Plasma Physics and Applications

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Week – 11

Lecture 54: Ambipolar Diffusion - II

Hello dear students. In this lecture, we are going to learn more important aspects about amipolar diffusion in plasma. In the last lecture, we have derived the amipolar diffusion coefficient for plasma which looks something like this. So, this is a function of temperature of electrons and ions, mass of ions and the collision frequency of ions. So, the basic understanding is despite the differences between electrons and ions in terms of mass, both velocities will be the same or both the particles will diffuse at the same rate. So, this kind of diffusion process is known as the am bipolar diffusion and this is a very peculiar process specific only to plasma.

So, when if the electrons and ions have the same temperature, we can say that the am bipolar diffusion coefficient and the plasma scale head have twice the value in comparison to neutral gas having the same molecules the size of ion. So, if you bring the diffusion coefficient, we know it is k B T by m nu. Now, what I am what I was saying is if the electron temperature is equal to the ion temperature, the am bipolar diffusion coefficient will be k B times 2 k B times T divided by m i nu i which is actually twice the. So, the am bipolar diffusion coefficient is twice the diffusion coefficient for normal neutral gas and if you consider the plasma scale height which is h p, the plasma scale height as k B times T e plus T i by m g.

$$D_{amb} = \frac{k_B(T_e + T_i)}{m_i \nu_i}$$

$$D_a = \frac{k_B T}{m \nu}$$

$$T_e = T_i$$

$$D_{amb} = \frac{2 k_B T}{m_i \nu_i} = 2 \underline{D}$$

$$H_p = \frac{k_B(T_e + T_i)}{m g}$$

$$T_e = T_i \qquad H_p = \underline{2H}$$

For electron temperature equal to the ion temperature, you can write the plasma scale height to be twice the scale height of an identical gas which has the same molecules or atoms but in the neutral state. So, one very important thing is the plasma the am bipolar diffusion is peculiar to plasma. Now, you can write the flux of particles. So, flux which can be written as gamma as n times u j number of particles per unit volume times the velocity. So, it is basically the fields that are set up when these electrons try to move faster are responsible for bringing the ions to larger velocities or both of them traveling together or both of them diffusing together.

So, this diffusion is what is responsible for the solar wind travel from sun to the earth. What we will do in this class is we will try to understand more aspects of this am bipolar diffusion or the same process explained in terms of different set of variables. So, solar wind as we know consists of electrons and ions both of them travel at the same speed if their masses are so much different then obviously the question is why they should travel with the same velocity and the answer is basically the am bipolar diffusion which is a very specific process which happens only in the plasma. So, the effect of electron is to double the scale height you see the velocity will be doubling the scale height. To understand this even further let us assume an equilibrium situation when the velocity W is equal to 0.

So, let us say the temperatures are equal T e is equal to T i let us say we call it as T and we know that the mass of electron will be much smaller than the mass of ion. So, we can neglect something that appears to be the multiple of mass of electron. So, we can write the simple equation as dP by dH minus m i g N plus e e N is equal to 0 or same equation as minus K T by N dN by dH minus m i g plus e e is equal to 0 or minus K T by N dN by dH minus e e is equal to 0. So, this is for obviously for ion and this is for electron. So, I am not writing temperature T i and T e because I assume both of them to be the same and in the electron equation I have not written the m e g term because the mass of electron being very very small we can neglect it.

Flux $\Gamma = n v_j$

$\underline{W=0}$ , $T_e = T_i = T$ , $m_e \ll m_i$

$$-\frac{dP}{dh} - m_i g n + E e n = 0$$

$$-\frac{kT}{n}\frac{dn}{dh} - m_i g + E e = 0 \quad \text{(ion)} \quad \text{—— (a)}$$

$$-\frac{kT}{n}\frac{dn}{dh} - E e = 0 \quad (e^-) \text{—— (b)}$$

So, let us say we call this equation as equation A and this as B. Adding both of them what we will get is 1 by N dN by dH is equal to minus m i g by 2 K T which is minus 1 by H P and this can be called as equation number C. Now we can use this equation number C in the electron equation and we can write what is equation B minus K T by N dN by dH minus e e is equal to 0 minus K B T by N dN by dH minus e e is equal to 0. So, P is equal to N K T, N is the number of particles per unit volume and if e is the electric field. So, we can write minus K B T times minus m i g 1 by dN by dH is m i g by

2 K T, m i g by 2 K T is equal to e e.

$$\frac{1}{n}\frac{dn}{dh} = -\frac{m_i g}{2kT} = -\frac{1}{H_p} \quad \text{—} \quad (c)$$

