w')}\underbrace{\frac{dT'}{dz}}_{\Gamma_s} - \underline{0.61T'}\frac{dw'}{dz}$$

Since $w'\downarrow\ z\uparrow$

$$\boxed{\Gamma_v' \simeq \Gamma_s}$$

$$\boxed{\Gamma_v' \approx \Gamma_d}$$

after LCL $\boxed{\Gamma_v' \approx \Gamma_s}$

So, let us say again we write the virtual temperature of the parcel as 1 plus 0.61 w prime T prime right. So, d T v prime by dz is simply 1 plus 0.61 w prime d T prime by dz plus 0.61 T prime dw prime by dz. Since w prime decreases with z we will write minus instead of plus right. So, we can say that the rate at which temperature changes inside the air parcel was previously gamma m or gamma can now be written as gamma s.

What is gamma s? The saturated adiabatic lapse rate because the air parcel has not touched the lifting condensation level. It has reached the LCL when it has reached the LCL it means that the temperature change here after considering the latent heat release should be gamma s right. So, let us say. So, we say that this quantity and this are almost of the same order.

So, we say that gamma v prime is approximately equal to gamma s. So, before reaching saturation or when the atmosphere was dry we wrote gamma v prime the rate of change of virtual temperature inside the air parcel almost equal to gamma d. Now after LCL we say that gamma v prime is approximately equal to gamma s. So, these two are two important things that keep in mind right.

So, now we can write again the stability conditions in terms of the lapse rates of environment in terms of saturated right.

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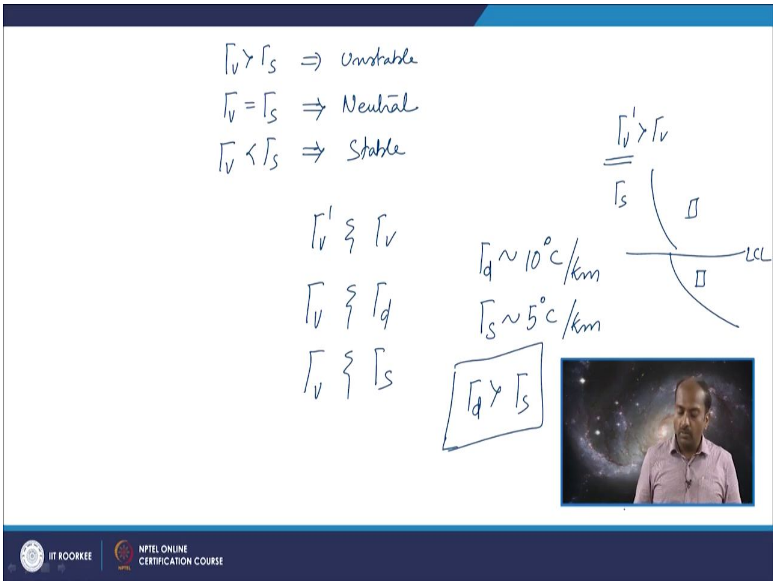

$\Gamma_v > \Gamma_s \Rightarrow$ Unstable

$\Gamma_v = \Gamma_s \Rightarrow$ Neutral

$\Gamma_v < \Gamma_s \Rightarrow$ Stable

$\Gamma_v' \gtrless \Gamma_v$

$\Gamma_v \gtrless \Gamma_d$

$\Gamma_v \gtrless \Gamma_s$

$\Gamma_d \sim 10^\circ C/km$

$\Gamma_s \sim 5^\circ C/km$

$\boxed{\Gamma_d > \Gamma_s}$

$\underline{\underline{\Gamma_v' > \Gamma_v}}$

So, we can write gamma v if it is greater than gamma s we have an unstable situation. This is still coming from gamma v prime greater than gamma v right. It is just that gamma v prime is now replaced in terms of gamma s right or gamma v is equals to gamma s neutral and gamma v less than gamma s is stable right.

So, now we have conditions between between gamma v prime and gamma v ofcourse, and gamma v and gamma d and then we have gamma v and gamma s right. So, depending on the height depending on the type of atmosphere depending on the saturation or condensation. Now we know that gamma d is approximately equal to 10 degree Celsius per kilometer right and gamma s the saturated adiabatic is approximately equal to 5 degree Celsius per kilometer that means, that gamma d is of course, greater than gamma s right.

Now, we can combine these conditions and write a conditions for an air parcel which is dry in comparison to the atmosphere and an air parcel which is saturated in comparison to the atmosphere because the rates are different. So, let us say if this is LCL some conditions are good here and some conditions are good here right. Now let us say if you take this air parcel here above the LCL the things are different right. So, we can combine these 3 conditions and rewrite the stability conditions in a more appropriate way.