Using (c) in (b)

$$-\boxed{\frac{kT}{n}\frac{dn}{dh}} - Ee = 0$$

$$-\cancel{kT}\left(\frac{-m_i g}{2\cancel{kT}}\right) = Ee$$

$$\boxed{\frac{m_i g}{2} = Ee}$$

So, what I have done I have 1 by N dN by dH this is what I have said. So, I can write cancelling out things that are appearing both sides we can write m i g by 2 is equals to e times e. What is e? e is the electric field small e is the charge m i is the mass of ion g is the gravitational pull. So, this force how does it come into existence e e this comes into existence because the electrons being lighter they move faster they set up an electric field this electric field will try to decelerate the electrons. But this electric field is the is building up a force against the electrons which is given by e e.

$$E = \frac{m_i g}{2e}$$

$$\frac{1}{n}\frac{dn}{dh}$$

$$n = n_0 \exp^{-h/H}$$

Oxygen ions

$$E = 10^{-6}\ V/m = \frac{\sigma}{\epsilon_0} = \frac{ne}{\epsilon_0}\Delta x$$

$$E = \frac{kT}{ne}\frac{dn}{dh}$$

$$\Delta x = \frac{\epsilon_0 k_B T}{ne^2} \cdot \frac{1}{n}\frac{dn}{dh}$$

$$\boxed{\Delta x = \frac{\lambda_D^2}{\delta}}$$

So, now we can see this equation and we can understand one very important aspect. So, the electric field e acts on the electron with a force which is equal to half m i g. So, this is for the electrons the equation that we have picked up is for the electrons. So, it appears the force on the right hand side is equal to as if the force is half the mass of ion times the gravitational pull. It is natural we can say that the electrons will behave as if they have a mass of m i by 2 you see this because of the opposing electric field to the electrons.

The electrons will behave as if they have a mass of m i by 2. So, this force is what is the direction of this force? This force is downwards along the gravitational pull. So, same force will naturally act in the upward direction for the ions. So, this is dN by dH you can use it in this equation and realize that this force will be exactly opposite to the ions. But the point is due to the electric field the electrons will behave as if they are moving with a mass of m i by 2 half the mass of ions.

Now the magnitude of this electric field E is m i g by 2 e is very small. Now if you take for instance the case of Earth's ionospheric plasma which consists of oxygen ions the oxygen ions m i I am referring to m i and e is the charge of electron. So, if you take for ionosphere of the earth where the majority positive charge carriers are oxygen ions in that case if you put that mass to perspective you will realize that the electric field that I am talking about is approximately of the order of 10 to the power of minus 6 units. So, this is the amount of electric field which is decelerating the electrons. So, correspondingly if you want to find out what is the surface charge density we can write the electric field as sigma by epsilon naught or number of charge carriers per unit volume multiplied by the charge times delta x.

$$mn\left[\frac{dv}{dt} + (v\cdot\nabla)v\right] = \pm enE - \nabla p - mn\nu v$$

$$\pm enE - \nabla p - mn\nu v = 0$$

$$v = \frac{1}{mn\nu}\left(\pm enE - k_B T\nabla n\right)$$

$$\boxed{v = \pm\frac{e}{m\nu}E - \frac{k_B T}{m\nu}\frac{\nabla n}{n}}$$

$\frac{e}{m\nu}$ : mobility

$\frac{k_B T}{m\nu}$ : Diffusion coefficient

So, we can use E as kT by N e dN by dH. Now delta x can be written as epsilon naught kB T divided by N e square times 1 by N dN by dH. So, this goes here and equating these two I written the relation for delta x. So, in terms of things that we know delta x is epsilon naught kT by N e square this is a very familiar term that we know very well which is nothing but lambda d square by some delta. So, where is lambda d square is nothing but the lambda d is nothing but the Debye s length and delta is just a distribution height.

$$\mu = \frac{qD}{K_B T}$$

$$\boxed{v = \pm \mu E - D \frac{\nabla n}{n}}$$

$$\boxed{\Gamma_j = n v_j = \pm \mu_j n E - D_j \nabla n}$$

$$\boxed{\Gamma = -D \nabla n}$$

Fick's Law

What is distribution height? This one 1 by N dN by dH. So, you take N is equals to N naught exponential minus H by H and then you take a derivative and do this you will realize what is the distribution height. So, what have we learnt? We learnt that how the electric field acts on the electron and what is the effect of this electric field, how the electrons will move as if they have a mass of half the mass of ions and typically what is the electric field magnitude. Now we will try to define ambipolar diffusion by introducing the mobility. What is mobility? Mobility is drift per unit electric field basically.

$$\left.\begin{aligned}\Gamma_i &= \mu_i n E - D_i \nabla n \\ \Gamma_e &= -\mu_e n E - D_e \nabla n\end{aligned}\right\}$$

$$E = \frac{D_i - D_e}{\mu_i + \mu_e} \cdot \frac{\nabla n}{n}$$

$$\Gamma_i = \mu_i n \left(\frac{D_i - D_e}{\mu_i + \mu_e}\right) \cdot \frac{\nabla n}{n} - D_i \nabla n$$

$$\Gamma_i = \frac{\mu_i D_i - \mu_i D_e - \mu_i D_i - \mu_e D_i}{(\mu_i + \mu_e)} \cdot \nabla n$$

So, for any species for charge species the fluid equations of motion are very well known to us we have discussed at very fine detail about this fluid equations at large lengths. So, dV by dT plus V dot del times V is equals to plus minus e n e minus del P minus m n eta V. So, this plus minus symbol is for accommodating both the charges. So, let us assume a steady state. What is steady state? In which velocity is not 0 velocity is actually a constant it is not changing with respect to time.

So, the forces are exactly balanced and velocity is a constant. So, in the steady state it will be forcing on the collisions. So, we can make an assumption that the collision frequency to be very large. So, in that case when the velocity is constant we have the entire left hand side becoming 0 and we can write plus minus e n e minus del P minus m n eta V is equals to 0. Or we can write that steady state velocity as 1 by m n nu times plus minus e n e minus k B T delta N.

$$\boxed{\Gamma = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \cdot \nabla n}$$

$$\boxed{D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}}$$

$$\mu \propto \frac{1}{m}$$

$$\mu_e \gg \mu_i$$

$$\mu_e + \mu_i = \mu_e$$

So, you can work out the algebra it is pretty much simple actually. So, I have written pressure gradient in terms of density gradient using the ideal gas equation in between. So, the velocity can further be simplified or arranged plus minus e by m nu times e minus k B T by m nu times delta N. Of course, this is still ambipolar diffusion but now we have brought in the idea of collisions. So, this can be treated as a velocity which the particles will achieve when there are many number of collisions.

$$D_a = \frac{\mu_i}{\mu_e} D_e + D_i \qquad \mu = \frac{qD}{kT}$$

$$\frac{D}{T} = k$$

$$\frac{D_e T_i}{D_i T_e} = \frac{\mu_e}{\mu_i}$$

$$\frac{\mu_i}{\mu_e} D_e = \frac{T_e}{T_i} D_i$$

$$D_a = \frac{T_e}{T_i} D_i + D_i$$

$$T_e = T_i$$

$$\boxed{D_a = D_i + D_i}$$

$$\boxed{D_a = 2D_i}$$

Now in this expression you see this e by m nu what is this? This is called as the mobility and you have k B T by m nu what is this? This is called as the diffusion coefficient. So, we can write mu the transport coefficients can be suitably adjusted and we can write mu as Q T by k B T. So, gotten rid of the collisional frequency in the mobility expression. So, the velocity V the steady state velocity which is achieved as a result of collisions constant collisions is this is some sort of a drift velocity actually minus D times delta N by m. You can just look at these equations write them down and build up the algebra that is there in between the intermediate algebra.

So, this is the velocity in terms of the mobility. So, now when you consider a particular species the flux with which the species is diffusing we write it as gamma j as N V j which is now going to be written as plus minus mu j N e minus the diffusion coefficient of the j th species multiplied by delta N. This is the flux of flux of the j th species. Now when you talk about flux we need to remember a very famous law which is a Fick s law of diffusion we can slightly modify and make a special case when the electric field is 0 and we can write we can get back the Fick s law as gamma is minus D times delta N. What is this is the Fick s law.

$$mn\frac{du_\perp}{dt} = q n\left(E + u_\perp \times B\right) - k_B T \nabla n - mn\nu u_\perp$$

$$\hookrightarrow \text{T.D,}\ \frac{\partial u_\perp}{\partial t} = 0$$

$$u_\perp$$

$$u_\perp = \mu_\perp E - D_\perp \frac{\nabla n}{n} + \omega_c^2 \frac{u_E + u_D}{\omega_c^2 \nu^2}$$

$$u_E = \frac{E \times B}{B^2}$$

$$u_D = \frac{-\nabla p \times B}{B^2}$$

So, what you do is what you see is the effect of ambipolar diffusion is to get a picture in which electrons are behaving as if they have a half the mass of ions and at the same time the ions have a diffusion coefficient which is twice or which have been smaller by a factor of 2 ideally. So, the effect of ambipolar diffusion is to increase the mobility of the ions by a factor of 2. Now we can get the flux in terms of the mobilities of electrons and ions separately let us see how we get it. So, we will write gamma from the earlier relation using this mobility mu N e plus minus mu N e minus dJ del N. So, the for ions it will be plus mu N e minus D delta N and for ions it will be exactly minus in the first term.