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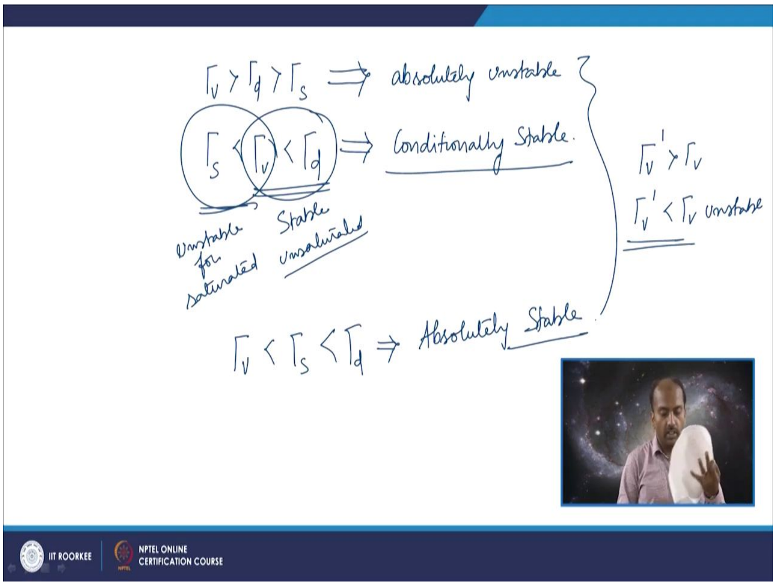

$\Gamma_v > \Gamma_d > \Gamma_s \Rightarrow$ absolutely unstable

$\Gamma_s < \Gamma_v < \Gamma_d \Rightarrow$ Conditionally Stable.

Unstable for saturated

Stable unsaturated

$\Gamma_v' > \Gamma_v$

$\Gamma_v' < \Gamma_v$ unstable

$\Gamma_v < \Gamma_s < \Gamma_d \Rightarrow$ Absolutely Stable

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So, for example, gamma v if it is gamma v is greater than gamma d. If the lapse rate of the environment is greater than gamma d; gamma d itself is almost 10 degree Celsius which is greater. So, it also means that it is also greater than gamma s. So, that temperature change with respect to height of the environment if it is greater than the dry adiabatic lapse rate. It also means that the it is also greater than gamma s. So, in that case, it is this condition is called as absolutely unstable.

Just to remember our condition was gamma v prime greater than gamma v for an for stable or gamma v prime less than gamma v for unstable. So, we have written this condition. So, gamma v is now larger right. So, this condition is known as unstable. The second condition is if gamma s the saturated adiabatic lapsed rate is less than gamma v less than gamma d. This condition results in what is called as conditionally stable.

What does it mean? So, gamma v is less than gamma d right. Let us go back saying if gamma v is less than gamma d.if gamma v is less than gamma d gamma d is less than gamma v and gamma d this is what I am talking about right. So, gamma v is less than gamma d. So, this was stable right. So, this is stable for unsaturated parcel if we have an air parcel such that it is lapse rate is less than the virtual lapse rate then it is stable.

So, this air parcel is stable for unsaturated parcels stable for unsaturated parcel and at the same time if you break the conditions into 2 it is unstable for saturated parcels. So, the same condition written in two different ways. This if this satisfied then it is stable for unsaturated parcel and this part is unstable for saturated parcel right.

So, this is called as the conditionally stable criteria. And thirdly so if gamma v is less than gamma s. So, it is less than 5 degrees Celsius per kilometer itself. So, it is obviously less than the 10 kilometers per kilometer right. So, this condition is called as absolutely stable right. So, we have been able to combine gamma v , gamma d and gamma s right.

So, and we have now we have the most appropriate or most inclusive stability conditions which are called as the absolutely stable, absolutely unstable and conditionally stable. So, we can also rewrite these conditions of stability in terms of a potential temperature. So, which we have already dealt with the derivations.

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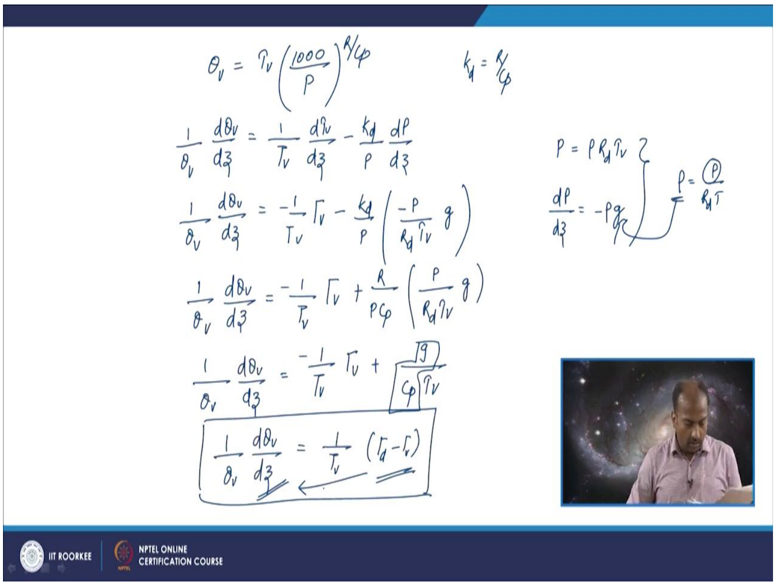