So, gamma is mu i N e minus d i delta N which is mu e N e minus d delta N. So, from these two equations the electric field e can be written as d i minus d e divided by mu i plus mu e times delta N by N. We can use this electric field in any of the flux of ions or electrons to write gamma i as mu i times N times d i minus d e divided by mu i plus mu e delta N by N minus d i delta N. So, slightly arranging the terms gamma i is mu i d i minus mu i d e. Mu is gamma is minus mu i d e plus mu e d i cancelling out all the terms mu i plus mu e times delta N.

What is this? This is the flux. Now this is again the Fick s law we have gotten back to the Fick s law nothing else, but the diffusion coefficient is slightly modified now. The entire thing that is this is the diffusion coefficient this is the amepolar diffusion coefficient which is mu i d e plus mu e d i divided by mu i plus mu e this is the diffusion coefficient. Now let us see some features of this diffusion coefficient. What is this called as? This is called as the amepolar diffusion coefficient. Now we know that the mobility is proportional to 1 by mass larger the particle smaller will be its mobility.

So, the mobility of electron will be much larger than the mobility of ion and it is natural for us to expect mu e plus mu i should just be equal to mu e because the mobility of ion will be very small. If you put this into this expression the amepolar diffusion coefficient which we have written in many forms so far is mu i divided by mu e times d e plus d i. You see this I just substituted mu i plus mu e as just mu e that is it. So, once we have mu only mu e in the denominator in the numerator we are not changing anything mu e will get cancelled in the second term and as a result only d i will remain. So, the amepolar diffusion coefficient is now a ratio of the mobilities multiplied by the diffusion coefficient of electron plus the diffusion coefficient of ions.

Now where do we go ahead of this from this? So, mobility from basic definition is Q times d divided by k B T. So, for the remaining things to be constant so the diffusion coefficient by temperature is a constant. So, dE T i divided by d i T e will be a constant and this constant is going to be mu e by mu i. So, from this we can get this ratio of mu i by mu e d e rearranging the terms will be T e divided by T i times d i. So, we can bring this here and write the ampere polar diffusion coefficient is nothing but T e divided by T i times d i plus d e so we have d i.

$$\mu_\perp = \frac{q/m\nu}{1+\omega_c^2\tau^2}$$

$$D_\perp = \frac{k_B T/m\nu}{1+\omega_c^2\tau^2}$$

So, by doing this we are not just eliminated the ratio of mobility we written the ratio of mobilities in terms of the temperatures and also eliminated one diffusion coefficient.

Now the earlier version of the ampere polar diffusion coefficient has two electron and ion diffusion coefficient now we have only ion. Now in a situation when the electron temperature is equal to the ion temperature the ampere polar diffusion coefficient dA simply becomes d i plus d i C or dA is twice the diffusion coefficient of ion. So, now we can summarize all these things. So, the effect of ampere polar electric field is to increase the diffusion of ions by a factor of 2 and make electrons move as if they have a mass of m i by 2.

So, these two factors are important m i by 2 is going to be the mass of electron and the diffusion coefficient of is twice the diffusion coefficient of ion. So, the effect of ampere polar diffusion or ampere polar electric field is to increase the diffusion of ions by a factor of 2 that is how they will start moving with the same velocity. Now let us consider the diffusion of weakly ionized plasma in the presence of a magnetic field. So, so far we have considered general nature of ampere polar diffusion having understood all these things let us consider the diffusion of weakly ionized plasma in the presence of a magnetic field. So, diffusion parallel to the magnetic field will not be influenced by the magnetic field.

So, we will only consider direction which is perpendicular to that. So, we will start with the with our basic equation m n d u perpendicular by dt is q n e plus u perpendicular cross B minus k B T del n minus m n nu u perpendicular. So, this has to be a total derivative. So, the partial derivative will become 0 considering a steady state. So, what will be left with is the advection term.

Since these things are very much familiar to you or have discussed multiple number of times. So, what we are interested is to find out an expression for u perpendicular. So, u perpendicular so this has to be expanded to total derivative and steady state means dou u perpendicular by dou t becoming 0. So, solving for u perpendicular we can get u perpendicular as mu perpendicular times E minus d perpendicular delta n by n plus omega c square times u e plus u d by omega c square mu square. So, this u e is nothing but the E cross B drift velocity u d is the diamagnetic drift velocity u e is E cross B by B square and u d is minus del P cross B by B square.

So, we can with the help of this by just doing some simple algebra we can define the perpendicular mobility mu perpendicular q by mv divided by 1 plus omega c square tau square and the perpendicular diffusion coefficient as k B T by mu divided by 1 plus omega c square tau square. So, what we can see is that the perpendicular motion of particle contains two parts one the usual drift which is which are impeded by collisions and mobility and diffusion drifts along the gradient in the potential and number densities. So, with this we conclude the ambipolar diffusion in the subsequent lectures we will try to understand various applications of plasma physics or how plasma can be used as a tool in material science or in photonics or many other things. Thank you very much.