$$\theta_v = T_v\left(\frac{1000}{P}\right)^{R/c_p} \qquad k_d = R/c_p$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = \frac{1}{T_v}\frac{dT_v}{dz} - \frac{k_d}{P}\frac{dP}{dz}$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = -\frac{1}{T_v}\Gamma_v - \frac{k_d}{P}\left(\frac{-P}{R_d T_v}g\right)$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = -\frac{1}{T_v}\Gamma_v + \frac{R}{Pc_p}\left(\frac{P}{R_d T_v}g\right)$$

$$\frac{1}{\theta_v}\frac{d\theta_v}{dz} = -\frac{1}{T_v}\Gamma_v + \frac{\boxed{g}}{\boxed{c_p}T_v}$$

$$\boxed{\frac{1}{\theta_v}\frac{d\theta_v}{dz} = \frac{1}{T_v}(\Gamma_d - \Gamma_v)}$$

$$P = \rho R_d T_v$$

$$\frac{dP}{dz} = -\rho g \qquad \rho = \frac{\boxed{P}}{R_d T}$$

So, you can say the virtual potential temperature theta v is T v times 1000 by p into r by cp. So, we have so far defined conditions of stability in terms of temperatures. We will define the stability conditions in terms of the potential temperature right. So, take a logarithmic derivative. So, it will be 1 by theta v d theta v by dz is equals to 1 by Tv d T v by dz minus kd by p into dp by dz. By the way kd is r by cp.

If I just expand it. So, it will be d theta v by dz is equals to minus 1 by T v gamma v minus kd by p. So, p is equals to rho rd T v and hydrostatic equation is dp by dz goes to minus rho g. Combining these 2 rho is equals to p by r d T. So, I will write rho into this equation. I can write kd by P into minus P by rd T v times g or 1 by theta v d theta v by dz is minus 1 by T v gamma v plus r by P cp into P rd T v simple algebra. I just rearranging the terms minus 1 by T v gamma v plus g by cp T v.

So, g by cp is the dry adiabatic lapse rate. So, which is 1 by theta v T theta v by dz is 1 by T v times gamma d minus gamma right. So, here gamma v. So, now, we have all the stability conditions defined in terms of the difference gamma d minus gamma v. Now, we can use this difference to write our stability conditions in terms of the virtual potential temperatures changing with respect to height. So, simply these conditions from gamma d minus gamma v becoming less than 0, greater than 0 or 0 are being transferred to this derivative right.

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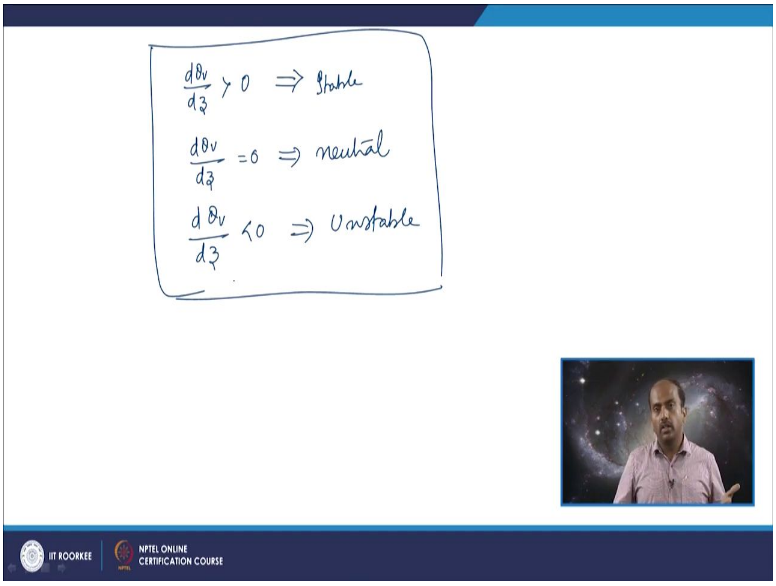

$\frac{d\theta_v}{dz} > 0 \Rightarrow$ Stable

$\frac{d\theta_v}{dz} = 0 \Rightarrow$ neutral

$\frac{d\theta_v}{dz} < 0 \Rightarrow$ Unstable

So, we can write that when d theta v by dz is greater than 0 the layer is stable. When d theta v by dz is equals to 0 we have neutral. So, this is still neutral we have defined what is conditionally stable right. So, the d theta v by dz z is less than 0 we have unstable equilibrium right.

So, this is how we write the various different types of stability criteria in terms of a various different types of lapse rates and temperatures right. So, eventually the stability of atmosphere is the concept which explains the formation of clouds. So, formation of clouds has to be at the expense of rising air parcels and reaching condensation level and then releasing heat or forming droplets right.

So, we will conclude here about the discussions on atmospheric stability. We will continue the discussions on probably like how do the clouds form, various different types of clouds, various different scales of clouds stuff like that.